

# **HIGHER MOMENTS RISK AND THE CROSS-SECTION OF STOCK RETURNS**

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# 1 Introduction

It is well known that excess kurtosis and negative skewness are standard characteristics of the stock return distributions. So it seems natural to take them into the account in asset pricing. The presence of the excess kurtosis in an index means that the market gives more probability to extreme observations than in normal distribution. Meanwhile, the appearance of negative skewness has the effect of highlighting the left tail of the distribution. In that case, market gives higher probability to decreases than increases in asset pricing.

It is by now well accepted that market skewness and kurtosis are important indicators of market risk, so the goal of this thesis is twofold: i) to analyze whether market skewness and kurtosis risks affect to the cross-section of stock returns, ii) to examine whether individual skewness and kurtosis are priced in the market. We use daily closing prices from the Eurostoxx market for which, to the best of our knowledge, this analysis has not been done.

The estimation method for higher moments is based on Gram-Charlier series expansion of the normal density function for the error term, as in León et al. (2004), where GARCH-type models are used allowing for time-varying volatility, skewness and kurtosis. These authors find a significant presence of conditional skewness and kurtosis in daily returns of stock indices and exchange rates.

We use a different estimation method for individual stocks, because estimating the parameters of the Exponentially weighted moving average or GARCH models for each stock has a high computational cost. Thus, we decide to calculate higher moments series for each stock as in *RiskMetrics* imposing a parameter  $\lambda = 0.94$  for all the cases.

Nevertheless, there are many methods for calculating higher moments. For example, recent studies by Chang et al. (2011) and Bams et al. (2015) show that such moments could be calculated using out-of-the-money European call and put options prices. High-frequency returns of a single day can also be used to compute the moments in a particular day as in Amaya et al. (2015). We could use the traditional technique of rolling window of daily returns.

We perform two types of empirical exercises to analyze if market higher moments risks affect to stocks' risk premium. First, we sort all stocks in Eurostoxx from 2000 to 2016 in quintiles based on their exposure of their returns to each moment's innovation, as in Chang et al. (2011) and Bams et al. (2015). These authors find that in down-markets, when investors are more risk-averse, the market volatility risk is priced significantly negative, while the effect disappears in up-markets. As for higher moments risk premium, skewness and kurtosis, they conclude that the risk premium for these moments are significantly negative and positive, respectively, but only when the investors risk aversion is low.

Similar results can be seen in Chang et al. (2011), where they also show that stocks with high exposure to innovations in market skewness have low returns on average. Nonetheless, the results are weaker for volatility and kurtosis, where stocks with high exposure in market volatility and

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kurtosis exhibit somewhat lower and higher returns on average, respectively.

We can find in the literature different techniques for portfolio construction: either equal-weighted portfolios, which are portfolios where all the stocks have the same weight, or value-weighted portfolios that are constructed according to the value they have in relation to the total of the portfolio. We use equal-weighted portfolios, and we find some evidence that market volatility is priced in the cross section of stocks, where stocks with high exposure to innovations in market volatility risk exhibit low returns on average. The results for market skewness and kurtosis are not so clear, but they show that the stocks with high exposure to these moments exhibit higher returns on average.

Therefore, factor portfolios for market volatility, skewness and kurtosis risk are constructed. For that, firstly, all stocks of Eurostoxx are sorted in terciles based on their exposure of their returns to innovations in either market moment (index return, volatility, skewness and kurtosis) and then these 12 groups are combined. We obtain that the average return on the market skewness and kurtosis risk factor portfolio are  $-0.10\%$  and  $-0.08\%$  per month, respectively, or  $-1.20\%$  and  $-0.98\%$  per year.

As a second approach, the prices of the market moment risks are also estimated, using *Fama and MacBeth* regressions as in Fama and MacBeth (1973). This two-step estimation approach is very popular. Amaya et al. (2015) use it to determine the significance on the cross-section of the stock returns of each market high moment individually and jointly.

Chang et al. (2011) use this methodology to estimate the price of market high moments risks. As in that work, we study whether market higher moments risks affect to the cross-section of stock returns and we use *Fama and MacBeth* methodology for calculating risk premiums. We find that the estimates of the premium for market volatility and kurtosis risk are negative and for market skewness risk is positive.

The remainder of the work is organized as follows. In section 2 the data is discussed. In section 3 the models used for computing the market moments' risk premia are discussed. Section 4 presents the methods used to extract higher moments from Eurostoxx index returns, as well as the extraction of the innovations. In section 5, there are presented the results for the stocks that are sorted into quintiles based on their exposure to innovations in market moments. Section 6 constructs factor portfolios and in section 7 estimates the price of market moments risk using Fama and MacBeth regressions. Section 8 presents the results for portfolios sorted on realized moments. Section 9 concludes.

## 2 Data

We use individual stocks from Eurostoxx which is a stock index of the Eurozone and it is compounded by 293 important firms. The data set includes daily closing prices from January 3, 2000 to April 7, 2016. If these period is considered there are just 203 companies because some of them are newer and enter in the index later. We also need the Eurostoxx index prices and they are obtained from *DataStream*, as well as the stocks data. The factor mimicking portfolio returns for size, book-to-market and momentum factors are obtained from the online data library of Ken French which can be found at [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

On the other hand, we also need the return of the risk-free asset, which is considered German 10-year bond and it is also obtained from *Datastream*.

The study is going to focus on stock returns rather than with prices, so given a series of asset prices  $P_0, P_1, \dots, P_T$  continuously compounded returns for period  $t$  are defined as  $R_t = \ln(P_t/P_{t-1})$ ,  $t = 1, 2, \dots, T$ .

## 3 The analytical framework

In this study, there are certain risk factors, which are market higher moments, namely volatility, skewness and kurtosis and the goal is to study empirically their effect to the cross section of stock returns. We use two strategies to investigate this and we use a multifactor representation of equilibrium returns, with the moments of the market return as state variables.

The first strategy is based on univariate sorting. The factors are moments of market return. So if we use a sample of returns and moments for a period  $t = 1, 2, \dots, T$ , the model is defined as:

$$R_{it} - R_{f,t} = \beta_{i0} + \beta_{i,MKT}(R_{m,t} - R_{f,t}) + \beta_{i,\Delta Vol} \Delta Vol_t + \beta_{i,\Delta Skew} \Delta Skew_t + \beta_{i,\Delta Kurt} \Delta Kurt_t + \epsilon_{it} \quad (3.1)$$

where  $R_{it}$  is the  $i$ th risky asset return,  $R_{f,t}$  is the return of the risk-free asset and  $R_{m,t}$  the market portfolio return. Furthermore,  $\Delta Vol_t = Vol_t - E_{t-1}[Vol_t]$ ,  $\Delta Skew_t = Skew_t - E_{t-1}[Skew_t]$  and  $\Delta Kurt_t = Kurt_t - E_{t-1}[Kurt_t]$ . The coefficients  $\beta_{i,MKT}$ ,  $\beta_{i,\Delta Vol}$ ,  $\beta_{i,\Delta Skew}$  and  $\beta_{i,\Delta Kurt}$  are the sensibilities or the measures of the  $i$ th risky asset's exposures to market excess return, market volatility, market skewness and market kurtosis risks, respectively. We also assume that  $\epsilon_{it}$  is homokedastic and independent of  $R_{m,t} - R_{f,t}$ ,  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$ .

There are different methods for the estimation of this model. A known alternative is rolling-windows, for which a time window has to be selected. With the data we have in the first time window, the coefficients with *Ordinary Least Squares (OLS)* method are estimated and then we

move the window forwards to obtain the next estimation. There are two ways to use rolling-windows, overlapping or non-overlapping them. When we use overlapping windows we could arise the risk of introducing too much autocorrelation, because the same data time and time again is used. The second option could be useful in order to avoid this problem, but it requires a larger sample.

To obtain  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$ , we need an appropriate structure for the time series of market moments that previously have been estimated and we can specific them observing the autocorrelogramas.

During the empirical study we build different portfolios. On the one hand, portfolios sorted on the stock's exposure to innovations in one of the market moments will be used. However, it is important to analyze the effects of each factor in pricing, so it would be convenient to study the implication of each market moment separately. So, we use another different criteria to construct different portfolios depending on the market moment, but in both cases would be required the estimates of the regression coefficients,  $\beta_{i,MKT}$ ,  $\beta_{i,\Delta Vol}$ ,  $\beta_{i,\Delta Skew}$  and  $\beta_{i,\Delta Kurt}$  so 3.1 model will be used.

The second set of results is based on multivariate sorts and cross-sectional regressions. In this part, we use the coefficients estimated in the previous equation to estimate the prices of the market moment risks,  $\lambda$ , from the cross-sectional relation:

$$R_{jt} - R_{f,t} = \lambda_{0t} + \lambda_{\Delta MKT,t} \beta_{j,MKT} + \lambda_{\Delta Vol,t} \beta_{j,\Delta Vol} + \lambda_{\Delta Skew,t} \beta_{j,\Delta Skew} + \lambda_{\Delta Kurt,t} \beta_{j,\Delta Kurt} + u_{jt} \quad (3.2)$$

The objective of this multifactor model is to analyze if the risk factors we are considering in each model are able to explain the changes in the cross-section of the stocks returns. For it, the parameters are obtained running *Fama and MacBeth* regressions, which is a well known methodology of estimating by *OLS* in two steps.

Economic theory provides little guidance on the signs of the prices of the market volatility, skewness and kurtosis risks obtained by this methodology. The volatility matters in the cross section of stock returns because this allows investors to hedge against changes in future opportunities. For instance, if high market volatility today is related with unfavourable investment tomorrow, then the stock that is related positively to the innovation in market volatility provides a hedge against a worsening in the investment opportunity set. In addition, previous studies have been found that when the volatility is high the returns are low. So, it is expected the sign of the price of market volatility risk to be negative.

However, for the case of the skewness and kurtosis theory does not provide much guidance, due to the fact that determining the expected sign would be a large empirical exercise. There are works that said the price of skewness risk is negative, while others conclude the opposite. Therefore, in this study the expected sign of the volatility risk premium is negative while for the case of skewness



and kurtosis it is not clear.

Recent studies by Amaya, Christoffersen, Jacobs and Vasquez (2015) and Chang, Christoffersen and Jacobs (2011) suggest the following. In the first one the authors determine that the relation between stock returns and market volatility is positive but statistically not significant. Regarding to skewness and kurtosis are statistically significant individually and jointly, and the signs are negative and positive, respectively. Whereas in the second work the results differ slightly. The authors show that market skewness risk indicate a robustly negative risk premium. However, the results on market volatility and kurtosis risk are sensitive to variations in empirical setup and across sample periods.

## 4 Market moments innovations

In this section, first of all we define the dynamic structure of the market returns. Then, we calculate the time series of volatility, skewness and kurtosis and finally, these series are modelled to define moment innovations.

Firstly, we define the structure of the Eurostoxx return series. It is daily data, so if there is autocorrelation it is supposed to be of first order. For that, autocorrelation function is studied. If does not exist any lags, returns are by themselves pure innovations. However, if a lag exists, we estimate AR(1) model and we analyze the residuals to know if they are autocorrelated. If not, it is concluded that returns fit AR(1) model. In this case, the results show that Eurostoxx has not any structure, concluding that it is by itself pure innovation.

### 4.1 Estimation of higher moments

There are many methods to estimate higher moments of the market returns. The traditional one and the most used in the literature is a rolling-window of daily returns using historical returns. However, for the implementation of this method the time window has to be chosen previously. Depending on the amplitude of the window you have chosen, the estimation is more or less precise, since it is well known that shorter windows capture better returns properties.

An interesting alternative to estimate market higher moments could be to use high-frequency returns of a single day to compute the moments in a particular day as we can see in Amaya et al. (2015). This way, daily realized moments are aggregated to obtain weekly realized volatility, skewness and kurtosis measures. The authors obtain results that are similar to what is often found in the literature, the returns skewness distribution is very fat-tailed and strongly peaked around zero and has a kurtosis above 3, suggesting fat-tailed returns. However, this would require having many data that is not easily accessible.

In our thesis, we use GARCH-type models for the estimation of high moments as we can see in León et al.(2004) work. These models allow for time-varying volatility, skewness and kurtosis jointly. GARCHSK and NAGARCHSK are the models.

First, we present the models and the estimation procedure.

#### 4.1.1 GARCHSK

This model resembles the model known as GARCH(1,1), but it also takes into account the conditional skewness and kurtosis. These moments are calculated as the conditional variance, by the GARCH(1,1) structure, but instead of using returns residuals, we need standardized residuals. The model is defined as

$$\begin{aligned} r_t &= E_{t-1}[r_t] + \epsilon_t \text{ where } \epsilon_t \sim (0, \sigma_\epsilon^2) \\ \epsilon_t &= h_t^{1/2} \eta_t; \quad \eta_t \sim (0, 1); \quad \epsilon_t | I_{t-1} \sim (0, h_t) \\ h_t &= \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 h_{t-1} \\ s_t &= \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 s_{t-1} \\ k_t &= \delta_0 + \delta_1 \eta_{t-1}^4 + \delta_2 k_{t-1} \end{aligned}$$

where  $E_{t-1}[\eta_t] = 0$ ,  $E_{t-1}[\eta_t^2] = 1$ ,  $E_{t-1}[\eta_t^3] = s_t$  and  $E_{t-1}[\eta_t^4] = k_t$ .  $\eta_t$  are standardized residuals and  $E_{t-1}[\bullet]$  denotes the conditional expectation on an information set up to period  $t - 1$ .

#### 4.1.2 NAGARCHSK

This model is very similar to the previous one, but presents an added difficulty at the time making an estimation, as there is one additional parameter and when estimating eight or nine parameters the difference is significative. It is defined as

$$\begin{aligned} r_t &= E_{t-1}[r_t] + \epsilon_t \text{ where } \epsilon_t \sim (0, \sigma_\epsilon^2) \\ \epsilon_t &= h_t^{1/2} \eta_t; \quad \eta_t \sim (0, 1); \quad \epsilon_t | I_{t-1} \sim (0, h_t) \\ h_t &= \beta_0 + \beta_1 (\epsilon_{t-1} + \beta_3 h_{t-1}^{1/2})^2 + \beta_2 h_{t-1} \\ s_t &= \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 s_{t-1} \\ k_t &= \delta_0 + \delta_1 \eta_{t-1}^4 + \delta_2 k_{t-1} \end{aligned}$$

where  $E_{t-1}[\eta_t] = 0$ ,  $E_{t-1}[\eta_t^2] = 1$ ,  $E_{t-1}[\eta_t^3] = s_t$  and  $E_{t-1}[\eta_t^4] = k_t$ .  $\eta_t$  are standardized residuals and  $E_{t-1}[\bullet]$  denotes the conditional expectation on an information set up to period  $t - 1$ .

Next, we study how these two models are estimated. First of all, we need an appropriate structure for the market returns, so we observe the autocorrelogramas. Once we have the structure

of the market returns, we have to estimate the parameters, and it is assumed that the distribution is based on a *Gram-Charlier* series expansion since we are considering skewness and kurtosis.

Using a *Gram-Charlier* series expansion of the normal density function and truncating at the fourth moment, we obtain the following density function for the standardized residuals,  $\eta_t$ , conditional on the information available in  $t - 1$ :

$$g(\eta_t | I_{t-1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\eta_t^2}{2}} \left[ 1 + \frac{s_t}{3!} (\eta_t^3 - 3\eta_t) + \frac{k_t - 3}{4!} (\eta_t^4 - 6\eta_t^2 + 3) \right] = \phi(\eta_t) \psi(\eta_t)$$

where  $\phi(\bullet)$  denotes the density function of the standard normal distribution.

However, we have to realize that this is not really a density function due to the integral in all  $\mathbb{R}$  is not equal to 1. So in León et al. (2004) [7] they suggest a new density function and they obtain  $f(\bullet)$ , by transforming  $g(\bullet)$ . See in Appendix A the proof that this function is really a density function that integrates to one.

$$f(\eta_t | I_{t-1}) = \frac{\phi(\eta_t) \psi^2(\eta_t)}{\Gamma_t} \quad (4.1)$$

where

$$\Gamma_t = 1 + \frac{s_t^2}{3!} + \frac{(k_t - 3)^2}{4!}$$

Thus, this way we have a density function for standardized residuals,  $\eta_t$ , that will be used in GARCHSK and NAGARCHSK models. Next, we need a likelihood function for the residuals,  $\epsilon_t$ . It is known that  $\epsilon_t = h_t^{-1/2} \eta_t$ , so the density function is:

$$g(\epsilon_t | I_{t-1}) = h_t^{-1/2} f(\eta_t | I_{t-1}) = h_t^{-1/2} \frac{1}{\Gamma_t} \phi(\eta_t) \psi^2(\eta_t)$$

Consequently, the likelihood function is given by:

$$L = \prod_{i=1}^T g(\epsilon_t | I_{t-1}) = h_t^{-T/2} (\Gamma_t)^{-1} \phi(\eta_t)^T (\psi^2(\eta_t))^T$$

Taking logarithms:

$$l = \ln L = -\frac{T}{2} \ln(h_t) - T \ln(\Gamma_t) + T \ln(\phi(\eta_t)) + T \ln(\psi^2(\eta_t))$$

After omitting unessential constants, the logarithm of the likelihood function for the conditional distribution  $\epsilon_t$  is:

$$l = -\frac{\ln(h_t)}{2} - \ln(\Gamma_t) - \frac{\eta_t^2}{2} + \ln(\psi^2(\eta_t))$$

In fact, this likelihood function is similar to the standard normal case plus two terms where time-varying skewness and kurtosis are considered. Moreover, note that the density function based on *Gram-Charlier* series expansion (4.1) is like the normal density function when  $s_t = 0$  and  $k_t = 3$ .

After using this methodology for the calculation of market higher moments it is expected to obtain results similar to the literature, heavy-tailed distributions with negative skewness and kurtosis greater than 3.

To estimate GARCHSK model we impose *variance targeting*, since thus the number of parameters to be estimated is three less, one in the variance equation, another in the skewness and the last from the kurtosis equation. The method for the variance equation consists in fixing a long term volatility level (in this case the sample variance) and using its analytical expression, fixing the constant of the model depending on the variance and the other two parameters, that is,

$$\beta_0 = \sigma^2(1 - \beta_1 - \beta_2) \quad (4.2)$$

As  $\sigma^2$  it is taken the variance of the whole sample, however, for the initial variance,  $h_0$ , we use the first subsample of 180 observations, and begins to extrapolate from the following observation on 181. The changes in the other equations are slightly different, but the idea is the same. For the first case is,

$$\gamma_0 = s(1 - \gamma_1\sigma^3 - \gamma_2) \quad (4.3)$$

where  $s$  and  $\sigma$  are the long term skewness and standard deviation, respectively. For the kurtosis the change is,

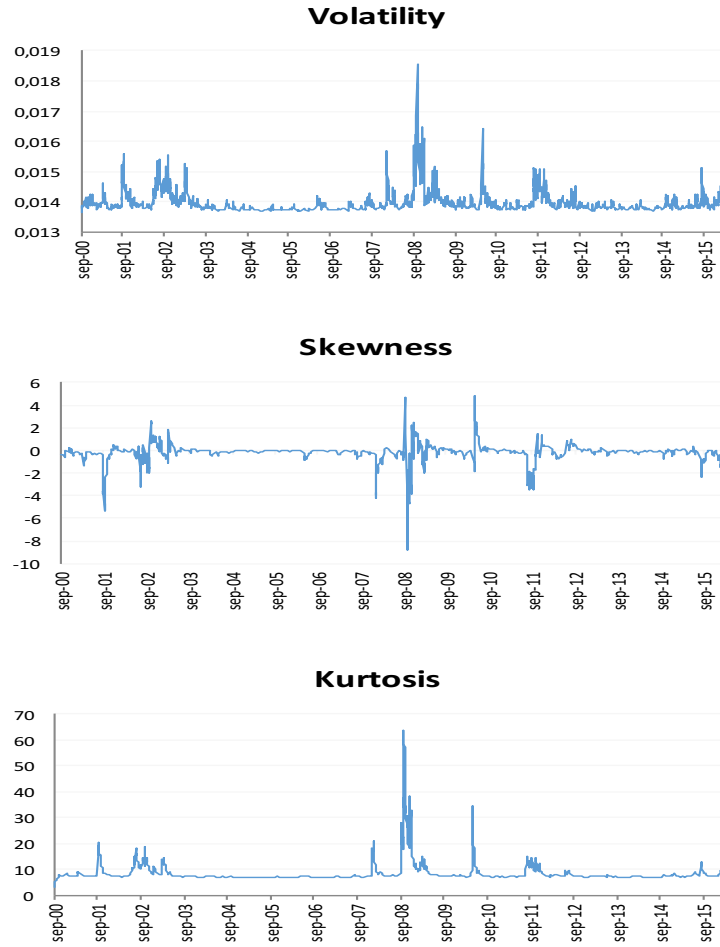
$$\delta_0 = k(1 - \delta_1\sigma^4 - \delta_2) \quad (4.4)$$

where  $k$  and  $\sigma$  are long term kurtosis and standard deviation. To initiate these time series has been used the same procedure as in the variance equation. The first value is calculated using 180 observations and then begins to extrapolate from the following observation on 181. The results for this model are shown in figure 1.

All three time series vary significantly through time, and specially at the beginning, in 2001 and around 2007. The period between 1997 and 2000 is known as the era of *dot-com bubble or Internet bubble*. During this years, there was a strong growth of economic values of firms related to Internet, causing a strong economic bubble that took to bankruptcy to a lot of companies. In 2001 the bubble went deflating rapidly, most of the firms stopped their activities when as there were no profits and there was no source of investment available. This could be the reason for the increases in volatility, skewness and kurtosis that can be seen in figure 1.

The changes observed around 2007 could be caused by the global financial crisis that started in that year. It started by the collapse of the housing bubble in the United States in 2006, and still continues in many countries.

To continue, we estimate the second model, NAGARCHSK. In this model, we also impose the condition of *variance targeting*, since otherwise the number of parameters to estimate would increase considerably. For that, we use the equations (4.3) and (4.4) for skewness and kurtosis,



**Figure 1:** Daily moments of Euro Stoxx index returns obtained with GARCHSK model.

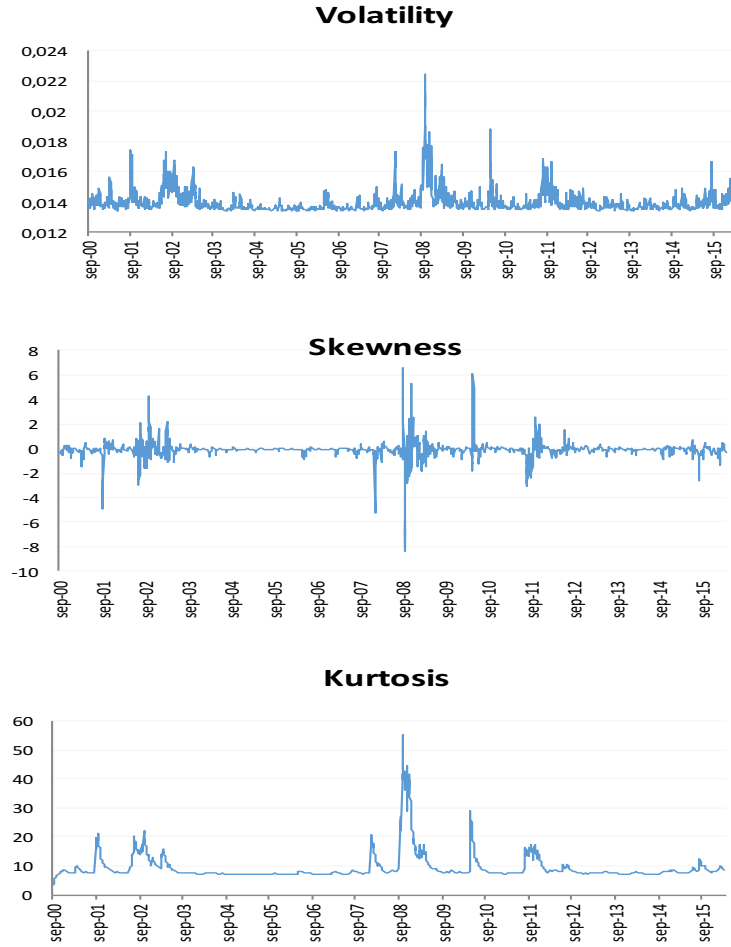
but the variance equation is:

$$\beta_0 = \sigma^2(1 - \beta_1(1 + \beta_3^2) - \beta_2)$$

The results for this model are shown in figure 2. All three time series are very similar to figure 1, where they are same-period variations. In addition, the skewness is always around zero and the kurtosis is larger than three.

## 4.2 Innovations in market moments

To obtain estimates of innovations in market moments, we fit an appropriate autoregressive moving average (ARMA) model to the time series for each moment calculated with two models. In the case of GARCHSK, we calculate the autocorrelation functions of different models. Firstly, we calculate the autocorrelation function of the original time series and then the autocorrelation function for first differences and ARMA(1,1) residuals for different market moments. The results are shown in appendix B, where we can observe that for volatility and skewness ARMA(1,1) models are needed



**Figure 2:** Daily moments of Euro Stoxx index returns obtained with NAGARCHSK model.

to remove the autocorrelation. However, for the kurtosis, taking the first difference remove most of the autocorrelation in the data, but ARMA(1,1) is more suitable.

For NAGARCHSK model the results are very similar so they are not reported. The resulting measures of innovations in market moments are obtained using the following models:

- GARCHSK model:

$$\Delta Vol_t = Vol_t - 0.002 - 0.887Vol_{t-1} + 0.147\Delta Vol_{t-1} \quad (4.5)$$

$$\Delta Skew_t = Skew_t + 0.013 - 0.908Skew_{t-1} + 0.050\Delta Skew_{t-1} \quad (4.6)$$

$$\Delta Kurt_t = Kurt_t - 0.312 - 0.963Kurt_{t-1} + 0.026\Delta Kurt_{t-1} \quad (4.7)$$

- NAGARCHSK model:

$$\Delta Vol_t = Vol_t - 0.003 - 0.770Vol_{t-1} + 0.140\Delta Vol_{t-1} \quad (4.8)$$

$$\Delta Skew_t = Skew_t + 0.020 - 0.802Skew_{t-1} + 0.019\Delta Skew_{t-1} \quad (4.9)$$

$$\Delta Kurt_t = Kurt_t - 0.242 - 0.972Kurt_{t-1} + 0.067\Delta Kurt_{t-1} \quad (4.10)$$

The innovations of volatility we obtain from these equations are very small in magnitude comparing with the innovations of skewness and kurtosis. However, if we analyse the ratio between the standard deviation of the innovations and the standard deviation of these series (volatility, skewness and kurtosis), we see that in all three cases the proportion of total variation of outcomes explained by the model is bigger than 60%. These small volatility innovations take us to estimate  $\beta_{\Delta Vol}$  much greater than  $\beta_{\Delta Skew}$  and  $\beta_{\Delta Kurt}$ , as we see later.

Once the ARMA(1,1) residuals for *Vol*, *Skew* and *Kurt* are estimated, in table 1 we present the correlation between them, and also the correlations with  $R_m - R_f$ , *SMB*, *HML* and *MOM* factors. The data for these factors are monthly, so the correlations are calculated with monthly innovations.

In order to obtain monthly innovations, firstly, we consider the first and the last closing data in each month and we calculate monthly market returns. Once we have them, we re-estimate GARCHSK model to obtain time series for volatility, skewness and kurtosis. Then we detect their structure, which coincide with daily data, ARMA(1,1), and we use the residuals of those.

**Table 1:** Correlations between monthly innovations for market moments and  $R_m - R_f$ , *SMB*, *HML* and *MOM* factors.

Risk factors	Correlation		
	$\Delta Vol$	$\Delta Skew$	$\Delta Kurt$
$\Delta Vol$	1	-0.90	0.89
$\Delta Skew$		1	-0.98
$\Delta Kurt$			1
$R_m - R_f$	-0.25	0.27	-0.25
<i>SMB</i>	-0.07	0.10	-0.13
<i>HML</i>	-0.08	0.08	-0.03
<i>MOM</i>	0.08	-0.11	0.07

In table 1 we can see that the correlation between all factors are very high, so to separate the effect from one factor on the other, we decide to orthogonalized them. In this way any multicollinearity problem is avoided.  $\Delta Vol$  will leave as it is and  $\Delta Skew$  and  $\Delta Kurt$  will be orthogonalized. So, first we orthogonalize  $\Delta Skew$  regressing it on  $\Delta Vol$  and after we regress  $\Delta Kurt$  on  $\Delta Vol$

and  $\Delta Skew$ . From now, throughout the work we use residuals from these regressions as  $\Delta Skew$  and  $\Delta Kurt$ .

## 5 Portfolios sorted on market moments

In this section we show three similar empirical exercises. In each case, the stocks are sorted into quintiles based on their exposure to innovations in market volatility, market skewness and kurtosis. In this way, we test if stocks with different exposures to innovations in market volatility, skewness or kurtosis exhibit different returns on average.

Portfolio construction can be done by different opinions. It is frequent to construct according to market beta, nevertheless, in this study we are going to analyze other relevant factors that are the higher moments of the market. So we are going to sort portfolios in different ways depending on all these factors. In each case, we sort the cross-section of stock returns into quintiles based on the stock's exposure to innovations in one of the market moments.

Finally, we briefly report some robustness exercises.

### 5.1 Portfolios sorted on market volatility exposure

First, we construct portfolios based on stocks exposure to  $\Delta Vol$  and then we compare the average returns and alphas of these portfolios. To construct the portfolios we use the following equation,

$$R_{it} - R_{f,t} = \beta_{i0} + \beta_{i,MKT}(R_{m,t} - R_{f,t}) + \beta_{i,\Delta Vol}\Delta Vol_t + \epsilon_{it} \quad (5.1)$$

On the other hand, it is also interesting to investigate how different the results are when we consider  $\Delta Skew$  and  $\Delta Kurt$ , so in the second specification the model is

$$R_{it} - R_{f,t} = \beta_{i0} + \beta_{i,MKT}(R_{m,t} - R_{f,t}) + \beta_{i,\Delta Vol}\Delta Vol_t + \beta_{i,\Delta Skew}\Delta Skew_t + \beta_{i,\Delta Kurt}\Delta Kurt_t + \epsilon_{it} \quad (5.2)$$

We calculate betas using a non-overlapping one-month window, and because the data is daily returns, we consider 21 days in a month.

Once we have the betas, the stocks are sorted into quintiles based on the coefficients of the regression,  $\beta_{\Delta Vol}$ . Thus, the first quintile contains stocks with the lowest beta, while in the fifth quintile we find the ones with the highest beta. This way to construct portfolios we can see in Chang et al. (2011) and Bams et al. (2015). In both works, the authors use one month time window and the windows are non overlapping.

In this work, there are 203 stocks in total and 5 portfolios, so we decide that quintile 1 and 5 have 40 stocks and 41 rest of them. On the other hand, in Chang et al. (2011), the authors form value-weighted portfolios using the capitalization of each stock as weighting in the portfolio, but in this work this information was not available, so the portfolios are equal-weighted.



After portfolio formation, we repeat the process for the next month moving the window forward one month. At the end of the procedure, we obtain time series of daily returns and  $\beta_{\Delta Vol}$  for each quintile portfolio.

Then, monthly returns have been calculated and for that we consider the first and the last closing data of each month. We obtain these information calculating the daily prices of portfolios, where the price of the portfolio is the sum of the prices of different stocks.

We also compute *Jensen's Alpha* using *Carhart* model, to assess if the effect of volatility persists after controlling for other well known factors as excess return, size, book-to-market and momentum. The *Carhart* four-factor model is an extension of *Fama and French* model, including Momentum factor. *Fama and French* is also an extension of the *CAPM*, but it includes more common factors which explain the asset's expected return better.

$$R_{it} - R_{f,t} = \alpha_{i0} + \beta_{i,MKT}(R_{m,t} - R_{f,t}) + \beta_{i,SMB}R_{SMB,t} + \beta_{i,HML}R_{HML,t} + \beta_{i,MOM}R_{MOM,t} + \epsilon_{it}$$

where *SMB* (*Small Minus Big*) is the portfolio that replicates the risk factor associated with the size understood as market capitalization, obtained as the difference between returns of small market capitalization companies versus returns of big market capitalization companies. *HML* (*High Minus Low*) is also a portfolio that replicates *Book-to-Market* (*BM*)<sup>1</sup> factor, and it can be obtained as the difference between companies with high ratio versus companies with low ratio. *MOM* (*Momentum*) is a factor that reflect the trend in differences between average return of portfolios of high returns and low returns. In the formation of this variable, the price a year ago has been considered as well as the fluctuation of the return of the previous month. So Momentum in a stock is described as the tendency for a stock's price to continue rising after an increase and vice versa.

The results are reported in table 2, in which the average of  $\beta_{\Delta Vol}$ , the monthly returns average and Carhart four-factor alpha can be seen. We also report the results of a portfolio that is long the highest quintile portfolio and short in the lowest quintile, and it is denoted as 5-1.

If the innovation in market volatility is a priced risk factor that should be taken into consideration, we expect to see that the average returns of the portfolios as well as the alphas have a decreasing pattern when the quintile is higher. This is expected because high volatility is generally related with a deterioration in the investment opportunity set. For this reason it is also expected to observe a negative average return in the 5-1 portfolio.

In table 2, we can see that the dispersion of *Carhart* four-factor model alphas between quintiles 5 and 1 is  $-2.04\%$  per month in the univariate case, while for the average return we show a smaller negative dispersion of  $-1.59\%$ . In both cases they are significant and although the pattern is not entirely monotonic across quintiles, generally it is decreasing. The results for the multivariate case

<sup>1</sup>It is a ratio used to find the value of a company by comparing the book value of a firm to its market value. It is calculated using firm's historical cost.

**Table 2:** Sorting on  $\beta_{\Delta Vol}$  for GARCHSK model. The table reports the average beta, average monthly returns for each portfolio in percent and monthly alphas in percent. Univariate is referred to (5.1) and the multivariate to (5.2). In parentheses the t-statistics are shown and the significant ones are bolded with 90% confidence.

	Quintile portfolio					
	1	2	3	4	5	5-1
Volatility beta (univariate)	-146.990 <b>(-16.86)</b>	-48.433 <b>(-17.03)</b>	-3.178 <b>(-3.78)</b>	41.954 <b>(16.25)</b>	137.871 <b>(17.19)</b>	
Average return	0.39 (0.60)	0.68 (1.43)	0.00 (0.00)	-0.17 (-0.43)	-1.20 <b>(-1.75)</b>	-1.59 <b>(-2.63)</b>
Carhart four-factor alpha	0.13 (0.19)	0.13 (0.25)	-0.54 (-1.20)	-0.63 (-1.46)	-1.91 <b>(-2.60)</b>	-2.04 <b>(-3.11)</b>
Volatility beta (multivariate)	-7145.4 <b>(-7.06)</b>	-2140.6 <b>(-6.87)</b>	173.9 <b>(2.28)</b>	2443.4 <b>(6.78)</b>	7388.9 <b>(7.34)</b>	
Average return	0.00 (-0.01)	0.47 (1.01)	-0.07 (-0.15)	-0.24 (-0.51)	-0.34 (-0.52)	-0.34 (-0.56)
Carhart four-factor alpha	-0.69 (-1.03)	0.23 (0.44)	-0.73 (-1.39)	-0.76 (-1.51)	-0.86 (-1.30)	-0.17 (-0.24)

are slightly different, since the dispersion of average return and the alphas are also negative, as was expected, but they are not significant. In addition, the pattern for the alphas is almost monotonic decreasing across quintiles as well as for the average returns.

In general, some evidence exists about  $\Delta Vol$  being a price risk factor with a negative price of risk, but the estimates are significant when  $\Delta Skew$  and  $\Delta Kurt$  are not added.

## 5.2 Portfolios sorted on market skewness exposure

In this subsection the strategy for portfolio construction is the same as for the volatility, except that stocks are sorted on  $\beta_{Skew}$ . So the regression (5.1) is replaced by the following,

$$R_{it} - R_{f,t} = \beta_{i0} + \beta_{i,MKT}(R_{m,t} - R_{f,t}) + \beta_{i,\Delta Skew}\Delta Skew_t + \epsilon_{it} \quad (5.3)$$

The results of these portfolios are reported in table 3. The multivariate case is referred to results obtained by equation (5.2).

The results from table 3 show that the dispersion of the alphas and the average return between quintiles 5 and 1 are negative for the multivariate case and positive for the univariate. In both cases they are not significant, but for the univariate case the alpha is significantly different from zero. Although the average return dispersion is not significant, we can observe that is not zero.

**Table 3:** Sorting on  $\beta_{\Delta Skew}$  for GARCHSK model. The table reports the average beta, average monthly returns for each portfolio in percent and monthly alphas in percent. Univariate is referred to (5.3) and the multivariate to (5.2). In parentheses the t-statistics are shown and the significant ones are bolded with 90% confidence.

	Quintile portfolio					
	1	2	3	4	5	5-1
Skewness beta (univariate)	-0.336 <b>(-10.86)</b>	-0.106 <b>(-10.18)</b>	0.003 (1.06)	0.113 <b>(10.27)</b>	0.354 <b>(10.14)</b>	
Average return	-1.03 (-1.57)	0.36 (0.78)	0.17 (0.38)	0.26 (0.61)	-0.13 (-0.20)	0.91 (1.48)
Carhart four-factor alpha	-1.74 <b>(-2.47)</b>	-0.17 (-0.35)	-0.35 (-0.70)	-0.07 (-0.16)	-0.53 (-0.80)	1.20 <b>(1.71)</b>
Skewness beta (multivariate)	-0.886 <b>(-8.40)</b>	-0.263 <b>(-8.13)</b>	0.025 <b>(3.10)</b>	0.308 <b>(8.33)</b>	0.939 <b>(8.76)</b>	
Average return	-0.46 (-0.71)	0.18 (0.38)	0.23 (0.59)	0.39 (0.80)	-0.80 (-1.24)	-0.34 (-0.54)
Carhart four-factor alpha	-1.20 <b>(-1.71)</b>	-0.36 (-0.67)	-0.14 (-0.33)	-0.09 (-0.17)	-1.23 <b>(-1.85)</b>	-0.03 (-0.04)

The pattern of the returns and alphas across quintiles are not monotonic, because it seem to be increasing but in the last quintile decrease a lot, specially for the multivariate case.

Overall, it is concluded that little evidence exists that  $\Delta Skew$  is a priced risk factor.

### 5.3 Portfolios sorted on market kurtosis exposure

The strategy for portfolio construction is the same as for the volatility, except that stocks are sorted on  $\beta_{Kurt}$ . So the regression is the following,

$$R_{it} - R_{f,t} = \beta_{i0} + \beta_{i,MKT}(R_{m,t} - R_{f,t}) + \beta_{i,\Delta Kurt}\Delta Kurt_t + \epsilon_{it} \quad (5.4)$$

The results of these portfolios are reported in table 4. The multivariate case is referred to results obtained by equation (5.2).

The results in table 4 show that portfolios sorted on  $\beta_{\Delta Kurt}$  exhibit almost a monotonically increasing pattern in average returns and *Carhart* four-factor model alphas for the univariate case. In addition, the spread in average returns and alphas between the fifth and first quintile are positive, 0.83% and 1.09%, respectively. While for the multivariate case they are negative,  $-0.16\%$  and  $-0.07\%$ , respectively.

**Table 4:** Sorting on  $\beta_{\Delta Kurt}$  for GARCHSK model. The table reports the average beta, average monthly returns for each portfolio in percent and monthly alphas in percent. Univariate is referred to (5.4) and the multivariate to (5.2). In parentheses are shown the t-statistics and the significant ones are bolded with 90% confidence.

	Quintile portfolio					
	1	2	3	4	5	5-1
Kurtosis beta (univariate)	-0.024 <b>(-20.78)</b>	-0.007 <b>(-19.49)</b>	0.001 <b>(3.74)</b>	0.008 <b>(20.33)</b>	0.026 <b>(20.20)</b>	
Average return	-0.94 (-1.54)	-0.15 (-0.33)	0.34 (0.86)	0.53 (1.23)	-0.11 (-0.15)	0.83 (1.34)
Carhart four-factor alpha	-1.65 <b>(-2.52)</b>	-0.57 <b>(-1.22)</b>	-0.14 <b>(-0.35)</b>	0.12 (0.26)	-0.56 <b>(-0.73)</b>	1.09 <b>(1.66)</b>
Kurtosis beta (multivariate)	-1.111 <b>(-7.28)</b>	-0.334 <b>(-7.11)</b>	0.026 <b>(2.27)</b>	0.379 <b>(6.97)</b>	1.150 <b>(7.56)</b>	
Average return	0.01 (0.02)	0.09 (0.17)	0.07 (0.17)	-0.26 (-0.61)	-0.15 (-0.24)	-0.16 (-0.30)
Carhart four-factor alpha	-0.61 <b>(-0.98)</b>	-0.36 <b>(-0.57)</b>	-0.43 <b>(-0.96)</b>	-0.73 <b>(-1.66)</b>	-0.68 <b>(-0.99)</b>	-0.07 <b>(-0.11)</b>

In addition, the alpha spread in the univariate case is the only significant one, since for the spread of average returns are not. However, in spite of the average return spread not being significant, we can see that it is not equal to zero in the univariate case.

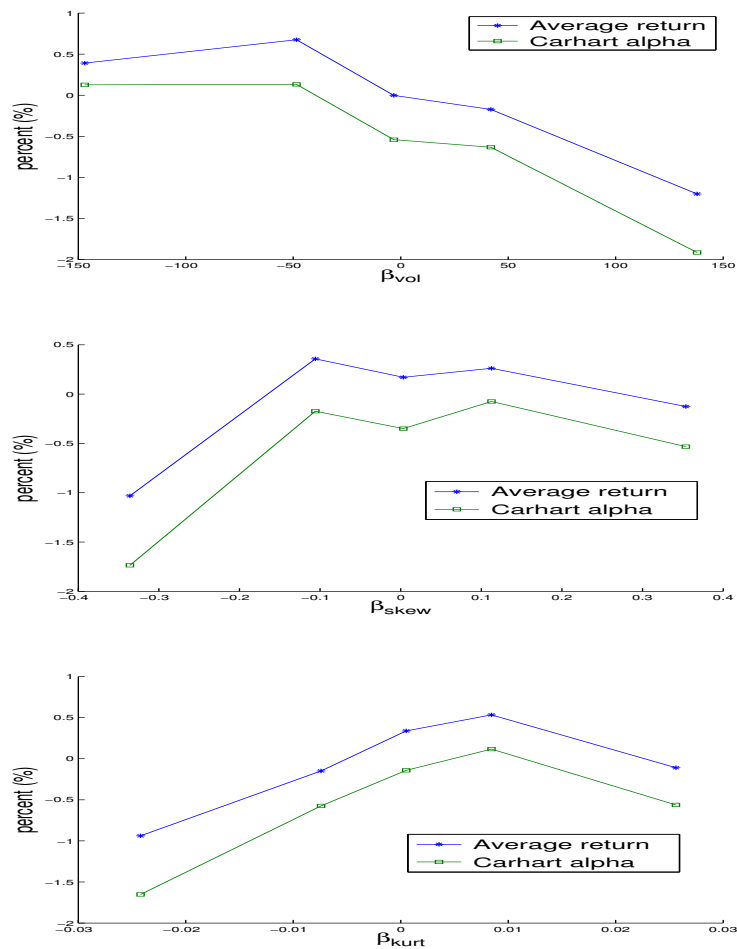
In general, little evidence exist to support that  $\Delta Kurt$  is a priced risk factor.

Comparing the results in tables 2, 3 and 4, the exposure to volatility risk seems quantitatively larger than the exposure to skewness and kurtosis risk.

The results from tables 2, 3 and 4 are summarized in figure 3. There, it is only the univariate case where the average monthly returns and the monthly alphas of each portfolio are on the vertical axis and the monthly betas in the horizontal axis. The top panel shows the results for the quintiles that are sorted on  $\Delta Vol$ , the middle panel portfolios sorted on  $\Delta Skew$  and the bottom panel shows the results for the portfolios sorted on  $\Delta Kurt$ .

The figure 3 exhibits that the results for volatility are stronger than for skewness and kurtosis, due to the fact that in the top panel the results are as expected and in the others the pattern is no monotonic.

In these empirical exercises, we decide to have 5 portfolios to continue with Chang et al. (2011) work, but we know that if we consider more portfolios the pattern of the average returns and alphas could be more representative or accurate.



**Figure 3:** Average return and alphas versus moment betas for each quintile.

We repeat the same exercises with 10 portfolios instead of 5, to see if the patterns of average returns and *Carhart* four-factor alphas are more representative. That is, to see if we can appreciate a clear tendency. We also include the univariate cases. The results are reported in appendix C. We report the results on a figure, to summarize better.

We observe that in the top panel, the tendency is decreasing as we can see with 5 portfolios. In addition, the spread of the average returns and alphas between portfolios 10 and 1 are significant (not reported), so the results for portfolios sorted on market volatility exposure are the same.

The results for the portfolios sorted on market skewness and kurtosis are similar to ones for 5 portfolios. That is, in the last portfolio the average return increases a lot respect to the previous, although the tendency of both patterns is increasing. Besides, the average return and alpha for the high-low portfolio are not significant in any cases. Thus, these results are consistent with those we have and indicate that if we consider more portfolios the results do not differ a lot.

## 5.4 NAGARCHSK model

The results shown until now are for the innovations estimated using the GARCHSK model, that are seen in figure 1. The procedure is repeated using the moments that are obtained by NAGARCH model. The results corresponding to tables 2, 3 and 4 for NAGARCHSK model are reported in D.

In general, we can see that the results are very similar to GARCHSK model, except that in portfolios sorted on  $\beta_{\Delta Skew}$ . For the univariate case, before the average returns and alphas spread were positive, while for NAGARCHSK innovations are negative, but they are not significant. So these results are consistent with the findings in tables 2, 3 and 4.

Therefore, seeing these results that are so similar to the previous ones, we decide to continue the study using the innovations estimated by GARCHSK model.

## 5.5 Results on subperiods

The period from 2007 is known as the era of global financial crisis. Its effects can be seen in figures 1 and 2, where the time series of volatility, skewness and kurtosis at that time are more variable. In order to analyze if the results are not affected by these circumstances, we repeat the procedure on two different subperiods: jan 2000-jul 2007 and aug 2007-apr 2016. The results are obtained using one-month beta estimation period and the multivariate case, equation 3.1, which includes innovations in all market moments. The results are reported in table 5.

For the period jan 2000-jul 2007 we can see that the results are similar to the ones obtained until now, where the estimates of dispersion of the average return for portfolios sorting on volatility beta are negative and significantly different from zero. When sorting on skewness beta and kurtosis beta the spread of average returns and alphas are not significant, but the negative sign in the dispersion of portfolios sorted on kurtosis are consistent with the multivariate case in table 4. In panel A the pattern of average returns and alphas are not monotonic in either case.

In Panel B we can see that the results obtained do not have any conclusive evidence, since there is nothing significant and also they are not as expected. In any case the patterns of average returns and alphas are monotonic. The positive sign in the spread of average return and alpha for portfolios sorted on skewness beta and kurtosis beta can also be seen in tables 4 and 3 in the univariate case.

These results could indicate that the effect of  $\Delta Vol$  was more considerable in jan 2000-jul 2007 period and the effect of this factor is not so strong for the second period, when the global financial crisis affected whole Europe.

**Table 5:** Sorting on different moments in subperiods. The equation used is 3.1. The table reports the average beta, average monthly returns for each portfolio in percent and monthly alphas in percent. In parentheses the t-statistics are shown and the significant ones are bolded with 90% confidence.

Sorting statistic	Quintile portfolio					5-1 portfolio	
	1	2	3	4	5	Estimate	t-statistic
Panel A: jan 2000-jul 2007							
Volatility beta	-2230.7	-647.8	35.6	721.7	2174.4		
Average return	0.53	0.50	0.33	1.02	-0.73	-1.25	<b>(-2.12)</b>
Carhart four-factor alpha	0.93	0.62	0.23	1.52	0.28	-0.65	<b>(-0.94)</b>
Skewness beta	-0.972	-0.319	-0.015	0.291	1.008		
Average return	0.25	0.55	0.48	-0.04	0.34	0.09	(0.17)
Carhart four-factor alpha	1.26	1.08	0.76	-0.04	0.65	-0.60	<b>(-0.92)</b>
Kurtosis beta	-0.782	-0.228	0.014	0.255	0.773		
Average return	0.46	0.37	0.19	0.98	-0.19	-0.65	<b>(-1.10)</b>
Carhart four-factor alpha	1.05	0.68	0.22	1.52	0.35	-0.70	<b>(-0.81)</b>
Panel B: agu 2007-apr 2016							
Volatility beta	-2568.8	-762.5	42.3	807.0	2438.7		
Average return	-0.74	-0.36	0.14	0.20	-0.64	0.10	(0.09)
Carhart four-factor alpha	-0.67	-0.88	-0.39	0.06	-0.84	-0.16	<b>(-0.15)</b>
Skewness beta	-0.627	-0.174	0.016	0.207	0.611		
Average return	-1.28	-0.08	0.53	0.37	-1.06	0.22	(0.18)
Carhart four-factor alpha	-1.44	-0.55	0.22	-0.04	-1.00	0.44	<b>(0.34)</b>
Kurtosis beta	-0.644	-0.192	0.010	0.201	0.613		
Average return	-1.07	0.03	0.26	0.13	-0.47	0.60	(0.54)
Carhart four-factor alpha	-1.05	-0.35	-0.12	-0.02	-0.79	0.26	<b>(0.23)</b>

## 5.6 Using rolling window

We repeat our procedure using market higher moments,  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$ , estimated from rolling window instead of GARCH-type models. We do this exercise using different amplitudes overlapping rolling windows. It is difficult to choose an appropriate amplitude, because with a short window we can represent better now situation, but the series are also more volatile because of the difficulty of estimating it with precision. However, a larger window provides smoother estimations. We use one moth and three month windows. The estimation of these series are reported in appendix E. In that series we see that the results for three-month window are more smoothers than for one-month.

Once we have estimated the series, we analyse the structure of them with the autocorrela-

tion functions, where we see that for the estimates of one-month windows all the moments fit to ARMA(1,1) model. But for the estimates of three-month windows, for volatility ARMA(1,1) model is needed to remove the autocorrelation, while for the skewness and kurtosis, taking the AR(1) model is enough. We repeat the same procedure of tables 2, 3 and 4 to construct portfolios. The results are reported in appendix E.

In general, we have not reached any robust conclusions. The signs of average returns and alphas for the high-low portfolios are the expected, but almost in no case are significant. For portfolios sorted on  $\beta_{Vol}$ , the results for one or three rolling window do not change. In both cases the patterns across quintiles are not monotonic, but the tendency is increasing.

For the portfolios sorted on  $\beta_{Skew}$  and  $\beta_{Kurt}$ , there is any clear tendency for average returns and alphas across quintiles, and only the results for three-month window for the kurtosis are significant with negative signs. Thus, whereas some results are consistent with the findings in tables 2, 3 and 4, such findings are hardly meaningful given the lack of robustness.

## 6 Factor portfolios

Results from the previous section show the effect of  $\Delta Vol$  on the cross section of the stock returns maintained across different sample periods. However, the effect of  $\Delta Kurt$  when different periods are considered changed as well as for the  $\Delta Skew$  effect.

As we can see in Chang et al. (2011), the results obtained until now could be difficult to interpret because of the correlation between different market moments, so it is important to analyze factors' implications individually. In addition, it is important to analyze the effects of each factor in pricing, so it would be convenient to study the implication of each market moment separately.

To this end, we consider another sorting criterion, which depends on the market moment and that also requires the betas estimated in equation 3.1. At the end of each month, we calculate betas and construct tercile portfolios based on  $\beta_{MKT}$ ,  $\beta_{\Delta Vol}$ ,  $\beta_{\Delta Skew}$  and  $\beta_{\Delta Kurt}$  (the lowest in tercile 1 and the highest in tercile 3). In that way there are 3 portfolios for each risk factors, which are four, and using the intersection of these four sorting criteria, we construct 81 portfolios. Thus, in each of the 81 portfolios, there is a tercile of each of the four market moment factors. Therefore, with all possible combinations we have 81 portfolios.

As in the previous section, there are 203 stocks in total and 3 portfolios, so we decide that tercile 1 and 3 have 68 stocks and 67 the second tercile. On the other hand, in Chang et al. (2011) value-weighted portfolios are formed using the capitalization of each stock as weighing in the portfolio, but in this work this information was not available, therefore the portfolios are equal-weighted.

After portfolio formation, we employ the procedure used in section 5 again to obtain the time series of beta, daily average returns and *Carhart* four-factor model alphas for each of the 81



portfolios. We also calculate monthly returns and for that, we consider the first and the last closing data of each month. We obtain these information calculating the daily prices of portfolios, where the price of the portfolio is the sum of the prices of different stocks.

This could be a great amount of information to process, consequently, we make groups to summarize all details. We construct these groups according to high (H), medium (M) or low (L) exposure to each of the factors one at a time and averaging over the 27 portfolios in each group. That is, when we combine the 81 portfolios, each tercile is in 27 of the portfolios. So, to simplify the results, we construct equal-weighted portfolios with these 27 portfolios. In this way, H portfolios show the terciles with the stocks with high exposure to a factor, M the terciles with stocks with medium exposure to a factor and L the ones with low exposure. This method of grouping allows to differ portfolios in exposure to one factor but are neutral in other factors.

The results are reported in table 6. We also bring out outcomes of H-L portfolios. That indicates portfolios which are long 27 high exposure portfolios and short 27 low exposure portfolios with regard to a given factor.

In table 6 we can see that the average return of the  $\beta_{\Delta Vol}$  high-low portfolio is  $-0.13\%$  per month but it is not significantly different from zero, as well as for the monthly alpha. These results are similar to table 2 where the signs for the alphas and average returns dispersion were also negative. In addition, the patterns are decreasing, consistent with earlier results.

The estimate of average return and Carhart four-factor model alpha of the  $\beta_{\Delta Skew}$  and  $\beta_{\Delta Kurt}$  high-low portfolios are negatives and not significant in all cases. In addition, no monotonic pattern is detected in any case, as in earlier results, in all cases they first increase and then decrease.

In these two last cases, the spread in average return and alpha between the high and low portfolios are negative and not significant in either case, but it could not be deduced that they are equal to zero, because their value has changed across portfolios.

In summary, these results are quite similar to earlier results observed in tables 2, 3 and 4, where we see that market higher moments are considered in asset pricing.

Continuing with the work, we use the returns of factor portfolios constructed above as proxies for risk factors,  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$ . So we consider volatility factor portfolio, skewness factor portfolio and kurtosis factor portfolio. They correspond to the high-low portfolios in table 6 and are defined as follows

$$\begin{aligned}
 FVol &= \frac{1}{27}(R\beta_{\Delta Vol H} - R\beta_{\Delta Vol L}) \\
 FSkew &= \frac{1}{27}(R\beta_{\Delta Skew H} - R\beta_{\Delta Skew L}) \\
 FKurt &= \frac{1}{27}(R\beta_{\Delta Kurt H} - R\beta_{\Delta Kurt L})
 \end{aligned} \tag{6.1}$$

**Table 6:** Portfolios sorted on exposure to  $R_m - R_f$ ,  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$ . The equation used to calculate the betas is 3.1. We assign each stock into three different groups, high (H), medium (M) and low (L), based on their exposures ( $\beta$ ) to each of the four market moments. Then, the stocks that belong to the same groups based on all four factors are put together to form value-weighted portfolios. The table reports the average beta, average monthly returns for each portfolio in percent and monthly alphas in percent. In parentheses the Newey and West t-statistics with 12 lags for the difference in average returns and alphas between the high and low exposure groups are shown and the significant ones are bolded with 90% confidence.

Portfolio	Average return (%)	Carhart four-factor alpha (%)	$\beta_{MKT}$	$\beta_{\Delta Vol}$	$\beta_{\Delta Skew}$	$\beta_{\Delta Kurt}$
<i><math>\beta_{MKT}</math></i>						
L	-0.39	-0.98	0.725	37.246	0.062	0.005
M	-0.32	-0.95	0.845	147.166	0.026	0.023
H	-0.35	-0.99	0.985	251.301	-0.013	0.040
H-L	0.04	-0.01				
t-statistic	(0.35)	(-0.07)				
<i><math>\beta_{\Delta Vol}</math></i>						
L	-0.30	-0.89	0.858	-599.43	-0.048	-0.093
M	-0.33	-0.97	0.843	148.899	0.026	0.023
H	-0.43	-1.05	0.855	886.245	0.097	0.137
H-L	-0.13	-0.15				
t-statistic	(-1.39)	(-1.51)				
<i><math>\beta_{\Delta Skew}</math></i>						
L	-0.34	-0.99	0.925	-409.178	-0.069	-0.063
M	-0.28	-0.87	0.843	152.705	0.026	0.023
H	-0.44	-1.06	0.788	692.186	0.119	0.106
H-L	-0.10	-0.07				
t-statistic	(-0.88)	(-0.48)				
<i><math>\beta_{\Delta Kurt}</math></i>						
L	-0.35	-1.00	0.858	-599.019	-0.047	-0.093
M	-0.28	-0.86	0.843	148.823	0.026	0.023
H	-0.43	-1.06	0.856	885.909	0.096	0.138
H-L	-0.08	-0.06				
t-statistic	(-0.81)	(-0.53)				

where  $R\beta_{\Delta Vol H}$  and  $R\beta_{\Delta Vol L}$  denote the sum of the returns on the 27 portfolios with the highest and lowest exposure to  $\Delta Vol$ , respectively.  $R\beta_{\Delta Skew H}$ ,  $R\beta_{\Delta Skew L}$ ,  $R\beta_{\Delta Kurt H}$  and  $R\beta_{\Delta Kurt L}$  are defined in the same way.

The return of these portfolios, FVol, FSkew and FKurt, represent the compensation for assuming the risk of market volatility, skewness and kurtosis. On the other hand, it is interesting to analyze if results obtained until now are economically significant, so we compare the average returns of  $R_m - R_f$ , *SMB*, *HML* and *MOM* with them in the same period.

Table 7 reports the average returns and corresponding t-statistics of the factor portfolios from table 6 defined in equations 6.1 and the means for the *Fama and French* factors and Momentum

**Table 7:** There are reported the average monthly returns (in percent) of the factor portfolios,  $FVol$ ,  $FSkew$  and  $FKurt$ . These portfolios correspond to H-L portfolios that can be seen in table 6, which are long 27 high exposure portfolios and short 27 low exposure portfolios with respect to a given factor. There are also reported the average monthly returns (in percent) of  $R_m - R_f$ ,  $SMB$ ,  $HML$  and  $MOM$  portfolios and their correlations with  $FVol$ ,  $FSkew$  and  $FKurt$ . In parentheses the Newey and West t-statistics with 12 lags are shown and the significant ones are bolded with 90% confidence.

Factors	Average return	t-statistic	Correlation						
			$FVol$	$FSkew$	$FKurt$	$R_m - R_f$	$SMB$	$HML$	$MOM$
$FVol$	-0.13	(-1.39)	1.00	0.37	0.78	-0.05	-0.07	-0.06	0.14
$FSkew$	-0.10	(-0.88)		1.00	0.38	0.09	-0.03	0.05	-0.15
$FKurt$	-0.08	(-0.81)			1.00	0.01	-0.03	-0.06	0.00
$R_m - R_f$	0.39	(0.81)				1.00	-0.08	0.25	-0.48
$SMB$	0.19	(1.54)					1.00	-0.03	0.12
$HML$	0.47	(1.45)						1.00	-0.18
$MOM$	0.89	<b>(2.60)</b>							1.00

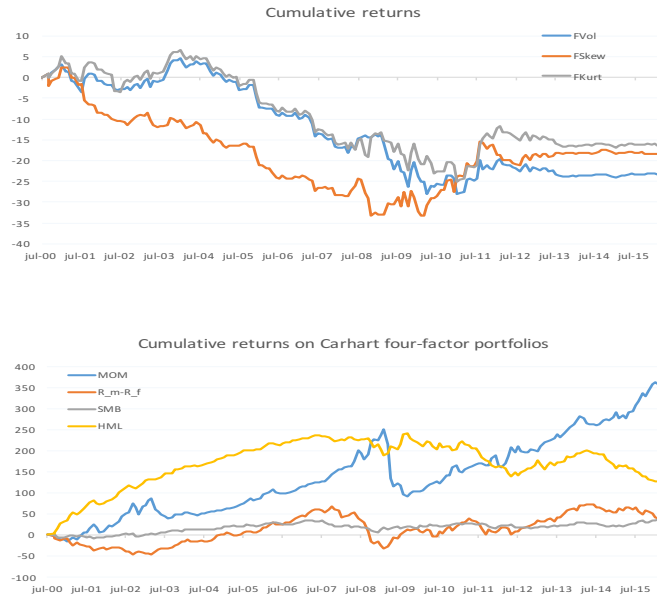
factor.

We find that the average return only on momentum factor portfolio is significantly different from zero. It is also the largest, 0.87%, and for volatility, skewness and kurtosis portfolios are negative, as it was known, but without being as large as the momentum factor portfolio. In addition, the correlations of the volatility, skewness and kurtosis portfolio with other risk factors are extremely low.

On the other hand, once terciles are built for each beta moment,  $\beta_{MKT}$ ,  $\beta_{\Delta Vol}$ ,  $\beta_{\Delta Skew}$  and  $\beta_{\Delta Kurt}$ , they are combined to have the 81 portfolios. As a result there are some stocks that are more than once in a portfolio, which should be considered only once and this implies that some of the 81 portfolios contain few stocks.

Figure 4 shows the cumulative returns of the seven factor portfolios. At the top there are the cumulative returns on three moment factor portfolios,  $FVol$ ,  $FSkew$  and  $FKurt$ , and in the bottom there are the cumulative returns on the  $R_m - R_f$ ,  $SMB$ ,  $HML$  and  $MOM$  factor portfolios. We observe that the returns for the *Carhart* four-factor portfolios are larger, and the momentum factor has the largest, but it is also more volatile.

Three moment factor portfolios have smaller cumulative returns and are negative for all cases, but they are more stable over time. Of these last three, all portfolios have similar cumulative return, specially  $FVol$  and  $FKurt$ , which was expected because of the high correlation between them.



**Figure 4:** Cumulative return on factor portfolios. In the top there are the cumulative returns on three moment factor portfolios and in the bottom there are the cumulative returns on the  $R_m - R_f$ ,  $SMB$ ,  $HML$  and  $MOM$  factor portfolios.

## 7 Exploring the risk premiums

In the previous section, we construct factor portfolios, which have been used as risk factors and their premiums are estimated calculating the average monthly returns. Thus, we see that the premium on market volatility risk is the largest with  $-0.13\%$  per month, since market skewness risk and market kurtosis risk are smaller in magnitude, at  $-0.10\%$  and  $-0.08\%$  per month, respectively.

However, in this section, we calculate the price of the risk using the cross-sectional regressions for different portfolios and different regression procedures. To this end, we use the 81 portfolios constructed in the previous section because they have the maximum spread in their exposures to  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$ . Due to this characteristic we will obtain good estimates for the prices of market volatility, market skewness and market kurtosis risk. After that, we corroborate the robustness of these results using other sets of testing portfolios: 25 Fama and French portfolios sorted on size and book-to-market ratio.

### 7.1 Fama and MacBeth regressions on the 81 factor portfolios

We apply the two-pass regressions of Fama and MacBeth to the 81 portfolios from section 6. It is a method of estimating by *Ordinary Least Squares (OLS)* in two steps. In the first one, the betas of the factors are estimated in time series and using previous data to the following stage.

For example, if we consider all factors, in this step the model to estimate is,

$$\begin{aligned}
R_{it} - R_{f,t} = & \beta_{i0} + \beta_{i,MKT}(R_{m,t} - R_{f,t}) + \beta_{i,\Delta Vol}\Delta Vol_t + \beta_{i,\Delta Skew}\Delta Skew_t + \beta_{i,\Delta Kurt}\Delta Kurt_t \\
& + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,MOM}MOM_t + \epsilon_{it}
\end{aligned}
\tag{7.1}$$

In the second step, a cross-sectional regression is done to explain assets return for every moment of the time. To do this the betas we have estimated in the previous step are used as regressors. The model for this second step is defined as,

$$\begin{aligned}
R_{it} - R_{f,t} = & \lambda_{0t} + \lambda_{MKT,t}\beta_{i,MKT} + \lambda_{\Delta Vol,t}\beta_{i,\Delta Vol} + \lambda_{\Delta Skew,t}\beta_{i,\Delta Skew} + \lambda_{\Delta Kurt,t}\beta_{i,\Delta Kurt} \\
& + \lambda_{SMB,t}\beta_{i,SMB} + \lambda_{HML,t}\beta_{i,HML} + \lambda_{MOM,t}\beta_{i,MOM} + u_{it}
\end{aligned}
\tag{7.2}$$

After the second stage, a monthly time series of the price of risk is estimated and the final estimate is obtained by averaging the series.

We run Fama and MacBeth regressions for different models. We consider  $R_m - R_f$ ,  $SMB$ ,  $HML$ ,  $MOM$ ,  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$  as factors, but they are combined in different ways. The factor models considered are (i) CAPM +  $\Delta Vol$ , (ii) CAPM +  $\Delta Skew$ , (iii) CAPM +  $\Delta Kurt$ , (iv) CAPM +  $\Delta Vol$  +  $\Delta Skew$ , (v) CAPM +  $\Delta Vol$  +  $\Delta Kurt$ , (vi) CAPM +  $\Delta Skew$  +  $\Delta Kurt$ , (vii) CAPM +  $\Delta Vol$  +  $\Delta Skew$  +  $\Delta Kurt$ , (viii) Carhart four-factor +  $\Delta Vol$ , (ix) Carhart four-factor +  $\Delta Skew$ , (x) Carhart four-factor +  $\Delta Kurt$ , (xi) Carhart four-factor +  $\Delta Vol$  +  $\Delta Skew$ , (xii) Carhart four-factor +  $\Delta Vol$  +  $\Delta Kurt$ , (xiii) Carhart four-factor +  $\Delta Skew$  +  $\Delta Kurt$  and (xiv) Carhart four-factor +  $\Delta Vol$  +  $\Delta Skew$  +  $\Delta Kurt$ .

We estimate these models differently, because for the  $SMB$ ,  $HML$  and  $MOM$  factors, the data is monthly, whereas the other factors are daily. Therefore, in models where both types of data are found, first we obtain monthly innovations of market moments.

For this, the GARCHSK model must be re-estimated with monthly market returns. Once we have obtained the time series of market volatility, skewness and kurtosis, their dynamic structure has to be detected. In this case, it coincides with daily data,  $ARMA(1, 1)$ , so we use the residuals of those.

After that, in the first stage of the method, we calculate the betas using an overlapping rolling window of 5 years. To finish with, in the second stage, we use those series of beta to estimate monthly prices of risk.

In the first 7 models, where the factors included are daily data, the estimation differs. In the first stage, we estimate the betas using non overlapping rolling windows of one month of daily returns. Thus, we obtain monthly betas and these series are used for monthly estimates of the price of risk. Then, those are averaged to get the final estimate. The results of Fama and MacBeth regressions are reported in table 8.

After the estimation, we see that the estimate of the price of market skewness risk,  $\lambda_{\Delta Skew}$ , is

**Table 8:** Price of risk factors. There are reported the estimated prices of risk for different multifactor models where  $R_m - R_f$ ,  $SMB$ ,  $HML$ ,  $MOM$ ,  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$  are considered as factors. For each model the prices of risk are calculated using the two-pass regressions of Fama and MacBeth to the returns of the 81 portfolios calculated in the previous section. The Newey and West t-statistics are also reported in parenthesis and the significant ones are bolded with 90% confidence.

	Factor							
	Constant	$R_m - R_f$	$\Delta Vol$	$\Delta Skew$	$\Delta Kurt$	$SMB$	$HML$	$MOM$
(i)	-0.001 (-0.11)	-0.003 (-0.37)	0.000 (0.21)					
(ii)	0.001 (0.15)	-0.003 (-0.40)		-0.058 (-0.24)				
(iii)	0.009 (1.42)	-0.011 <b>(-1.74)</b>			-0.745 (-0.88)			
(iv)	0.003 (0.50)	-0.005 (-0.84)	0.000 (-0.92)	-0.067 (-0.32)				
(v)	0.009 (1.37)	-0.010 <b>(-1.73)</b>	0.000 (-0.67)		-0.559 (-0.91)			
(vi)	0.005 (1.11)	-0.008 (-1.39)		-0.039 (-0.24)	0.561 (0.85)			
(vii)	0.006 (1.41)	-0.008 (-1.62)	0.000 (-1.18)	-0.098 (-0.60)	0.045 (0.10)			
(viii)	0.004 (0.41)	-0.003 (-0.30)	0.000 (0.49)			-0.080 (-0.17)	-0.380 (-0.83)	-0.898 (-1.42)
(ix)	0.005 (0.58)	-0.004 (-0.41)		-0.003 (-0.14)		-0.229 (-0.36)	-0.349 (-0.92)	-0.787 (-1.23)
(x)	0.012 (1.16)	-0.010 (-0.91)			-0.017 (-1.03)	-0.236 (-0.45)	-0.417 (-1.04)	-0.863 (-1.30)
(xi)	0.006 (0.53)	-0.004 (-0.32)	0.000 (0.29)	0.006 (0.33)		-0.261 (-0.51)	-0.305 (-0.64)	-0.829 (-1.41)
(xii)	0.003 (0.33)	-0.002 (-0.17)	0.000 (-0.07)		-0.006 (-0.53)	-0.120 (-0.23)	-0.355 (-0.81)	-0.906 (-1.41)
(xiii)	0.008 (0.82)	-0.007 (-0.66)		0.007 (0.32)	-0.013 (-0.78)	-0.300 (-0.72)	-0.281 (-0.62)	-0.755 (-1.34)
(xiv)	0.007 (0.70)	-0.006 (-0.46)	0.000 (0.47)	0.006 (0.30)	-0.013 (-0.91)	-0.458 (-1.00)	-0.313 (-0.65)	-0.851 (-1.44)

negative in some models, but it is not significant in any case. The price of market kurtosis risk,  $\lambda_{\Delta Kurt}$ , is not significant in none of the models, moreover, in most of the models is negative.

In the case of the estimate of the price of market volatility risk, it is not significant in any model. But we can see that in all cases the estimate is 0.000 in spite of the t-statistics are not equal to 0. This implies that the standard deviation of the price of risk are very reduced, so the estimation precision is very high.

For *SMB*, *HML* and *MOM* factors all price of risk have negative price but they are not significant.

## 7.2 Fama and MacBeth regressions on the other portfolios

In this subsection, we check the robustness of the previous section estimates running the same Fama and MacBeth regression on other portfolios, namely, 25 Fama and French portfolios sorted on size and book-to-market ratio and distinguished according to value weighted and equal weighted. This exercise is interesting for analysis, because as said before, some of the 81 portfolios used until now are composed of few stocks, what can lead to confusing estimations.

In this case, the estimated model is only the last one (xiv), where we consider all factors. The results for these regressions are reported in table 9. There, in Panel A, we report the results using 5 years' (60 months) window to calculate the betas, while in Panel B, the betas are calculated using a 90 months window.

For the 25 portfolios sorted on size and book-to-market, the price of risk estimated are not very significant. We find that the estimate of  $\lambda_{\Delta Vol}$  is significantly negative in four cases, consistent with the results in Chang et al. (2011). In other two cases, it is not significant, but as said before, although the estimate is small the t-statistic is high, so the estimation precision is very high.

The estimate of the price of skewness is significant when the window is 60 months. In addition, it is positive, different from most of the models in table 8, where the estimated price was negative, but they were not significant.

In relation to  $\lambda_{\Delta Kurt}$ , it is not significant in any case, but we see that the estimate sign is negative in all cases, consistent with the results in table 8.

The traditional pricing factors such as *SMB*, *HML* and *MOM* are not estimated significantly, even when we use 25 size and book-to-market portfolios, which should give an advantage to *SMB* and *HML* factors, as they are built considering these factors. So the estimates of the price of risk for the moment factors must be interpreted carefully.

## 7.3 Interpreting the sign of the price of market moments risk

In previous works, such as in Chang et al. (2011), the authors find that the price of market volatility risk is negative, and suggest that an increase in the market volatility indicate a deterioration on

**Table 9:** Price of risk factors. There are reported the estimated prices of risk for different multifactor models where  $R_m - R_f$ ,  $SMB$ ,  $HML$ ,  $MOM$ ,  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$  are considered as factors. The prices of risk are calculated using the two-pass regressions of Fama and MacBeth to the 25 Fama and French portfolios sorted on size and book-to-market and distinguished according to value weighted (VW) and equal weighted (EW). In Panel A the  $\lambda$ s are estimated by running a time series regression of 5 years (60 months) of monthly returns and in Panel B, the rolling window is 90 months. The Newey and West t-statistics are also reported in parenthesis and the significant ones are bolded with 90% confidence.

	Price of risk						
	$\lambda_{MKT}$	$\lambda_{\Delta Vol}$	$\lambda_{\Delta Skew}$	$\lambda_{\Delta Kurt}$	$\lambda_{SMB}$	$\lambda_{HML}$	$\lambda_{MOM}$
Panel A: 60 months beta							
81 portfolios	-0.006 (-0.46)	0.000 (0.47)	0.006 (0.30)	-0.013 (-0.91)	-0.458 (-1.00)	-0.313 (-0.65)	-0.851 (-1.44)
25 size and book-to-market portfolios (VW)	0.006 (0.79)	-0.001 <b>(-2.22)</b>	0.083 <b>(2.77)</b>	-0.025 (-1.00)	0.234 (1.48)	-0.360 (-1.08)	-1.25 <b>(-1.83)</b>
25 size and book-to-market portfolios (EW)	0.011 (0.85)	-0.002 <b>(-2.70)</b>	0.063 <b>(3.67)</b>	-0.013 (-0.53)	0.152 (0.91)	-0.453 (-1.16)	-0.963 (-1.33)
Panel B: 90 months beta							
81 portfolios	-0.004 (-0.37)	0.000 (-0.62)	-0.009 (-0.73)	-0.003 (-0.34)	0.167 (0.64)	-0.369 (-1.10)	-0.229 (-0.84)
25 size and book-to-market portfolios (VW)	0.006 (0.80)	-0.001 <b>(-1.94)</b>	0.031 (0.62)	-0.011 (-0.34)	0.121 (0.56)	-0.518 (-1.03)	-1.192 (-1.56)
25 size and book-to-market portfolios (EW)	0.019 (1.84)	-0.002 <b>(-2.26)</b>	0.035 (1.37)	-0.002 (-0.08)	0.072 (0.33)	-0.632 (-1.07)	-0.382 (-0.76)

the future investment opportunities. Because investors want to hedge against future risks, they prefer stocks with high return when the market volatility is higher than expected. So it is said that the price of market volatility risk is negative.

In addition, this negative relation we can see in table 1, where we observe that the correlation between the market excess and  $\Delta Vol$  is negative. In addition, during the thesis we observe that in some cases it seems to be also negative, but in other cases the estimations of the price of market volatility risk are very small, so the sign is not very well appreciated.

As for the skewness, we observe that the estimation of the price of the market skewness risk is negative and not significantly different from zero in most of the models. While for 25 Fama and French portfolios sorted on size and book-to-market the estimation of  $\lambda_{\Delta Skew}$  is positive and significant for 60 months window.

However, the first results are not consistent with the table 1, where the correlation between the



**Table 10:** Time series relationship between  $\Delta Vol$ ,  $\Delta Skew$ ,  $\Delta Kurt$  and  $R_m - R_f$ . The Newey and West t-statistics are also reported in parenthesis and the significant ones are bolded with 90% confidence.

Regressors	Dependent variable: $R_m - R_f$							
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
$\Delta Vol$		-3.764			-3.959	-3.74		-3.94
		(-0.65)			(-0.86)	(-0.62)		(-0.74)
$\Delta Skew$			0.030		0.030		0.030	0.030
			<b>(7.82)</b>		<b>(7.83)</b>		<b>(7.58)</b>	<b>(7.54)</b>
$\Delta Kurt$				-0.002		-0.002	-0.002	-0.002
				(-1.13)		(-1.09)	(-1.02)	(-0.96)
$R_m - R_f(t - 1)$	-0.008	-0.012	-0.020	-0.006	-0.025	-0.011	-0.019	-0.024
	(-0.41)	(-0.65)	(-1.20)	(-1.13)	(-1.47)	(-0.59)	(-1.21)	(-1.41)

excess market return and  $\Delta Skew$  is positive. The recent work by Chang et al. (2011) shows that the market skewness risk indicates a negative risk premium, while the results on market volatility and kurtosis risk are sensitive to variations.

Nevertheless, as said above, determining the sign of the price of market skewness and kurtosis risk is largely an empirical exercise due to the fact that we can do different interpretations. For example, in Bates (2000), Pan (2002) and Doran et al. (2007), the authors use option-implied skewness, but they conclude that a positive innovation in option-implied market skewness indicates a decreasing risk in the stocks, which would be related to an improved investment opportunity. So it would be expected a positive relation between the excess market return and  $\Delta Skew$ . On the other hand, the stocks with low exposure to innovations in market skewness indicate that they would have to hedge against downside risk of the stock market. Hence, a positive price of market skewness risk would be expected.

For all of this, we estimate some models where the dependent variable is the market excess return, and we check the relation between the market moments residuals and the market excess return. The results are reported in table 10, where it is also included the lagged market excess return, although its effect is not meaningful.

The results from table 10 indicate that the relation between  $\Delta Skew$  and excess market return is positive and significantly different from zero. These results are consistent with the previous ones for 25 Fama and French portfolios, but not with the results in table 8.

The relation between the market excess return and  $\Delta Vol$  and  $\Delta Kurt$  are not relevant in any of the models. But the signs are the expected ones, consistent with all previous results.

However, these results of  $\Delta Skew$  and  $\Delta Kurt$  must be interpreted carefully, since as we observe the estimations are very small comparing with the t-statistic, which imply that the standard

**Table 11:** Forecast regressions for market return. Monthly excess return of Eurostoxx index are used. The Newey and West t-statistics are also reported in parenthesis and the significant ones are bolded with 90% confidence.

Regressors	Dependent variable: $R_m - R_f(t)$							
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
$\Delta Vol(t-1)$		-3.202 <b>(-2.23)</b>			-4.203 <b>(-2.76)</b>	-3.257 <b>(-1.79)</b>		-5.148 <b>(-3.07)</b>
$\Delta Skew(t-1)$			0.089 <b>(3.05)</b>		0.108 <b>(3.29)</b>		0.086 <b>(3.23)</b>	0.121 <b>(3.85)</b>
$\Delta Kurt(t-1)$				-0.042 <b>(-0.58)</b>		0.004 <b>(0.06)</b>	-0.018 <b>(-0.39)</b>	0.066 <b>(1.58)</b>
$R_m - R_f(t-1)$	0.072 <b>(0.76)</b>	-0.020 <b>(-0.26)</b>	0.021 <b>(0.25)</b>	0.104 <b>(0.92)</b>	-0.011 <b>(-1.13)</b>	-0.025 <b>(-0.22)</b>	0.036 <b>(0.37)</b>	-0.195 <b>(-1.52)</b>

deviation of the estimation is very low and the precision with which is estimated high.

The opposite happen for volatility, where we observe that the estimations are quite high comparing with the t-statistics. Consequently, the standard deviation of the estimations are high, and the precision is low, which leads to conclude that the statistics are not significant.

On the other hand, we see that the signs of the relations between the market excess return and the innovations for market moments observed in table 1, in table 10 are identical.

After all these results, we conclude that market higher moments risks affect to stocks risk premium. The results on the market skewness risk is not clear if the risk premium is positive or negative, because the results show opposite conclusions. The results on market volatility and kurtosis risk indicate a negative risk premium.

Furthermore, in Chang et al. (2011) the authors investigate forecasting capacity of  $\Delta Vol$ ,  $\Delta Skew$  and  $\Delta Kurt$  to see if these factors are able to forecast market forthcoming changes. Continuing with their work, we also analyze these results. For that, we use monthly data. The results are reported in table 11.

The results indicate that lagged market volatility and skewness are significant predictors of future market returns. Moreover, the signs are the expected ones, negative and positive, respectively. Lagged kurtosis sometimes is estimated with positive or negative sign, but in none of the cases is statistically significant. Thus, lagged market kurtosis can not forecast future market returns. Therefore, we can intuit the future market returns seeing lagged market volatility and skewness, but these results ought to be interpreted carefully because they are sensitive to outliers.

## 8 Sorting stock returns on different moments

In previous sections we investigate that the behaviour of market higher moments affect to the cross-section of the stocks. Thus, in this section we analyze the importance of higher moments of the own stocks. We examine whether the higher order moments of each stock are priced in the Eurostoxx market. We follow Amaya et al. (2015), where portfolios are formed according to their realized volatility, skewness and kurtosis. In fact, we analyze the relation between stock returns for the current month and the previous month's realized volatility, skewness and kurtosis.

First of all, we need estimates of time series of volatility, skewness and kurtosis of each stock. It might be appropriate to estimate the parameters of the Exponentially weighted moving average or GARCH models for each stock, but this has a high computational cost and it is left for further research. So, we decide to calculate those series as in *RiskMetrics* imposing a parameter  $\lambda = 0.94$  for all the cases. Therefore, the time series of variance, skewness and kurtosis are calculated with the following models,

$$\begin{aligned} r_t &= E_{t-1}[r_t] + \epsilon_t \text{ where } \epsilon_t \sim (0, \sigma_\epsilon^2) \\ \epsilon_t &= \sigma_t \eta_t; \quad \eta_t \sim (0, 1); \quad \epsilon_t | I_{t-1} \sim (0, \sigma_t^2) \\ \sigma_t^2 &= \lambda \sigma_{t-1}^2 + (1 - \lambda) \epsilon_{t-1}^2 \\ s_t &= \lambda s_{t-1} + (1 - \lambda) \eta_{t-1}^3 \\ k_t &= \lambda k_{t-1} + (1 - \lambda) \eta_{t-1}^4 \end{aligned}$$

where  $E_{t-1}[\eta_t] = 0$ ,  $E_{t-1}[\eta_t^2] = 1$ ,  $E_{t-1}[\eta_t^3] = s_t$  and  $E_{t-1}[\eta_t^4] = k_t$ .  $\eta_t$  are standardized residuals and  $E_{t-1}[\bullet]$  denotes the conditional expectation on an information set up to period  $t - 1$ .

After analyzing autocorrelogramas for each return series, we conclude that for each stock, they either follow an AR(1) model or they are by themselves a pure innovation. We use the sample moments computed with the subsample of the first 180 observations as initial conditions, and we begin to extrapolate over time starting with observation 181.

After calculating the time series of the moments of each stock, every month stocks are ranked into quintiles according to their realized moment at the beginning of the month. As in section 5, there are 203 stocks in total, so we decide that quintile 1 and 5 have 40 stocks and 41 rest of them. We construct value-weighted portfolios, so we calculate daily prices of portfolios, where the price of the portfolio is the sum of the prices of different stocks. Then, monthly returns have been calculated using the first and the last closing data of each month.

We also report the *Carhart* four-factor alphas, and we calculate them as in the previous sections.

The results of different portfolios are reported in table 12, where panel A shows the results for the realized volatility, panel B for realized skewness and panel C for realized kurtosis.

We observe that the volatility and the skewness of the own stocks is more valued in the asset pricing, as we see in both cases the results are significant and the t-statistics are relatively high. We

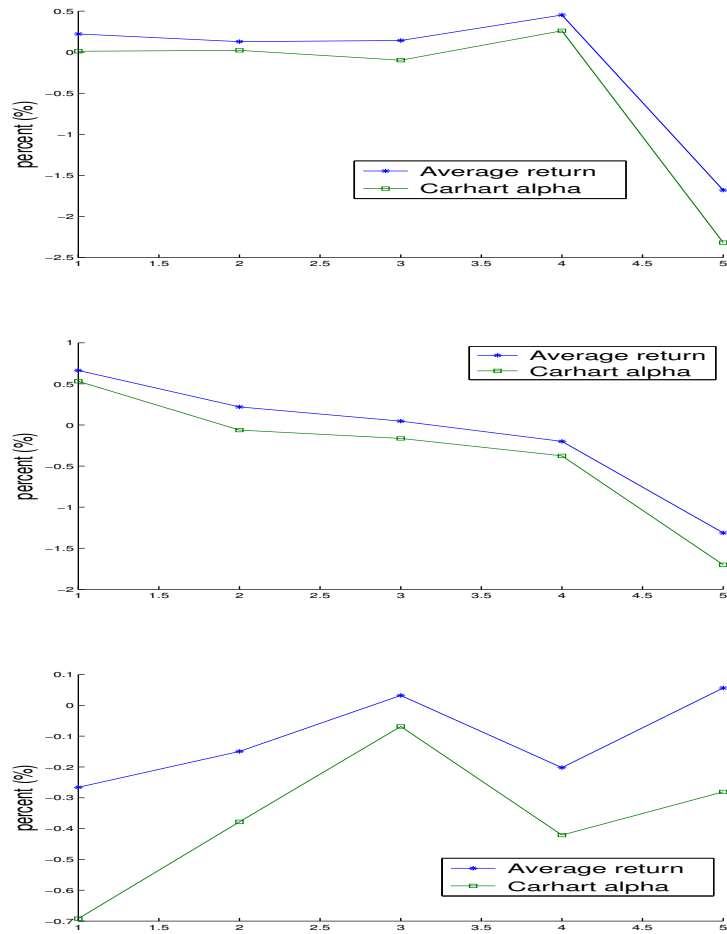
**Table 12:** Realized moments and the cross-section of stock returns. The table reports the average monthly returns for each portfolio in percent and monthly alphas in percent. In parentheses the t-statistics are shown and the significant ones are bolded with 90% confidence.

	1	2	3	4	5	5-1
Panel A: Realized volatility						
Average return	0.22 (0.63)	0.13 (0.35)	0.14 (0.30)	0.45 (0.78)	-1.68 <b>(-1.71)</b>	-1.90 <b>(-2.10)</b>
Carhart four-factor alpha	0.01 (0.03)	0.02 (0.06)	-0.10 (-0.19)	0.26 (0.41)	-2.32 <b>(-2.20)</b>	-2.33 <b>(-2.43)</b>
Panel B: Realized skewness						
Average return	0.66 (1.14)	0.22 (0.42)	0.05 (0.09)	-0.20 (-0.43)	-1.31 <b>(-2.18)</b>	-1.98 <b>(-3.48)</b>
Carhart four-factor alpha	0.53 (0.87)	-0.06 (-0.11)	-0.16 (-0.32)	-0.38 (-0.73)	-1.70 <b>(-2.43)</b>	-2.23 <b>(-3.07)</b>
Panel C: Realized kurtosis						
Average return	-0.27 (-0.53)	-0.15 (-0.29)	0.03 (0.06)	-0.20 (-0.34)	0.06 (0.11)	0.32 (0.67)
Carhart four-factor alpha	-0.69 (-1.22)	-0.38 (-0.71)	-0.07 (-0.13)	-0.42 (-0.64)	-0.28 (-0.52)	0.41 (0.66)

observe that when the returns volatility and skewness are larger, the portfolios returns are smaller, so the price of the risk are negative. In the volatility case, the pattern of the returns across quintiles is not monotonic, but we see that it has a decreasing trend. We find strong evidence of negative cross-sectional relation between realized volatility and kurtosis and future stock returns.

Kurtosis might not be such significant in asset pricing. The returns pattern is not monotonic and there is no clear tendency either. So, we conclude that in our sample, realized kurtosis and the next week's stock returns are not related.

The results from table 12 are summarized in figure 7, where the average monthly returns and the monthly alphas of each portfolio are on the vertical axis and the quintiles in the horizontal axis. The top panel shows the results for the quintiles that are sorted on volatility, the middle panel portfolios sorted on skewness and the bottom panel shows the results for the portfolios sorted on kurtosis.



**Figure 5:** Average return and alphas in each quintile.

## 9 Conclusion

We examine how the cross section of stock returns is affected by the behaviour of the higher moments of the market returns. To do so, we estimate market volatility, skewness and kurtosis using GARCHSK and NAGARCHSK models. We use individual stocks from Eurostoxx as well as the Eurostoxx index and the data sample contains daily closing prices from 2000 to 2016.

We use two different strategies, so our thesis is divided in two parts. The first one is based on the univariate sorting, where we construct different portfolios depending on the exposure of stocks returns to innovations in one of the market moments. Additionally, we also construct factor portfolios to separate the pricing effects of different factors. The risk premium of these portfolios is estimated by computing their average return.

The second part of our analysis is based on the multivariate sorting and cross-sectional regressions. Here, we compute the price of market volatility, skewness and kurtosis risk using *Fama and MacBeth* regressions to different test portfolios.

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We find that the stocks with high exposure to innovations in market volatility have low returns on average. This suggests that market volatility risk receives a negative risk premium, consistent with the results in the second part of our analysis.

The results for skewness and kurtosis are not so evident. Average returns and alphas of portfolios with different sensitivity to skewness and kurtosis do not display a strictly monotonic pattern, although stocks with higher exposure to innovations in market skewness and kurtosis exhibit somewhat higher returns on average. The risk premium associated to market kurtosis risk is also negative, although it does not seem to be statistically significant, possibly because of the difficulty of estimating it with precision. The evidence on market skewness risk is not very clear, although it seems to receive a positive and statistically significant risk premium, although this type of risk seems to receive a positive and statistically significant premium, as shown by the analysis on 25 Fama and French portfolios sorted on size and book-to-market ratio.

Similar work was done by Chang, Christoffersen and Jacobs (2011) using the stocks included in the Center for Research in Securities Prices NYSE/Amex/Nasdaq file, reaching somewhat different results. These authors find that stocks with high exposure to innovations in implied market skewness exhibit low returns on average and they also conclude that market skewness risk indicates a robustly negative risk premium. In our case, the results for the market skewness risk are not so evident.

For the case of volatility and kurtosis, their results are weaker and more sensitive to variations in the empirical setup and across sample periods. However, they also conclude that the price of volatility and kurtosis risk are negative and positive, respectively. Hence, our results only coincide with those of Chang, Christoffersen and Jacobs (2011) just in the negative sign of volatility risk premium.

On the other hand, we also analyze whether the higher moments of each stock are priced in the Eurostoxx market. To do so, we calculate the series of these higher moments with *RiskMetrics* imposing a parameter  $\lambda = 0.94$  for all cases. Once we have the series, we construct portfolios sorting stocks according to their value at the beginning of the month to each higher moment, in that way, we re-calculate the portfolios every month.

The results show that higher moments of each stock are priced in the Eurostoxx market, even more than higher moments of market returns. Specially, stocks volatility and skewness, where we find a significant negative relation between these moments and the next week's stock returns, so it is important to consider them in asset pricing.

Similar work was done by Amaya et al. (2015) using every listed stock in the Trade and Quote database, and they reach somewhat different results. These authors find a reliable and significant negative relation between realized skewness and next week's stock, consistent with our findings. However, they find a little evidence of a reliable relation between realized volatility and the cross-

section of next week's stock returns.

The general conclusion is that to fully understand the behaviour of the cross section of stock returns it is necessary to consider the time evolution of the higher moments of market returns and each stock returns.

## A Appendix 1

Here we show that the function  $f(\eta_t | I_{t-1})$  is really a density function, and its integral in all  $\mathbb{R}$  is equal to one. In order to obtain that, we rewrite  $\psi(\eta_t)$  as:

$$\psi(\eta_t) = 1 + \frac{s_t}{\sqrt{3!}} H_3(\eta_t) + \frac{k_t - 3}{\sqrt{4!}} H_4(\eta_t)$$

where  $\{H_i(x)\}_{i \in \mathbb{N}}$  represent some polynomials such that  $H_0(x) = 1$ ,  $H_1(x) = x$  and for  $i \geq 2$  follow the next expression:

$$H_i(x) = \frac{xH_{i-1}(x) - \sqrt{i-1}H_{i-2}(x)}{\sqrt{i}}$$

Therefore, it is verified that the family of polynomials  $\{H_i(x)\}_{i \in \mathbb{N}}$  is an orthonormal basis satisfying that:

$$\int_{-\infty}^{\infty} H_i(x)\phi(x)dx = 1 \quad (\text{A.1})$$

$$\int_{-\infty}^{\infty} H_i(x)H_j(x)\phi(x)dx = 0 \quad \forall i \neq j \quad (\text{A.2})$$

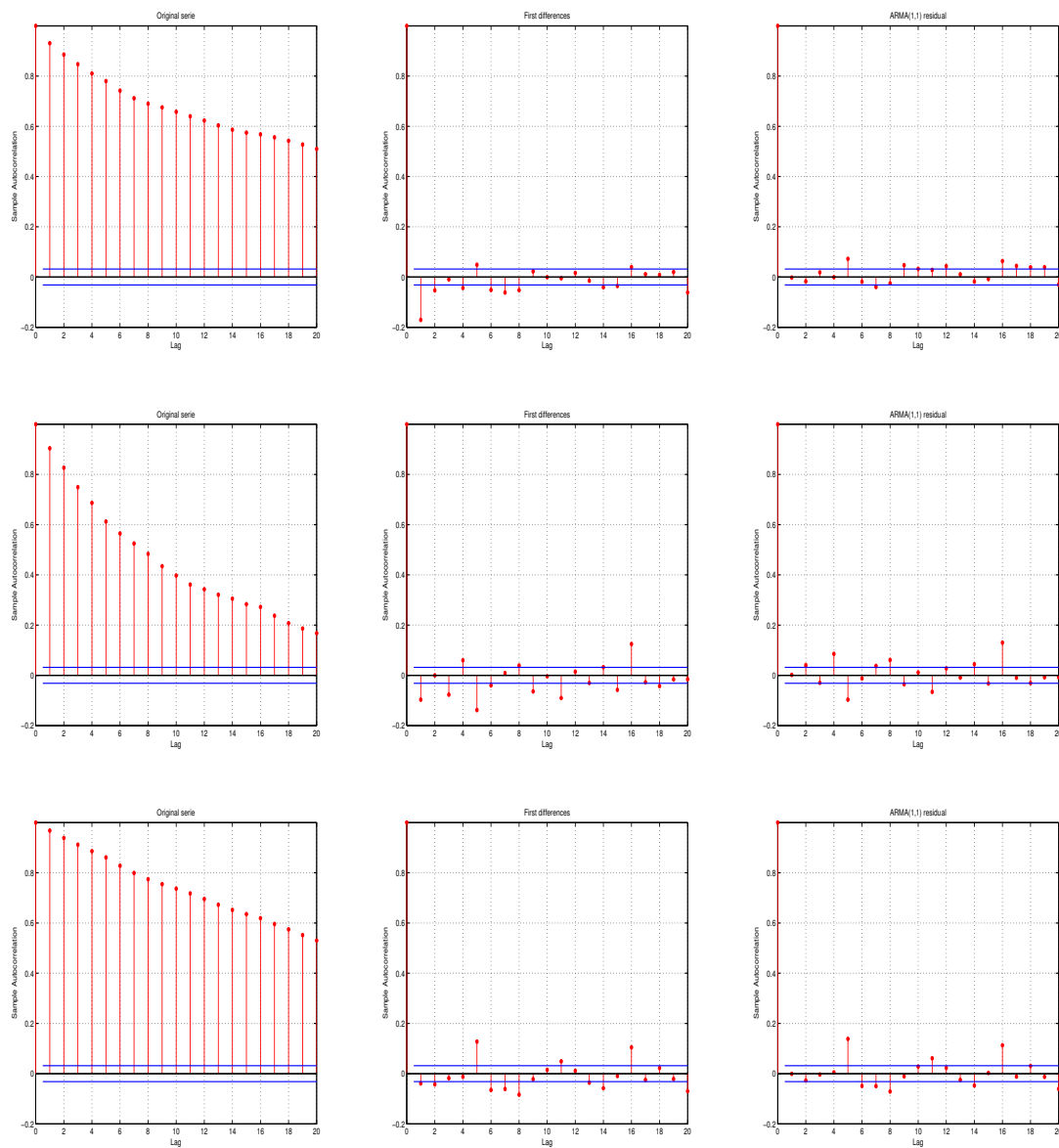
where  $\phi(\bullet)$  is a density function of  $N(0, 1)$  distribution. So if we integrate the function (4.1), using (A.1) and (A.2) conditions:

$$\begin{aligned} & \left( \frac{1}{\Gamma_t} \right) \int_{-\infty}^{\infty} \phi(\eta_t) \left[ 1 + \frac{s_t}{\sqrt{3!}} H_3(\eta_t) + \frac{k_t - 3}{\sqrt{4!}} H_4(\eta_t) \right]^2 \\ &= \left( \frac{1}{\Gamma_t} \right) \left[ \int_{-\infty}^{\infty} \phi(\eta_t) d\eta_t + \frac{s_t^2}{3!} \int_{-\infty}^{\infty} H_3^2(\eta_t) \phi(\eta_t) d\eta_t + \frac{(k_t - 3)^2}{4!} \int_{-\infty}^{\infty} H_4^2(\eta_t) \phi(\eta_t) d\eta_t \right] \\ &= \left( \frac{1}{\Gamma_t} \right) \left[ 1 + \frac{s_t^2}{3!} + \frac{(k_t - 3)^2}{4!} \right] = 1 \end{aligned}$$



## B Appendix 2

### B.1 Autocorrelation functions of market moments with GARCHSK



**Figure 6:** Autocorrelation functions of market moments. There can see sample autocorrelations of the original time series (first column), their first differences (second column) and ARMA(1,1) residuals (third column) for volatility (first row), skewness (second row) and kurtosis (third row). The horizontal blue lines around the zero indicate 95% confidence interval.

## C Appendix 3

### C.1 Results for 10 portfolios

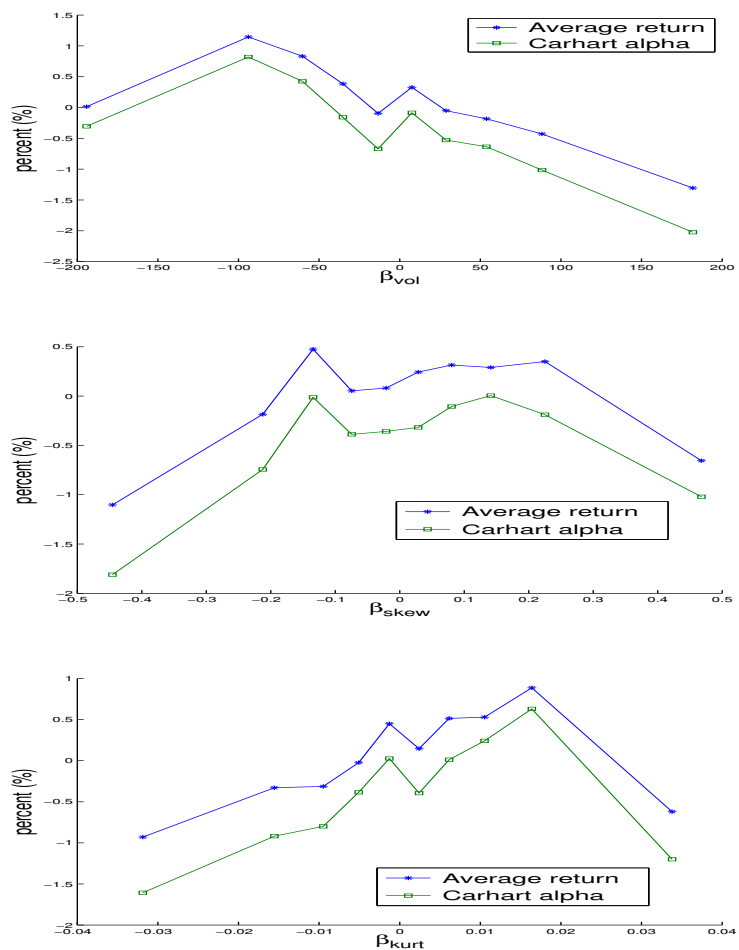


Figure 7: Average return and alphas in each decile.

## D Appendix 4

### D.1 Results for NAGARCHSK model

**Table 13:** Sorting on  $\beta_{\Delta Vol}$  for NAGARCHSK model. The table reports the average beta, average monthly returns for each portfolio in percent and monthly alphas in percent. Univariate is referred to (5.1) and the multivariate to (5.2). In parentheses the t-statistics are shown and the significant ones are bolded with 90% confidence.

	Quintile portfolio					
	1	2	3	4	5	5-1
Volatility beta (univariate)	-63.117 <b>(-17.50)</b>	-20.353 <b>(-17.72)</b>	-0.862 <b>(-2.66)</b>	18.810 <b>(16.42)</b>	61.289 <b>(17.19)</b>	
Average return	0.47 (0.73)	0.43 (0.91)	-0.20 (-0.47)	-0.01 (-0.02)	-1.03 (-1.51)	-1.50 <b>(-2.43)</b>
Carhart four-factor alpha	0.18 (0.27)	-0.10 (-0.18)	-0.72 (-1.61)	-0.46 (-1.01)	-1.75 <b>(-2.38)</b>	-1.93 <b>(-2.92)</b>
Volatility beta (multivariate)	-1348.2 <b>(-8.09)</b>	-414.9 <b>(-7.91)</b>	63.0 (0.36)	436.0 <b>(6.93)</b>	1382.9 <b>(7.51)</b>	
Average return	-0.12 (-0.17)	0.46 (1.00)	0.30 (0.69)	-0.24 (-0.56)	-0.66 (-1.02)	-0.54 (-0.86)
Carhart four-factor alpha	-0.59 (-0.78)	-0.02 (-0.04)	-0.14 (-0.32)	-0.81 <b>(-1.78)</b>	-1.22 <b>(-1.72)</b>	-0.63 (0.85)

**Table 14:** Sorting on  $\beta_{\Delta Skew}$  for NAGARCHSK model. The table reports the average beta, average monthly returns for each portfolio in percent and monthly alphas in percent. Univariate is referred to (5.3) and the multivariate to (5.2). In parentheses the t-statistics are shown and the significant ones are bolded with 90% confidence.

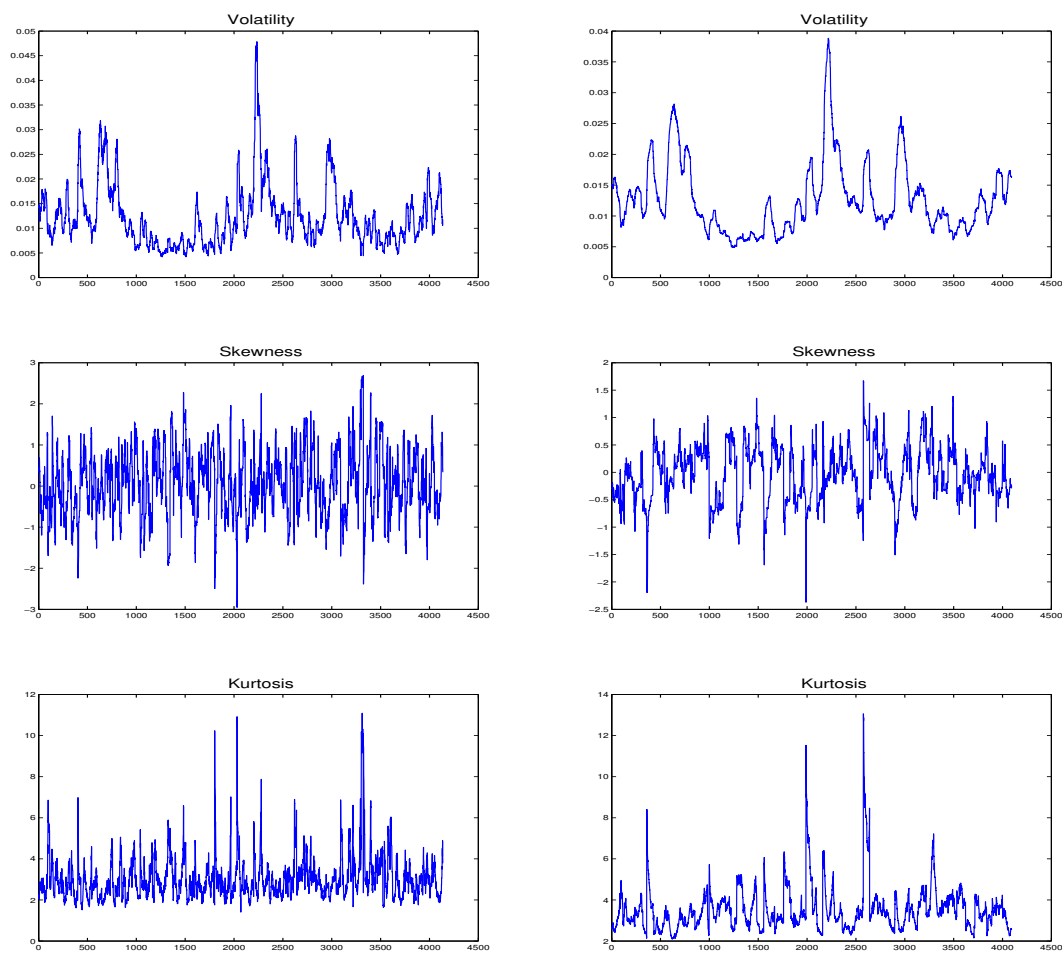
	Quintile portfolio					
	1	2	3	4	5	5-1
Skewness beta (univariate)	-0.163 <b>(-14.75)</b>	-0.050 <b>(-14.17)</b>	0.001 (0.797)	0.053 <b>(14.62)</b>	0.1565 <b>(15.58)</b>	
Average return	-0.05 (-0.07)	0.18 (0.40)	0.07 (0.17)	0.02 (0.04)	-0.43 (-0.65)	-0.38 (-0.60)
Carhart four-factor alpha	-0.65 (-0.93)	-0.25 (-0.52)	-0.45 (-0.99)	-0.50 (-1.08)	-0.87 (-1.23)	-0.22 (-0.30)
Skewness beta (multivariate)	-0.786 <b>(-9.26)</b>	-0.237 <b>(-9.66)</b>	0.009 (1.01)	0.253 <b>(8.09)</b>	0.799 <b>(8.47)</b>	
Average return	-0.30 (-0.45)	0.00 (-0.01)	0.20 (0.46)	0.19 (0.42)	-0.63 (-0.95)	-0.33 (-0.51)
Carhart four-factor alpha	-1.08 (-1.46)	-0.49 (-1.01)	-0.27 (-0.55)	-0.16 (-0.33)	-1.09 (-1.58)	-0.01 (-0.63)

**Table 15:** Sorting on  $\beta_{\Delta Kurt}$  for NAGARCHSK model. The table reports the average beta, average monthly returns for each portfolio in percent and monthly alphas in percent. Univariate is referred to (5.4) and the multivariate to (5.2). In parentheses the t-statistics are shown and the significant ones are bolded with 90% confidence.

	Quintile portfolio					
	1	2	3	4	5	5-1
Kurtosis beta (univariate)	-0.035	-0.011	0.001	0.011	0.036	
	<b>(-22.37)</b>	<b>(-20.76)</b>	<b>(-1.92)</b>	<b>(22.23)</b>	<b>(22.39)</b>	
Average return	-0.59	-0.17	0.47	0.20	-0.29	0.30
	<b>(-0.97)</b>	<b>(-0.39)</b>	<b>(1.21)</b>	<b>(0.41)</b>	<b>(-0.41)</b>	<b>(0.49)</b>
Carhart four-factor alpha	-1.24	-0.67	0.16	-0.27	-0.81	0.43
	<b>(1.91)</b>	<b>(-1.45)</b>	<b>(0.37)</b>	<b>(-0.51)</b>	<b>(1.12)</b>	<b>(0.65)</b>
Kurtosis beta (multivariate)	-0.771	-0.237	0.005	0.248	0.788	
	<b>(-8.66)</b>	<b>(-8.49)</b>	<b>(0.49)</b>	<b>(7.70)</b>	<b>(7.92)</b>	
Average return	-0.20	0.25	0.08	0.00	-0.44	-0.24
	<b>(-0.30)</b>	<b>(0.60)</b>	<b>(0.19)</b>	<b>(-0.02)</b>	<b>(-0.68)</b>	<b>(-0.37)</b>
Carhart four-factor alpha	-0.94	-0.19	-0.36	-0.47	-0.97	-0.03
	<b>(-1.26)</b>	<b>(-0.42)</b>	<b>(-0.83)</b>	<b>(-0.99)</b>	<b>(-1.39)</b>	<b>(-0.04)</b>

## E Appendix 5

### E.1 Results for rolling window



**Figure 8:** Estimates using one-month rolling window. **Figure 9:** Estimates using three-month rolling window.

**Table 16:** Sorting on different moments. We use the univariate case for all of them. The table reports the average beta, average monthly returns for each portfolio in percent and monthly alphas in percent. In parentheses the t-statistics are shown and the significant ones are bolded with 90% confidence.

Sorting statistic	Quintile portfolio					5-1 portfolio	
	1	2	3	4	5	Estimate	t-statistic
Panel A: One-month rolling window							
Volatility beta	-10.681	-3.374	-0.110	3.151	10.077		
Average return	-0.59	0.64	0.33	0.02	-0.96	-0.37	(-0.63)
Carhart four-factor alpha	-1.16	0.36	-0.06	-0.55	-1.62	-0.46	(-0.63)
Skewness beta	-0.030	-0.010	0.000	0.010	0.030		
Average return	-0.63	0.25	0.26	0.26	-0.56	0.07	(0.12)
Carhart four-factor alpha	-1.17	-0.12	-0.21	-0.25	-1.24	-0.08	(-0.11)
Kurtosis beta	-0.02	-0.008	0.000	0.009	0.027		
Average return	-0.26	0.27	0.16	-0.18	-0.61	-0.35	(-0.55)
Carhart four-factor alpha	-1.08	-0.27	-0.31	-0.57	-0.95	0.13	(0.18)
Panel B: Three-month rolling window							
Volatility beta	-31.955	-10.386	-0.695	8.994	29.592		
Average return	-0.29	0.35	0.05	-0.09	-0.51	-0.22	(-0.38)
Carhart four-factor alpha	-0.75	-0.07	-0.49	-0.52	-1.22	-0.47	(-0.76)
Skewness beta	-0.099	-0.031	0.001	0.032	0.102		
Average return	-0.45	0.44	0.07	0.26	-0.57	-0.12	(-0.19)
Carhart four-factor alpha	-1.05	0.06	-0.38	-0.21	-1.17	-0.12	(-0.17)
Kurtosis beta	-0.061	-0.019	0.000	0.020	0.064		
Average return	-0.03	0.12	0.29	0.42	-1.14	-1.11	<b>(-2.10)</b>
Carhart four-factor alpha	-0.64	-0.25	-0.18	0.02	-1.83	-1.18	<b>(-1.92)</b>

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