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TEMA 3: Ecuaciones en derivadas parciales *

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Resolver las EDP siguientes:

1. //Oteo//

| | |
|-----|-------------------------------------------------------|
| EDP | $u_t = u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(1, t) = 0$ |
| CI | $u(x, 0) = 1 \quad (0 < x < 1)$ |

2. //Oteo//

| | |
|-----|-------------------------------------------------------|
| EDP | $u_t = u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(1, t) = 0$ |
| CI | $u(x, 0) = x^2 - x \quad (0 < x < 1)$ |

3. //Oteo//

| | |
|-----|---------------------------------------------------------------------------------------------|
| EDP | $u_{tt} = u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(L, t) = 0$ |
| CI | $u(x, 0) = \sin(3\pi x/L) \quad (0 < x < L)$ $u_t(x, 0) = (3\pi\alpha/L) \sin(3\pi x/L)$ |

4. //Oteo// Problema de la cuerda de guitarra vibrando

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|-----|----------------------------------------------------------------------------------------------------------------------|
| EDP | $u_{tt} = u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(1, t) = 0 \quad (0 < x < 1)$ |
| CI | $u(x, 0) = \begin{cases} 2hx & x \leq 1/2 \\ 2h(1-x) & 1/2 < x \leq 1 \end{cases}$ $u_t(x, 0) = 0 \quad h : cte.$ |

5. //Pablo Z. [Carlos M.]//

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|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| EDP | $u_{tt} = c^2 u_{xx}, \quad (0 < t < \infty)$ |
| CI | $u(x, 0) = \sqrt{1 - x^2}, \quad -1 < x < 1; \quad u(x, 0) = 0, \quad \text{si no}$ $u_t(x, 0) = cx/\sqrt{1 - x^2}, \quad -1 < x < 1; \quad u_t(x, 0) = 0, \quad \text{si no}$ |

6. //Adrián R. [Jorge G.]//

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|-----|---------------------------------------------------------------------------------------------------|
| EDP | $u_{tt} = c^2 u_{xx}$ |
| CI | $u(x, 0) = x \quad (-\infty < x < \infty)$ $u_t(x, 0) = 1/(1 + \exp x) \quad (0 < t < \infty)$ |

*Preguntas y soluciones contrastadas por [...]

7. //Roser [Cristina]//

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|-----|---------------------------------------------------------------------------------------------|
| EDP | $u_{tt} = c^2 u_{xx}$ |
| CI | $u(x, 0) = x^2 \quad (-\infty < x < \infty)$ $u_t(x, 0) = \tan x \quad (0 < t < \infty)$ |

8. //Cristina [Roser]//

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|-----|---------------------------------------------------------------------------------------------------|
| EDP | $u_{tt} = c^2 u_{xx}$ |
| CI | $u(x, 0) = \sin 2x \quad (-\infty < x < \infty)$ $u_t(x, 0) = x \exp x \quad (0 < t < \infty)$ |

9. //Torroba [Jorge P.]//

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|-----|-------------------------------------------------------------------------------------------------------|
| EDP | $u_{tt} = c^2 u_{xx}$ |
| CI | $u(x, 0) = 1/(1 + x) \quad (-\infty < x < \infty)$ $u_t(x, 0) = \exp(-x) \quad (0 < t < \infty)$ |

10. //Luis G. [Gonzalo]//

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|-----|---------------------------------------------------------------|
| EDP | $u_{tt} = 16u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(\pi, t) = 0$ |
| CI | $u(x, 0) = 8 - 15x^2 \quad (0 < x < \pi)$ $u_t(x, 0) = 23$ |

11. //Andrés U. [Aitor]//

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|-----|-------------------------------------------------------|
| EDP | $u_{tt} = c^2 u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(L, t) = 0$ |
| CI | $u(x, 0) = 0 \quad (0 < x < L)$ $u_t(x, 0) = 2x$ |

12. //Jorge P. [Carlos S.]//

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|-----|----------------------------------------------------------|
| EDP | $u_{tt} = c^2 u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(1, t) = 0$ |
| CI | $u(x, 0) = x - x^2 \quad (0 < x < 1)$ $u_t(x, 0) = 0$ |

13. //Chimo [Victoria]//

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|-----|--------------------------------------------------------------------------------------------------------------------------------|
| EDP | $u_{tt} = \alpha^2 u_{xx}$ |
| CC | $u(0, t) = h \quad (0 < t < \infty), \quad (0 < x < L)$ $u(L, t) = 0$ |
| CI | $u(x, 0) = h - 2hx/L \quad (\text{si } x < L/3); \quad u(x, 0) = h/2 - hx/2L \quad (\text{si } x \geq L/3)$ $u_t(x, 0) = 0$ |

14. //Victoria [Chimo]//

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|-----|---------------------------------------------------------|
| EDP | $u_t = \alpha^2 u_{xx} - \phi u_x$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u_x(1, t) = 1$ |
| CI | $u(x, 0) = 0 \quad (0 < x < 1)$ |

15. //Gonzalo [Luis]//

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|-----|-------------------------------------------------------|
| EDP | $u_t = \alpha^2 u_{xx}, \quad \alpha = 2$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(1, t) = 0$ |
| CI | $u(x, 0) = 42x \quad (0 < x < 1)$ |

16. //Pablo C. [Leo]//

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|-----|-------------------------------------------------------|
| EDP | $u_t = \alpha^2 u_{xx}$ |
| CC | $u(0, t) = 2 \quad (0 < t < \infty)$ $u(1, t) = 0$ |
| CI | $u(x, 0) = x^2 + 2 \quad (0 < x < 1)$ |

17. //Carlos F. [Jorge L.]//

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|-----|-------------------------------------------------------|
| EDP | $u_t = \alpha^2 u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(1, t) = 0$ |
| CI | $u(x, 0) = 4x \quad (0 < x < 1)$ |

18. //Adrián [Jorge G.]//

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|-----|-------------------------------------------------------|
| EDP | $u_t = \alpha^2 u_{xx}$ |
| CC | $u(0, t) = 2 \quad (0 < t < \infty)$ $u(1, t) = 1$ |
| CI | $u(x, 0) = 4 - x \quad (0 < x < 1)$ |

19. //Jorge L. [Carlos F.]//

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|-----|-------------------------------------------------------|
| EDP | $u_t = \alpha^2 u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(1, t) = 0$ |
| CI | $u(x, 0) = x \quad (0 < x < 1)$ |

20. //Jorge G. [Adrián]//

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|-----|----------------------------------------------------------|
| EDP | $u_t = \alpha^2 u_{xx}$ |
| CC | $u(0, t) = 5 \quad (0 < t < \infty)$ $u(\pi, t) = 10$ |
| CI | $u(x, 0) = 5x \quad (0 < x < \pi)$ |

21. //Jorge P. [Carlos S.]//

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|-----|---------------------------------------------------------|
| EDP | $u_t = \alpha^2 u_{xx}$ |
| CC | $u(0, t) = 7 \quad (0 < t < \infty)$ $u(12, t) = -5$ |
| CI | $u(x, 0) = 2x \quad (0 < x < 12)$ |

22. //Leo [Pablo C.]//

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|-----|----------------------------------------------------------------------------------------------------------------|
| EDP | $u_{tt} = c^2 u_{xx}$ |
| CC | $u(0, t) = 0 \quad (0 < t < \infty)$ $u(1, t) = 0 \quad (0 < x < 1)$ |
| CI | $u(x, 0) = 0$ $u_t(x, 0) = \begin{cases} -2hx & x \leq 1/2 \\ 4h(x - 1/2) - h & 1/2 < x \leq 1 \end{cases}$ |