1.- Integral indefinida de la función sen ωt

$$\int \text{sen}\ \omega t\ dt = -\frac{\cos \omega t}{\omega} + C$$

2.- Integral indefinida del cuadrado de la función sen ωt

$$\int \text{sen}^2\ \omega t\ dt = \int \frac{1 - \cos 2\,\omega t}{2}\, dt = \int \frac{1}{2}\, dt - \int \frac{1}{2} \cos 2\,\omega t\ dt = \frac{t}{2} - \frac{\text{sen}\ 2\,\omega t}{4\omega} + C$$

3.- Integral de la función sen ωt entre 0 y T/2

$$\int_0^{T/2} \text{sen}\ \omega t\ dt = -\left|\frac{\cos \omega t}{\omega}\right|_0^{T/2} = -\frac{1}{\omega}\left(\cos\ \omega \frac{T}{2} - \cos 0\right) = -\frac{T}{2\pi}\left(\cos \frac{2\pi}{T}\frac{T}{2} - 1\right) =$$

$$= -\frac{T}{2\pi}\left(\cos\pi - 1\right) = -\frac{T}{2\pi}(-1-1) = \frac{T}{\pi}$$

$$\int_0^{T/2} \text{sen}\ \omega t\ dt = \frac{T}{\pi}$$

4.- Integral del cuadrado de la función sen ωt entre 0 y T/2

$$\int_0^{T/2} \text{sen}^2 \omega t\ dt = \int_0^{T/2} \frac{1 - \cos 2\,\omega t}{2}\, dt = \left|\frac{t}{2} - \frac{\text{sen}\ 2\,\omega t}{4\omega}\right|_0^{T/2} = \frac{T}{4} - \frac{T}{8\pi}\left(\text{sen}\ 2 \frac{2\pi}{T}\frac{T}{2} - \text{sen}\ 0\right) =$$

$$= \frac{T}{4} - \frac{T}{8\pi}\left(\text{sen } 2\pi - \text{sen}0\right) = \frac{T}{4}$$

$$\int_{0}^{T/2} \text{sen}^{2}\omega t \, dt = \frac{T}{4}$$

5.- Integral de la función sen ωt entre t_1 y t_2 siendo

$0 < t_1 < t_2 < T/2$; $t_t + t_2 = T/2$; $\omega t_1 = \text{arcsen}\frac{V_b}{V_0}$; $\omega t_2 = \pi - \omega t_1$; con $V_b < V_0$

$$t_1 = \frac{T}{2\pi}\text{arcsen}\frac{V_b}{V_0} \qquad t_2 = \frac{T}{2} - \frac{T}{2\pi}\text{arcsen}\frac{V_b}{V_0} \qquad t_2 - t_1 = \frac{T}{2} - \frac{T}{\pi}\text{arcsen}\frac{V_b}{V_0}$$

$$\int_{t1}^{t2} \text{sen } \omega t \, dt = -\left|\frac{\cos \omega t}{\omega}\right|_{t1}^{t2} = -\frac{1}{\omega}\left(\cos \omega t_2 - \cos \omega t_1\right) = -\frac{T}{2\pi}\left(\cos\left(\pi - \omega t_1\right) - \cos \omega t_1\right) =$$

$$= -\frac{T}{2\pi}\left(-\cos\omega t_1 - \cos\omega t_1\right) = \frac{T}{\pi}\cos\left(\text{arcsen}\frac{V_b}{V_0}\right)$$

$$\int_{t1}^{t2} \text{sen } \omega t \, dt = \frac{T}{\pi}\cos\left(\text{arcsen}\frac{V_b}{V_0}\right)$$

6.- Integral del cuadrado de la función sen ωt entre t_1 y t_2

$$\int_{t1}^{t2} \text{sen}^{2}\omega t \, dt = \int_{t1}^{t2} \frac{1 - \cos 2\omega t}{2} dt = \left|\frac{t}{2} - \frac{\text{sen } 2\omega t}{4\omega}\right|_{t1}^{t2} = \frac{(t_2 - t_1)}{2} - \frac{T}{8\pi}\left(\text{sen } 2\omega t_2 - \text{sen } 2\omega t_1\right)$$

$$\int_{t1}^{t2} \text{sen}^{2}\omega t\,dt == \frac{T}{4} - \frac{T}{2\pi}\arcsen\frac{V_b}{V_0} + \frac{T}{2\pi}\frac{V_b}{V_0}\cos\left(\arcsen\frac{V_b}{V_0}\right)$$

En la anterior demostración se han hecho los siguientes cálculos intermedios:

$$\text{sen}\,2\,\omega t_2 = 2\text{sen}\,\omega t_2 \cos\omega t_2 = 2\text{sen}(\pi - \omega t_1)\cos(\pi - \omega t_1) = -2\text{sen}\,\omega t_1 \cos\omega t_1$$

$$\text{sen}\,2\,\omega t_2 - \text{sen}\,2\,\omega t_1 = -2\text{sen}\,\omega t_1 \cos\omega t_1 - 2\text{sen}\,\omega t_1 \cos\omega t_1 = -4\text{sen}\,\omega t_1 \cos\omega t_1 = -4\frac{V_b}{V_0}\cos\left(\arcsen\frac{V_b}{V_0}\right)$$

$$\text{sen}\,\omega t_1 = \text{sen}\left(\arcsen\frac{V_b}{V_0}\right) = \frac{V_b}{V_0}$$

$$t_2 - t_t = \frac{T}{2} - \frac{T}{\pi}\arcsen\frac{V_b}{V_0}$$