Lossy Network Correlated Data Gathering with High-Resolution Coding

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Abstract

We consider a sensor network measuring correlated data, where the task is to gather all data from the network nodes to a sink. We consider the scenario where data at nodes is lossy coded with high-resolution, and the information measured by the nodes has to be reconstructed at the sink within both certain total and individual distortion bounds. First, assuming fixed node positions, we consider the problem of finding the optimal transmission structure and the rate-distortion allocations at the various spatially located nodes, such as to minimize the total power consumption cost of the network. We prove that the optimal transmission structure is the shortest path tree and that the problems of rate and distortion allocation separate in the high-resolution case, namely, we first find the distortion allocation as a function of the transmission structure, and then, for a given distortion allocation, the rate allocation is computed. Then, we also study the case when the node positions can be chosen, by finding the optimal node placement for two different targets of interest, namely total power minimization and network lifetime maximization. Finally, we propose a node placement solution that provides a tradeoff between the two metrics.

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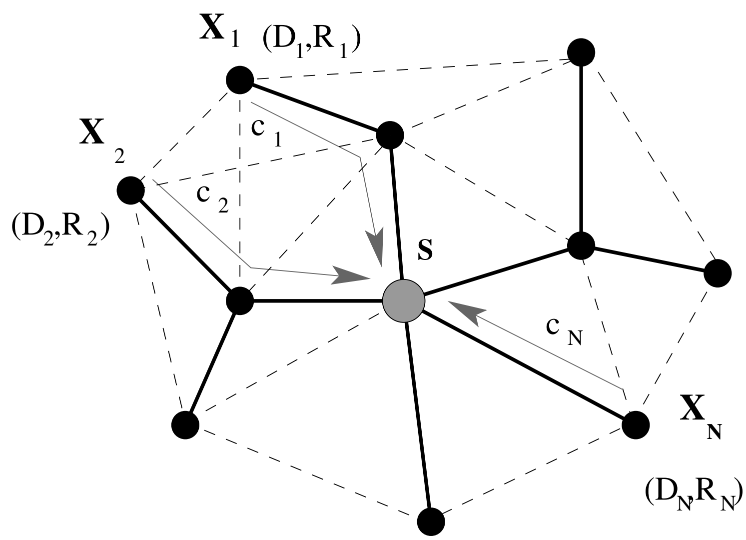


Fig. 1. In this example, data from nodes $X_1, X_2, \ldots, X_N$ need to arrive at sink S. A rate supply R_i is allocated to each node X_i, and the distortion at that node is D_i. In thick solid lines, a chosen tree transmission structure is shown. In thin dashed lines, the other possible links are shown. The weight of the path from node i to the sink is c_i, and the corresponding path is shown in gray line.

I. Introduction

A. Lossy Correlated Data Gathering

Consider a network of sensors measuring a spatially correlated data field. An important task in such a scenario is data gathering, where the goal is to gather data from all the nodes to the base station, or the *sink*, via a subset of the links of the network. An example is shown in Fig. 1, where there are N nodes with sources $X_1, \ldots, X_N$, a sink denoted by S, and a representation graph of connectivity with edges connecting certain nodes. The transmission topology in our model is assumed to be an undirected connected graph with nodes connected within a certain distance, assuming a certain transmission range, with point-to-point links. Data at nodes have to be encoded, and for proper reconstruction of the field at a sink node, they need to be available at the sink within a certain prescribed total distortion (in Fig. 1, we denote the distortion and rate at node i by D_i and R_i, respectively). In some specific scenarios, where a minimum accuracy is necessary for individual measurements across network nodes, there are also individual distortion upper bounds imposed on the accuracy of data transmitted from the nodes.

In addition to the encoding of the data by the nodes, these data also need to be transmitted over the network from the sources to the sink, which results in certain transmission costs (in Fig. 1, the total weight of the path from node i to the sink is denoted by c_i). It is thus important to study the interplay between the rate-distortion allocations at nodes, and the transmission structure used to transport the data. We consider a joint treatment of the rate allocation and the chosen transmission structure, by means of cost functions that are functions of both of them. The cost functions, usually found in practice, separate the rate term from the path weight term [5]. For instance, the [rate] $\times$ [path weight] cost function measures the power consumption in sensor networks, where [rate] is the number of transmitted bits, and [path weight] is a supraunitary power κ (usually $\kappa = 2$ or $\kappa = 4$) of the Euclidean distance over which the transmission is done. Typical goals in such settings are (a) the minimization of the total network power consumption, defined as the total sum of powers consumed by nodes, and (b) maximization of the network lifetime, defined as the time until the first node fails (it runs out of power). In this work, we consider jointly the optimization of both rate and distortion allocation at nodes, transmission structure, and node placement, in the context of sensor networks with correlated data at nodes.

Consider a network of N nodes (see Fig. 1). Let $\mathbf{X} = (X_1, \ldots, X_N)$ be the vector formed by the random variables representing the sources measured at nodes $1, \ldots, N$. The samples taken at nodes are spatially correlated. We assume that the random variables are continuous and that there is a high-resolution multidimensional quantizer (e.g. lattice quantizer) in each sensor. A rate-distortion allocation $\{(R_1, D_1), \ldots, (R_N, D_N)\}$ (bits) has to be assigned at the nodes so that the quantized measured information samples are described within a certain total distortion D and potentially also with individual distortion upper bounds $D_i^{max}, i = 1, \ldots, N$. That information has to be transmitted through the links of the network to the designated sink node[1] (see Fig. 1). In [8], [10], the problem of lossless coding of the data at nodes with Slepian-Wolf coding [19] has been studied. In this work, we extend that problem by allowing nodes to code their data in a *lossy* manner, such that the information received by the sink describes the data at nodes within certain prescribed distortion constraints. Namely, the rate allocation at nodes in our lossy data gathering scenario essentially depends on the differential entropy of the non-quantized data that

[1]Due to the interaction between the rate-distortion region and the transmission structure optimization, our results can be easily generalized to the case of multiple sinks, as we will show in this paper.

is measured and on the distortion levels of quantization that are assigned to each of the sources located at nodes.

Moreover, we study the problem of optimal node placement for the tasks of first, minimizing the total network power consumption, and second, maximizing the network lifetime. Note that the placement solutions resulting from these two corresponding optimizations are different, since optimizing the lifetime of the network means equalizing power consumption at nodes, which does not necessarily coincide with the optimal placement solution when the sum of power consumptions has to be minimized. Interestingly, for the correlation functions we consider in this paper, when total network power consumption is minimized, the bottleneck node in the optimal positioning is the one at the *extremity* of the network, rather than the node closest to the sink. This is both due to the coding strategy, that assigns low rates to nodes far from the sink, and to the inter-node distance optimization, as we show in this paper.

The general distributed rate-distortion region for coding correlated data at general quantization resolution levels (e.g. low rates) is not known even for the Gaussian case [16]. Thus, we will restrict our study to the case of data gathering with lossy *high-resolution* quantization [3], [20], in which case, the rate-distortion region is known. Namely, the rate-distortion region of coding with high-resolution for arbitrarily correlated sources has been found in [20]. In general (e.g. low resolution) this region provides an outer bound, which becomes tighter as the resolution increases. Thus, the results we obtain for the data gathering scenario in the high-resolution case provide a lower bound on the performance of *any* other rate-distortion allocation strategy, in terms of power efficiency, for a network that performs data gathering of correlated data. Moreover, our results on the problem of joint rate-distortion allocation and transmission structure optimization can be generalized should good bounds for low resolution rate-distortion curves become available.

B. Related Work

Recent work that exploit correlation in the data in the context of sensor networks include [2], [11], [15], [17]. A setting related to the problem we consider in this paper, namely optimizing correlated data gathering with communication costs for the case of lossless coding of finely quantized data with the same scalar (resolution) quantizer at each node, has been studied in [8], where Slepian-Wolf [6], [19] bounds are used, and in [9], where a simple coding technique with explicit communication is considered; in this work, while assuming high-rate quantization, lattice quantizers are used where the resolutions of the quantizers can be adapted across nodes.

A similar problem of data compression by opportunistic aggregation was considered in [14], where no collaboration among nodes is assumed. In this work we consider collaboration among nodes by *lossy* coding of jointly correlated data at nodes, and we provide the optimal allocation within the rate-distortion region given in [20], that minimizes the transport cost to perform the data gathering to a sink. Moreover, we consider the effect of this coding technique on the optimal placement of the nodes for gathering lossy coded data when both total network power efficiency and lifetime optimization are considered.

C. Main Contributions and Outline

We solve the total power efficiency optimization problem by finding the transmission structure and the rate-distortion allocations across nodes in a closed form, for high-resolution coding of correlated data. Namely, we show that (a) the shortest path tree (SPT) is the optimal transmission structure, and (b) the solution of the rate-allocation optimization can be decoupled in two steps: we first find the rate allocation, and then we use the closed-form solution for the rate allocation to determine the distortion allocation. Our result also provides a general lower bound in terms of power efficiency for the cost of data gathering of lossy coded data in sensor networks with a given connectivity graph. Further, when the positions of the nodes can be chosen, we determine the optimal node placing for optimizing two metrics of interest, namely the total power consumption on one hand, and the network lifetime on the other hand. Finally, we propose a solution for the node placement which provides a tradeoff between the two metrics.

The rest of the paper is structured as follows. In Section II, we formulate the optimization problem we consider in this paper, and briefly introduce a family of Gaussian random fields that we use as an example of correlation fields. In Section III, we show that the optimization problem separates, and we sequentially find the optimal transmission structure, rate allocation, and distortion allocation under various settings with total and individual distortion constraints. Further, in Section IV, we find the optimal placement in a one-dimensional scenario for two optimization goals, namely network total power and network lifetime, and we present numerical simulations to illustrate our results. We also propose a simple solution for the node placement that provides a tradeoff between the two metrics. We conclude with Section V.

II. Problem Setting

A. Optimization Problem

For the optimization of rate-distortion allocation, and transmission structure, we use a family of cost functions that separate the rate and the total path weights. These power cost functions are relevant for various problems in sensor networks related to power efficiency optimization [5]. More specifically, we assume that the power to transmit a bit through a link distance w_e is $w_e^\kappa, \kappa \geq 2$. Our results also apply for other models for the power dependence on the distance [12].

Note that, as opposed to the Slepian-Wolf scenario considered in [8], here, since we consider the case of lossy coding of data at the nodes, this implies an additional number of optimization variables, namely the values of the *distortions* $\{D_i\}_{i=1}^N$ at nodes. This increases importantly the complexity of the problem. Also, additional constraints are introduced by the rate-distortion region [20]. In the case of high resolution coding ($D_i \rightarrow 0$, for any $i \in V$), that we consider in this paper, the rate-distortion region derived in [20] becomes tight. In general, this region provides an outer bound which becomes tighter as the rates become large. The expressions for the rate/distortion region for high-resolution data rates found in [20] are similar in form to the Slepian-Wolf region [19], although they include additional terms related to the various distortions. Also, as in the Slepian-Wolf scenario, an important feature of the rate-distortion region in [20] is that no communication is allowed among the nodes for performing the data coding, although, of course, in our data gathering problem, communication is needed and used only to transport the coded data from nodes to the sink. As a result of this, the problems of transmission structure optimization, rate and distortion allocation separate, as we will see in detail in Section III.

Assume a given node placement such as the one illustrated in Fig. 1. Denote R_i and D_i, $i = 1 \ldots N$, the allocated rate and distortion at node i. The most general form of our optimization problem is given as follows:

$$\{R_i^*, D_i^*, ST^*\}_{i=1}^N = \arg \min_{\{R_i, D_i, ST\}_{i=1}^N} \sum_{i=1}^N c_i R_i \tag{1}$$

under the constraints

$$\sum_{i \in \mathcal{X}} R_i \geq h(\mathcal{X}|V \backslash \mathcal{X}) - \log \left((2\pi e)^{|\mathcal{X}|} \Pi_{i \in \mathcal{X}} D_i \right), \forall \mathcal{X} \subset V \tag{2}$$

$$\sum_{i=1}^N D_i \leq D \tag{3}$$

$$D_i \leq D_i^{\max}, i = 1 \ldots N. \tag{4}$$

where ST is the spanning tree to be found, which defines the transmission structure; $c_i, i = 1 \ldots N$ are the total weights of the path from node i to the sink on the spanning tree ST, thus $c_i = \sum_{e \in \mathcal{E}_i} w_e^\kappa$, where $e \in \mathcal{E}_i$, and $\mathcal{E}_i$ is the set of edges linking node i to the sink S on ST, and w_e is the Euclidean distance of edge e; $h(\cdot)$ denotes the differential entropy. The constraints given by (2) express the rate-distortion region constraints given in [20], namely that the sum of rates for any given subset of nodes has to be larger than the entropy of the random variables measured at those nodes, conditioned on the random variables measured at all other nodes. In constraint (3), D is the maximum allowed total distortion and in (4), $D_i^{\max}, i = 1 \ldots N$ are the maximum individual constraints.

The optimization problem (1) assumes a fixed placement of nodes. However, in practice, notice that a well chosen placement of nodes (namely, allowing adjustable values of c_i, under the constraint that the desired measured field is fully covered by the nodes) can drastically increase the total efficiency of the network, as shown in [13]. In the general two-dimensional setting, the problem (1) is hard to formalize and solve when the node placement is introduced as a variable in the optimization, since a proper constraint set for the node placements, that preserve coverage, needs to be defined. Thus, in Section III, where we study the general optimization for a two-dimensional network, we will consider that the node placement is fixed. However, in Section IV, we will particularize the optimization (1) for the one-dimensional case, for which we solve completely the placement problem too. Namely, we provide optimal node placements for (a) minimization of the total power consumed by the network, and (b) maximization of the lifetime of the network.

B. Example: Correlated Gaussian Random Field

For the sake of clarity, in our numerical simulations we assume a *jointly Gaussian model* for the spatial data $\mathbf{X}$ measured at nodes, with an N-dimensional multivariate normal distribution $\mathbf{X} \sim \mathcal{N}^N(\mu, \mathbf{K})$:

$$f(\mathbf{X}) = \frac{1}{\sqrt{2\pi}\det(\mathbf{K})^{1/2}} e^{-\left(\frac{1}{2}(\mathbf{X}-\mu)^T \mathbf{K}^{-1}(\mathbf{X}-\mu)\right)}$$

where $\mathbf{K}$ is the covariance matrix of $\mathbf{X}$, and μ the mean vector. The diagonal elements of $\mathbf{K}$ are the variances $K_{ii} = \sigma_i^2$. The rest of K_{ij} depend on the distance between the corresponding nodes (e.g. $K_{ij} = \sigma_{ii}\sigma_{jj}\exp(-ad_{i,j}^{\beta})$ or $K_{ij} = \frac{\sigma_{ii}\sigma_{jj}}{1+ad^{\beta}}$, with $a > 0$ and $\beta \in \{1, 2\}$), where $d_{i,j}$ is the distance between nodes i and j [7], [10]. Without loss of generality, we will restrict our analysis to unit variance $\sigma_{ii} = 1, i = 1 \ldots N$ and zero-mean $\mu = 0$ processes: $\mathbf{X} \sim \mathcal{N}^N(0, \mathbf{K})$.

Notice that although the numerical evaluations will be performed using the Gaussian random field model, our results are valid for any spatially correlated random processes, whose correlation decreases with the distance.

III. Optimization

A. The Optimal Communication Tree Structure is SPT

Constraints (2) imply that nodes can code (asymptotically at high resolution) with any rate that obeys the rate-distortion region *without* explicitly communicating data with each other. As a consequence of the fact that it is possible to perform joint source coding without nodes communicating among them, we can state the following Proposition:

Proposition 1 – *Separation of the joint optimization of source coding and transmission structure:*

The overall joint optimization (1) can be achieved by first optimizing the transmission structure with respect to only *the path weights c_i, and then optimizing the distortion and rate allocation under the remaining constraints, for the given transmission structure.*

Proof: The joint cost function we consider is separable as the product of a function that depends only on the rate and another function that depends only on the link weights of the transmission structure. Once the rate allocation is *fixed*, the best way (least cost) to transport any amount of data from a given node i to the sink S does not depend on the value of the rate

R_i. Since this holds for any rate allocation, it is also true for the minimizing rate allocation and the result follows. ■

Proposition 1 implies that the optimal transmission structure that optimizes (1) is the shortest path tree: ST^*=*SPT*. Denote by c_i^* the total weight of the path from node i to the sink S on the *SPT*. Suppose without loss of generality that nodes are ordered in a list with increasing values of the weights corresponding to the shortest paths from each node to the sink, that is, $c_1^* \leq c_2^* \leq \cdots \leq c_N^*$. Then, by using Proposition 1, the optimization (1) becomes:

$$\begin{aligned} \{R_i^*, D_i^*\}_{i=1}^N &= \arg \min_{\{R_i, D_i\}_{i=1}^N} \sum_{i=1}^N c_i^* R_i \\ \text{under the constraints} & \\ (2), (3), (4). & \end{aligned} \tag{5}$$

Note that Proposition 1 also implies that our results can be easily extended to the case of multiple sinks. Namely, if data from a given node is required at more than one sink, then there are two possible scenarios for the rate allocation at a node: (a) to code its data with a different rate for each sink, or (b) to code its data with the same rate for all sinks. In the first scenario, the optimal transmission structure corresponding to each sink will be the shortest path from the node to that sink, and there will be a separate optimal rate-distortion allocation solution across the network, for each sink. In the second scenario, the shortest path from a node to the sink will be replaced by the weight of the minimal Steiner tree[2] spanning the node and the sinks that require data from that node. All the results of this paper naturally generalize to the case of multiple sinks by replacing the SPT with a superpositions of Steiner trees corresponding to the optimal transmission structure of each node; this will also determine the corresponding optimal weights. Therefore, we will restrict our analysis to the case of a single sink.

B. Optimal Rate Allocation

First, we show that, regardless of constraints (3), (4), the optimal rate allocation has to obey only constraints (2), that is, the rate allocations do not depend on either the upper bounds on

[2]The Steiner problem is a computationally difficult approximation problem, however good approximation algorithms are known to exist [1].

the total distortion or on the individual distortions. Moreover, for any set of distortion values $\{D_i\}_{i=1}^{N}$, the rate allocation is given by:

Theorem 1 *Optimal rate allocation:*

$$\begin{aligned} R_1^* &= h(X_1) - \log 2\pi e D_1 \\ R_2^* &= h(X_2|X_1) - \log 2\pi e D_2 \\ &\dots \\ R_N^* &= h(X_N|X_{N-1},\dots,X_1) - \log 2\pi e D_N. \end{aligned} \tag{6}$$

Proof: First, we prove that (6) is a feasible solution for (5). Consider any constraint from (2), for some subset $\mathcal{X} \subset V$. Denote by $M = |\mathcal{X}|$ the number of elements in $\mathcal{X}$. Order the indices of $X_i \in \mathcal{X}$ as $i_1, i_2, i_3, \dots i_M$, with i_1 indexing the closest node and i_M the furthest node from the sink on the SPT.

If we rewrite the left-hand-side of (2) in terms of the solutions (6) that we provide in the statement of the theorem, we have:

$$\begin{aligned} \sum_{i \in \mathcal{X}} R_i &= h(X_{i_M}|X_{i_M-1},\dots,X_1) - \log 2\pi e D_{i_M} + \qquad (7) \\ &\quad h(X_{i_{M-1}}|X_{i_{M-1}-1},\dots,X_1) - \log 2\pi e D_{i_{M-1}} + \\ &\quad \dots + h(X_{i_1}|X_{i_1-1},\dots,X_1) - \log(2\pi e) D_{i_1}. \end{aligned}$$

Expanding the right-hand-side terms with the chain law for conditional entropies, we obtain:

$$\begin{aligned} h(\mathcal{X}|\mathcal{X}^C) &= h(X_{i_M}|\mathcal{X}^C \cup \{\mathcal{X} - \{X_{i_M}\}\}) + h(X_{i_{M-1}}|\mathcal{X}^C \cup \{\mathcal{X} - \{X_{i_M}, X_{i_{M-1}}\}\}) + \\ &\quad \dots + h(X_{i_1}|\mathcal{X}^C \cup \{\mathcal{X} - \{X_{i_M},\dots,X_{i_1}\}\} - \log(2\pi e)^{|\mathcal{X}|} D_{i_1} D_{i_2} \dots D_{i_M} \qquad (8) \\ &= h(X_{i_M}|V - \{X_{i_M}\}) - \log 2\pi e D_{i_M} + \\ &\quad h(X_{i_{M-1}}|V - \{X_{i_M}, X_{i_{M-1}}\}) - \log 2\pi e D_{i_{M-1}} + \\ &\quad \dots + h(X_{i_1}|V - \{X_{i_M}, X_{i_{M-1}},\dots,X_{i_1}\}) - \log 2\pi e D_{i_1}. \end{aligned}$$

Consider the terms on the right-hand-side of (7) and (8). For any $i_k \in \mathcal{X}$, the term corresponding to X_{i_k} in (8) is at most equal to its counterpart in (7), because the set of nodes on which the entropy conditioning is done for each term in (7) is a subset of its counterpart in (8). Since the choice of $\mathcal{X}$ was arbitrary, then any constraint in (2) is satisfied by the assignment (6). Note also that the rate allocation in (6) satisfies with equality the constraint on the total sum of rates:

$$\sum_{i \in V} R_i \geq h(X_1, \ldots, X_N) - \log(2\pi e)^N \Pi_{i=1}^N D_i. \tag{9}$$

We prove now by induction that the assignment in (6) makes the expression to be minimized in (5) smaller than any other valid assignment. Note first that we cannot allocate to X_N less than $h(X_N|X_{N-1}, X_{N-2}, \ldots, X_1) - \log 2\pi e D_N$ bits, due to the particular constraint in (2) corresponding to $\mathcal{X} = \{X_N\}$. Assume now that a rate allocation solution with $h(X_N|X_{N-1}, X_{N-2}, \ldots, X_1) - \log 2\pi e D_N$ bits to X_N is not optimal, and X_N is assigned $h(X_N|X_{N-1}, \ldots, X_1) - \log 2\pi e D_N + b$ bits. Due to (9), at most b bits in total can be extracted from the rates assigned to some of the other nodes. But since c_N^* is the largest coefficient in the optimization problem (5), it is straightforward to see that any such change in rate allocation increases the cost function in (5). Thus assigning $R_N = h(X_N|X_{N-1}, \ldots, X_1) - \log 2\pi e D_N$ bits to X_N is optimal.

Consider now the rate assigned to X_{N-1}. From the rate constraint corresponding to $\mathcal{X} = \{X_{N-1}, X_N\}$, it follows that:

$$\begin{aligned} R_N + R_{N-1} &\geq h(X_N, X_{N-1}|X_{N-2}, \ldots, X_1) - \log(2\pi e)^2 D_N D_{N-1} \\ &= h(X_N|X_{N-1}, X_{N-2}, \ldots, X_1) - \log 2\pi e D_N + \\ &\quad\; h(X_{N-1}|X_{N-2}, \ldots, X_1) - \log 2\pi e D_{N-1}. \end{aligned}$$

Since for optimality of the solution R_N must be given by $R_N = h(X_N|X_{N-1}, X_{N-2}, \ldots, X_1) - \log 2\pi e D_N$, thus $R_{N-1} \geq h(X_{N-1}|X_{N-2}, \ldots, X_1) - \log 2\pi e D_{N-1}$. Following a similar argument as for X_N, the optimal solution allocates $R_{N-1} = h(X_{N-1}|X_{N-2}, \ldots, X_1) - \log 2\pi e D_{N-1}$. The rest of the proof follows similarly by considering successively the constraints corresponding to subsets $\mathcal{X} = \{X_i, X_{i+1}, \ldots, X_N\}$, with $i = N-2, N-3, \ldots 1$. ■

C. Total Distortion Constraint

In this section, we consider optimization of (1) for the case where we assume that the constraints given by (4) are not active. By Theorem 1, if the positions of the nodes are fixed, $\{R_i^*\}_{i=1}^N$ *only* depends on $\{D_i\}_{i=1}^N$. Therefore, at this point, we can insert in (5) the values for $\{R_i^*\}_{i=1}^N$ given by Theorem 1, thus obtaining an optimization problem having as argument the set of distortions $\{D_i\}_{i=1}^N$ only, that is:

$$\{D_i^*\}_{i=1}^N = \arg\min_{\{D_i\}_{i=1}^N} \sum_{i=1}^N c_i^* \cdot (h(X_i|X_{i-1},\ldots,X_1) - \log 2\pi e D_i)$$

under the constraint (10)

$$\sum_{i=1}^N D_i \leq D.$$

Note that the differential entropy terms in (10) do not depend on the distortions D_i. Thus, (10) can be equivalently written as:

$$\{D_i^*\}_{i=1}^N = \arg\max_{\{D_i\}_{i=1}^N} \sum_{i=1}^N c_i^* \log 2\pi e D_i$$

under the constraint (11)

$$\sum_{i=1}^N D_i \leq D.$$

Denote $\sum_{i=1}^N c_i^* = C$. The solution of the optimization problem (11) is easily obtained, using Lagrange multipliers, giving a linear distribution of distortions:

$$D_i^* = D \cdot \frac{c_i^*}{C}, \quad i = 1 \ldots N. \tag{12}$$

By combining (6) and (12), we obtain that the rate-distortion allocation at nodes is given by:

$$R_i^* = h(X_i|X_{i-1},\ldots,X_1) - \log\left(\frac{2\pi e D c_i^*}{C}\right), \quad i = 1 \ldots N. \tag{13}$$

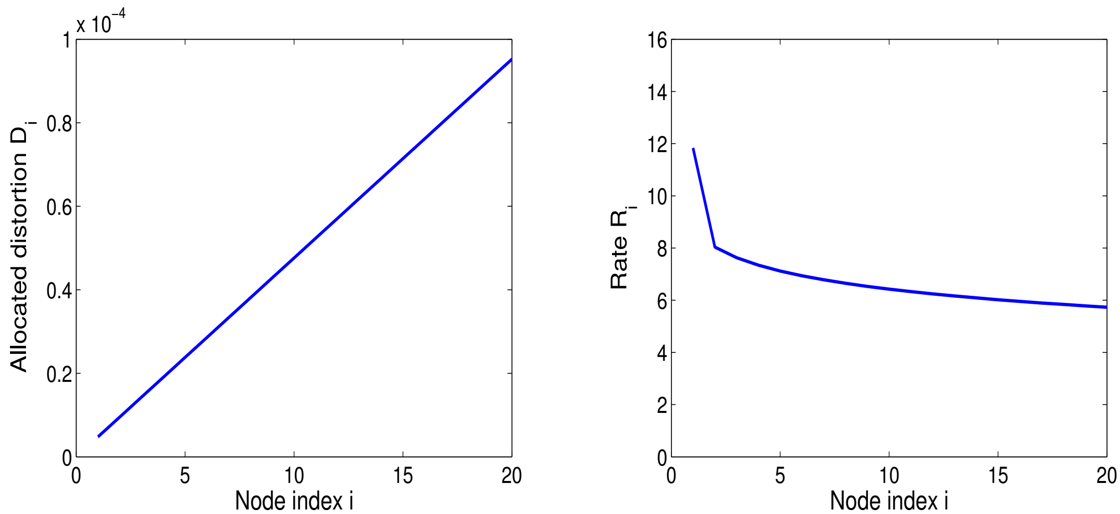
Note that for correlation functions that depend only on the distance among nodes, as we consider in this paper, and for an uniform placement of nodes, the differential entropy monotonically decreases as the number of nodes on which conditioning is done increases. Also, by definition, the sequence $\{c_i^*\}_{i=1}^N$ is monotonically increasing with i. As a result, the rate allocation R_i in (13) is a function that monotonically decreases with the node index i.

Further, (13) essentially depends only on the path weights ordering of the nodes on the SPT, given by $\{c_i^*\}_{i=1}^N$. For example, in the case of correlated Gaussian random fields, (13) can be written as:

$$R_i^* = \log \frac{\det(\mathbf{K}(1,\ldots i))}{\det(\mathbf{K}(1,\ldots,i-1))} \frac{C}{c_i^* \cdot D}, \quad i = 1 \ldots N, \tag{14}$$

where $\mathbf{K}(1,\ldots,i)$ is the correlation matrix corresponding to nodes $1,\ldots,i$.

In Fig. 2, we illustrate the distortion and rate allocations provided by (14) for the one-dimensional grid, shown in Fig. 3, with uniform inter-node distances, measuring a correlated



(a) Distortion allocation as a function of node index.

(b) Rate allocation as a function of node index.

Fig. 2. One-dimensional grid with constraint on the total distortion. Nodes are labelled with increasing indexes, with node 1 closest to the sink and node N farthest. The correlation model is $\exp(-ad_{i,j})$.

Fig. 3. One-dimensional grid network.

Gaussian random field with $\beta = 1$ (which corresponds to sampling a continuous Gauss-Markov process). The same analysis holds for arbitrary two-dimensional networks. For clarity in our numerical experiments, we keep the values of the model parameters constant along this paper: Gaussian random processes with $\alpha = 10^{-3}$, $\beta = 1$, and $N = 20$, $D = 10^{-3}$, $D_i^{\max} = 0.7 \cdot 10^{-3}$, $\kappa = 2$.

These results are intuitively expected: at the extremities of the network (where the cost c_i^* of the path from node i to the sink is large), small rates are allocated, meaning large allocated distortions and that conditioning is done on many nodes that are closer to the sink on the *SPT*; on the contrary, large rates, meaning small distortions and that conditioning is done on fewer

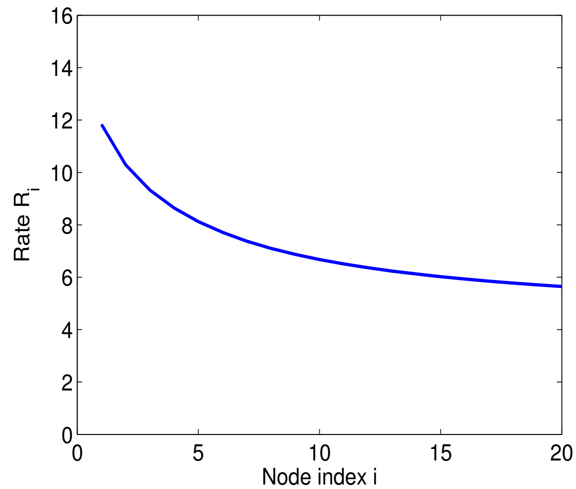


Fig. 4. Optimal rate allocation for the one-dimensional grid with constraint on the total distortion. The correlation model is $\exp(-ad_{i,j}^2)$.

nodes closer to the sink on the *SPT*, are allocated at nodes near the sink (where the cost c_i^* is small). For the nodes in-between, there is a monotonically increasing set of rates as we move towards the sink.

Note that for $\beta = 1$, the resulting continuous process is Gauss-Markov, which means that if the nodes are equally-spaced then the differential entropies are equal in (13), and thus the decrease in rate in Fig. 2(b) is due only to the increase in allocated distortion.

Similar results are obtained for many other network parameter settings. For instance, for a process with $\beta = 2$, the result is a less abrupt decay of the rate when the node index increases, as a result of both the smoother decrease of the differential entropy and of the increase in allocated distortion in (14), with the number of nodes on which conditioning is done (see Fig. 4, for $N = 20, D = 10^{-3}, \alpha = 0.1, \kappa = 2, \beta = 2$). For the sake of clarity, in the next sections we only show numerical results for processes with correlation function $\exp(-a|d|)$; the curves corresponding to all our results are similar in shape and behavior for processes with correlation function $\exp(-a|d|^2)$ or $\frac{1}{1+a|d|^\beta}$.

D. Individual Distortion Constraints

Let us now additionally introduce the individual constraints (4). For the sake of clarity, assume first that all individual constraints are equal, namely $D_i^{\max} = T$. Using again Lagrange optimization, it is simple to show that the solution in this case can be obtained as follows:

- Consider the solution for the distortion allocation given by (12), without introducing individual distortion constraints.
- Let $m+1$ be the smallest node index for which $D_i \geq T, i = m+1 \dots N$. Allocate $D_i^* = T, i = m+1, \dots, N$, that is, we make the last $N-m$ constraints alive. To find the rest of the distortion values $D_i, i = 1, \dots, m$, we solve:

$$\begin{aligned} \{D_i^*\}_{i=1}^m &= \arg \max_{\{D_i\}_{i=1}^m} \sum_{i=1}^m c_i^* \log 2\pi e D_i \\ &\text{under the constraint} \\ \sum_{i=1}^m D_i &\leq D - (N-m)T. \end{aligned} \tag{15}$$

  which is exactly the same type of problem as in (11). Thus, the complete solution is readily given by:

$$\begin{aligned} D_i &= T, i = m+1 \dots N \\ D_i &= c_i^* \frac{D-(N-m)T}{C_m} \end{aligned}$$

  where $C_m = \sum_{i=1}^m c_i^*$. Fig. 5 shows the distortion and rate allocations that are obtained in this case.

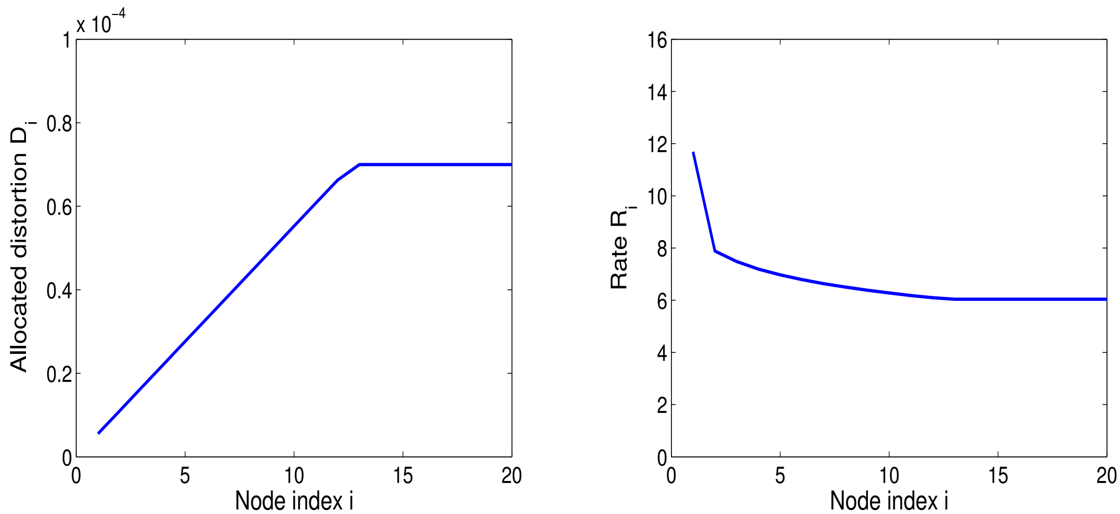
In Fig. 5(b), we see that when individual distortions are considered, the rates allocated at the nodes of the extremity of the network are equalized (as compared to Fig. 2(b)). As a result of this rate load added to nodes at the extremity of the network, the rates at the nodes closer to the sink can be correspondingly decreased.

In the case where the individual constraints are different, a similar procedure can be used, that is, a subset of the constraints will be made active and then again a problem similar to (11) can be easily solved.

IV. Placement Optimization for the One-Dimensional Network

A. Total Power Minimization

In some scenarios, the positions of the nodes are not fixed in advance, but it is possible to place the nodes optimally so as to minimize various resources [13]. Since the study in this paper is concerned with power efficient scenarios, one possible task to be achieved when the node



(a) Distortion allocation as function of node index.

(b) Rate allocation as a function of node index.

Fig. 5. One-dimensional grid with constraints on the total *and* individual distortions. Nodes are labelled with increasing indexes, with node 1 closest to the sink and node N farthest.

placement can be chosen is the total power efficiency. Namely, we consider the optimization (1) where the positions of the nodes are additional variables, and the optimization is done additionally over the weights of the paths from the nodes to the sink:

$$\begin{aligned}\{R_i^*, D_i^*, c_i^*\}_{i=1}^N &= \arg \min_{\{R_i, D_i, c_i\}_{i=1}^N} \sum_{i=1}^N c_i R_i \\ \text{under the constraints} \quad & (2), (3), (4),\end{aligned} \tag{16}$$

with additional constraints on $\max\{c_i\}_{i=1}^N$ (coverage constraint) and on $\max\{c_i - c_{i-1}\}_{i=2}^N$ (inter-node space constraint). The coverage constraint imposes that the entire area is covered. The inter-node space constraint ensures that any given point in the measured area is close enough to a sensor node, such that the data corresponding to that point can be reconstructed with a certain minimal accuracy at the sink, by approximating it with the value measured by the closest sensor.

For the sake of simplicity, in this section, we consider a one-dimensional scenario (see Fig. 3). A similar analysis can be performed in the two-dimensional case, since the rate and distortion allocations only depend on the ordering of the nodes on the computed SPT; however, in the two-dimensional case, an additional model describing how nodes can be moved from their initial positions is required (e.g. on two-dimensional grids), which makes the problem more complicated. One possible solution is to use a radial structure for the node placement, as in [13]

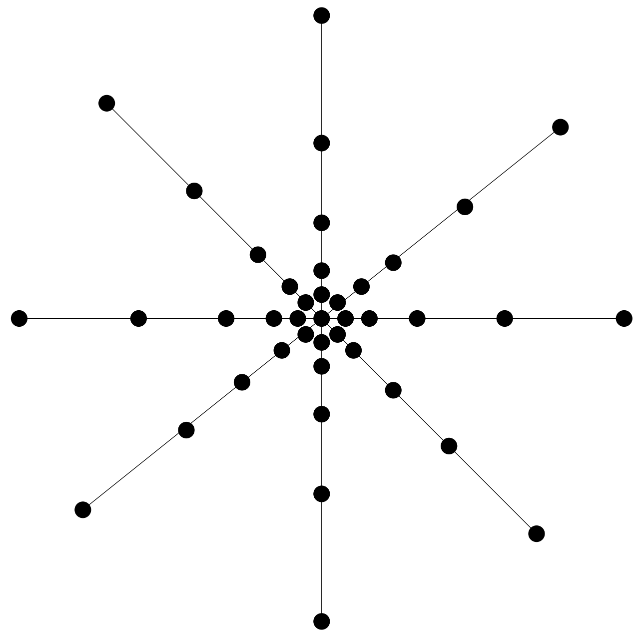


Fig. 6. A *wheel* two-dimensional placement, with the sink in the middle.

(see Fig. 6), where the placement of nodes for each spoke is as in the optimal one-dimensional placement presented in this section. A complete study of the optimal number of spokes and number of nodes per spoke for a given N is beyond the scope of this paper.

Finding analytically the solution for (16) by partial derivatives is complicated, since the expressions for the rates $\{R_i^*\}_{i=1}^N$ obtained in (13) depend on both $\{D_i^*\}_{i=1}^N$ and $\{h(X_i|X_{i-1},\ldots,X_1)\}_{i=1}^N$; moreover, these sets of values depend themselves on the values of $\{c_i\}_{i=1}^N$. Instead, we use the following iterative algorithm for optimization:

Algorithm 1: Optimal placement (see Fig. 7).

- Step 1: Initially, the values of c_i are chosen such that the nodes are equally spaced (as in Fig. 3)[3].
- Step 2: **until** convergence **do**:
  - (a) Solve (16) with parameters $\{c_i\}_{i=1}^N$ to obtain $\{R_i\}_{i=1}^N$ as in (13).
  - (b) Solve (16) with parameters $\{R_i\}_{i=1}^N$ to obtain $\{c_i\}_{i=1}^N$.

[3]Note that the placement uniquely determines the ordering of nodes. Namely, once an order is set among the nodes, the same order will hold along the optimization, while only the distances between the nodes will change.

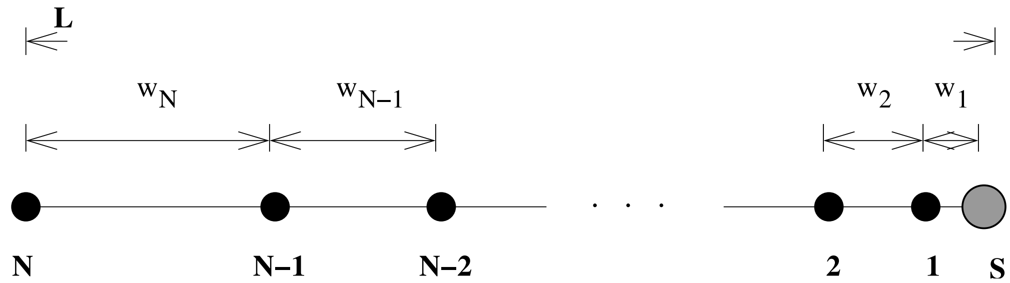


Fig. 7. Optimized placement for the one-dimensional network.

The convergence of the algorithm is ensured by the convexity of the cost function and the constraint sets [4]. Moreover, since there is only one local minimum, the convergence of the algorithm to the global minimum is ensured.

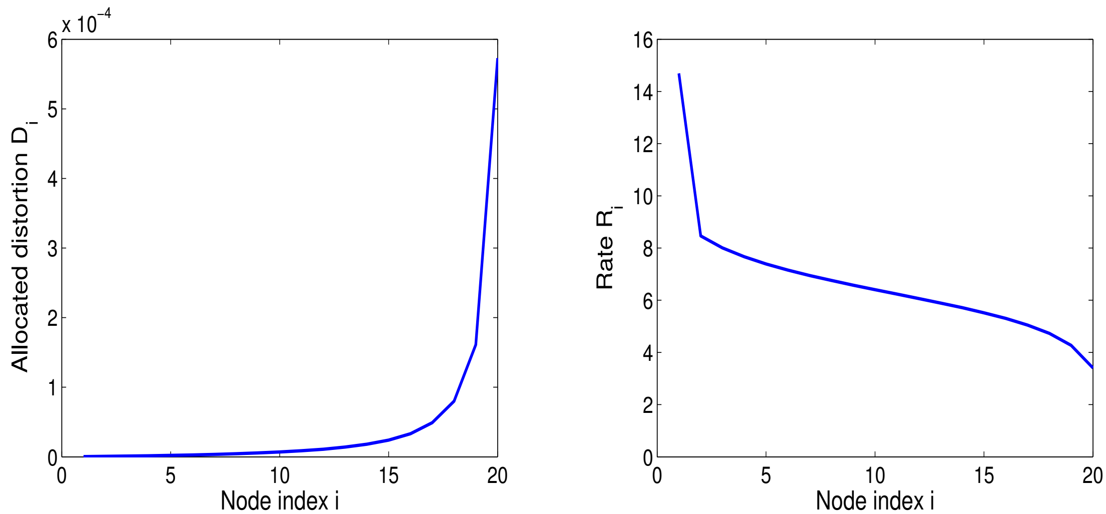
The optimization in Step 2(a) of the Algorithm has been presented in Section III. We describe now the optimization in Step 2(b). For the sake of clarity, let us first ignore the inter-node constraints in (16). Denote $w_i^\kappa = c_i - c_{i-1}, i = 1 \dots N$, where w_i is the Euclidean distance between node i and node $i-1$, and κ is the power coefficient of the distance[4]. Thus, $c_i = \sum_{j=1}^{i} w_i^2$. The values of R_i^* are computed as in (13), using as initialization $w_i = \frac{L}{N}$, corresponding to equally spaced nodes. Notice that the expression of the solution for R_i^* is independent of the actual value of the placement. Therefore, we can now rewrite the optimization (16) including the unknowns $\{w_i\}_{i=1}^N$ as follows:

$$\begin{aligned} \{w_i^*\}_{i=1}^N &= \arg \min_{\{w_i\}_{i=1}^N} \sum_{i=1}^{N} (\sum_{j=i}^{N} R_j^*) w_i^\kappa \\ &\text{under the constraint:} \\ \sum_{i=1}^{N} w_i &= L. \end{aligned} \tag{17}$$

The solution of (17) is easily obtained using Lagrange multipliers. For $\kappa = 2$, it is possible to obtain the closed-form solution:

$$w_i^* = \frac{L}{\sum_{j=i}^{N} R_j^* \left(\sum_{l=1}^{N} \frac{1}{\sum_{j=l}^{N} R_j^*} \right)}, \quad i = 1 \dots N. \tag{18}$$

[4]For the sake of the simplicity of analysis, we will use $\kappa = 2$ in solving the optimization problems, however our results can be easily extended to arbitrary values of κ; it is hard to obtain closed-form solutions for arbitrary $\kappa > 2$.



(a) Distortion allocation as a function of node index.

(b) Rate allocation as a function of node index.

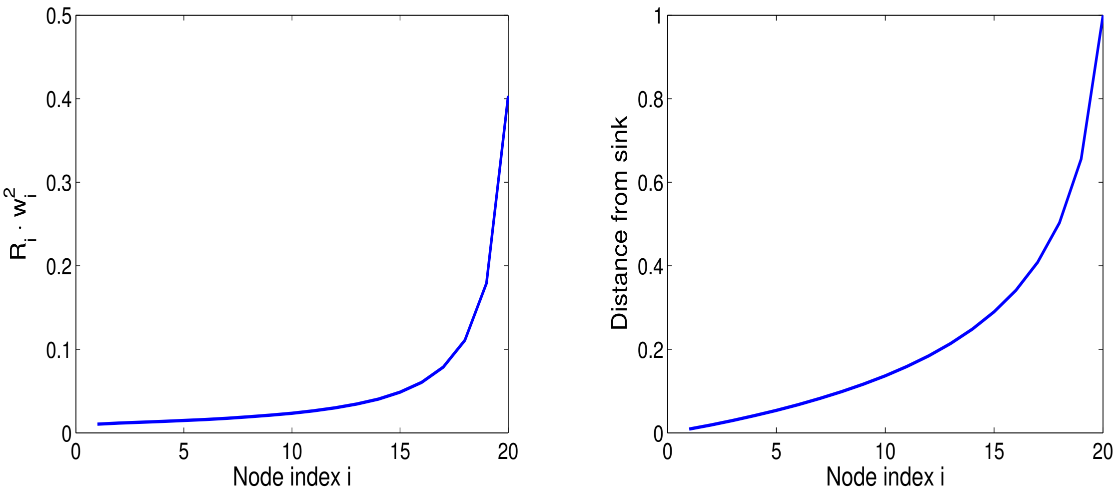
Fig. 8. Optimized placement for the one-dimensional network for the case of minimization of total consumed power.

Note that by placing the nodes as given by (18) the rate allocation changes as a function of $\{w_i^*\}_{i=1}^N$, since both the differential entropies and optimal distortions in (13) depend on the values of c_i. Next, for these values of $\{c_i^*\}_{i=1}^N$, we optimize over $\{R_i^*\}_{i=1}^N$ (Step 2(a) of Algorithm 1). We repeat this iterative procedure until the algorithm converges. The resulting distortion and rate allocation are shown in Fig. 8.

It is complicated to study in a closed-form the monotonicity of the rate allocation (13) for a placement which is not uniform, due to the complex interplay between node positions, distortion allocations and conditional differential entropies. However, our numerical simulations show monotonicity of the rate allocation in the case of power efficient optimal placement, as well.

The convergence of Algorithm 1 is ensured by the convexity of the cost function and the constraint sets. For the instances we used ($N = 20$, $\alpha = 10^{-3}$, $\kappa = 2$, $\beta = 1$, no constraints on the inter-node distances), the algorithm converges in at most 4 steps. Our numerical results show that nodes at the extremity of the network are largely spread, and nodes near the sink are closer to each other (see Fig. 9(b)).

Note that by using a similar technique we can solve the problem that additionally considers the constraints imposed on the maximum inter-node distances. In the resulting solution, the distances



(a) Power consumption as a function of node index.

(b) Distance from sink as a function of node index.

Fig. 9. Placement optimization for minimizing the total power.

between the nodes would become more even.

B. Network Lifetime Maximization

The plot in Fig. 9 shows the power consumption at each node and the distances from the nodes to the sink for the scenario in Section IV-A, as a result of solving problem (16). Moreover, in Fig. 10 we plot the variance of power consumption per node as the size of network increases. We see that the decrease in variance slows down as the size of the network increases.

The power consumption of a node is a quantity inversely proportional to the lifetime of a node. Sometimes, in practice, a different target of interest is to maximize the lifetime of the network. For instance, in Fig. 9, the last node N will be the first to die. Let us now separately consider another set of constraints related to the lifetime of the network, namely we impose that all nodes consume the same amount of power. This new set of constraints are aimed at maintaining a large number of nodes alive. Namely, our problem now amounts to solving the

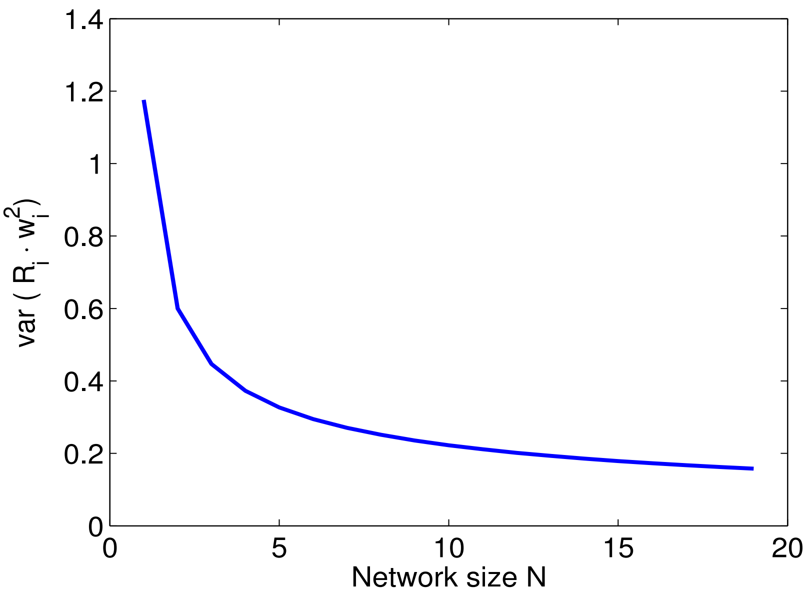


Fig. 10. Variance of the power consumption with the increase of the network size.

following set of equations:

$$\begin{aligned}\sum_{i=1}^{N} w_i &= L \qquad (19)\\ \left(\sum_{j=i}^{N} R_j^*\right) w_i^2 &= \left(\sum_{j=k}^{N} R_j^*\right) w_k^2, i, k = 1 \dots N\\ \sum_{i=1}^{N} D_i &= D.\end{aligned}$$

The solution is easily found to be:

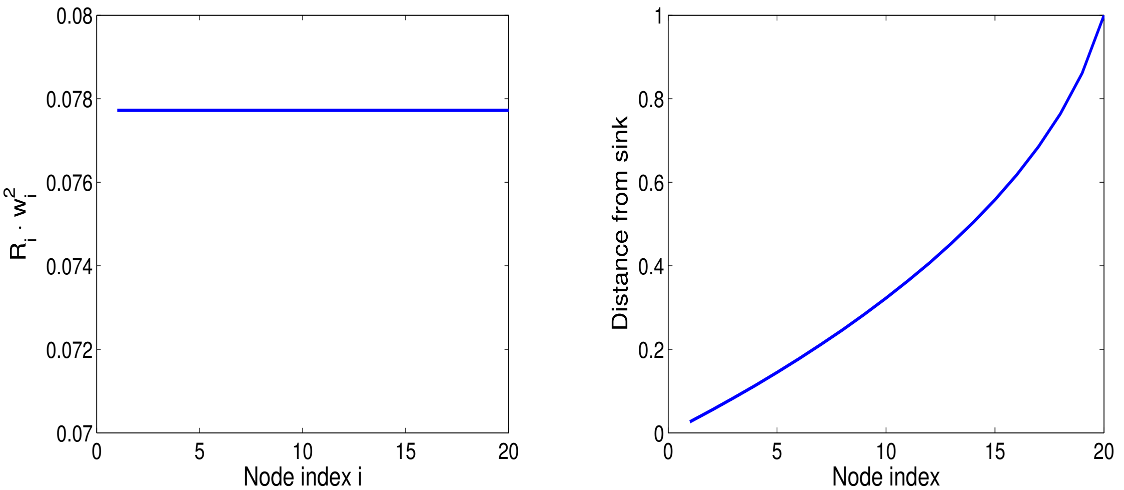
$$w_i^* = \frac{L}{\sqrt{\sum_{j=i}^{N} R_j^*}\left(\sum_{l=1}^{N} \sqrt{\frac{1}{\sum_{j=l}^{N} R_j^*}}\right)}. \qquad (20)$$

A similar result can be easily obtained for an arbitrary value of the distance power coefficient κ, namely the square roots in (20) are replaced by the κth roots.

Note that $\{R_i^*\}_{i=1}^N$ depend on $\{w_i^*\}_{i=1}^N$ as in (13), thus we can implement a similar iterative procedure to find the optimal weights $\{w_i^*\}_{i=1}^N$:

Algorithm 2: Lifetime optimization.

- **until** convergence **do**:



(a) Power consumption as a function of node index.

(b) Distance from sink as a function of node index.

Fig. 11. Placement for lifetime optimization.

- (a) Given $\{w_i\}_{i=1}^N$, express $\{R_i\}_{i=1}^N$ as (13).
- (b) Solve (19) with parameters $\{R_i\}_{i=1}^N$ to obtain $\{w_i\}_{i=1}^N$ as (20).

Fig. 11 shows the distance from sink, and power consumption at each node, when the optimization of node placement is done as in (20).

For the same instance, the total power consumed with the optimized placement of Section IV-A is 1.17 [bit $\cdot$ m^2], whereas with the scenario in this section, the total power is 1.55 [bit $\cdot$ m^2]. Note that a placement that optimizes both the total power consumption of the network while maintaining the same individual consumption levels at nodes should be a tradeoff of the results of the two optimization problems.

However, it is complicated to provide an exact optimal solution for the node placement that optimizes one of the target metrics under constraints on the other target metric. This requires starting from the node placement solution that optimizes one of the metrics, and moving each and every node until the constraint on the other quantity is satisfied. This optimization is clearly not scalable, thus, we provide a simple sub-optimal solution that still provides a good behavior, in terms of trading-off nicely both costs.

Namely, we propose a solution for the node placement that is obtained by computing a weighted combination of the two separate placement solutions. Namely, if we denote the place-

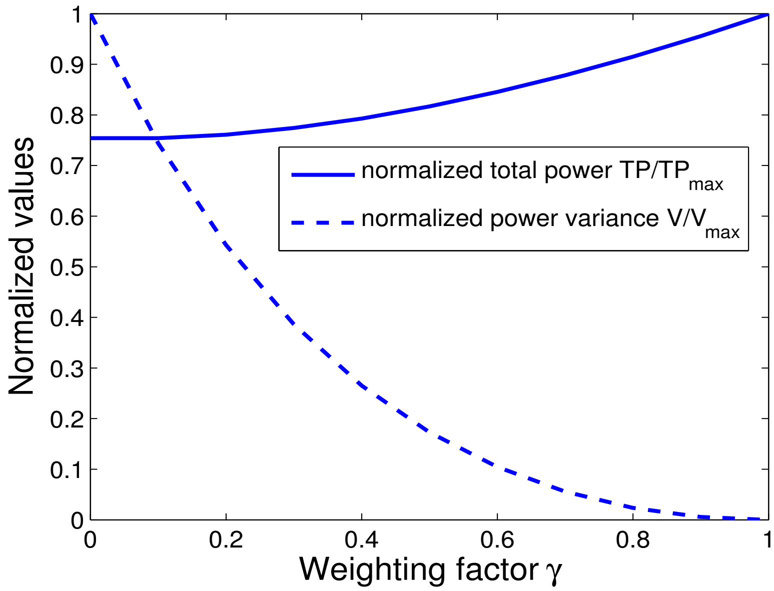


Fig. 12. Normalized total power and power variance for the tradeoff placement, as a function of the weighting factor γ.

ment solution for total power minimization in (17) by $\{w_i'\}_{i=1}^N$, and the placement solution for network lifetime maximization in (20) by $\{w_i''\}_{i=1}^N$, then our proposed tradeoff solution is:

$$w_i^* = \gamma \cdot w_i'' + (1-\gamma) \cdot w_i', \quad i = 1 \ldots N. \tag{21}$$

In Fig. 12, we plot the normalized values for the total power and respectively power variance, for $\gamma \in [0,1]$, when node positioning is done as in (21). Note that, as expected, the choice of γ provides a tradeoff between having a small power consumption for the network versus equalizing the individual power at nodes.

V. Conclusions

We considered the problem of power efficient data gathering in sensor networks, with high-resolution coding, under distortion constraints. We found the rate and distortion allocations in a closed-form, and illustrated our results with numerical experiments on Gaussian correlated random fields. We also studied the problem of optimal placement for two power efficiency targets of interests, namely total power and network lifetime, and compared the tradeoffs involved. Finally, we proposed a node placement solution that provides a tradeoff for the optimization of the two metrics.

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