

Panel discussion

Language in Mathematics Education

Panellists:

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About the Panel

The aim is to promote a discussion of language in mathematics education from different perspectives. The panellists have chosen a research protocol, which is transcribed next, and have each prepared a statement to pose issues concerning language and mathematics education.

At the beginning of the plenary session, each panellist will expand on their initial position. These initial statements will be followed by a discussion drawing on the protocol as a common ground where issues raised by the panellists can be discussed and other ideas about language in mathematics education can be raised.

All PME participants are asked to read the protocol and the contributions to this session before the session so that they share a frame of reference from which the discussion might start. Ideas presented by the panellists, research about the topic of the panel, and gaps in the positions put forth can be starting points.

Putting maths into language and language into maths

The task

The pupils were asked to solve the following problem: “Two year five classes had 75 books to share between the two classes. However, one class had more pupils than the other and the head teacher decided that it would get 13 books more than the other. How many books will each class receive?” After the pupils had solved the problem, they received the following instructions:

“Your friends will now solve the same problem you just solved and you want to help them. You will write a message with instruction for them on how to solve the problem. But there is something about the message you are going to write for them. Their problem will have different numbers from those in your problem. That means that your message cannot use your numbers in the explanation. You have to explain to them how to solve the problem without using any numbers in the message. ”

Solution

The pupils wrote all the information on one page and then decided to divide 75 by 2, obtaining 37 and a half; they checked the calculation over and again had 37 and a half as the result. (The names used here are fictitious).

1. Jane: *Thirty seven and a half?* (Both laugh). *Thirty seven and one book left.*
2. Ann: *Take away 13.*
3. Jane: *No, that won't work. That's 14 books left.*
4. Ann: *Because we did the take away?*
5. Jane: *Yes.*
6. Ann: *35 and 35 is 70.*
7. (They re-check the division sum).
8. Jane: *Thirty seven and a half, right? When you divide 75 by 2* (looking at the interviewer).
9. Interviewer (Int.): *Did the classes have the same number of books?*
10. Jane: *No, we have to take away some and give to the other class.*
11. Ann: *From where?*
12. Jane: *I know, we'll take the 13 books away first* (crumples up page). *Seventy five take away thirteen.*
13. Ann: (does)
$$\begin{array}{r} 75 \\ - 13 \\ \hline \end{array}$$

14. Jane: *Then one class has half that. Thirty one.*
15. Ann: (writes 31 under the 62 without indicating the computation and says) *31.*
16. Jane: *One class gets 31. And the other gets 13 more.*
17. Ann: (does)
$$\begin{array}{r} 31 \\ + 13 \\ \hline 44 \end{array}$$
18. Jane: *44 and 31 is the answer.*
19. Int.: *Are you sure?*
20. Jane (to Ann): *Do they add up to 75?*
21. Ann: (does)
$$\begin{array}{r} 44 \\ + 31 \\ \hline 75 \end{array}$$
22. Both: *Yes* (emphatically and laughing).

Putting the solution into language

The pupils approached the task by looking at the computations and writing a description of what they had done as the instructions. They continuously checked where they were with respect to the computations. (Underlined portions were spoken and written; pupils took turns at writing)

The page with computations looked as below.

| | |
|--|--|
| $\begin{array}{r} 75 \\ - 31 \\ \hline 44 \end{array}$ | |
| $\begin{array}{r} 31 \\ + 13 \\ \hline 44 \end{array}$ | $\begin{array}{r} 44 \\ + 31 \\ \hline 75 \end{array}$ |

23. Jane: *Get the number of books and ... take away ...*
24. Ann: *Take away what?*
25. Jane: *Take away.*
26. Ann: *Take away what?*
27. Jane: *Take away the amount.*
28. Ann: *Take away the number that one class had more.*

29. Jane: *Yes. (Reads what was written) Yes. Take away the amount that is bigger.*
30. Ann (seems to interrupt): *Take away (crosses out number) the amount of how many more books one class had.*
31. Jane: *Add.*
32. Ann (shows computation on paper): *Split into half.*
33. Jane: *Oh, yes, split the number, the amount into half and add the number of how many more.*
34. Ann: *Of how many more ... you want to put here?*
35. Jane: *Of how many more books - or how many books more? We've got that far (pointing at computations used in solving the problem; adds books to the text) one class had.*
36. Ann: *We're here (pointing at previous solution).*
37. Jane: *Then see if you are right, add up the two answers to see if it comes to the amount in the beginning.*
38. Ann: *The amount you started with.*
39. Jane: *Of books you started with.*

When the message was finished, the interviewer gives the new set of instructions:

40. Int.: *That is brilliant, that is very good indeed. Now, you wrote your message in English and I want you to try to write it in mathematics. Using signs like in mathematics.*
41. (The pupils ask how and the interviewer continues).
42. Int.: *Now, because you don't know the numbers they will have in their problem, you can use a letter in its place, like a code. You can start like this, instead of saying get the number of books (reading from the pupils' message) you tell them that the total number of books will be called A in your message.*
43. Pupils: *What? The number of books will be called A?*
44. Int.: *Yes, you see, instead of saying the number of books, you say A, like a code, and you explain to them what the code means. You can call the total number of books A.*
45. Jane: *Oh, I get it. Call the number of books they have A. (To interviewer) Like this? And what?*
46. Int.: *You can tell them to call the difference - do you know what difference is?*
47. Both: *Yes.*
48. Jane: *Call the difference between the classes?*

49. Int.: *Yes (encouragingly).*
50. Jane: *What? B?*
51. Int.: *Yes, good. Then you can tell them what sums to do. Have you done this in your classroom?*
52. Ann: *No.*
53. Int.: *Do you do formulas in science?*
54. Ann: *No.*
55. Int.: *Or in maths?*
56. Ann: *No.*
57. Jane: *I know, call the difference between the two classes B.*
58. Int.: *Now you can tell them what mathematics to do.*
59. (Jane wants to change the old message but is asked to do a new one).
60. Ann: *Get A and take away B.*
61. Jane: *Split B.*
62. Ann: *Split A.*
63. Jane: *A?*
64. Ann: *A? Take away B and split.*
65. Jane: *Oh yes, get A and take away B (reading from message), and call A take away B = C.*
66. Ann: *Split in half.*
67. Jane: *Yes. Split C in half. And then get the number of how many more books (reading from old message).*
68. Ann (pointing to computations for solution): *What's that?*
69. Jane: *That's A.*
70. Ann: *That's B. I'm up to there.*
71. Jane (following with Ann and also looking at earlier message): *Split C in half (from present message) and then add the amount of how many more books one class had (looks at old message) and add B.*
72. Ann: *To what?*
73. Jane: *To C.*
74. Ann: *No, C is that one (points at earlier solution).*
75. Jane: *To D.*
76. Ann: *What's D?*

77. Jane: *Get the second answer. Call the second answer. Half of C.*
78. Ann: *What is the second answer? Half of C?*
79. (The pupils check everything up to then, reading the message and comparing it to the computation).
80. Jane: $C : 2 = D$.
81. Ann: *Call the answer.*
82. Jane: *Call the answer to B and $D = E$.*
83. Ann: *Call the answer to B and $D = E$.*
84. Jane: *Then add E and D and see if you can get A.*
85. Int.: *How will they know which ones are their answers?*
86. Jane: *What do you mean?*
87. Int.: *How many books each class gets.*
88. Jane: *These* (points to paper).
89. Int.: *Which?*
90. Jane: *D.*
91. Ann: *E.*
92. Int.: *Do you want to tell them that then?*
93. Ann: *One class has D and the other has E.*

The interviewer finally asks them which message they think will be the best for their friends to use and they agree immediately that the second one is the best. The message was written on three different pieces of paper and the pupils arranged them on the table for their friends to use later.

Total time: approximately 21 minutes.

Solving the problem with the message

The task for the second pair of pupils was to use the message to solve the problem. The interviewer read the problem to them and they wrote down 75 and 13 underneath. The first pair had put the three pages on the table and the message read from left to right like this:

94. Left page: Call the answer to B and $D = E$.
95. Centre page: Call the total number of books A.
96. Call the difference between the two classes B.
97. Call A take away B equals C.

98. $C : 2 = D$
99. Right page: Get A and take away B.
100. Split C in half
101. and add B to D.
102. Add E and D to get A.
103. One class has B and one has E.

The pupils read the message in this order, skipping “ $C : 2 = D$ ”. They re-read the centre and the page on the right (not shown on video to save time).

104. Lynne: (Inaudible).
105. Carey: *The difference between the two classes? A. Get A and take away B.*
106. Lynne: *Sixty two.*
107. Carey: *Sixty two.*
108. Lynne: *Sixty two. There.*
109. Carey: (Writes 62 and B next to it).
110. Lynne: *The difference between the two classes.*
111. Int.: *I don't think I will be able to hear.*
112. Carey: *OK.*
113. Carey: *Sixty two. B.*
114. Int.: *So, do you know what they say you should call A?*
115. Carey: *Yes, that* (points to the number; then writes and checks over) *A. Call the difference between the two classes B. No, that's not B* (showing 62 and crossing out the B next to it), *that's B* (showing 13, writes B next to it).
116. Lynne: *Take away, they said here.*
117. Carey: *Take away B. Call A. Call A take away B equals C. That's sixty two, isn't it?*
118. Lynne: *Yes.*
119. Carey: *So, sixty two is C* (writes 62 C).
120. Lynne: *Yes.*
121. (Both read together: Get A take away B equals C).
122. Carey: *Split C in half, 31, then add B to D. (Pause) Call the answer to B and D E. What?*
123. Lynne (to interviewer): *We don't know what's D.*

124. Carey: *What's D?*
125. Lynne (to interviewer): *Call the answer to B and D E.*
126. Carey: *Then add D and E - What's E? - to get A. One class has D and one class has E (reading from message).*
127. Lynne (to interviewer): *We don't know what E is.*
128. Int.: *I see.*
129. Carey: *What's E?*
130. Int.: *Did you read both sides? Perhaps they wrote on both sides.*
131. Carey: *Call the answer ... Both sides? (Turns page over) No. Because.*
132. Int.: *Oh, there it is. Call the answer to B and D equal E. B and D. Is what?*
133. Carey: *B and D. Is forty four.*
134. Lynne: *Yes.*
135. Carey: *Forty four.*
136. Int.: (Unintelligible).
137. Carey: *Forty four. E. Forty four is E. Oh! Then. Add B and D to get A.*
138. Lynne: *Seventy five.*
139. Carey: *One has D and one has E. (Speaks and writes) Forty four one.*
140. Int.: *One class, right?*
141. Carey: *And the other thirty one.*
142. Int.: *Do you think it was hard?*
143. Both: *No.*
144. Carey: *Once we knew what B and D were.*