Strategic Delegation with Multi-Product Firms: A General Demand Approach

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Abstract

This paper examines the strategic incentives of a multi-product manufacturer to delegate the sales of the full product line, and thus employ a different distribution channel for each of the products. It faces the following tradeoff: there is a strategic effect associated with delegation but if both products' sales are delegated, intra-firm competition is not internalized. By delegating the sales of one product and selling the other one itself, the multi-product manufacturer strikes just the right compromise: the externalities between its own products are partially internalized while a strategic advantage is achieved against its rival (single-product) manufacturer. Previous results on delegation do not generalize when a manufacturer sells multiple products (even in the case of a multi-product retailer).

Keywords: multi-product manufacturer, distribution channels, delegation.

JEL Classification System: D21, L29.

1 Introduction

A question much analyzed in the literature on vertical relations has been whether producers wish to distribute their products through independent retailers or remain vertically integrated. There is a strategic effect associated with delegation of sales but complete control over how to market a product is lost. This paper investigates such a tradeoff when one of the manufacturers sells multiple products. In this setting, we examine whether a multi-product manufacturer may strategically decide not to delegate the sales of all products and thus employ different distribution channels for different products.

The Nash equilibrium distribution structures in a duopoly have been studied by McGuire and Staelin (1983), Vickers (1985) and Bonanno and Vickers (1988). These papers identified upstream firms' strategic incentive to decentralize their distribution and pricing decisions. Oligopolistic producers will delegate sales to retailers for the homogeneous and differentiated products case when the contract is a two-part tariff; delegation also occurs in equilibrium for the linear contract case as long as there is a sufficient degree of product substitutability.[1] A variation, provided that sales are delegated, is to determine whether retail distribution should involve the use of exclusive or common retailers, as done by Bernheim and Whinston (1998), Lin (1990) and O'Brien and Shaffer (1993), only to mention a few. But in all of these studies a manufacturer is allowed to offer just one product.

Most industries are characterized by firms each producing a range of differentiated products, a setting in which product line decisions become an additional strategic variable. Mussa and Rosen (1978) provided a first analysis in which a monopolist chooses the range of qualities to be sold, i.e. its product line, and

[1] Also important is the observability and renegotiation of intra-channel agreements - see e.g. Coughlan and Wernerfelt (1989) and Katz (1991). An excellent survey on the value of precommitment in vertical chains is Irmen (1998).

then their prices. Champsaur and Rochet (1989) have developed the oligopoly case and shown that firms' product lines do not overlap. The extension of multi-product competition to the context of manufacturer-retailer relations is not an easy task. A first attempt has recently been made by Villas-Boas (1998). By comparing the decentralized and the coordinated channel outcomes, Villas-Boas (1998) identifies a further coordination problem in the bilateral monopoly case: the manufacturer finds it optimal to widen its product line to ensure that the retailer carries the full line and targets each market segment with the right product.

Shaffer (1991) also examines a bilateral monopoly and studies optimal marketing strategies for a multi-product monopolist when the retailer's shelf space is limited. The manufacturer cannot recover his first-best profits unless, in addition to two-part tariff contracts, vertical restraints are imposed on retailers.[2] In both Shaffer (1991) and Villas-Boas (1998) there is a stage at which the retailer decides how many products to carry. However, and since the manufacturer plays before the retailer, the manufacturer can either design a contract or widen its product line in a way that the retailer accepts to carry the full line. These authors therefore focus on coordination problems and how to recover the integrated channel profits. Our paper complements their analyses by considering the duopoly case where one of the manufacturers produces two differentiated products and, in contrast with them, we are rather interested in the strategic incentives of a multi-product manufacturer to delegate the sales of the full product line. If retailers are not employed then the manufacturer is allowed to sell the products directly to consumers. In the marketing literature the distinction is made between sales

[2]There are a number of contributions in both the industrial organization and marketing literatures devoted to study multi-product competition and product line pricing. Among these, it is worth mentioning the papers by Lal and Matutes (1989), Dobson and Waterson (1996), and those by Oren et al. (1984), Reibstein and Gatignon (1984) and recent work by Shugan and Desiraju (2001). However, vertical relations and associated strategic decisions are not examined.

through factory-owned or privately-owned dealerships. Hence, it may occur that, in equilibrium, different products are distributed through different channels. In other words, we wish to study whether previous results on delegation with single-product firms generalize to the case with multi-product firms.

To this end we propose a non-cooperative multi-stage game with observed actions. The game consists of three stages. In the first stage, the multi-product and the single-product manufacturers simultaneously decide whether to delegate sales or wish to market the products themselves, i.e. they choose their distribution channels. In the second stage, and depending on their earlier choice, manufacturers select the terms of payment, a two-part tariff, for their contract with their respective retailers. There is Cournot competition in the last stage of the game. By structuring the problem in this way, we may examine why a multi-product manufacturer strategically selects a particular channel for each of the products. It is shown that it delegates the sales of just one of the products thereby employing a different distribution channel for each of them. By appropriately setting the wholesale price levels together with the decision of not delegating the sales of both products, the multi-product manufacturer optimally combines the tradeoff between the intensity of intra-brand and intra-firm competition to achieve the highest possible equilibrium output.

The recent remarkable growth of the Internet provides a timely example of cross-channel competition for multi-product manufacturers, that is, online sales versus conventional retail sales. Computer goods are the single largest category of retail goods sold online. Computer manufacturers, such as Dell and Gateway, employ two main methods of selling to residential customers. One is through distribution networks such as computer stores like CompUSA, general retailers or catalog merchants. The other method is direct sales to consumers through an Internet site or ads in computer magazines. Other categories of goods sold online are books, airline tickets, clothing, cars and mutual funds.[3]

Section 2 sets out the model. Different subsections are devoted to completely characterize the equilibrium at each stage of the game. Section 3 presents the class of examples corresponding to the vertical differentiation case. Some brief concluding remarks close the paper.

2 The Model

There are two manufacturers and a competitive supply of retailers who play the following multi-stage game G with observed actions. In the first stage, manufacturers, M_1 and M_2 decide simultaneously and independently whether to delegate sales to retailers or sell directly the product to consumers. In the second stage, manufacturers decide the terms of the two-part tariff contract to be signed with retailers if appropriate; otherwise no action is taken at this stage. Finally, all the active agents (manufacturers and/or retailers) play a quantity game à la Cournot.

We assume that there is a market with two differentiated products, A and B. The marginal costs of production are constant, c_A and c_B, respectively. It is assumed that M_1 is a multi-product firm that produces both A and B, whereas M_2 is a firm that produces just one product, say product B. Inverse demand functions are linear and are denoted by $p_A = p_A(Q_A, Q_B)$ and $p_B = p_B(Q_A, Q_B)$, where p_A and p_B are the prices and Q_A and Q_B stand for total output. Let p_i^j be $\frac{\partial p_i}{\partial Q_j}$ for $i,\ j = A,\ B$. All of these partial derivatives are assumed to be negative, and each own effect may either strictly dominate or coincide with the

[3] There is little empirical work on direct competition between retail and Internet commerce. These pieces of evidence can be found in recent work by Brynjolfsson and Smith (2000) and Goolsbee (2000, 2001).

corresponding cross effect in absolute terms. However, product homogeneity is not allowed, i.e. both own effects are not equal to their corresponding cross effects simultaneously. Finally, the smallest own effect is greater than or equal to the greatest cross effect in absolute terms. This assumption is sufficient to guarantee that, with multi-product sellers, the second order conditions are always satisfied. Despite linearity in demands, the assumptions above do cover the most widely employed models for differentiated products (see Table 1 below). On the one hand, it incorporates the non-address or representative consumer approach, where all consumers gain utility from consuming a variety of brands (such as a variety of books, CDs, software, etc...). The asymmetric case refers to the case of different own effects in each of the (inverse) demand functions. On the other hand, the address approach is comprised too: each consumer buys only one brand (such as one computer, one car, one airline ticket, etc...), but consumers have different preferences for their most preferred brand. The assumed demand structure is valid for vertical product differentiation - take product A to be of a higher quality than product B. In particular, for the models of Mussa and Rosen (1978) and Gabszewicz and Thisse (1979) in the uncovered market case, that is, when a fraction of consumers do not purchase the good. The difference between these two models is whether cross effects are equal or not. Further note that the models displayed below do not exhaust all the possibilities considered.[4]

[4]Note that in the Hotelling (horizontal differentiation) model, brands are not uniformly ranked by consumers. Firms sell an identical product and it is their location what makes the product different to consumers. In order to avoid the price equal to marginal cost case that would occur with Bertrand competition when two products are located at the same place, three locations (and demand functions) would be required. The three-product case with price competition will be discussed at the end of Section 2.3.

Type of approach	
a) representative consumer	
symmetric	$\lvert p_A^A\rvert = \lvert p_B^B\rvert > \lvert p_A^B\rvert = \lvert p_B^A\rvert$
asymmetric	$\lvert p_A^A\rvert \neq \lvert p_B^B\rvert$, $\lvert p_A^B\rvert = \lvert p_B^A\rvert$ and $\min\{\lvert p_A^A\rvert, \lvert p_B^B\rvert\} > \lvert p_A^B\rvert$
b) address	
Mussa-Rosen	for A high and B low quality: $\lvert p_A^A\rvert > \lvert p_B^B\rvert = \lvert p_A^B\rvert = \lvert p_B^A\rvert$
Gabszewicz-Thisse	for A high and B low quality: $\lvert p_A^A\rvert > \lvert p_B^B\rvert = \lvert p_B^A\rvert > \lvert p_A^B\rvert$

Table 1: Models of product differentiation included in the analysis.

At stage one, M_1 may decide either: a) to produce and sell himself both products (action N), or b) to hire a retailer for distributing product A, and sell himself product B (action A), or c) the reverse of b (action B), or d) to delegate the sales of both products to independent and different retailers (action AB).[5] Actions N and AB involve the use of the same types of channel for both products, whereas actions A and B suppose a different type of distribution channel for each of the products. The rival manufacturer M_2 also chooses the way its product will be distributed: either sold by the manufacturer itself (action N) or through a retailer (action B).

With multi-production and regardless of the distribution channel, there appear three outputs in the market, q_A, q_{B1} and q_{B2} where the second subindex stands for the manufacturer who produces the good. Then $Q_A = q_A$ and $Q_B =$

[5] The assumption of two independent retailers is not unrealistic. Exclusive dealership clauses, as happens with other vertical restraints, are extensively employed in e.g. the fragrances and cosmetics industries. Also the whole range of some health and beauty aids is not available at discount stores. In clothing, French firm Lacoste has its own franchise shops where clothes with its distinctive "alligator" sign are sold. Lacoste, possibly for reputation reasons, does not wish other lower quality clothes (of his own) to be marketed through the same channel. Examples can also be found where the sales of a variety of products are delegated to a common retailer. The case of a common multi-product retailer will be discussed below.

$q_{B1} + q_{B2}$. Thus, we may identify inter-brand competition (between q_A and both q_{B1} and q_{B2}), intra-brand competition (between q_{B1} and q_{B2}) and intra-firm competition (between q_A and q_{B1}). For example, when the multi-product firm decides not to delegate sales at all, it directly internalizes intra-firm competition. Under delegation of both products through different retailers, the intensity of intra-firm competition is maximal. In our setting, the multi-product manufacturer has two decision variables to control for the intensity of intra-firm competition: a) the wholesale price(s), which is also a means to gain sales vis a vis its rival, and b) the choice of the distribution channel. We will show that the multi-product manufacturer obtains higher payoffs when both products' sales are delegated than when it sells directly to consumers. In other words, the multi-product manufacturer prefers to lose the ability to completely internalize any externalities between its own products. The question is, and this is the objective of the paper, whether it could do better by using a different distribution channel for each of the products. By delegating the sales of one product and selling the other one itself, the multi-product manufacturer strikes just the right compromise: the externalities between its own products are partially internalized while a strategic advantage is achieved against the rival manufacturer. The remainder of the Section is organized as follows. We characterize the comparative statics of the third stage of the game depending on whether there are two or three active sellers in the market. To illustrate the above tradeoff an analysis is made for the cases when the multi-product manufacturer delegates the sales of both products', just one of them or none. Then the main result is proven. We finally elaborate on delegation of both products sales to one common retailer.

2.1 The choice of quantities

The game is solved in the standard backward way and we begin by characterizing the choice of quantities by the active sellers in the third stage of the game. The eight subgames can be classified into two types according to the number of active sellers: subgames with two sellers, one of which is a multi-product seller, the (N, N) and (N, B) subgames, and the remaining six subgames with three single-product sellers. Such a distinction is relevant concerning the first order conditions and the corresponding comparative statics which will be used in the sequel. For the sake of the exposition we develop only one of each type.

First consider the (AB, B) subgame, one with three sellers. The contract linking a manufacturer and a retailer is a two-part tariff contract. It consists of an up-front fixed fee, F_h, independent of the amount of output sold, and a per unit wholesale price, w_h, for $h = A,\ B1, B2$. Retailers choose quantities that maximixe the following payoffs π_h:

$$\begin{aligned}\max_{q_A} \pi_A &= (p_A(Q_A, Q_B) - w_A)q_A - F_A \\ \max_{q_{B1}} \pi_{B1} &= (p_B(Q_A, Q_B) - w_{B1})q_{B1} - F_{B1} \\ \max_{q_{B2}} \pi_{B2} &= (p_B(Q_A, Q_B) - w_{B2})q_{B2} - F_{B2}\end{aligned} \tag{1}$$

The following first order conditions are obtained:

$$p_A^A q_A + p_A(Q_A, Q_B) - w_A = 0 \tag{2}$$

$$p_B^B q_{B1} + p_B(Q_A, Q_B) - w_{B1} = 0 \tag{3}$$

$$p_B^B q_{B2} + p_B(Q_A, Q_B) - w_{B2} = 0 \tag{4}$$

It is easy to check that the choice variables are strategic substitutes. Second order conditions for a maximum and the required stability conditions are satisfied, given the assumptions on the demand schedule. These equations implicitly define the best-reply functions. Solving the system (2), (3) and (4) yields the equilibrium quantities, denoted by an asterisk, as functions of wholesale prices, w_A, w_{B1}, and w_{B2}. Total differentiation yields the comparative statics displayed in Table 2 below, where $\Delta = 2p_B^B(3p_A^A p_B^B - p_A^B p_B^A) < 0$.

$\frac{dq_A^*}{dw_A} = \frac{-3(p_B^B)^2}{-\Delta} < 0$	$\frac{dq_A^*}{dw_{B1}} = \frac{dq_A^*}{dw_{B2}} = \frac{p_A^B p_B^B}{-\Delta} > 0$	$\frac{dq_{B1}^*}{dw_A} = \frac{dq_{B2}^*}{dw_A} = \frac{p_B^A p_B^B}{-\Delta} > 0$
$\frac{dq_{B1}^*}{dw_{B1}} = \frac{dq_{B2}^*}{dw_{B2}} = \frac{p_A^B p_B^A - 4p_A^A p_B^B}{-\Delta} < 0$	$\frac{dq_{B1}^*}{dw_{B2}} = \frac{dq_{B2}^*}{dw_{B1}} = \frac{2p_A^A p_B^B - p_A^B p_B^A}{-\Delta} > 0$	

Table 2: Comparative statics for the case of three single-product sellers.

Note that the solution to any other subgame can be characterized from the above construction, for instance, the first order conditions corresponding to the (A, N) subgame follow by substituting w_{B1} and w_{B2} for c_B.

Next we examine a subgame with two sellers, one of which is a multi-product seller, the (N, B) subgame. The multi-product manufacturer sells products A and B directly to consumers, i.e. it remains vertically integrated. Denote these payoffs by Π_1. The rival single-product manufacturer delegates sales. The optimization problem is given by,

$$\begin{aligned} \max_{q_A, q_{B1}} \Pi_1 &= (p_A(Q_A, Q_B) - c_A)q_A + (p_B(Q_A, Q_B) - c_B)q_{B1} \\ \max_{q_{B2}} \pi_{B2} &= (p_B(Q_A, Q_B) - w_{B2})q_{B2} - F_{B2} \end{aligned} \tag{5}$$

By setting $\partial\Pi_1/\partial q_A$, $\partial\Pi_1/\partial q_{B1}$ and $\partial\pi_{B2}/\partial q_{B2}$ equal to zero we have,

$$
\begin{aligned}
p_A^A q_A + p_B^A q_{B1} + p_A(Q_A, Q_B) - c_A &= 0 \qquad (6) \\
p_A^B q_A + p_B^B q_{B1} + p_B(Q_A, Q_B) - c_B &= 0 \qquad (7) \\
p_B^B q_{B2} + p_B(Q_A, Q_B) - w_{B2} &= 0 \qquad (8)
\end{aligned}
$$

The first order conditions (2)-(3) are different from (6)-(7). The equilibrium quantities, denoted by an upper bar, are therefore different and so are comparative statics, as shown by Table 3, where $\bar{\Delta} = p_B^B(6p_A^A p_B^B - 4p_A^B p_B^A - (p_A^B)^2 - (p_B^A)^2) < 0$.

$\frac{d\bar{q}_A}{dc_A} = \frac{-3(p_B^B)^2}{-\bar{\Delta}} < 0$	$\frac{d\bar{q}_A}{dc_B} = \frac{p_B^B(p_A^B+2p_B^A)}{-\bar{\Delta}} > 0$	$\frac{d\bar{q}_A}{dw_{B2}} = \frac{p_B^B(p_A^B-p_B^A)}{-\bar{\Delta}} \gtreqless 0$
$\frac{d\bar{q}_{B1}}{dc_A} = \frac{p_B^B(2p_A^B+p_B^A)}{-\bar{\Delta}} > 0$	$\frac{d\bar{q}_{B1}}{dc_B} = \frac{(p_A^B p_B^A-4p_A^A p_B^B)}{-\bar{\Delta}} < 0$	$\frac{d\bar{q}_{B1}}{dw_{B2}} = \frac{2p_A^A p_B^B-p_A^B(p_A^B+p_B^A)}{-\bar{\Delta}} > 0$
$\frac{d\bar{q}_{B2}}{dc_A} = \frac{-p_B^B(p_A^B-p_B^A)}{-\bar{\Delta}} \gtreqless 0$	$\frac{d\bar{q}_{B2}}{dc_B} = \frac{(2p_A^A p_B^B-p_B^A(p_A^B+p_B^A))}{-\bar{\Delta}} > 0$	$\frac{d\bar{q}_{B2}}{dw_{B2}} = \frac{(p_A^B+p_B^A)^2-4p_A^A p_B^B}{-\bar{\Delta}} < 0$

Table 3: Comparative statics when one agent is a multi-product seller.

It should be noted that, contrary to the case of three single-product sellers, there are some signs that can go either way. For example, an increase in the marginal cost of product A may lead to an increase, a decrease or have no effect in the output of product B by the single-product manufacturer. The sign depends on the particular demand schedule; $\frac{d\bar{q}_{B2}}{dc_A}$ is negative under the Gabszewicz-Thisse specification, whereas it is zero both under the Mussa-Rosen and the representative consumer specifications.

2.2 The choice of the terms of payment

For any given pair of first-stage actions, manufactures decide simultaneously and independently on the wholesale price and the up-front fixed fee to be charged to their respective retailers. We proceed by first presenting the (AB, B) subgame.

By subgame perfection, the third-stage equilibrium quantities are functions of wholesale prices, $q_A^*(w_A, w_{B1}, w_{B2})$, $q_{B1}^*(w_A, w_{B1}, w_{B2})$ and $q_{B2}^*(w_A, w_{B1}, w_{B2})$. Manufacturers' payoffs are:

$$\begin{aligned}
\Pi_1 &= (w_A - c_A)q_A^*(w_A, w_{B1}, w_{B2}) + F_A \\
&\quad +(w_{B1} - c_B)q_{B1}^*(w_A, w_{B1}, w_{B2}) + F_{B1} \\
\Pi_2 &= (w_{B2} - c_B)q_{B2}^*(w_A, w_{B1}, w_{B2}) + F_{B2}
\end{aligned} \tag{9}$$

Since there is a competitive supply of retailers and sales are delegated to three separate retailers, the equilibrium up-front fixed fee set by manufacturers will be a fully extracting fee, that is, F_h is set equal to the variable profit of each retailer as follows: $F_A = (p_A - w_A)q_A^*(w_A, w_{B1}, w_{B2})$, $F_{B1} = (p_B - w_{B1})q_{B1}^*(w_A, w_{B1}, w_{B2})$ and $F_{B2} = (p_B - w_{B2})q_{B2}^*(w_A, w_{B1}, w_{B2})$. Then the manufacturers´optimization problem becomes:

$$\begin{aligned}
&\max_{w_A, w_{B1}} \Pi_1 = (p_A - c_A)q_A^*(w_A, w_{B1}, w_{B2}) + (p_B - c_B)q_{B1}^*(w_A, w_{B1}, w_{B2}) \\
&\max_{w_{B2}} \Pi_2 = (p_B - c_B)q_{B2}^*(w_A, w_{B1}, w_{B2})
\end{aligned} \tag{10}$$

By the envelope theorem ($w_A = p_A^A q_A + p_A$, $w_{B1} = p_B^B q_{B1} + p_B$ and $w_{B2} = p_B^B q_{B2} + p_B$), the system of first order conditions is given by:

$$\frac{\partial \Pi_1}{\partial w_A} = (w_A - c_A)\frac{dq_A^*}{dw_A} + (w_{B1} - c_B)\frac{dq_{B1}^*}{dw_A} + \alpha q_{B1}^* + \beta q_A^* = 0 \tag{11}$$

$$\frac{\partial \Pi_1}{\partial w_{B1}} = (w_A - c_A)\frac{dq_A^*}{dw_{B1}} + (w_{B1} - c_B)\frac{dq_{B1}^*}{dw_{B1}} + \gamma q_{B1}^* + \delta q_A^* = 0 \tag{12}$$

$$\frac{\partial \Pi_2}{\partial w_{B2}} = (w_{B2} - c_B)\frac{dq_{B2}^*}{dw_{B2}} + (p_B^B \frac{dq_{B1}^*}{dw_{B2}} + p_B^A \frac{dq_A^*}{dw_{B2}})q_{B2}^* = 0 \tag{13}$$

where the following simplifying notation has been used:
$\alpha = p^B_B \frac{dq^*_{B2}}{dw_A} + p^A_B \frac{dq^*_A}{dw_A} = \frac{-2p^A_B(p^B_B)^2}{-\Delta} > 0$, $\beta = p^B_A \frac{dq^*_{B2}}{dw_A} + p^B_A \frac{dq^*_{B1}}{dw_A} = \frac{2p^B_A p^A_B p^B_B}{-\Delta} < 0$, $\gamma = p^B_B \frac{dq^*_{B2}}{dw_{B1}} + p^A_B \frac{dq^*_A}{dw_{B1}} = \frac{2p^A_A(p^B_B)^2}{-\Delta} < 0$ and $\delta = p^B_A \frac{dq^*_{B2}}{dw_{B1}} + p^B_A \frac{dq^*_{B1}}{dw_{B1}} = \frac{-2p^B_A p^A_A p^B_B}{-\Delta} > 0$.
The equilibrium wholesale prices that solve (11)-(12)-(13) are denoted by w^{ABB}_h, for $h = A$, $B1$ and $B2$ where superscripts will be used to identify the subgame. By the comparative statics in Table 3 the second term in (13) is negative. Since $\frac{dq^*_{B2}}{dw_{B2}} < 0$, it is the case that the wholesale equilibrium price w_{B2} set by the single-product manufacturer is below unit production cost c_B. Whenever the equilibrium wholesale prices differ from unit production costs, manufacturers and retailers do not face the same incentive structure. This is a standard result in the literature of strategic delegation with single-product firms when the third stage choice variables are strategic substitutes and the contract is a two-part tariff. Then, reaction functions are downward sloping in wholesale prices space. Other things equal, a wholesale price below unit production costs produces a shift-out of the firm's reaction function. Thus, by setting $w_{B2} < c_B$, the manufacturer induces the retailer to sell more than under vertical integration, i.e. sales are incentived to strategically gain market share against its rival.

We now show that the multi-product manufacturer never sets *both* wholesale prices below or above the corresponding unit production costs when each product's sales are delegated to independent retailers. This is regardless of the choice of channel, delegation or not, made by the rival single-product manufacturer. This finding is very important to understand what follows. By delegating both products' sales the multi-product manufacturer gives up the fully internalization of intra-firm competition. With only wholesale prices to control for intra-firm competition, it will set one of them above and the other one below unit production cost because in this way the multi-product manufacturer pushes the output ratio $\frac{q^*_A\left(w^{ABB}_A, w^{ABB}_{B1}, w^{ABB}_{B2}\right)}{q^*_{B1}\left(w^{ABB}_A, w^{ABB}_{B1}, w^{ABB}_{B2}\right)}$ in a particular direction. Thus $\frac{d(\frac{q^*_A}{q^*_{B1}})}{dw_A} < 0$ and $\frac{d(\frac{q^*_A}{q^*_{B1}})}{dw_{B1}} > 0$ and by setting $\{w_A < c_A,\, w_{B1} > c_B\}$ the manufacturer increases the market share of product A relative to that of product B. That is, it points at market A. The opposite happens for $\{w_A > c_A,\, w_{B1} < c_B\}$ and the manufacturer points at market B. Which product's sales are incentived is related to whether the ratio of outputs by the multi-product manufacturer lies above or below a ratio measuring the degree of product differentiation. In fact, p_B^B/p_A^B relates the own and the cross effects of product B in the whole demand system and therefore it relates intra-brand and inter-brand competion parameters. If these parameters are such that both ratios are equal, then the multi-product manufacturer will set wholesale prices equal to the corresponding unit production costs, that is, as if it employed a sell-out contract with retailers.

Proposition 1 *At the second stage subgames where the multi-product manufacturer delegates both products sales, it incentives at most one retailer's sales, that is, either*

$\{w_A^{ABB} < c_A,\, w_{B1}^{ABB} > c_B\}$ *iff* $\frac{q_A^*\left(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB}\right)}{q_{B1}^*\left(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB}\right)} > \frac{p_B^B}{p_A^B}$ *or*

$\{w_A^{ABB} > c_A,\, w_{B1}^{ABB} < c_B\}$ *iff* $\frac{q_A^*\left(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB}\right)}{q_{B1}^*\left(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB}\right)} < \frac{p_B^B}{p_A^B}$ *or*

$\{w_A^{ABB} = c_A,\, w_{B1}^{ABB} = c_B\}$ *iff* $\frac{q_A^*\left(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB}\right)}{q_{B1}^*\left(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB}\right)} = \frac{p_B^B}{p_A^B}$.

Proof. See the Appendix.

For the remaining subgames, we distinguish those where the multi-product manufacturer delegates one of its products, that is the (A, B), (B, B), (A, N) and (B, N) subgames from those where it sells directly to consumers, the (N, B) and (N, N) subgames. For expositional purposes, we develop the (B, B) subgame.

Manufacturers' payoffs are[6]:

$$\begin{aligned}\Pi_1 &= (p_A - c_A)q_A^*(c_A, w_{B1}, w_{B2}) + (w_{B1} - c_B)q_{B1}^*(c_A, w_{B1}, w_{B2}) + F_{B1} \\ \Pi_2 &= (w_{B2} - c_B)q_{B2}^*(c_A, w_{B1}, w_{B2}) + F_{B2}\end{aligned} \quad (14)$$

Since, at equilibrium $F_{B1} = (p_B - w_{B1})q_{B1}^*(c_A, w_{B1}, w_{B2})$ and $F_{B2} = (p_B - w_{B2})q_{B2}^*(c_A, w_{B1}, w_{B2})$, then the manufacturers' optimization problem is given by:

$$\begin{aligned}&\max_{w_{B1}} \ \Pi_1 = (p_A - c_A)q_A^*(c_A, w_{B1}, w_{B2}) + (p_B - c_B)q_{B1}^*(c_A, w_{B1}, w_{B2}) \\ &\max_{w_{B2}} \ \Pi_2 = (p_B - c_B)q_{B2}^*(c_A, w_{B1}, w_{B2})\end{aligned} \quad (15)$$

which, by the envelope theorem, yields the following system of first order conditions:

$$\frac{\partial \Pi_1}{\partial w_{B1}} = (w_{B1} - c_B)\frac{dq_{B1}^*}{dw_{B1}} + \gamma q_{B1}^* + \delta q_A^* = 0 \quad (16)$$

$$\frac{\partial \Pi_2}{\partial w_{B2}} = (w_{B2} - c_B)\frac{dq_{B2}^*}{dw_{B2}} + (p_B^B \frac{dq_{B1}^*}{dw_{B2}} + p_B^A \frac{dq_A^*}{dw_{B2}})q_{B2}^* = 0 \quad (17)$$

Then, three possibilities arise $w_{B1}^{BB} > c_B$ or $w_{B1}^{BB} < c_B$ or $w_{B1}^{BB} = c_B$ which are consistent with (16). The first one applies if and only if $\frac{q_A^*}{q_{B1}^*} > \frac{-\gamma}{\delta} = \frac{p_B^B}{p_A^B}$, the second one if and only if $\frac{q_A^*}{q_{B1}^*} < \frac{p_B^B}{p_A^B}$ while the third one is satisfied if and only if $\frac{q_A^*}{q_{B1}^*} = \frac{p_B^B}{p_A^B}$. Note that from (17) we conclude that $w_{B2}^{BB} < c_B$. A similar reasoning can be done for the (A, B) subgame with a parallel conclusion, $w_A^{AB} < c_A$ or $w_A^{AB} > c_A$ or $w_A^{AB} = c_A$ are all compatible with the first order condition for w_A; they apply if and only if $\frac{q_A^*}{q_{B1}^*} > \frac{-\alpha}{\beta} = \frac{p_B^B}{p_A^B}$, $\frac{q_A^*}{q_{B1}^*} < \frac{p_B^B}{p_A^B}$ and $\frac{q_A^*}{q_{B1}^*} = \frac{p_B^B}{p_A^B}$, respectively. It is also the case that $\hat{w}_{B2}(A, B) < c_B$. For the (A, N) and (B, N) subgames, and

[6] To keep notation as simple as possible an asterisk denotes the third stage equilibrium outputs for all subgames with three active sellers. The arguments of those equilibrium quantities identify the particular subgame analyzed.

since the single-product manufacturer does not delegate sales, the multi-product manufacturer is the only agent taking a decision at this stage of the game. The same conclusions as in the (A, B) and (B, B) subgames arise, respectively, in terms of wholesale prices.

No action is taken in the second stage in the (N, N) subgame. Finally, in the (N, B) subgame, the single-product manufacturer maximizes $\Pi_2 = (p_B - c_B)\bar{q}_{B2}(c_A, c_B, w_{B2})$ with respect to w_{B2} yielding the first order condition:

$$\frac{\partial \Pi_2}{\partial w_{B2}} = (w_{B2} - c_B)\frac{d\bar{q}_{B2}}{dw_{B2}} + (p_B^B \frac{d\bar{q}_{B1}}{dw_{B2}} + p_B^A \frac{d\bar{q}_A}{dw_{B2}})\bar{q}_{B2} = 0 \tag{18}$$

which is different from those above since there are two active sellers in the third stage of the game. By developing $p_B^B \frac{d\bar{q}_{B1}}{dw_{B2}} + p_B^A \frac{d\bar{q}_A}{dw_{B2}}$ it follows that its sign is the opposite to the sign of $2p_A^A p_B^B - (p_B^A)^2 - (p_A^B)^2$. The latter expression is positive since we have assumed that the smallest own effect is greater than or equal to the greatest cross effect in absolute terms. Since $\frac{d\bar{q}_{B2}}{dw_{B2}} < 0$ we conclude that $w_{B2}^{NB} < c_B$.

Several conclusions can be extracted from the previous analysis of the second-stage equilibrium. At any subgame where the single-product manufacturer delegates sales, it sets the equilibrium wholesale price below unit production cost. In doing so, the single-product manufacturer commits to a greater output in the ensuing stage. We have thus generalized a well-known result in the literature on delegation to obtain that such a strategic effect shows up even if the rival is a multi-product firm. However, there appear some qualitative changes when upstream firms are not all single-product and are allowed to sell more products. Specifically, the multi-product manufacturer may set the wholesale above, below or equal to unit production cost when the sales of only one product are delegated. If it employs two separate retailers, then both wholesale prices are not simultaneously set above or below unit production cost. Hence, *we can conclude that in* *the presence of an upstream multi-product firm, the strategic effect of delegation is not necessarily sales enhancing.*

2.3 The equilibrium choice of distribution channels

In this Section we solve for the first-stage of the game where both manufacturers decide on the delegation profile. It must have been noted that, regardless that the multi-product manufacturer's first-stage action be either AB, or A or B, one of the possible second stage equilibrium involves setting the corresponding wholesale price equal to unit production cost. In other words, delegation occurs but a retailer's sales are not incentived. The rival manufacturer's best response is the same to any of the aforementioned first-stage actions - the system of first order conditions (11)-(12)-(13) yields the same solution as the system formed by (16)-(17). Therefore, equilibrium outputs and manufacturers' payoffs coincide. This happens when $q_A^*(c_A, c_B, \cdot)/q_{B1}^*(c_A, c_B, \cdot) = p_B^B/p_A^B$.

Remark 1 *All subgames where the multi-product manufacturer delegates without giving sales incentives are payoff equivalent.*

Then the issue for the multi-product manufacturer is just a matter whether to employ retailers or not because delegating both products' sales or the sales of only one product leaves him indifferent at equilibrium. In such an eventuality, and for expositional reasons, we assume that one product, say product B, is sold through a retailer. Having said that, the next Proposition can be stated.

Proposition 2 *At the first stage equilibrium of game G, the multi-product manufacturer uses a different channel for each product, one is delegated to an independent retailer while the other is sold directly to consumers. The single-product manufacturer will always employ an independent retailer. In particular, either*
a) (A, B) is the equilibrium distribution pattern. The equilibrium wholesale prices are set below the corresponding unit production costs, or
b) (B, B) is the equilibrium distribution pattern. The equilibrium wholesale prices are never set above the corresponding unit production costs.

Proof. See the Appendix.
This finding adds to the literature on strategic delegation by allowing one of the manufacturers to sell multiple products. To see the intuition suppose that the multi-product manufacturer is constrained to only choose between two first-stage actions, AB and N. The equilibrium obtained would entail delegation of sales by both manufacturers. Delegation is a commitment to a greater output in the next stage. From the point of view of the multi-product manufacturer, note that there is a strategic advantage associated with delegation but it loses the complete internalization of any externalities between its own products. In fact, the former effect outweighs the latter. As shown in Proposition 1 above, the equilibrium wholesale prices would not be simultaneously set above or below unit production costs. Further note that, with multi-production, the outcome when wholesale prices equal unit production costs does not coincide with that under vertical integration since intra-firm competition is not internalized. Thus, whatever the case that may happen, delegating both products' sales is a commitment to a greater output by the multi-product firm.

Then the question arises, can the multi-product manufacturer improve by delegating the sales of just one product? Consider the (AB, B) subgame with $w_A^{ABB} > c_A$ and $w_{B1}^{ABB} < c_B$. We argue that a move to action B by the multi-product manufacturer, and hence to the equilibrium in the (B, B) subgame, makes it better off. In so doing, it will compete with cost c_A, a lower cost than under action AB. Other things being equal, and by strategic substitution[7] this will induce an increase in both wholesale prices of product B, specifically, $w_{B1}^{ABB} < w_{B1}^{BB} < c_B$ and $w_{B2}^{ABB} < w_{B1}^{BB} < c_B$. Therefore, the move produces an increase in output q_A, which has a positive effect on profits, and a decrease in q_{B1}, a negative effect. The Proposition shows that the former effect offsets the latter thus meaning that the multi-product manufacturer is better off by employing a different channel for each of the products. When delegating the sales of just one product, it commits to a greater output in two ways; firstly, by dispensing with the full internalization of intra-firm competition and, secondly, by not setting any equilibrium wholesale price strictly above unit production cost. Consequently, if market conditions are such that it pays the multi-product manufacturer to point at market A, then it only delegates the sales of product B. The argument applies mutatis mutandis when $w_A^{ABB} < c_A$ and $w_{B1}^{ABB} > c_B$ and then the profitable move is to delegate only the sales of product A. To sum up, by delegating the sales of just one of the products, the multi-product manufacturer exploits the strategic advantage in one of the markets while partially internalizing the externalities between its own products.

With price competition at the third stage of the game and two-part tariff contracts, Bonanno and Vickers (1988) have shown that (single-product) manufacturers have a unilateral incentive to delegate sales; they opt for vertical separation and obtain profits above those under vertical integration. Suppose then that there is competition in prices in the last stage of game G and assume some differentiation among all three products. Certainly, the above mentioned effects associated with delegation of both products' sales show up. On the one hand,

[7] As proven in the Appendix, wholesale prices are strategic substitutes.

intra-firm competition is not internalized. On the other, there is the strategic advantage effect. There is however a key difference with competition in quantities. The decision variables, prices and wholesale prices, are now strategic complements - reaction functions are upward sloping. In equilibrium, wholesale prices exceed unit production costs which reduces competition. That is, as in Bonanno and Vickers (1988), delegation of both products' sales leads to a more collusive outcome. If one of the products were sold directly to consumers, then the multi-product manufacturer would compete with a lower cost for that product thus resulting in a more competitive outcome. Note that, since variables are strategic complements any change of one strategic variable induces a change of the other strategic variables in the same direction; therefore, the outcome under delegation of just one product's sales is more competitive than under delegation of both products' sales. We conjecture that it is in the interest of both manufacturers to coordinate in the most collusive outcome and then, at equilibrium, the sales of the three products will be delegated to retailers.[8] With quantity competition, a multi-product manufacturer does not want delegate both products' sales because this will exacerbate the competition between the two products. With price competition, it wishes to delegate both products' sales because then competition is less intense. Confronting the results under strategic substitutability and complementarity leads us to a testable implication.

2.4 Delegation to a common retailer by the multi-product manufacturer

Once having analyzed that earlier results in the literature on delegation do not extend when there is a multi-product upstream firm, it seems natural to generalize the analysis to consider multi-product retailers. Put differently, suppose that the multi-product manufacturer employs one common retailer when it decides to

[8] This point was raised to us by a referee.

delegate the sales of both products. Are the results of the existing literature recovered? or, is it still possible that products be sold through different channels? Consider the game G already described where now M_1's first-stage action AB implies delegation of both products' sales to the same retailer. There are two opposing effects at work. On the one hand, intra-firm competition is fully internalized, though by a different agent. This would leave us solely with the strategic effect of delegation. On the other hand, the retailer has the power to credibly threaten the multi-product manufacturer with dropping one of the products. Such power stems from its discretion over brand choice because both products are substitutes. Therefore, the retailer will not accept a contract with product specific fully extracting fees. In other words, the most up-front fixed fee the multi-product manufacturer can elicit is each product's marginal contribution to the retailer's profit. Therefore, the retailer earns strategic rent, i.e. the foregone profit from the reduced sales of substitute products. Our analysis of subgame (AB, B) can be viewed as an extension of Shaffer (1991) and O'Brien and Shaffer (1993).[9] The following result is proven in the Appendix.

Result 1 *The common retailer selling both multi-product manufacturer's products earns a positive strategic rent. Therefore the multi-product manufacturer cannot fully extract the retailer payoffs by using a product specific two-part tariff contract.*

Given this result is is a priori unclear whether the multi-product manufacturer will prefer to delegate both products' sales or just one of them. To analyze this question, consider the new (AB, B) subgame. Retailers' third stage first order conditions are :

[9] A positive strategic rent occurs regardless that the two products belong to same manufacturer, as in Shaffer (1991), or to different single-product manufacturers, as in O'Brien and Shaffer (1993).

$$p_A^A q_A + p_B^A q_{B1} + p_A(Q_A, Q_B) - w_A = 0 \quad (19)$$

$$p_A^B q_A + p_B^B q_{B1} + p_B(Q_A, Q_B) - w_{B1} = 0 \quad (20)$$

$$p_B^B q_{B2} + p_B(Q_A, Q_B) - w_{B2} = 0 \quad (21)$$

The multi-product manufacturer offers the common retailer the two-part tariff contract $\{w_A, F_A, w_{B1}, F_{B1}\}$ which is product specific, while the single product one offers $\{w_{B2}, F_{B2}\}$. Second stage manufacturers' payoffs are now as in (9) noting that the equilibrium quantities are $\bar{q}_h(w_A, w_{B1}, w_{B2})$, $h = A, B1, B2$. The fixed fee F_{B2} is a fully extracting fee as before, yet the fixed fees F_A and F_{B1} are now equal to the corresponding product's marginal contribution to the retailer's profits. Specifically,

$$F_A = (p_A - w_A)\bar{q}_A(w_A, w_{B1}, w_{B2}) + (p_B - w_{B1})\bar{q}_{B1}(w_A, w_{B1}, w_{B2}) - (\tilde{p}_B - w_{B1})\tilde{q}_{B1}(w_{B1}, w_{B2})$$

$$F_{B1} = (p_A - w_A)\bar{q}_A(w_A, w_{B1}, w_{B2}) + (p_B - w_{B1})\bar{q}_{B1}(w_A, w_{B1}, w_{B2}) - (\tilde{p}_A - w_A)\tilde{q}_A(w_A, w_{B2})$$

The notation is the following: $\tilde{p}_A$, is product $A's$ inverse demand function when the common retailer drops product $B1$, and $\tilde{p}_B$, is product $B's$ inverse demand function when it drops product A. The first two terms in F_A and F_{B1} are the retailer's profits of carrying both products while the third terms are the profits when one of the products is dropped. Note that the wholesale prices serve an additional purpose, i.e. to control for the amount of the strategic rent that can be captured from the retailer. If the retailer threatens with dropping product A the multi-product manufacturer may use w_{B1} to reduce the magnitude of the third term; it then has an incentive to increase w_{B1}.

Then manufacturers' optimization problem is given by:

$$\begin{aligned}
&\max_{w_A, w_{B1}} (p_A - c_A)\bar{q}_A(w_A, w_{B1}, w_{B2}) + (p_B - c_B)\bar{q}_{B1}(w_A, w_{B1}, w_{B2}) \\
&+(p_A - w_A)\bar{q}_A(w_A, w_{B1}, w_{B2}) + (p_B - w_{B1})\bar{q}_{B1}(w_A, w_{B1}, w_{B2}) \\
&-(\tilde{p}_A - w_A)\tilde{q}_A(w_A, w_{B2}) - (\tilde{p}_B - w_{B1})\tilde{q}_{B1}(w_{B1}, w_{B2}) \\
&\max_{w_{B2}} (p_B - c_B)\bar{q}_{B2}(w_A, w_{B1}, w_{B2})
\end{aligned} \tag{22}$$

We focus on how the multi-product manufacturer sets wholesale prices. The first order conditions, by the envelope theorem and after some manipulation, can be written as

$$w_A - c_A = \frac{\left(\frac{p_A^B p_B^A + 4p_A^A p_B^B}{p_B^B}\right)\bar{q}_A - 4p_A^A\tilde{q}_A + (2p_A^B + 3p_B^A)\bar{q}_{B1} - \frac{4(2p_A^B + p_B^A)}{3}\tilde{q}_{B1}}{2} \tag{23}$$

$$w_{B1} - c_B = \frac{(3p_A^B + 2p_B^A)\bar{q}_A - \left(\frac{4p_A^A p_B^B(p_A^B + 2p_B^A)}{4p_A^A p_B^B - p_B^A p_A^B}\right)\tilde{q}_A + 5p_B^B\bar{q}_{B1} - 4p_B^B\tilde{q}_{B1}}{2} \tag{24}$$

We are interested in the situation where both $w_A - c_A$ and $w_{B1} - c_B$ are positive. This clearly happens as long as the numerators in (23)-(24) are positive. A sufficient condition for this result is the ratios $\frac{\tilde{q}_A}{\bar{q}_A}$ and $\frac{\tilde{q}_{B1}}{\bar{q}_{B1}}$ to be large enough. In particular, $\frac{\tilde{q}_A}{\bar{q}_A} > \frac{(4p_A^A p_B^B - p_B^A p_A^B)(3p_A^B + 2p_B^A)}{4p_A^A p_B^B(p_A^B + 2p_B^A)} > 1$ and $\frac{\tilde{q}_{B1}}{\bar{q}_{B1}} > \max\{\frac{5}{4}, \frac{3(2p_A^B + 3p_B^A)}{4(2p_A^B + p_B^A)}\} > 1$. Since it can be checked that these ratios are greater tan one, the above conditions will be more likely met the less differentiated the products are. Both ratios increase as products become more homogeneous. And also, the right hand side of the inequalities is smaller the more homogeneous products are. To sum up, little product differentiation is sufficient to have equilibrium wholesale prices above unit production costs.

Having identified such a theoretical characterization, we may argue in the following terms as compared with delegation of only one product's sales. Delegation to a common retailer implies a decrease in output by the multi-product manufacturer, given that intra-firm competition is internalized and that retailer's sales are disincentived. Since manufacturers' profits are increasing with output *we conclude that delegation of only one product's sales will dominate delegation of both products sales to a common retailer.*

Finally, one possibility for the above conclusion to be reversed is that the multi-product manufacturer can effectively implement fully extracting fees. This would require the introduction of a vertical restraint in the contract, such as full-line forcing, a clause that does eliminate the retailer's ability to make its threat a credible one. In such a case, the first order conditions corresponding to the multi-product manufacturer's second stage equilibrium reduce to:

$$w_A - c_A = \frac{\left(\frac{p_B^A}{p_B^B}\right)[p_A^B \bar{q}_A + p_B^B \bar{q}_{B1}]}{2} \tag{25}$$

$$w_{B1} - c_B = \frac{[p_A^B \bar{q}_A + p_B^B \bar{q}_{B1}]}{2} \tag{26}$$

which obviously imply that at equilibrium $w_A < c_A$ and $w_{B1} < c_B$. Since sales would be incentived, the strategic effect supposes an output increase which goes in the opposite direction to the output decrease due to the internalization of intra-firm competition. This leaves open the possibility for delegation of both products to arise at equilibrium.

3 Vertical differentiation and channel choice

The purpose of this Section is to illustrate the previous general presentation by means of a class of examples, those of vertical product differentiation. There are some papers that deal with manufacturer-retailer relations in models of vertical differentiation, as those by Bolton and Bonanno (1988), Winter (1993) and Villas-Boas (1998). These papers look into coordination problems in a setting with one manufacturer and one or several retailers. Our analysis may thus be seen as complementary to theirs in that the multi-product manufacturer may delegate the sales of only one product and that there is a competing manufacturer. Bolton and Bonanno (1988) show that, if retailers choose which quality they want to offer, franchise fees or resale price maintenance are not enough to recover the profits of the vertically integrated structure. In Winter (1993) retailers are differentiated by their location and provide service, which reduces the time it takes to purchase a good. He examines how vertical restraints correct coordination problems in a channel with both horizontal and vertical differentiation. Villas-Boas (1998) studies product line issues in the bilateral monopoly case. In contrast, we examine whether there exists a strategic rationale behind product quality and the channel through which it is distributed. Next, we show how the characterization above spells out in a model with vertical product differentiation.[10]

Consider the demand specification initially proposed by Gabszewicz and Thisse (1979). There is a market with two firms, firm H produces a high-quality good whereas firm L produces a low-quality good. Both these qualities are exogenously given. Let $T = [0, 1]$ represent the set of consumers. A consumer of type $t \in T$ has an initial income given by $R(t) = R_1 + R_2 t$, with $R_1 > 0$ and $R_2 \geq 0$. All consumers have identical preferences and their utility function is defined by,

$U(0, R(t)) = u_0 R(t)$, in case of no purchase, $U(H, R(t)-p_H) = u_H(R(t)-p_H)$, if the consumer buys the high-quality product and pays p_H for it, and $U(L, R(t)-p_L) = u_L(R(t)-p_L)$, if the low-quality product is bought and pays p_L for it. The scalars u_0, u_H and u_L are positive and verify $u_H > u_L > u_0 > 0$. This means that

[10] The provision of cum-sales or post-sales services by retailers can give rise to a situation of vertical differentiation, as in Bolton and Bonanno (1988) and Winter (1993). On the other hand, there is evidence on a positive relationship between higher quality products and their market shares. The variation in market shares is closely related to the business cycle which abounds on the role played by disposable income in the purchasing decision. The empirical literature includes quality, quality variability, income, and the number of brands among the regressors in their analyses (see e.g. Hoch and Banerji, 1993).

all consumers agree that the high-quality product is preferred to the low-quality product which in turn is preferred to not buying. Purchases are mutually exclusive. Then, although consumers agree on the quality ranking, each consumer has a different reservation price since they have different income.

The market T can be partitioned between those consumers who buy the high-quality product, those who buy the low-quality product and those who buy neither of them. The general presentation above is valid for the demand configuration where both qualities have positive demand but there are unserved consumers. The demand expressions obtained are $q_L = \frac{u_H p_H - u_L p_L}{(u_H - u_L)R_2} - \frac{u_L p_L}{(u_L - u_0)R_2}$ and $q_H = 1 - \frac{u_H p_H - u_L p_L}{(u_H - u_L)R_2} + \frac{R_1}{R_2}$, for the low and the high-quality product, respectively. To study quantity competition, we invert the above demand system to obtain, $p_L = \frac{u_L - u_0}{u_L}(R_1 + R_2(1 - q_H - q_L))$ and $p_H = \frac{u_H - u_0}{u_H}(R_1 + R_2) - \left(\frac{u_H - u_0}{u_H}\right) R_2 q_H - \left(\frac{u_L - u_0}{u_H}\right) R_2 q_L$. This inverted demand system can be written in the following convenient way,

$$p_L = a_L - d_L q_L - d_L q_H \tag{27}$$

$$p_H = a_H - d_L b q_L - d_H q_H \tag{28}$$

where it is verified that $a_H > a_L$, $d_H > d_L$ and that $a_H > d_H$, and $a_L > d_L$. The parameter b is the relative marginal utility of income for quality, i.e. $\frac{u_L}{u_H}$. It is the case that $0 < b < 1$ and that $d_H > bd_L$.

Now assume that one of the firms is a multi-product manufacturer. In particular, firm H produces both the high and the low-quality products and firm L remains a single-product manufacturer. This corresponds with manufacturer M_1 that produces both A and B and manufacturer M_2 that produces B in our earlier presentation. To see the connection with our general construction, let us write $p_A = a_A - d_B b(q_{B1} + q_{B2}) - d_A q_A$ and $p_B = a_B - d_B q_A - d_B(q_{B1} + q_{B2})$. The

Cournot-Nash third-stage quantities in the (AB, B) subgame are:

$$q_A^* = \frac{3a_A - 2ba_B - 3w_A + bw_{B1} + bw_{B2}}{2(3d_A - bd_B)} \tag{29}$$

$$q_{B1}^* = \frac{2a_B d_A - a_A d_B + d_B w_A - (4d_A - bd_B)w_{B1} + (2d_A - bd_B)w_{B2}}{2d_B(3d_A - bd_B)} \tag{30}$$

$$q_{B2}^* = \frac{2a_B d_A - a_A d_B + d_B w_A - (4d_A - bd_B)w_{B2} + (2d_A - bd_B)w_{B1}}{2d_B(3d_A - bd_B)} \tag{31}$$

These equilibrium quantities are substituted back into manufacturers' profits to compute the second-stage equilibrium. Setting $\partial \Pi_1 / \partial w_A$ and $\partial \Pi_1 / \partial w_{B1}$ equal to zero and solving for w_A and w_{B1} allows us to write the following margins as a function of the parameters and w_{B2}

$$\begin{aligned} w_A - c_A \quad = \quad & -\frac{(1+3b)(a_A - c_A)d_B + c_B(4d_A + (b-1)bd_B - 2a_B(d_A + b^2 d_B)}{(8d_A - (1 + b(6+b))d_B} \\ & + \frac{(2d_A - b(1+b))w_{B2}}{(8d_A - (1 + b(6+b))d_B} \end{aligned} \tag{32}$$

and

$$w_{B1} - c_B = -(w_A - c_A) \tag{33}$$

By substitution of (32)-(33) in the equilibrium quantities, the output ratio by the multi-product manufacturer q_A^*/q_{B1}^* can be written as

$$\frac{q_A^*}{q_{B1}^*} = \frac{-\left(d_B(4a_A - (1+3b)a_B - 4c_A + 2(1+b)c_B + (b-1)w_{B2}\right)}{2(1+b)d_B(a_A - c_A) + (8d_A - 4bd_B)c_B + (4d_A + (b-1)b)a_B - (4d_A - b(3+b)d_B)w_{B2}}$$

The ratio p_B^B/p_A^B is now equal to $1/b$. Therefore, the condition $q_A^*/q_{B1}^* > p_B^B/p_A^B$

stated in Proposition 1 above amounts to

$$(1+3b)(a_A-c_A)d_B+c_B(4d_A+(b-1)bd_B-2a_B(d_A+b^2d_B)-(2d_A-b(1+b))w_{B1}>0 \tag{34}$$

which is precisely minus the numerator in (32). This illustrates the characterization in Proposition 1 above for the case of vertical product differentiation. If (34) holds, then at equilibrium $w_A - c_A$ is negative whereas $w_{B1} - c_B$ is positive. Hence, the multi-product manufacturer only incentives the sales of the high-quality product. A parallel reasoning can be made in the subgames when just the high or the low-quality product is delegated. The equilibrium variables and profits corresponding to each of the eight subgames can be computed and then find the equilibrium of the game G. This has been studied in a companion paper (Moner-Colonques et al, 2000) from which we extract the following result. It translates Proposition 2 to the case of vertical differentiation.

Proposition 3 *The first stage equilibrium of game G is unique. Depending on the size of the relative profitability ratio of both markets, either (A, B) or (B, B) is the equilibrium distribution pattern. The high quality is delegated whenever the profitability ratio of both markets is big enough (i.e. $\frac{a_A-c_A}{a_B-c_B} > \max\{1, \frac{4d_A^2-2b(1-3b)d_Ad_B-2b^3d_B^2}{d_B(2(1+4b)d_A-b(1+3b)d_B)}\}$); the low quality is delegated otherwise. Then, at equilibrium the single-product manufacturer delegates sales while the multi-product manufacturer only delegates the sales of one of the qualities. Furthermore, the wholesale prices established by manufacturers at the equilibrium path are set below the corresponding unit production costs.*

The relative market profitability ratio is the relation between $(a_A - c_A)$, the unitary profitability of the high-quality product, and $(a_B - c_B)$, the unitary profitability of the low-quality product. This ratio is big enough for a low degree of inter-quality competition and for a low valuation of the purchase of the low-quality product. Suppose that $\frac{a_A - c_A}{a_B - c_B}$ is very large. When both the high and the low-quality products are delegated the multi-product manufacturer sets $w_A^{ABB} < c_A$ and $w_{B1}^{ABB} > c_B$. This is done in an effort to dampen intra-firm competition. Note, however, that its low-quality product would compete at a disadvantage against the rival single-product firm, which sets a wholesale price below c_B. Thus, by choosing to delegate only the sales of the high-quality product instead of delegating both, the multi-product manufacturer achieves two things. In the first place, it induces a higher sales effort from its retailer in the high-quality market, which is a very profitable market. Secondly, it is able to partially compensate for the internalization loss of intra-firm competition when both qualities are delegated to two independent retailers. By selecting different channels, intra-firm competition is weaker and it competes with a cost c_B rather than with $w_{B1}^{ABB} > c_B$, though still at a competitive disadvantage with the rival single-product manufacturer.

Finally, we also employ the particular case of vertical product differentiation to compute the multi-product manufacturer's profits when both products' sales are delegated to a common retailer. Then, profits are compared with those under delegation of only one product's sales.

Result 2 *Suppose* $c_A = 12$, $c_B = 2$, $R_1 = 5$, $R_2 = 40$, $u_0 = 1$, $u_A = 40$. *For every subgame, the conditions for positive equilibrium quantities and total output less than one are satisfied iff* $u_B \in (20.6, 24.5)$. *Then the multi-product manufacturer delegates the sales of only product B.*

This numerical example is aimed at emphasizing that the central result of the paper is robust to a relaxation of the assumption that retailers cannot carry more than one product. A further interesting aspect to be mentioned is that the multi-product manufacturer's equilibrium wholesale prices are set above the corresponding unit production costs. This illustrates the conclusion stated in the above Section claiming that delegation only one product's sales dominates delegation of both products' sales to a common retailer.

4 Concluding Remarks

This paper has investigated whether a multi-product manufacturer will strategically employ a different distribution channel for each of the products. By letting it delegate the sales of only one product, it has been proven that previous results on delegation are not robust when a firm sells multiple products. This result holds even in the case of a multi-product retailer instead of using a separate retailer for each product. The multi-product firm faces the following tradeoff. On the one hand, there is the strategic effect of delegation. On the other, the internalization of intra-firm competition is lost when both products' sales are delegated. By appropriately setting the wholesale price levels together with the decision of not delegating the sales of both products, the multi-product manufacturer strikes just the right compromise: the externalities between its own products are partially internalized while a strategic advantage is achieved against its rival single-product manufacturer.

While the proposed model provides a theoretical answer to some observed marketing tactics, other remain unexplained. Our model could be extended to address the issue of competition between national brands and private labels. Mills (1995) compares a successive monopoly with the case where the retailer introduces a low-quality product and also sells the manufacturer's high-quality product. An interesting avenue for future research is to consider such a strategy on the retailer's side in an oligopolistic upstream and downstrean structure. Then, one might study how the retailer improves its bargaining position and whether a private label is always introduced.

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A Proofs

Proof of Proposition 1.

We first show that only three combinations of wholesale prices relative to unit production costs are compatible with equations (11) and (12). Assume that w_{B1}^{ABB} is the equilibrium wholesale price. Next check the sign of $\frac{\partial \Pi_1}{\partial w_A}$ evaluated at $w_A = c_A$. From (12) we know that $(w_{B1}^{ABB} - c_B)\frac{dq_{B1}^*}{dw_{B1}} + \gamma q_{B1}^* + \delta q_A^* = 0$ and therefore $q_{B1}^* = -\frac{\delta q_A^*}{\gamma} - \frac{(w_{B1}^{ABB} - c_B)\frac{dq_{B1}^*}{dw_{B1}}}{\gamma}$, which upon substitution in (11) yields

$$\left.\frac{\partial \Pi_1}{\partial w_A}\right|_{w_A=c_A} = \frac{(\beta\gamma - \delta\alpha)}{\gamma}q_A^* + \frac{(w_{B1}^{ABB} - c_B)}{\gamma}(\gamma\frac{dq_{B1}^*}{dw_A} - \alpha\frac{dq_{B1}^*}{dw_{B1}})$$

but since $\beta\gamma = \delta\alpha$, $sign[(\gamma\frac{dq_{B1}^*}{dw_A} - \alpha\frac{dq_{B1}^*}{dw_{B1}})] = -sign[\Delta] > 0$ and $\gamma < 0$ then $sign[\left.\frac{\partial\Pi_1}{\partial w_A}\right|_{w_A=c_A}]$ is $-sign[w_{B1}^{ABB} - c_B]$.

Therefore, if $w_{B1}^{ABB} < (>) c_B$ then $\left.\frac{\partial\Pi_1}{\partial w_A}\right|_{w_A=c_A} > (<) 0$ which means that an increment (reduction) in w_A is profitable for the multi-product manufacturer. Finally, for $w_{B1}^{ABB} = c_B$, the equilibrium is attained at $w_A = c_A$.

Further we can identify the particular combination that shows up as a function of the ratio $\frac{q_A^*}{q_{B1}^*}$. From (11) and (12) we know that if $w_A = c_A$ and $w_B = c_B$ it must be the case that $\frac{q_A^*}{q_{B1}^*} = \frac{-\alpha}{\beta} = \frac{-\gamma}{\delta} = \frac{p_B^B}{p_A^B}$, and viceversa. If $w_A < c_A$ and $w_B > c_B$ equations (11) and (12) verify iff $\frac{q_A^*}{q_{B1}^*} > \frac{p_B^B}{p_A^B}$. A similar reasoning applies to $w_A > c_A$ and $w_B < c_B$.

Proof of Proposition 2:

We begin by proving that the single-product manufacturer always employs a retailer and incentives sales. Note that for a single-product firm who employs a retailer through a two-part tariff contract, non-delegation is equivalent to delegation with a wholesale price equal to unit production cost. Thus if we prove that the second stage equilibrium w_{B2} is different from c_B then we will conclude that N is not a first stage equilibruim action for the single-product manufacturer. To see this, the second stage first order condition when it delegate sales is either:

$$\frac{\partial \Pi_2}{\partial w_{B2}} = (w_{B2} - c_B)\frac{dq_{B2}^*}{dw_{B2}} + (p_B^B\frac{dq_{B1}^*}{dw_{B2}} + p_B^A\frac{dq_A^*}{dw_{B2}})q_{B2}^* = 0$$

when the multi-product manufacturer first-stage action is A, B or AB, or:

$$\frac{\partial \Pi_2}{\partial w_{B2}} = (w_{B2} - c_B)\frac{d\bar{q}_{B2}}{dw_{B2}} + (p_B^B\frac{d\bar{q}_{B1}}{dw_{B2}} + p_B^A\frac{d\bar{q}_A}{dw_{B2}})\bar{q}_{B2} = 0$$

when the multi-product manufacturer does not employ retailers.

In the former case, w_{B2} must be smaller than c_B since $\frac{dq^*_{B2}}{dw_{B2}} < 0$ and $(p^B_B \frac{dq^*_{B1}}{dw_{B2}} + p^A_B \frac{dq^*_A}{dw_{B2}})q^*_{B2} < 0$.

In the latter case, w_{B2} must also be smaller than c_B since $\frac{d\bar{q}_{B2}}{dw_{B2}} < 0$ and $(p^B_B \frac{d\bar{q}_{B1}}{dw_{B2}} + p^A_B \frac{d\bar{q}_A}{dw_{B2}}) = \frac{p^B_B(2p^A_A p^B_B - p^A_B p^A_B - p^B_A p^B_A)}{-\Delta} < 0$, given that $(2p^A_A p^B_B - p^A_B p^A_B - p^B_A p^B_A) > 0$, by the assumption that the smallest own effect is greater than or equal to the greatest cross effect in absolute terms. Therefore, we conclude that the single-product manufacturer will always employ one independent retailer and will incentive its sales.

Given that manufacturer M_2 has action B as a dominant strategy, we need only concentrate on four subgames and prove that either action A or action B are $M_1's$ first-stage equilibrium action. The following Lemma will be useful in proving which is the first-stage equilibrium action for the multi-product manufacturer.

Lemma 1 *First, the variables w_A, w_{B1} and w_{B2} are strategic substitutes for the single-product manufacturer. Second, the variables w_A, w_{B1} and w_A, w_{B2} are strategic substitutes for the multi-product manufacturer. Finally, the variables w_{B1}, w_{B2} are strategic substitutes if either $|p^B_B| > |p^B_A|$ or $|p^B_B| = |p^B_A| > |p^A_B|$, while they are independent if $|p^B_B| = |p^B_A| = |p^A_B|$.*

Proof. Firstly, note that $\frac{\partial^2 \Pi_2}{\partial w_{B2} \partial w_{B1}} = (p^B_B \frac{dq^*_{B1}}{dw_{B2}} + p^A_B \frac{dq^*_A}{dw_{B2}})\frac{dq^*_{B2}}{dw_{B1}}$ and $\frac{\partial^2 \Pi_2}{\partial w_{B2} \partial w_A} = (p^B_B \frac{dq^*_{B1}}{dw_{B2}} + p^A_B \frac{dq^*_A}{dw_{B2}})\frac{dq^*_{B2}}{dw_A}$ are both negative since both p^B_B and p^A_B are negative while $\frac{dq^*_{B1}}{dw_{B2}}, \frac{dq^*_A}{dw_{B2}}, \frac{dq^*_{B2}}{dw_{B1}}$ and $\frac{dq^*_{B2}}{dw_A}$ are positive as established in Table 2 in the text.

Next,

$\frac{\partial^2 \Pi_1}{\partial w_A \partial w_{B2}} = (p^B_B \frac{dq^*_{B2}}{dw_A} + p^A_B \frac{dq^*_A}{dw_A})\frac{dq^*_{B1}}{dw_{B2}} + (p^B_A \frac{dq^*_{B2}}{dw_A} + p^B_A \frac{dq^*_{B1}}{dw_A})\frac{dq^*_A}{dw_{B2}}$ whose sign, by straightforward computations is given by $sign[p^A_B(2p^A_A p^B_A - (p^B_A)^2 + 2p^A_A p^A_B - p^B_A p^A_B)]$ which is negative provided that own effects weakly dominate cross effects. Now

$$\frac{\partial^2 \Pi_1}{\partial w_{B1} \partial w_A} = \frac{\partial^2 \Pi_1}{\partial w_A \partial w_{B1}} = \frac{dq^*_A}{dw_{B1}} + (p^B_B \frac{dq^*_{B2}}{dw_{B1}} + p^A_B \frac{dq^*_A}{dw_{B1}})\frac{dq^*_{B1}}{dw_A} + (p^B_A \frac{dq^*_{B2}}{dw_{B1}} + p^B_A \frac{dq^*_{B1}}{dw_{B1}})\frac{dq^*_A}{dw_A}$$

whose sign is given by $sign[p^A_B((p^B_A)^2 + p^A_A p^B_B)]$, which is negative. Finally,

$\frac{\partial^2\Pi_1}{\partial w_{B1}\partial w_{B2}} = (p_B^B\frac{dq_{B2}^*}{dw_{B1}}+p_B^A\frac{dq_A^*}{dw_{B1}})\frac{dq_{B1}^*}{dw_{B2}}+(p_A^B\frac{dq_{B2}^*}{dw_{B1}}+p_A^B\frac{dq_{B1}^*}{dw_{B1}})\frac{dq_A^*}{dw_{B2}}$ whose sign is given by $sign[(p_A^B)^2(p_A^B-p_B^A)+2(p_A^A)^2(p_B^B-p_A^B)]$ which is negative provided that $|p_B^B| > |p_A^B|$ and that own effects weakly dominate cross effects. In the case of $|p_B^B| = |p_A^B|$ the $sign[\frac{\partial^2\Pi_1}{\partial w_{B1}\partial w_{B2}}] = sign[(p_A^B - p_B^A)]$, which can only be either negative when $|p_A^B| > |p_B^A|$ or zero when $|p_A^B| = |p_B^A|$. The possibility of $|p_A^B| < |p_B^A|$ is not possible because together with $|p_B^B| = |p_A^B|$ would imply that $|p_B^B| < |p_B^A|$ which is ruled out.■

Concerning the multi-product manufacturer's first-stage action,

a)We first show that action AB is weakly dominated by either action A or B. Since there are three possible second stage equilibria for action AB, i. e.$\{w_A > c_A$, $w_{B1} < c_B\}$, $\{w_A < c_A$, $w_{B1} > c_B\}$ and $\{w_A = c_A$, $w_{B1} = c_B\}$, we consider them separately.

a.1)Consider $\{w_A > c_A$, $w_{B1} < c_B\}$. As has been proven above the first stage equilibrium of the single-product manufacturer involves delegation of sales.

Step 1: we prove that $\Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{BB}) > \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB})$

This is equivalent to finding whether:

$$\int_{w_{B2}^{ABB}}^{w_{B2}^{BB}} \frac{\partial\Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2})}{\partial w_{B2}} dw_{B2} > 0$$

By the envelope theorem we know that $w_{B1} = p_B^B q_{B1} + p_B$ and $w_A = p_A^A q_A + p_A$,then $\frac{\partial\Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2})}{\partial w_{B2}}$ can be written as:

$$\frac{\partial\Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2})}{\partial w_{B2}} = \sigma q_{B1}^* + \mu q_A^* + (w_A^{ABB} - c_A)\frac{dq_A^*}{dw_{B2}} + (w_{B1}^{ABB} - c_B)\frac{dq_{B1}^*}{dw_{B2}}$$

where $\sigma = (p_B^B\frac{dq_{B2}^*}{dw_{B2}} + p_B^A\frac{dq_A^*}{dw_{B2}}) = \frac{2p_A^A p_B^B(p_B^A - 2p_B^B)}{(-\Delta)} > 0$, $\mu = (p_A^B\frac{dq_{B2}^*}{dw_{B2}} + p_A^B\frac{dq_{B1}^*}{dw_{B2}}) = \frac{-2p_A^A p_B^B p_A^B}{(-\Delta)} > 0$, $\frac{dq_A^*}{dw_{B2}} = \frac{p_B^B p_A^B}{(-\Delta)} > 0$ and $\frac{dq_{B1}^*}{dw_{B2}} = \frac{2p_A^A p_B^B - p_A^B p_A^B}{(-\Delta)} > 0$. Finally, from the reaction functions for w_A and w_{B1} in the (AB, B) subgame, we know that

$w_{B1}^{ABB} - c_B = \frac{p_B^B q_{B1} - p_A^B q_A}{2}$ and $w_A^{ABB} - c_A = \frac{-p_B^A q_{B1} + p_A^B q_A}{2}$, which upon substitution above yields:

$$\frac{\partial \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2})}{\partial w_{B2}} = \frac{-p_B^B[p_A^A(3p_B^B - p_B^A)q_{B1}^* + p_A^B(3p_A^A - p_A^B)q_A^*]}{-\Delta} > 0$$

which is in fact satisfied for any of the three possible second stage equilibrium of the (AB, B) subgame. Therefore, $\Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{BB})$ is greater than, smaller than or equal to $\Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB})$ as long as w_{B2}^{BB} is greater than, smaller than or equal to w_{B2}^{ABB}, the integration limits.

Step 2: we now prove that $\Pi_1(c_A, w_{B1}^{BB}, w_{B2}^{BB}) > \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{BB})$. This is equivalent to proving that:

$$\int_{w_{B1}^{ABB}}^{w_{B1}^{BB}} \int_{w_A^{ABB}}^{c_A} \frac{\partial^2 \Pi_1}{\partial w_A \partial w_{B1}} dw_{B1} dw_A > 0$$

Since we depart from the $w_A^{ABB} > c_A$, a move from action AB to action B implies $w_{B1}^{BB} > w_{B1}^{ABB}$ and $w_{B2}^{BB} > w_{B2}^{ABB}$ and also $\frac{\partial^2 \Pi_1}{\partial w_A \partial w_{B1}} < 0$. Therefore, the above inequality is satisfied. We conclude that:

$$\Pi_1(c_A, w_{B1}^{BB}, w_{B2}^{BB}) > \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{BB}) > \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB})$$

The multi-product manufacturer is better off with delegation of only product B rather than with delegation of both products.

a.2) Had we departed from the second stage equilibrium with $\{w_A^{ABB} < c_A$, $w_{B1}^{ABB} > c_B\}$ the above profits ranking does not hold since in step 1 we will have that the best response to an increment in w_A will be $w_{B2}^{BB} < w_{B2}^{ABB}$. The appropriate deviation for the multi-product manufacturer will be to delegate the sales of only product A. Firstly,

$$\Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{AB}) > \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB}) \text{ since } \int_{w_{B2}^{ABB}}^{w_{B2}^{AB}} \frac{\partial \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2})}{\partial w_{B2}} dw_{B2} > 0$$

and secondly,

$$\Pi_1(w_A^{AB}, c_B, w_{B2}^{AB}) > \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{AB}) \text{ given that } \int_{w_{B1}^{ABD}}^{c_B} \int_{w_A^{ABB}}^{w_A^{AB}} \frac{\partial_1^2 \Pi}{\partial w_A \partial w_{B1}} dw_{B1} dw_A > 0.$$

a.3) Finally, had we departed from the second stage equilibrium $\{w_A^{ABB} = c_A, w_{B1}^{ABB} = c_B\}$, a deviation by M_1either to action A with $w_A^{AB} \leq c_A$ or to action B with $w_{B1}^{BB} \leq c_B$ will not be unprofitable. For example, if we consider action A with $w_A^{AB} \leq c_A$, then

$$\Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{AB}) \geq \Pi_1(w_{B1}^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB})$$

since $\frac{\partial \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2})}{\partial w_{B2}} = \sigma q_{B1}^* + \mu q_A^* > 0$ and $w_{B2}^{AB} \geq w_{B2}^{ABB}$, by strategic substitution. It turns out that:

$$\int_{w_A^{ABB}=c_B}^{c_B} \int_{w_A^{ABB}=c_A}^{w_A^{AB}} \frac{\partial_1^2 \Pi}{\partial w_A \partial w_{B1}} dw_{B1} dw_A \geq 0$$

and therefore the ranking profits is

$$\Pi_1(w_A^{AB}, c_B, w_{B2}^{AB}) \geq \Pi_1(w_A^{ABB}, w_{B1}^{ABB}, w_{B2}^{AB}) \geq \Pi_1(w_{B1}^{ABB}, w_{B1}^{ABB}, w_{B2}^{ABB})$$

This ends the proof that action AB is weakly dominated by either action A or B.

b)We finally prove that action N is also dominated.

First note that, contrary to what happens with single-product manufacturers, not to delegate sales is not equivalent to delegate sales of both products to independent separate dealers using a two-part tariff contract, when transfer prices for each product are set equal to their respective unit production cost. The difference is precisely the lack of internalization of intra-firm competition. This is why the multi-product firm's equilibrium outputs under action N are smaller than those under action AB for $w_A = c_A$, $w_{B1} = c_B$. Also, by strategic substitution, the single-product firm's equilibrium output is greater under N. We have to establish that, departing from action N, an output increase by the multi-product firm leads to a profits increase. That is:

$$\begin{aligned}\frac{d\Pi_1}{dq_A} &= \frac{\partial\Pi_1}{\partial q_A} + \frac{\partial\Pi_1}{\partial q_{B1}}\frac{dq_{B1}}{dq_A} + \frac{\partial\Pi_1}{\partial q_{B2}}\frac{dq_{B2}}{dq_A} \\ \frac{d\Pi_1}{dq_{B1}} &= \frac{\partial\Pi_1}{\partial q_{B1}} + \frac{\partial\Pi_1}{\partial q_A}\frac{dq_A}{dq_{B1}} + \frac{\partial\Pi_1}{\partial q_{B2}}\frac{dq_{B2}}{dq_{B1}}\end{aligned}$$

are both positive since $\frac{\partial\Pi_1}{\partial q_A} = \frac{\partial\Pi_1}{\partial q_{B1}} = 0$ by the envelope theorem, $\frac{\partial\Pi_1}{\partial q_{B2}} = p_A^B\bar{q}_A + p_A^B\bar{q}_{B1} < 0$ and q_A, q_{B1} and q_{B2} are strategic substitutes. Therefore, the multi-product firm will prefer to delegate sales of both products with $w_A = c_A$ and $w_{B1} = c_B$ rather than not to delegate sales. By the same token, the multi-product firm will prefer to delegate the sales of only one product, say either A with $\{w_A < c_A \text{ , } w_{B1} = c_B\}$ or B with $\{w_A = c_A, \, w_{B1} < c_B\}$ rather than not to delegate sales because output increases.

Proof of Result 1

If the multi-product manufacturer wants the common retailer to distribute its two products it must offer a two-part tariff contract that pays the common retailer. Assume that a retailer accepts the following contract $\{w_A, F_A, w_{B1}, F_{B1}\}$ to distribute the two multi-product manufacturer's products, and another retailer accepts the contract $\{w_{B2}, F_{B2}\}$ to distribute the single-product manufacturer product. Denote by $\hat{q}_A, \hat{q}_{B1}$ and $\hat{q}_{B2}$ the equilibrium quantities. Let $\tilde{p}_B(0, q_{B1} + q_{B2})$ denote product $B's$ inverse demand function when the common retailer does not sell product A and let $\tilde{p}_A(q_A, q_{B2})$ be product $A's$ inverse demand function when the common retailer does not sell product B. Then the common retailer accepts the contract if and only if:

$$(p_A(\hat{q}_A, \hat{q}_{B1} + \hat{q}_{B2}) - w_A)\hat{q}_A + (p_B(\hat{q}_A, \hat{q}_{B1} + \hat{q}_{B2}) - w_{B1})\hat{q}_{B1} - F_A - F_{B1} \geq$$
$$\max\{\max_{q_A}(\tilde{p}_A(q_A, q_{B2}) - w_A)q_A - F_A), \max_{q_{B1}}(\tilde{p}_B(0, q_{B1} + q_{B2}) - w_{B1})q_{B1} - F_{B1})$$

If F_A and F_{B1} are fully rent-extracting then the left-hand side of the above expression will be zero while the right-hand side is not zero given that $\max_{q_A}(\tilde{p}_A(q_A, q_{B2}) - w_A)q_A - F_A \geq (\tilde{p}_A(\hat{q}_A, q_{B2}) - w_A)\hat{q}_A - F_A =$

$(\tilde{p}_A(\hat{q}_A, q_{B2}) - w_A)\hat{q}_A - (p_A(\hat{q}_A, \hat{q}_{B1} + \hat{q}_{B2}) - w_A)\hat{q}_A =$

$(\tilde{p}_A(\hat{q}_A, q_{B2}) - p_A(\hat{q}_A, \hat{q}_{B1} + q_{B2}))\hat{q}_A > 0.$

Similarly for the case of product B. In fact, the retailer is able to earn a positive rent from each product, since had the multi-product manufacturer offered a contract with fully rent extraction on only one product, the common retailer could threaten again with dropping precisely this product.