MODELLING DEFAULT RISK CHARGE (DRC): INTERNAL MODEL APPROACH

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Abstract

The Fundamental Review of Trading Book (FRTB), known as Basel 4, introduces capital requirements for market risk in bank's trading book for which is necessary model default events. The FRTB overhaul trading book capital rules in order to obtain a more coherent and consistent framework. The main aims of this review of previous Basel regulation are to consolidate existing measures and to reduce variability in capital level across banks.

All elements of the new regulation in market risk capital requirements should be fully tested before framework is introduced. There are several reasons that explain the importance of modelling and following up the new measures of the regulation hereinabove. Ambiguous rules about the way of developing models for the new measures can flow into divergence on domestic rules, so the huge variability across firms could not decrease. Moreover, a lack of consistency in the model could lead on an inappropriate high capital levels for some business lines that will reduce liquidity and increase cost for end users and, as a consequence, in this scenario some business lines will become uneconomic. There are two approaches in capital requirements of Basel 4: standardized approach and internal model approach. This article focuses in a specific measure under de internal approach: the Default Risk Charge measure.

Keywords: Fundamental Review of the Trading Book, Default Risk Charge, Market Risk

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1. Motivation

"The fault, dear Brutus, is not in our stars,
But in ourselves, that we are underlings."
Julius Caesar, Act I, Scene III, L. 140-141

A consistent framework for the financial regulation and cautious measures for the risk of the financial institutions are one of the most important goals of the economic science. In order to be aware of this, the analogy that Édouard Daladier[1] made between the financial system and the human body could be quite explanatory. While the heart pumps the blood into the different part of the body, the financial system injects liquidity into the different sectors of the economy, and a failure in any of them could be mortal.

Historically, when the financial structure has been built on weak pillars the consequences that have been suffered were outrageous. The results have been reflected in our cultural background inspiring books and films such as *The Graves of Wrath* where Steinbeck reveals the consequences of an appalling financial regulation and the lack of awareness of the risks that banks had taken in the previous decade. The financial crisis inquiry report of United States published in 2011 tries to raise awareness of the importance of the financial regulation, pointing out that the subprime crisis had not been consequence of an awful fortune, but it had been the result of not paying enough attention to the risk positions that the financial sector had begun to take. The quote of Shakespeare that opens this introduction hits in the target of this idea. This idea motivated the recent review of the bank regulation guidelines proposed by the Basel Committee and known as the FRTB (Fundamental Review of Trading Book) or Basel 4. This idea also impregnates the present thesis in order to get a right model approach for a particular measure called Default Risk Charge (DRC).

The aim of this Master‘s thesis is focused on comparing different methodological approaches, always having concern of the recommendation made by the Committee, and given some assumptions studying the variability of the DRC measure. More precisely, this Master‘s thesis is structured in four chapters.

The following chapter analyzes the meaning of market risk in management and as regulation capital. Therefore in this part of the article the evolution of market risk capital requirement is handled in the different approach that Basel Committee on Banking

[1] Édouard Daladier was the French prime minister in the early thirties.

Supervision has applied over time and the reasons that explain the enhancement of the measures until the nowadays regulation.

Chapter 3 introduces a broadly concept of the DRC, its goals and challenges. Chapter 4 introduces DRC measure under the perspective of the internal model approach, taking into account the requirements with the purpose of modeling the DRC. This chapter is, at the same time, subdivided into two parts. The first part is oriented at modeling probabilities of default while the second part is focused on assessment of loss given default under different models. The goal of this section is to regard different ways of generate a P&L (profit and loss) distribution that would allow to perform the assessment of the DRC.

Chapter 5 is connected with chapter four; due to this is an empirical approach of the different methods to calculate DRC measure presented in chapter 3 given some portfolios.

Finally, chapter 6 outlines the main ideas of the article and sum up the objectives of this measure, challenges and possible lines of work that could be developed.

2. Regulation background

2.1. Basel I

An accurate economic regulation is one of the main requirements to allow a sustainable economic growth. The existence of moral hazard and other market failures make essential regulation in financial field. Basel commitments are the backbone of overarching financial regulation. Basel I commitment in 1988 was the first major step in addressing regulation of internationally active banks. Unfortunately, Basel I suffered from its limited scope. The exclusion of market and operational risk in Basel I can serve as proof of the lack of scope of this first approach to bank regulation guidelines. Market risk can be defined as the risk of losses in on and off-balance sheet positions arising from adverse movements in market prices, while the operation risk can be understood as the risk of loss resulting from inadequate or failed internal processes, people and systems, or for external events.

Bank‘s regulation is always triggered by shocks and recessions that bring weaknesses out. In Basel, the introduction of capital requirements for market risk was the consequence of the collapse of Barings in 1995. The result was the 1995/1996 market risk amendment to Basel I. This bankruptcy clears up the importance of market risk that was out of the scope of the initial Basel proposal. The assessment of market risk in Basel included all instruments in the trading book including those that were out of balance sheet, and was focused on internal models developed by own institutions in order to manage the risks that they had taken.

2.2 Basel II

Basel I commitment quickly became outdated due to its simplicity that did not take into account the existence of bank guarantee neither distinguish the risk firm profile when the capital was fixed. Basel II commitment began to negotiate in 1999 and is finally published in 2006. The structure of capital regulation framework increased thanks to this review of bank‘s guideline regulation. Basel II was built on three pillars: minimum capital for credit, market and operational risk, a supervisor process both internal as self-assessment and external that oversaw the perform of the firm, and disclosed information in order to let market discipline reward banks that carry out a sensible criteria management. Basel II furthered in terms of self-regulation, allowing banks to assess their own risks, as a result of developing an internal approach for sophisticated banks that requires complex and advanced estimation methodologies.

The review of Basel II arose as a consequence of the series of shortcomings displayed by the 2008 financial crisis in the aftermath of Lehman Brothers collapse. The insufficient capital requirement for trading book exposures prior to the 2007-2008 period of the financial crisis was in part responsible for the difficulties that some firms suffered. An ambiguous definition of the regulatory boundary between the banking book and the trading book left the way open to regulatory capital arbitrage in order to reduce the financial buffer of capital for unexpected losses fixed by the regulation. In addition, risk measurement methodologies were not sufficiently robust. In fact, internal models for market risk requirements relied on risk drivers determined by banks, which has not led to sufficient capital for the banking system as a whole. The main difference between both books is that trading book is hold for speculation proposes while banking books consist of portfolios that are supposedly hold until maturity.

2.3 Basel 2.5

The Basel 2.5 reforms included requirements for banks to hold additional capital against default and rating migration risks, and also required banks to calculate an additional value-at-risk (VaR) capital charge in stressed economic conditions, in order to alleviate the inadequate capital requirement established by the previous regulation. This reform published in 2009 also removed most securitization exposures form internal models due to the lack of model‘s robustness for these assets, and required such exposures to be treated as if held in the banking book.

2.3.1. Shortcomings

In spite of the set of revisions to the market risk framework that was introduced by the Basel Committee as part of the Basel 2.5 package of reforms, a number of structural flaws in market risk framework remained unaddressed. That is the reason why on January 2016, the Basel Committee on Banking Supervision (BCBS) published its revised capital requirements for market risk, also known as the Fundamental Review of the Trading Book (FRTB). The FRTB looks for mitigate the flaws that Basel 2.5 did not fixed correctly.

2.3.2. Boundary between trading book and banking book

The regulatory boundary between books was not fully specified in Basel 2.5. The Basel Committee made only minor amendments to the specification of instruments that should be excluded from, or included in, the trading book. Subjective criterions of banks were key determinant when a portfolio had to be associated to trading or banking book, so capital arbitrage continued. The new boundary between the trading book and banking book will limit the potential for regulatory arbitrage. The FRTB has stringent rules for internal transfers between trading and banking book with the purpose of eliminating capital arbitrage. In order to achieve this goal, a list of assets that should be placed in the trading book is also specified, and unless a justifiable reason that must be its place. Moreover, new regulation will limit the institution‘s ability to move illiquidity assets from its trading book to its banking book. Furthermore, if the capital charge on an instrument is reduced as a consequence of a movement of this instrument between books. The difference charge measured in the switching is imposed as a fixed, additional and disclosed capital charge. The Basel Committee on Banking Supervision has designed this range of measure to reduce incentives for arbitrage between the regulatory banking and trading books.

2.3.3 Flaws of VaR-type measure

Another example of the drawbacks of the Basel 2.5 reform was the weaknesses that VaR-type measures framework presented. The VaR metric does not correctly capture exposures to some risks and also creates wrong incentives as the result of being focus just in a percentile of the profit and loss (P&L) distribution. Particularly, the fact that VaR measure is not subadditive implies that the diversification is penalized.

The VaR-based metric used to capitalize trading book exposures was inadequate for capturing credit risk inherent in trading exposures. The hasty growth in the market for traded credit implied that banks held large exposures to undercapitalized credit-related instruments in their regulatory trading book. Also market illiquidity was inadequately captured, due to the fact that previous framework made the assumption that banks can exit or hedge their trading book exposures over a 10-day period without affecting market prices. However, when the banking system holds similar exposures in traded instruments, the market where those instruments are traded is likely to quickly turn illiquid in times of stress, as 2008 financial crisis shows us. In FRTB framework, the concept of liquidity horizon is introduced, that can be defined as the time required to exit or hedge a risk position without materially affecting market prices in stressed market conditions.

The VaR-based framework just takes in consideration a percentile, without looking beyond that percentile, as a consequence this measure fails to capture ―tail risks”. Ohran and Karaahmet (2009) found out that VaR performs correctly when the economy experiment smoothly moves, but fails during times of economic stress, because VaR hasthe feature of being insensitive to the size of loss beyond the pre-specified threshold. For instance, in the pre-crisis phase, providing insurance against certain tail events was recognized as a ―risk-less” strategy according on prevailing regulatory capital requirements at that time. Large unexpected losses when these tail events have occurred emanate from this incorrect use of VaR measure.

VaR measure has received generalized criticism due to its drawback that is consequence of paying attention just in a certain percentile. Artzner, Delbaen, Eber, & Heath (1999; 1997) stand out the lack of sub-additivity as the most notable flaw of this measure, whereas Pflug (2000) point out that Conditional Value at Risk (CVaR), also known as Expected Shortfall does not have this inconvenient property by dint of looking at losses beyond VaR. This is the main reason why previous most of VaR measures of Basel 2.5 are changed by CVaR measures in Basel 4, with the exception of DRC model.

In the FRTB initiative is exposed a new and costly measure to capture internal models‘ tail risk. The final framework replaces Value at Risk (VaR) and Stressed VaR (SVaR) measures for capturing risk with the Expected Shortfall (ES). This ensures capture of tail risks that are not accounted for in the existing VaR measures, and that implies higher capital requirements under the final framework. Furthermore, tail risk capture comes at a significant cost, as data requirements and operational complexities of the ES measure are likely to be extensive.

2.3.3. Excessive recognition of diversification benefits

Under the Basel 2.5 framework there were generous recognition of the risk-reducing effects of hedging and diversification, due to the fact that the estimation of correlations derived from historical data under ―normal” market conditions. Hedging benefits proved illusory as correlation assumptions broke down over periods of market stress. The revised framework establishes that ES must be calibrated to a period of significant financial market stress. Moreover, a series of changes under the revised regulation serve to constrain the capital-reducing effects of hedging and diversification in the internal models approach. The ES capital charge for modellable risk factors does not allow unconstrained recognition of diversification benefits, but limited the cross-risk class diversification benefits.

2.3.4. Lack of sensitivity of the standardized approach

Additionally, under the Basel 2.5 regulation framework, the standardized approach was not a credible fallback to the internal models approach, because it does not embed a clear link between the models-based and standardized approaches either in terms of calibration or the conceptual approach to risk measurement. Moreover, standardized approach endures a lack of risk sensitivity and insufficient capture of risks associated with complex instruments. Nipping these weaknesses of the design of the current framework in the bud is one of the main aims of the FRTB initiative. The revised standardized approach for market risk based on price sensitivities, which is intended to be more risk sensitive compared to the existing standard approach, and therefore reduce the gap between internal models and standard rules. It provides a fallback in the event that bank‘s internal model is deemed inadequate, including the use as a floor to an internal models-based charge. The framework‘s requirements are clearly designed to push firms towards the new standardized approach, which is consistent with the overall regulatory trend of moving away from internal model-based approaches. This idea is bolstered by the framework‘s more stringent requirements applicable to the use of internal models.

2.4. Basel 4

At this point, regulatory background is clearly exposed as a process of continuous enhancements driven by shocks and financial crisis up to the following regulation reform that nowadays is being published, the Fundamental Review of Trading Book (FRTB), also known as Basel 4[2]. The revised capital standard for market risk is focused on three key areas: the revised boundary between trading book and banking book, the revised standardized approach and the revised internal models approach.

2.4.1. The revised boundary between trading book and banking book

As stated above, FRTB imposes strict limits to firms‘ ability to move instruments between books, enhancing supervisory powers and reporting requirements. What is more, clearing which instruments should be stayed in each book and the treatment of internal risk transfers across the regulatory boundary. Nevertheless, it is not clear that the revised

[2] Basel 3 framework focused on capital, liquidity ratios (LCR, NSFR), Leverage ratio and counterparty credit risk modeling while Basel 4 focused on market risk.

boundary will be effective in reducing such capital arbitrage positions in all jurisdictions due to that national regulators are given discretion in defining their instruments lists.

2.4.2. The revised standardized approach

Concerning the revised standardized approach (SA), the overhaul seeks a risk-sensitive measure that can be useful as a credible fallback and floor to the internal model approach (IMA), while sophisticated treatment for market risk is not required under this approach.

2.4.2.1. The sensitivities based Method

The main components of the standardized capital requirement in the trading book are the followings. The Sensitivities Based Method tries to capture ―delta”, ―vega” and ―curvature” risks, extending the use of sensitivities to a much broader set of risk factors than current framework. These risks are related with changes in the instruments in relation to the underlying, the implicit volatility and the changes in the delta in relation to the underlying respectively.

2.4.2.2. The default risk charge (DRC)

The second component of the SA is the default risk charge (DRC). The aim of this measure is to reduce the potential discrepancy in capital requirements for similar risk exposures across the banking book and trading book. It is a floor and a fallback for the IMA default risk charge when the bank‘s supervisory authority does not approve explicitly this last one. Moreover DRC standardized approach must be applied for exposure in the trading book.

2.4.2.3. The residual risk add-on

The third component of the standardized approach is the residual risk add-on that captures any other risks beyond the main risk factors already captured in the two previous components. It provides for a simple and conservative capital treatment for complex/sophisticated instruments that would otherwise be inaccurately captured.

2.4.3. The revised internal models approach

Finally, regarding the internal model approach (IMA) capital requirements for non-securitizations in the trading book, the total capital requirement would be the sum of

Expected Shortfall (ES), the DRC and stressed capital add-on (SES) for non-modellable risks.

2.4.3.1. Expected Shortfall (ES)

The use of a Global Expected Shortfall (ES) is made up of equally weighted average of diversified ES and non-diversified partial ES capital charges for specified risk classes.

Expected Shortfall should be calculated as the expected losses of a portfolio conditional to the fact that the loss is greater than the 97.5th percentile of the loss distribution. [3]This risk measure must be computed on a daily basis and at a base liquidity horizon of ten days.[4]

2.4.3.2. The default risk charge (DRC)

In addition to this measure, DRC captures default risk of credit and equity trading book exposures with no diversification effects allowed with other market risks (including credit spread risk). A default risk measure in the capital requirements for market risk could be rather confusing given the fact that might be associated with credit risk in a first glance. Traditionally, trading book portfolios consisted of liquid positions easy to trade or hedge. However, developments in banks' portfolios have led to an increase in the presence of credit risk and illiquid positions not suited to the original market capital framework. That is the reason why, a measure as DRC, or the previous Incremental Risk Charge (IRC), is essential in order to address these flaws in the traditional understanding of market risk.

Despite the fact that both DRC and IRC are designed to capture default risk, DRC is an enhanced measure of the IRC, correcting some flaws that IRC, a measure of Basel 2.5, had presented. IRC dealt with long term changes in credit quality and in its application huge discrepancies in risk measures had emerged. The variability of market risk-weighted assets was that the more complex IRC models were a relatively large source of unwarranted variation. In response, the revised framework replaces the IRC with a DRC model, in other words, IRC was transformed in favour of a default-only risk capital charge without a migration feature, because it was an important source of variation. These changes are good news for the industry because it prevents from double-counted credit migration risk that in the Basel 2.5 framework is counted once as part of credit risk volatility and once as a stand-

[3]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 3, paragraph 181 (b)
[4]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 3, paragraph 181 (a) and (c)

alone modeled risk. Even though the undeniable superiority of DRC over IRC, the DRC measure introduces a series of new challenges as the modeling of a big number of issuers with low correlation as a result of the mandatory inclusion of equity products.

2.4.3.3. The stressed capital add-on (SES) for non-modellable risks

Lastly, Stressed capital add-on aggregates regulatory capital measure for non-modelable risk factors (NMRF) in model-eligible desks. This component creates incentives for banks to source high-quality data due to the fact that NMRF are subject to a conservative capital add-on. In the end, banks will need more data and stronger data analysis to meet the new risk measurement and reporting requirements. There will be a high cost of risk measures assessment, given the fact that the ES measure has operational complexities and the enormous data requirements.

To sum up, the current framework is not based on any overarching view on how risks from trading activities should be categorized and captured to ensure that the outputs reflect credible and intuitive capital outcomes. The Fundamental Review of the Trading Book overhauls trading book capital rules with a more coherent and consistent framework.

3. Default Risk Charge as a risk measure

3.1 Definition

Default Risk Charge is conceived as a measure to capture default risk of credit and equity trading book exposures with no diversification effects allowed with other market risks (including credit spread risk). This stiffness in the lack of diversification effects allowed with other market risks is due to the huge variability that previous measure, Incremental Risk Charge (IRC), presented. Choosing the migration feature in order to deal with changes in credit quality has entailed discrepancies in risk measures, causing huge risk weight asset variability across financial firms. Since 2012, the Basel Committee on Banking Supervision (FRTB) has being working on the update of the market risk global regulatory framework, transforming the IRC in favor of a default-only risk capital charge named DRC[5]. Moreover, additional differences with IRC are that DRC measure is extended to equity position and that a constant portfolio is assumed. The most relevant consequence of assuming a constant portfolio is that any benefit from recognizing a dynamic hedging strategy is denied.

The DRC tries to capture stress event in the tail of the default distribution, which may not be captured by credit spread shocks in mark-to-market risk. So the DRC measure gives different information from that one provided by credit spread risk. Due to the relationship between credit spread and default risk banks must seek approval for each desk with exposure to these risks, otherwise affected desk will be subject to the standardized capital framework.[6]

The default risk charge is wished-for to capture jump-to-default risk. The jump-to-default risk of each instrument is a function of notional amount (or face value) and market value of the instruments and the Loss Given Default (LGD)[7]. The default risk is the risk of direct loss due to an obligor‘s default as well as the potential for indirect losses that may arise from a default event.[8] These inputs are useful to obtain a profit and loss distribution

[5]In the FRTB consultative papers define the Incremental Default Risk measure, finally was called Default Risk Charge
[6]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (r)
[7]Loss Given Default is understood as one minus recovery rate.
[8]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (a)

(P&L) on the exposure, understanding P&L as cumulative mark-to-market loss on the position.

In the internal model approach (IMA) DRC can be defined as a VaR-type measure at 99.9% level of confidence and a one-year horizon. However in the case of equity sub-portfolios a minimum liquidity horizon of 60 days can be applied[9]. Concerning the risk horizon, it is necessary to point out the difference between the capital horizon and the liquidity horizon. Although the capital horizon for regulatory capital is set to one year, banks are allowed to specify different liquidity horizons or holding periods for instruments such as equity. For instance, for equity sub-portfolios do not seem to be adequate to assume that the portfolio will remain constant over a period like one year. That means that if the liquidity period is chosen shorter than one year, banks must assume that the risk during successive liquidity periods within the one-year capital horizon is identical. In other words, the constant portfolio assumption for equity portfolio is just supported during three months, and between three months period within the one-year capital horizon, a constant level of risk approach is allowed. Summing up, in the case of the portfolios, a minimum period is set to three months for the liquidity horizon whereas the capital horizon is always one year.

This issue has been developed in Martin et al (2011) and also Klaassen and Van Eeghen (2009) referring to the previous risk measure, the Incremental Risk Charge (IRC). Summing up, a constant position approach of one year is made for the main assets while for equity assets the constant position approach is reduced to three months, allowing using a constant level of risk approach after these three months. In practice, this approach could be setup employing a geometric scaling to obtain the default probability for each period. For instance, for a three-month period the probability of default for a given asset could be assessed as $PD_{three\ mont\,hs} = 1 - (1 - PD_{annual}\ )^{\frac{1}{4}}$. So in the case of a liquidity period lower than the capital horizon, a multistep model could be employed. The chance of changing the composition of the portfolio in order to keep stable the level of risk is the main advantage of using a liquidity period lower than the capital horizon. On the other hand the most important drawback is the possible unhedged positions (as a consequence of changing the composition of the portfolio.

There are some flaws in the choice of the constant level approach that have been analyse in Algorithmics (2009) about the IRC measure that could be extrapolate to the DRC case. For instance, the multi-period models are inappropiated if they do not preserve the

[9]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (b)

historically observed annual correlations. Multi-period models implicitly reduce the correlations when roll-overs are modeled. Any modeled rebalancing of the portfolio adds ―noise” to a continuous process over time. This effect of reducing the correlation between obligors as the number of steps in simulation increases is known as ―correlation wash-out”. An additional issue of multi-period models is the current contradiction from the principles of utility theory. As a consequence of the default losses, a bank should reinvest the same dollar amount into an identical level of risk asset. However, due to the losses, the investment is now a higher percentage of the wealth of the bank, so the firm is willing to take more risk for the same reward. In technical terms, assuming a constant level of risk entails a decreasing marginal utility of wealth as wealth goes down, which contradicts a fundamental axiom for the utility theory. Moreover, the concept of reinvesting at the same level risk should be questioned as a feature that is not realistic in ―tail events” conditions. In other words, the constant level of risk assumption is not a reliable feature in the context of extreme markets, due to the fact that in this kind of situations capital preservation and de-leveraging is the generalized industry‘s behavior. According to Martin et al (2011) neither of the two approaches is superior per se, so for the purpose of the article, a constant portfolio approach is employed.

The combination of the constant level of risk assumption, in the case of equity, and the one-year capital horizon reflects supervisors‘ assessment of the appropriate capital needed to support the risk in the trading portfolio. It also reflects the importance to the financial markets of banks having the capital capacity to continue providing liquidity to the financial markets in spite of trading losses. Consistent with a ―going concern” view of a bank, this assumption is appropriate because a bank must continue to take risks to support its income-producing activities. For regulatory capital adequacy purposes, it is not appropriate to assume that a bank would reduce its VaR to zero at a short-term horizon in reaction to large trading losses.[10]

Value at Risk (VaR) is a fundamental tool for managing market risks. It measures the worst loss to be expected of a portfolio over a given time horizon under normal market conditions at a given confidence level. The VaR assessment must be done weekly and the default risk charge model capital requirements is the maximum between the average DRC

[10] Guidelines for computing capital for incremental risk in the trading book - BCBS (2009) Chapter II, Section B, paragraph 15

measures over the previous twelve weeks (i.e. three months) and the most recent DRC assessment.[11]

The measure that focuses this article is a market risk measure, even though it is based on a dependent credit metric. A measure that tries to capture default risk in a market risk framework can be rather confusing in a traditional way of understanding trading book portfolios, where are composed by liquid assets that are easily hedged or traded. Nevertheless, in nowadays banks‘ portfolios are presenting credit risk and illiquid positions not suited to the original market capital framework. In this scenario is where measures as DRC can be understood in a market risk framework.

The instruments that are included to calculate P&L in order to set up DRC measure are all those which are not subject to standardized DRC (as is the case of non-securitization position) and whose valuation do not depend solely on commodity prices or foreign exchange rates. However, this last explicit consideration is just considerer in the 2013 Consultative Document[12], while in the capital requirements for market risk published by the Committee in early January 2016 has been suppressed. Besides, sovereign exposures (including agencies that are explicitly backed by the government), equity positions and defaulted debt positions must be included in the model. In relation to equity positions, the default of an issuer must be modeled as resulting in the equity price dropping to zero[13] and, as a consequence, the recovery rate is assumed zero.

3.2. Goals

The DRC model has the goal of correct the main drawback of the IRC measure, reducing the VaR variability. To achieve this aim, the risk of migration deterioration is not considered in the measure, because the migration feature was a source of variation. Furthermore, constrains on the modeling choices for internal model, limiting them to a two-

[11] Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (d)

[12] Basel Committee on Banking Supervision (2013), Fundamental Review of the Trading Book. A Revised Market Risk Framework, Consultative Document, October. Annex 1, Chapter C, Section 8, paragraph 186 (c)

[13] Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (c)

factor model correlation, are looking for reducing also the variability[14]. Apart from this, factors of default correlation must be based on listed equity or CDS (Credit Default Swap).[15].

3.3. Challenges

The most important challenges that this new measure bring along are the model risk due to the high confidence level of the risk measure and the long horizon, the disparities among correlation matrices because of the use of factors based on equity prices or CDS and the cliff effect along others. The cliff effect is called to the variation on the assessment that the measure can suffer as a consequence of small changes in the value of the exposure or other parameters (default probability for instance). An additional challenge is the questionable use of large pool approximation due to the features of the trading book positions such as actively traded positions, the presence of long-short credit risk exposures and potentially small and heterogeneous number of positions. Large pool approximation is referred to the assumption in the Vasicek loss portfolio distribution model that the basket of asset that compounds the portfolio is large and homogeneous enough to be considered as a perfectly diversified portfolio of identical assets. The consequence is that given the systematic factors, the loss of the portfolio is defined by the level of conditional default risk.

It is remarkable to point out that the Basel Committee has published a consultative document in March 2016 about the IRB (Internal Rating Based) approach, where some proposes are presented in order to improve this approach for determining the regulatory capital requirements for credit risk.[16]The most outstanding propose, that could affect our measure, is the recommendation of avoid using internal approach for exposures where the model parameters cannot be estimated sufficiently reliably for regulatory purposes, like banks and other financial institutions, large corporate and equities. For instance, in our article there is a lack of information about recovery rates for sovereign bonds due to the fact that it is a very unusual event, as a consequence the Loss Given Default could be inadequate measured and this could be the cause of a persistent high variability in this kind of VaR-type

[14]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (b)

[15]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (b)

[16]Reducing variation in credit risk-weighted assets – constraints on the use of internal model approaches. Consultative Document. BCBS (2016) Chapter 2

measure. This recommendation for credit risk measures could be inconsistent with the DRC measure due to the fact that low probability of default portfolios could lead to a DRC's assessment under the internal approach but in IRB approach is not proposed by the Committee, when both kinds of measures analyzed the default risk.

4. Default Risk Charge: a model approach

In this second chapter, an approach modelling for the DRC measure is proposed, always taking into account the requirements that the Basel Committee required for the internal approach. There are three main issues in this chapter concerning loss due to a default event: Exposure At Default (EAD), Probability of Default (PD) and the Loss Given Default (LGD).

4.1. Exposure At Default (EAD)

To begin with, a series of issues should be clarified on terms of the exposure at default in order to find a coherent framework with the standardized approach like the value that must be taking in account for apply the exposure at default, the definition of some basic concepts about this issue.

4.1.1. Making the EAD uniform between SA and IMA.

In the DRC under the standardized approach (SA) the EAD is approximated by the notional amount, in order to determine the loss of principal at default, and a mark-to-market loss that determines the net loss so as to not double-count the mark-to-market loss already recorded in the market value of the position.[17]The EAD should be, as a consequence, different from the notional value due to the fact that the loss market is subtracted from the notional value.

Under the internal approach, exposure at default will be modelled so the EAD on each simulation should be consistent with the standardized approach. In the case of bonds and CDS exposure at default under the standardized approach should be the notional amount that will be multiplied by the loss given default (LGD) and, afterward corrected by the mark-to-market profit and loss distribution (P&L). In the internal approach, the same point of view can be assumed, if a the underlying company of a call option defaults, the contribution of the call in the EAD of the DRC is zero, given the fact that the default would extinguish the call option‘s value and this loss would be captured through the mark-to-market P&L, so the option would not be exercised. This is the interpretation to be expected from the perspective

[17]Minimum capital requirements for market risk-BCBS (2016) Chapter B, Section 7, paragraph 145

of the incremental loss from default in excess of the mark-to-market losses already taken into account in the current valuation.[18]

4.1.2. Long/ short definition

Another issue to deem in terms of exposure at default is the definition of a long or short exposure. To be coherent with the SA, reducing the existing gap and given the fact that the standardized approach is a fallback and a floor to internal models, a common risk data infrastructure should be able to support both approaches. As a consequence, the definition according to the standardized approach is regarded. The determination of the long/short position must be established with respect to the underlying credit exposure.[19] For instance, a long exposure results from an instrument for which the default of the underlying obligor results in a loss. In the case of derivative contracts the long/ short direction of position is determined by whether the contract has long or short exposure to the underlying credit exposure as was previously stated. A result of the definition is the fact that CDS should be defined as a long or short position in relation if the underlying obligor default produces profit or loss, so the default of the seller of the CDS is not considered in the measure due to the fact that credit value adjustment (CVA) risk framework already regarded this subject.

4.1.3. EAD and CVA

A last consideration in the EAD is the existence of wrong or right way risk, i.e. the dependence between credit and derivative‘s underlying. Assuming independence between these two components can be unrealistic in some cases, however the way risk is really difficult to quantify because this dependence between credit and derivative‘s underlying is complex to determinate. Moreover, this is out of the scope of the measure that is presented in this Master‘s thesis and, if this subject must be analysed, should be done in the CVA risk framework of the Basel Committee.

4.2. Marginal PD

Secondly, the default simulation model must accomplish a series of requirements and has to be part of a coherent framework under IMA.

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[18]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (o)
[19]Minimum capital requirements for market risk-BCBS (2016) Chapter B, Section 7, paragraph 140

The internal model approach permit banks to use their modelling techniques once they were approval by the banks‘ supervisory authority. The first challenge on the assessment of DRC is the complexity of obtaining a reasonable probability of default (PD).

4.2.1. Types of PD

4.2.1.1. Depending on the way where are obtained

There are two types of probabilities of default depending on the way where are obtained: historical and risk-neutral. The historical or objective PD is obtained looking at the historical default across the time; while risk-neutral or implicit PD is obtained implicitly from market prices, as a consequence this PD embed market risk premium. The market risk premium that involves this PD means a relevant shortcoming because this bias the prediction of default frequency and the correction is quite difficult.

The difference between both PD is higher as company credit rating is lower. The value of implicit PD is higher than the value of objective PD due to the following reasons: the illiquidity of the bonds, the fact that the scenarios of depression thought by the investors are worse than the historical scenarios and the impossibility of diversification the non-systematic risk in a bond because of its skewness. Risk-neutral PD is more accurate for valuing credit derivatives or estimating the impact of default risk on the pricing of instruments while objective PD is more precise for carrying out scenario analysis to calculate potential future losses.

Intuitively, real world probability of default seems more accurate in order to assess the default risk charge.[20] The revised capital requirements for market risk establishes that PDs implied from market prices are not acceptable unless they are corrected to obtain an objective probability of default. Chan-Lau (2006), Ait-Sahalia and Lo (2000), Bakshi, Kapadia, and Madan (2003), Bliss and Panigirtzoglou (2004), and Liu, Shackleton, Taylor, and Xu (2004) among others have studied possible transformation from implicit to real PD based on strict assumptions such as a representative utility of wealth. However, this transformation is out of the scope of the objectives of this article, so that real PDs are going to be used in this Master‘s thesis due to the fact that supervisor authority must give an explicit approval to use such kind of transformation.

[20]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (s)

In addition to that, regulation also establishes that default risk must be measured for each obligor and the PDs are subject to a floor of 3 basis points.[21] Chourdakis and Jena (2013) perform a critical assessment on levels of default probability that can infer for events with few occurrences, such as sovereign default. Moreover, for some determined portfolios such as high grade sovereign debt portfolios a fixed floor of 3 basis points could carry out a bias risk measure compared to its real credit risk.

It is difficult for banks to obtain reliable estimates of PD for low-default exposures. This is because the lower the likehood of default, the more observations a bank needs to produce a reliable estimate. Moreover, given that each observation of an obligor will likely not result in a default event for low default exposures, obtaining reliable of LGDs are even more challenging. This issue is related with the main challenge explained in the previous section about the recommendation of the Committee about the use of the standardized approach better than the IRB approach for low probability of default portfolios for computing the capital requirements for credit risk. This recommendation could be extrapolated to the DRC measure, betting for the standardized approach instead of the advanced approach for the assessment of the DRC for banks, large companies, high rated sovereigns and other low probability of default obligors.

4.2.1.2. Depending on business cycle

The probability of defaults also can be classified given the business cycle in through-the-cycle PD and point-in-cycle PD depending on if the PD takes into account all the business cycle or just the economic conditions in which it is calculated. The point-in-time PD is more risk sensitive than the through-the-cycle PD due to the fact that this last measure reflects the current economic conditions and, as a consequence, it is a potentially pro-cyclical risk measure so the thought-the-cycle PD should be employed.

Relating to PD, Basel 4 established some conditions that PD must follow.[22] For instance, if an institution has approved PD estimates as part of the internal rating-based (IRB) approach (for credit risk), this data must be used. In our numerical example, that will be showed in the next chapter, PD provided by external sources are used, a choice let by FRTB. Also the thought-the-cycle PD is supposed to be used when the regulation holds the

[21]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (f)

[22]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (s)

following: PDs must be measured based on historical default data which should be based on publicly traded securities over a complete economic cycle. The minimum historical observation period for calibration permitted by the regulation review is five years.

4.3. Default across obligors

Once the main features of the PD applied in the DRC measure have been presented, a default simulation model could be developed.Industry approaches usually separate the problems of estimating PDs and the parameters describing the dependence of defaults.

4.3.1. Types of credit risk models

4.3.1.1. Depending on period of time examined

To begin with, credit risk models can be divided between static or dynamic. Dynamic models when the exact timing of the default is essential like in the analysis of credit securities. On the other hand, static models suits better the goal of this Master‘s thesis because credit risk management has an concern in the 99.9 percentile loss distribution of a portfolio over a fixed time period. Static models are focused on the loss distribution for the fixed time period rather than a stochastic process describing the evolution of risk in time like in dynamics models.

4.3.1.2. Depending on the formulation of the model

In addition to that approach of classifying credit risk models, the models also can be divided into structural or firm-value models on the one hand and reduced-form models on the other hand depending on their formulation. Structural models attempt to explain the mechanism by which default take place. For instance in this type of models default occurs whenever a stochastic variable (in static models, otherwise would be a stochastic process) generally representing an asset value falls below a threshold representing liabilities. This is the reason why structural models are also known as threshold models. On the other hand, the reduced-form models left unspecified the precise mechanism that steers default. An example of a reduced-form model is the mixture model where the default risk of an issuer is assumed to depend on a set of common factors, which are also stochastically modelled. Conditionally to the factors, defaults of individual firms are assumed to be independent. Dependence between defaults arises from the dependence of individual default probabilities on the common factors. Jarrow and Protter (2004) pointed out a relationship between firm-value models and reduced-form models. Essentially, they showed that, from the perspective of investors with incomplete accounting information, in other words, investor has not fully informed about assets or liabilities of the firm, a firm-value model becomes a reduced-form model.

4.3.2. Comparison: IRB vs. DRC default simulation model

A widely used formula is regarded in order to keeps a model structure according the previous Basel review. The regulatory approach, as for instance in Basel II Advanced Internal Ratings-Based (AIRB)[23], is based on the theoretical model of a mixture of normal distributions presented by Vasicek in 1987.

The mapping function used to derive conditional PDs from average PDs is derived from an adaptation of Merton‘s (1974) single asset model to credit portfolios. According to Merton‘s model, borrowers default if they cannot completely meet their obligations at a fixed assessment horizon (e.g. one year) because the value of their assets is lower than the due amount. Merton modelled the value of assets of a borrower as a variable whose value can change over time. He described the change in value of the borrower‘s assets with a normally distributed random variable. On the whole, in Merton model value of risky debt depends on firm value and default risk is correlated because firm values are correlated via a common dependence on systematic factor.

Vasicek (2002) showed that under certain conditions, Merton‘s model can naturally be extended to a specific asymptotic single-risk-factor (ASRF) credit portfolio model. Within the most relevant assumptions, apart from Gaussian dependence, is the large homogeneous portfolio and that the PD is assumed constant across firms. Vasicek model is basically the same as the intensity model when the intensity is the same for all the names, the number of names is huge, the investment is equally weighted across them and a Gaussian copula is deemed.

Under the ASRF model proposed by Vasicek, the asset returns can be decomposed in a systematic component and an idiosyncratic component.

$$r_i = \sqrt{\rho} * F + \sqrt{(1-\rho)} * \epsilon_i$$

Where:

$r_i \sim N(0,1)$ is the asset returns

[23] Basel Committee on Banking Supervision (2006)

$F \sim N(0,1)$ is the systematic factor, that is usually represented by a market factor

$\epsilon_i \sim N(0,1)$ is the idiosyncratic component of the asset returns

F and ϵ_i are uncorrelated random variables. Other factor copula models could be obtained by choosing F and ϵ_i to have other distribution. These distribution choices affect the dependence between the variables and will be analyzed in the section where the model is proposed.

ρ measures the sensitivity to the systematic risk, and as a consequence, runs the correlation between defaults, and as a result its value is between 0 and 1. Since correlation between two firms is the same and equal to ρ, that suppose a loading factor for each firm of $\sqrt{\rho}$.

The default event occurs when the asset returns fall below a threshold represented by $k = N^{-1}(PD_0)$ (i.e. the thought-the-cyclePD_0), so the probability of default conditional to the systematic factor, that is the same that the probability that asset returns fall belong the triggered threshold k conditional to the value of F, is the following expression:

$$P(D_i = 1|F) = P(r_i < k|F) = P\left(\sqrt{\rho} * F + \sqrt{(1-\rho)} * \epsilon_i < N^{-1}(PD_0)\middle|F\right) =$$

$$P\left(\epsilon_i < \frac{N^{-1}(PD_0) - \sqrt{\rho} * F}{\sqrt{(1-\rho)}}\middle|F\right) = N\left(\frac{N^{-1}(PD_0) - \sqrt{\rho} * F}{\sqrt{(1-\rho)}}\right)$$

This model is very attractive because of its simplicity. The idiosyncratic factors are assumed to be diversified away in a large portfolio and a transformation of the quantile of the systematic factor is the only requirement in order to obtain a quantile of the overall frequency of default. All PDs are defined as a function of only one factor, so the portfolio risk is just a monotone transformation of systematic factor. Additionally to the fact that the asymptotic loss probability of a portfolio is given by the probability than the systematic factor reaches a specific value, this model allows capital additivity. That is the reason why IRB approach is based on this model.

Indeed, the internal rating based (IRB) approach for credit risk is based on a single risk-factor model assuming that (a) there is one systemic risk factor; (b) risk components (e. g. PD and LGD) are independent; (c) loan portfolio is infinitely granular. Obviously, none of these assumptions have a sustainable base and, actually there have been several evidences of these unrealistic axioms. For instance, Gordy and Lütkebohmert (2013) show how portfolio finite granularity needs adjustment to capital charge. On the other hand, Folpmers (2012) has the evidence of positive PD-LGD correlation. Furthermore, adjustment to multi-risk factor is shown in Pykhtin (2004). According to the document about capital requirements for the market risk published by the Committee the default simulation model must have two types of systematic factors[24], and the recovery rate and the probability of default must incorporate the dependence on the systematic risk factors in order to reflect the economic cycle via the estimated loss.[25] In conclusion, the DRC measure under the internal approach for market risk is quite different from the default simulation model under the IRB approach for credit risk, although the approved PDs and LGDs estimated as part of the IRB approach could be used.[26] Again, a coherent framework between the IRB and the advanced DRC is necessary as has been pointed out in the challenge chapter on chapter two. It is essential to strengthen the link between both approaches polishing up the regulation in order to assume a common framework in terms of default risk.

4.3.3. Proposed model

Once the differences presented by the default simulation model of the DRC from the IRB approach model have been exposed, an extended model is proposed combining Witzany (2011), Phykhtin (2004) and the usual multi-factor default-mode Merton model.

The default event is modeled via the next function using a similar expression as in Wilkens and Predescu (2016) in order to prevent from cumbersome notation. Following this expression and as in the hereinabove paper, the default is triggered when the function (that represents a sort of creditworthiness index) V_i for obligor i falls below zero:

$$V_i = -\lambda * x_i + r_i \qquad \textbf{(1)}$$

Equation 1: General expression for the default triggered equation following Wilkens and Predescu (2015)

Where:

x_i is a indicator vector that values 1 in the case of type (either corporate or sovereign) and the credit rating of obligor i.

λ is a vector of length of 2xL, due to this vector represent the inverse of the normal distribution for each scalar that composed the PD vector. In other words: $\underset{1x(2xL)}{\lambda} =$ $(\lambda_{1,1}, \dots, \lambda_{1,L}, \lambda_{2,1}, \dots, \lambda_{2,L}) = (N^{-1}(PD_{1,1}), \dots, N^{-1}(PD_{1,L}), N^{-1}(PD_{2,1}), \dots, N^{-1}(PD_{2,L}))$

[24]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (b)

[25]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (m)

[26]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (s), (t)

PD is a vector of probabilities of default of length 2xL, due to there is L categories in the spectrum of rating, and two main categories in the type of obligor. The inverse of the normal distribution is employed in the PDs, given the fact that the PDs are in the range between 0 and 1 and as the asset return follow a standardized normal distribution. As in the Vasicek ASRF model, default is triggered if the asset return of obligor i (r_i) is lower than the inverse normal cumulative function of the probability of default of an obligor with the same rating and type as obligor i ($N^{-1}(PD_i)$). As a consequence, the inverse is introduced in the expression of overall asset return of obligor i (V_i) that clarifies when the default occurs. For every obligor, the failure happens when the overall asset return of the obligor is not positive.

$$r_i = \rho_i Y_i + \sqrt{1-\rho_i^2} * \epsilon_i \tag{2}$$

Equation 2: Asset-value changes under a similar metric that follows Pykhtin (2004)

Where borrower i's standardized asset return is driven by a combination of the systematic factors:

r_i is referred to the asset return of obligor i. It is in the determination of the models parameters where the limitations imposed by the Committee force to make a series of assumptions. The fact that the new regulation allows only the use of listed equity prices or CDS[27] for estimating default correlation forces to supposed that equity is the mayor component of the financial structure. In other words, the postulation of plausible factor-model for the mechanism generating default dependence by the industry is wobbly due to the fact that industry models are forced to calibrate the factor model by taking equity returns as a proxy for asset-value and fitting a factor model to equity returns. So as a consequence of the requirements of the Committee, the financial structure of the firms is supposed to be composed mainly by equity. However, a very high debt level and hence, following a model based on Merton approach, a high defaut probability will not be reflected in the factor model because of the use of equity and the forbidden of use accounting data such as debt information, asset values or actual default correlation. Actually, this is an important drawback of the DRC under the internal model approach that rules out structural type models and other approaches where not only listed price information is used, although that additional information could be publicly disclosed.

ρ_i is the sensibility to the systematic risk.

$\epsilon_i \sim N(0,1)$ is the idiosyncratic component of the asset returns.

[27] Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (b)

$Y_i \sim N(0,1)$ is known as the composite factor, that is the result of the combination of two systematic factors. The relation between the systematic factors and the composite factor is given by the next expression:

$$Y_i = \omega_{1,i} * F_1 + \omega_{2,i} * F_2 \tag{3}$$

Equation 3: Relation between systematic factors and the composite factor

Where the loadings $\omega_{i,1} and \omega_{i,2}$ must satisfy the relation $\omega_{1,i}^2 + \omega_{2,i}^2 = 1$ to ensure that the composite factor has unit variance given the fact that the systematic factors are uncorrelated, so for guarantee this condition each factor is divided by $\sqrt{\omega_{1,i}^2 + \omega_{2,i}^2}$. Gürtler *et al.* (2008) apply this model for economic capital assessment differencing so many risk factors as sectors, so for calculating the factor weights $\omega_{k,i}$a Choleski decomposition of the inter-sector correlation matrix is employed. It is relevant to point out that this approach is a common mathematical method to generate correlated normal random variables, although different types of copulas could be employed in this model.

Asset default correlation between obligors i and j would be given by:

$$corr_{i,j} = \rho_i * \rho_j * (\omega_{1,i} * \omega_{1,j} + \omega_{2,i} * \omega_{2,j}) => \tag{4}$$

$$corr_{i,j} = \rho_i * \rho_j * \left(\sum_{k=1}^{2} \omega_{k,i} * \omega_{k,j} \right)$$

If obligor i and j has the same second factor, otherwise the correlation would be:

$$corr_{i,j} = \rho_i * \rho_j * (\omega_{1,i} * \omega_{1,j})$$

Equation 4: Asset default correlation

Dependence between different firm defaults may exist since they are affected by common macroeconomic factors. The higher is the simultaneous default the greater is the portfolio risk concentration. Whereas, the lower is the default correlation; the greater is the portfolio diversification. Therefore, the dynamics of correlation of default is a critical issue in order to deal with the portfolio credit risk.

Another flaw of the factor model requirements of the DRC measure is the fact that asset correlation is employed as a proxy of default correlation, and in turn, equity correlation is employed as proxy for asset correlation. Put another way, an assumption rests in another assumption, and as a consequence, a proxy for a proxy seems quite unclear. Economic literature has point out that equity correlation overestimate asset correlation, as carry out Düllmann *et al.* (2008), and to be biased indicator of default correlation as was showed by

De Servigny and Renault (2002) and Qi *et al.* (2015). Frye (2008) pointed out that relaxing the assumption that set correlation in the credit risk model equal to asset correlation produces misleading statements of risk. However the restrictions imposed by the Basel Committee compelled to employ equity correlation, so this article is always concerned of the limitations of the underlying assumptions and the biased results of relaxing this axiom. Moreover, Zhou (2001) also makes the assumption of equity correlation being equal to asset correlation. This approximation is increasingly valid as ratings improve because higher-grade safer firms leverage ratio tends to drop significantly, and as a consequence, the difference between equity and asset gradually diminishes. Strictly speaking, when a firm has zero debt, its equity is identical to its asset. Thereupon, equity correlation has some features that would like to see also in default correlation.

$F_1 and\ F_2$ are the standardized normal distributed systematic factors. Since it is more convenient to work with uncorrelated factors, these factors in case of not being uncorrelated must be orthogonalized. Witzany (2011) and Kruger (2015) also admit a possible autocorrelation speaking of modeling, since the systematic factors are supposed to represent some sort of macroeconomic variables. However, in terms of annual returns, the persistence of the factors is generally speaking quite low so, an approximation following a white noise does not seem wrong.

4.3.3.1. Type of factors

There are a number of different approaches to the calibration of a factor model depending on whether or not the factors are observable or, on the contrary, are regarded to be latent or unobservable.

In an observable factor model, also called econometric approach model, stock indices or other observable economic time series are employed as factors, such as CDS changes, country index, industry index or interest rates. It is assumed that the appropriate factors for the return series have been identified in advance and data on these factors have been collected.

In a latent factor model, also called statistic factor model, appropriate factors are themselves estimated from the data. The main strategy for finding factors from the data is to use the method of principal components (PCA) to construct factors. The obtained factors, while they are explanatory in a statistical sense, may not have any obvious interpretation.

However, a classification of factor models into three types is more common in the econometrics literature;[28] these are macroeconomic factor models, fundamental factor models and statistical factor models. In this categorization, our observable factor model would be a macroeconomic factor model, where a series of stationarity assumptions are hold. The principal factors considered in this Master‘s thesis, apart from a global factor, are industry indices and country indices. Other possible factors that could be employed to extend the assessment could be size factor and a Book-To-Market factor (BTM). These two last factors follow the Fama and French (1992) model for describing equity return. The size factor tries to pick up the fact that small firms are more vulnerable than big firms and, therefore their equity is riskier, so they return should be higher. The BTM tries to gather up the fact that firms where their book value is higher to their market value are more sensitive to financial stress periods, and as a consequence should offer a higher return. An example of a possible factor made up from CDS spread instead of equity listed prices, could be the spread between the long term CDS corporate debt and CDS sovereign debt as a proxy of the business solvency.

On the other hand, fundamental factor models use specific firm‘s information such as accounting data. As this kind of data is not allowed by the Committee,[29] the last factor model will not be employed. It is necessary to point out that BTM is a factor based on equity listed prices, although in order to make up this factor it is necessary to have knowledge about their book value that is part of the accounting data. This information is just employed to distribute the returns of the firms in different portfolios, so technically accounting data is not explicitly used in the performing of this systematic factor.

All things considered, the function that triggered the default can be written as the combination of (1), (2) and (3):

$$V_i = -\lambda * x_i + r_i = -\lambda * x_i + \rho_i \left(\sum_{k=1}^{2} \omega_{k,i} * F_k \right) + \sqrt{1-\rho_i^2} * \epsilon_i \qquad (5)$$

Subject to the restriction: $\sum_{k=1}^{2} \omega_{k,i}^2 = 1$ or equivalent $\frac{\omega_{k,i}}{\sqrt{\sum_{k=1}^{2} \omega_{k,i}^2}}$

Equation 5: Default triggered equation under Gaussian copula

[28] see for instance Hamerle (2003)
[29] Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (m)

4.3.3.2. Rank correlation issue: another copula family is possible

Finally, it is a key issue to be aware of the type of correlation that is being employed. Default correlation is a key component when PDs are simulated. As a consequence of using a Gaussian copula, the only parameter that is estimated reflects the linear correlation or Pearson coefficient. While using other kind of copulas, such as Clayton or Gumbel copulas, could gather up the tail dependence of the variables. This is highly desirable due to the fact that Committee appeal to the default simulation model designed by firms under the IMA approach to reflect the effect of issuer and market concentrations that can arise during stressed conditions.[30] Quoting Alan Greenspan during the Joint Central Bank Research Conference in 1995:

> *Inappropriate use of the normal distribution can lead to an understatement of risk, which must be balanced against the significant advantage of simplification. From the central bank's corner, the consequences are even more serious because we often need to concentrate on the left tail of the distribution in formulating lender-of-last-resort policies. Improving the characterization of the distribution of extreme values is of paramount importance."*

Rank correlation is any scalar measure of dependence that depends only on the copula of a bivariate distribution and not on the marginal distributions, unlike linear correlation as Pearson correlation, which depends on both. There are two main varieties of rank correlation: Kendall‘s and Spearman‘s. Kendall‘s tau can be understood as a measure of concordance for bivariate random vectors (X;Y). Given a pair of observation of this bivariate vector (i.e. $(x_1, y_1); (x_2, y_2)$), is said that they are concordant if both pairs have the same sign (i.e. $(x_1 - x_2) * (y_1 - y_2) > 0$) and otherwise is said that are discordant. Additionally, Spearmann‘s correlation is applied to a set of observations of the same length that have a certain order.

Most default models used in industry use the Gaussian copula, so it is important to measure the sensitivity of the distribution of the number of defaults with respect to the Gaussian copula assumption due to the fact that choosing this kind of copula could underestimate the probability of joint large movements of risky factors, with severe implications for the performance of credit risk models. The Gaussian distribution copula is light-tailed and without tail dependence. On the other hand, Daul *et al.* (2003) employ a t

[30]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (k)

distribution copula that is heavy tailed and has tail dependence. However t-student copula supposes tail dependence in right and left tail and a copula with only left copula is more adequate.

The aspect of the light-tailed tails of the Gaussian distribution is not a real problem for our measure, due to the fact that the aim is simulating annual returns. As the interval of the returns is bigger the returns begin to look more identically distributed and independent, less heavy tailed and the volatility clusters disappears. Because of the sum of the daily logarithm returns to obtain an annual return, it is to be expected that some sort of central limit effect takes place, and as a consequence the distribution is becoming more normal and less leptokurtic. So in terms of the kurtosis of the distribution, a Gaussian distribution could be assumed coherent[31]. Nevertheless, an important aspect for the proposal of this article is the rank correlation between assets, issue that Gaussian copula does not reflect.

Speaking of tail dependence, when an equity index suffers a sharp drop, it is common that other indices follow it in the fall. That is the reason why in this article an approach such as applied Crook and Moreira (2011) is chosen. A specific copula family known as Clayton is adopted to represent the association between the systematic factors. The consequence of lower tail dependence of the Clayton copula is the tendency of this copula to generate joint extreme values in the lower corner, which is an attractive feature for our simulation. Extreme synchronized falls in financial markets occurs more frecuently than the assigned by the models, so the proposed model should be qualified to face the perfect storm, when concentration of risk happens. Several researchers as Schönbucher and Schubert (2001), Schönbucher (2002), Gregory and Laurent (2003), Rogge and Schönbucher (2003), Madan et al. (2004), Laurent and Gregory (2005), Schloegl and O'Kane (2005), Friend and Rogge (2005) have been considered this model in a credit risk, although it was considered in term of pricing credit derivatives.

Unlike Gaussian or t-Student copula, which are implicit copulas,[32] in order to obtain Arquimedean copulas is necessary to use the generating function. So for the Clayton copula there is the following generation function:

$$\psi(u) = \alpha^{-1}(u^{-\alpha} - 1); \alpha \neq 0$$

And the inverse of this generation function is:

$$\psi^{-1}(u) = (\alpha * u + 1)^{-\frac{1}{\alpha}}$$

[31] See McNeil et al (2005= pages 122 and 123

[32] Implicit copulas are those which can obtain their density copula using the inversion method, in other words, using their joint distribution function and their marginal distributions.

Given the fact that the copula using generating functions is defined as:

$$C(u_1, u_2, \dots, u_n) = \psi^{-1}(\psi(u_1), \psi(u_2), \dots, \psi(u_n)) = \psi^{-1}\left(\alpha^{-1} * \left(\sum_{i=1}^{n} u_i^{-\alpha} - n\right)\right)$$

So for the case of a bivariate vector would be:

$$C(u_1, u_2) = (u_1^{-\alpha} + u_2^{-\alpha} + 1)^{-\frac{1}{\alpha}}$$

The copula density function is quite useful when the parameters of the copula are estimated by maximum likelihood (ML). The density copula function is the derivative from marginal distribution function. For the bivariate Clayton copula this is:

$$c(u_1, u_2) = \frac{dC(F_1(x_1), F_2(u_2))}{dF_1(x_1) * dF_2(u_2)} \qquad (6)$$

$$= (\alpha + 1) * (u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-2-\left(\frac{1}{\alpha}\right)} * u_1^{-\alpha-1} * u_2^{-\alpha-1}$$

Equation 6: Clayton copula density function

In order to calibrate the Clayton copula first is necessary to explain the relationship between this copula and the Kendall's tau. Given the relation $\alpha = 2 * \left(\frac{\tau}{1-\tau}\right)$ can be related the parameter from the Clayton copula with the Kendall rank correlation measure, as was pointed out by Embrechts *et al.* (2003). This could be a first approach in order to estimate the parameter of the copula. Further strategies to obtain a value for this parameter would be the maximum likelihood (ML) estimation. Within ML estimation, a margin inference approach is based on estimating first the parameters of the marginal distribution and then the parameters of the copula, while canonical maximum likelihood estimates simultaneously all the parameters, which is more efficient but also more complex. Margin inference and canonical ML would be identical to obtain the relationship between systematic factors, given the fact that the Clayton copula is employed to simulate the combined behaviour of the two systematic factors, which have been orthogonalized so their correlation would be zero, and the marginal distribution of both factors is normal.

The idea is use a hierarchicalnested multivariate copula approach such as Otani (2013), where in a first level a Clayton copula for the systematic factors is employed, and then, a Gaussian copula high-level is used to combine the systematic component with the idiosyncratic component. In other words:

$$C^{Gaussian}\left(C^{Clayton}\left(N^{-1}(F_1), N^{-1}(F_2)\right), N^{-1}(\epsilon_i)\right) \qquad (7)$$

Equation 7: Hierarchical nested multivariate copula for the asset returns simulation

Therefore, combining (1), (2) and (7), the triggered default function would be:

$$V_i = -\lambda * x_i + r_i = -\lambda * x_i + \rho_i * F_{CN}\left(C^{Clayton}\left(N^{-1}(F_1), N^{-1}(F_2)\right)\right) + \sqrt{1-\rho_i^2} * \epsilon_i \quad (8)$$

Equation 8: Default triggered equation under a nested copula which employs Clayton copula and Gaussian in a upper level

Where F_{CN} represents the cumulative distribution of the composite factor

The following table represents the different levels in the nested multivariate copula under this alternative approach.

[Insert Figure 1 here]

4.3.3.3. Linear correlation issue

According to the new Basel commitment that is going to overhaul current regulation, correlation in the default simulation must be based on data covering a period of 10 years and also must be based on listed equity prices or on credit spreads.[33] Thereupon, as stated before, no accounting data can be use as input for the measure.

The model also has to recognize the impact of correlations between among obligors that obviously should come from the two-factor model. These correlations must be based on objective data and not chosen in an opportunistic way (i.e. high correlation for portfolios that combine long and short positions and low correlation for long portfolios). Additionally the correlations must be measured over a period over a liquidity horizon of one year and must be calibrated over a period of at least 10 years.[34] These two last conditions are good examples of the quantity and quality or relevant data required by the Committee and explain the costly data requirements for assessing this measure that could push some firms to standardized approach. The same assumption that Wilkens and Prescedu (2016) has made about correlation supports this article. Monthly non-overlapping returns are employed in order to obtain the model parameters. The assumption is that correlation measured over monthly and annual intervals are identical, and a good predictor for future one-year correlation.

The period of 10 years for calibration purpose of the correlation should include a period of stress, so it is supposed to point a period for which the DRC model provides the highest loss estimate, so in this study that period can be identified as the one with the highest correlation due to the fact average correlation rise when market is bearish and fall when market is bullish. This inverse relationship between market value and correlation has been

[33]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (b)

[34]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (i)

pointed out previously in the literature. Longin and Solnik (1995, 2001) found that correlations between country level indices are higher when the market is decreasing. Ang and Cehn (2002) found the same result for correlations between portfolios of U.S. stocks and the aggregate market. Also, as Pankaj Baag (2014) pointed out, Basel II Accord assumes that the average asset correlation moves with probability of default and is a good predictor of probability of default. Moreover, equity correlation could be employed as a valid proxy of asset correlation given the fact that the difference between equity and asset gradually diminishes as ratings improve due to that higher-grade safer firms leverage ratio tends to drop significantly.

To identify a stress period, a DECO correlation of the assets that make up the portfolio could be employed. This approach is more adequate than rolling windows due to the fact that this last approach suffers a bias problem compared to other models such as GARCH-DDC or GARCH-DECO. Rolling windows also presents indetermination of the measurement interval and the return windows that has to be chosen. Moreover, the fact that rolling windows is a mixture of conditional and unconditional moments is rather uncomfortable, due to the fact that all data inside the interval receives the same weight bringing on bias estimation because of outliers. These questions are not trivial and are the main reasons why the choice of a dynamic conditional correlation seems the most sensible.

As Engle (2002) pointed out, DECO correlations have some advantages than DCC does not. In the Dynamic Equicorrelation (DECO) model, all pairs of returns have the same correlation on a given day, but this correlation varies over the time. This eliminates the computational difficulties of high-dimension systems that DCC presents due to the fact that equicorrelated matrices have simple analytic inverses and determinants, becoming simpler and feasible likelihood calculation. Furthermore, the fact that for any pair of assets the correlation depends on the returns histories of all pairs allows DECO drawing on broader information set when formulating the correlation process of each pair. The drawback of DCC compared to DECO is it failure in capturing the information pooling aspect. Also, assuming all pairs of assets have the same correlation reduced estimation noise as Elton and Gruber (1973) carried out for asset allocation.

A matrix $R_t$is an equicorrelation matrix of $nx1$ vector of random variables if takes the following form:

$$R_t = (1 - \rho_t) * I_n + \rho_t * J_n \tag{9}$$

Equation 9: Equicorrelation matrix form

Where:

n represents the number of assets that are employed to calculate the equal correlation parameter

I_n denotes the n-dimensional identity matrix, and J_n is the nxnmatrix of ones.

Using the DECO correlation as was presented in 2002 by Engels simplify the expression of the inverse and the determinant. The expression of the inverse would be:

$$R_t^{-1} = \left(\frac{1}{1-\rho_t}\right) * I_n - \frac{\rho_t}{(1-\rho_t) * (1+(n-1) * \rho_t)} J_n \tag{10}$$

Equation 10: Inverse of an equicorrelation matrix

On the other hand, the calculation of the determinant of the equicorrelation matrix would be:

$$\det(R_t) = (1-\rho_t)^{n-1} * (1+(n-1) * \rho_t) \tag{11}$$

Equation 11: Determinant of an equicorrelation matrix

Using a quasi-maximum likelihood estimation with daily returns, DECO correlation would be obtained in a two steps estimation process where first the returns are standardized using a GARCH approach, then, in a second step DECO correlation would be the result of the maximization of the log-likelihood:

$$\max L(\{r_t\}, \hat{\theta}, \phi) = \frac{1}{T}\sum_{t=1}^{T} \log f_{2,t}(r_t, \hat{\theta}, \phi) \tag{12}$$

$$= \frac{1}{T}\sum_{t=1}^{T} (\log |R_t^{DECO}| + r_t^{st}{}' R_t^{DECO^{-1}} * r_t^{st})$$

Equation 12: Maximization of the log-likelihood for getting the parameter of the equicorrelation matrix

Where $\hat{\theta}$ is the solution to the GARCH process followed by each series of returns, and $\hat{\phi}$ is the second-stage maximize solving the above function given $\hat{\theta}$. It is important to remark that this two step estimation problem will be consistent as Engle (2002) has demonstrated. The correlation matrix of standardized returns R_t^{DECO} is given by:

$$Q_t = \bar{Q} * (1-\alpha-\beta) + \alpha * \tilde{Q}_{t-1}^{\frac{1}{2}} * r_{t-1}^{st} * r_{t-1}^{st} * \tilde{Q}_{t-1}^{\frac{1}{2}} + \beta * Q_{t-1}$$

$$R_t^{DCC} = \tilde{Q}_t^{-\frac{1}{2}} * Q_t * \tilde{Q}_t^{-\frac{1}{2}} = \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,n-1} & \rho_{1,n} \\ \rho_{1,2} & 1 & \cdots & \rho_{2,n-1} & \rho_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{1,n-1} & \rho_{2,n-1} & \cdots. & 1 & \rho_{n,n-1} \\ \rho_{1,n} & \rho_{2,n} & \cdots & \rho_{n,n-1} & 1 \end{pmatrix}$$

$$R_t^{DCC^L} = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ \rho_{1,2} & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \rho_{1,n-1} & \rho_{2,n-1} & \dots. & 0 & 0 \\ \rho_{1,n} & \rho_{2,n} & \dots & \rho_{n,n-1} & 0 \end{pmatrix}$$

$$\rho_t = \frac{(\rho_{1,2} + \cdots + \rho_{1,n-1} + \rho_{1,n} + \rho_{2,3} + \cdots \rho_{2,n} + \cdots + \rho_{n-1,n})}{\frac{n}{2} * (n-1)}$$

Where:

$\bar{Q}$ is the unconditional covariance matrix of standardized returns. An important feature that must be explicitly showed is that as a consequence of use daily returns, there is not supposed to be autocorrelation in the returns so, the daily returns could be model as innovation shock.

$\tilde{Q}_t$ replaces the off-diagonal elements of Q_t with zeros but retains its main diagonal.

R_t^{DCC} represents the correlation matrix of standardized returns and $R_t^{DCC^L}$ represents the lower triangular matrix of R_t^{DCC} without including the diagonal of ones. The equally weighted correlation would be the mean of these values. The values of the DECO correlation matrix would be the expression hereinabove in (9).

Combining (10), (11) and (12) the maximization of the log-likelihood would be:

$$\max L(\{r_t\}, \hat{\theta}, \phi) = \frac{1}{T}\sum_{t=1}^{T}\left[\log\left((1-\rho_t)^{n-1} * (1 + (n-1) * \rho_t)\right) + \left(\frac{1}{1-\rho_t}\right) * (\sum_i \left({r^{st}}_{i,t}^2\right) - \frac{\rho_t}{(1 + (n-1) * \rho_t)} \sum_i \left(r_{i,t}^{st}\right)^2\right] \tag{13}$$

Equation 13: Maximization of the log-likelihood explicit form for getting the parameter of the equicorrelation matrix

Where: $r_{i,t}^{st}$ is the standard return of asset I in t. A relevant remark is the fact that normal distribution with daily returns without any kind of structure is employed to obtain the correlation parameters of the DECO. The result of applying this method of obtaining dynamics correlation across the time for the main exchange index returns of the Eurozone is exhibited below.

[Insert Figure 2 here]

In this figure the equal correlation between the different exchange index returns from 1995 until 2015 can be observed. The most important last 20 years crises are included in the graph. The performance of the exchange index returns in stress conditions is similar; most of them suffer from sharp falls producing huge losses. As a consequence of the previously stated, the following day of the fall, newspapers print in the front page sentence like ―Black day in European stock markets" or ―Stock markets suffer huge drops across Europe". The correlation looks higher between European stock indexes in downturns periods, so correlation higher than the 75 percentile of the historical DECO correlation are considered for indentifying a stress period. Following this rule, the last ten years of the sample encompasses a period of stress, so could be adequate for estimation proposes.

The periods of stress from 2007 due to the financial crisis or the sovereign debt crisis in Europe

4.3.3.4. Quantile regression

In order to obtain the parameter ω of the distribution of equation (2), a quantile approach is considered.

Quantile regression is a technique for isolating a dataset into sections. Minimising the sum of symmetrically weighted absolute residuals yields the median where 50% (q=0.5) of observations fall either side. Similarly, other quantile functions are yielded by minimising the sum of asymmetrically weighted residuals. This technique allows robust estimation of extreme values of the database. Given the fact that there is an interest in realizing a good simulation of the lower returns, which are those that trigger default, this technique could be really helpful in order to achieve this goal. As mentioned before, quantile regression is more robust to the presence of outliers than other prediction methods such as Ordinary Least Squares. Given the fact that the goal is have robust simulation for the lower returns of the asset, this could be an interesting property when estimation of the $\omega_{1,i}$ and $\omega_{2,i}$ is deemed in equation (5).For further information about this technique is hardly recommended to consult Koenker and Bassett (1978) and Koenker and Hallock (2001).

For an asset i, and a two systematic factor model the following regression is regarded:

$$r_{i,t} = F_{1,t} * \omega_{1,i} + F_{2,t} * \omega_{2,i} + \epsilon_{i,t} \quad (14)$$

Equation 14: Regression of the asset in order to obtain the weights for equation (5)

The interest is deemed in the returns that rest in the lower values, due to the fact that those values are the responsible for the default event, so it would be appreciated if the model could perform robust simulation of those outliners. That is the reason why quantile regression is considered in this article. So equation (14) should be estimated the value of the parameters for low levels of q as is expressed in this equation:

$$\min_{(\omega_{1,i},\omega_{1,i})} q * \left(\sum_{r_{i,t} > (F_{1,t} * \omega_{1,i} + F_{2,t} * \omega_{1,i})} \left(r_{i,t} - \left(F_{1,t} * \omega_{1,i} + F_{2,t} * \omega_{1,i} \right) \right) \right) - (1-q) * \left(\sum_{r_{i,t} < (F_{1,t} * \omega_{1,i} + F_{2,t} * \omega_{1,i})} \left(r_{i,t} - \left(F_{1,t} * \omega_{1,i} + F_{2,t} * \omega_{1,i} \right) \right) \right) \quad (15)$$

Equation 15: minimization of the absolute error following a quantile regression

Once the weights $\omega_{1,i}$ and $\omega_{2,i}$ are obtained, the value of the composite factor Y_i could be calculated and in turn, the last parameter, ρ_i could be achieved based on the estimation of the correlation between the time series of the individual return and the composite factor. It is relevant to point out that, as has previously been stated, F_1 and F_2 are uncorrelated.

In order to exhibit the practical implications of choosing a quantile regression, the following three figures compare the OLS regression to different quantile-level regression. In figure 3, a time-serie data for the composite factor and the asset return have been simulated. Then, different regression have been employed, the interest resides in the line that represent the default regression. This line has been created supposing that the annual default probability of the firm is 2%. Afterwards, the normal inverse distribution is applied to obtain this default threshold, if the asset returns fall beyond this line, the firm will break. A detail of this regression is in figure 4, where the attention is focused in the section where the line regression cut the default threshold. In this last figure the systematic return has to fall sharper if a high quantile is employed than in a lower quantile. See, for instance that if the systematic return is -3%, following the quantile regression this q=0.1 the firm will default while for the other quantile regression (and also the OLS regression) the systematic returns must fall sharper in order to trigger the default event for the firm.

[Insert Figure 3 here]

[Insert Figure 4 here]

Employing a quantile regression with a lower quantile allows robust regression for the outliers in the left tail of the distribution, as it was stated before. The reason for that could be discerned in figure 5.As can be seen, for OLS regression the weight to the extreme values are high, regardless the sign of the error, while in the quantile regression extreme values weight is the same that the values more close to zero, the importance resides in the sign of the error. While in the quantile regression with q=0.5 the importance of both types of errors is the same, in the other two quantile regression are not. For instance, in quantile regression with a q=0.1, the negative errors have a higher importance in the optimization function that the positive error, that means that samples of asset returns which are higher than estimated asset returns ―suffer more punish”. This is the goal in the simulating issue, estimated returns given the systematic ones that could be as low as could be real asset returns.

[Insert Figure 5 here]

4.3.4. Simulation process

A previous step or a step zero is showed before explained the algorithm for the simulation process, in order to explain the approach that have been employed with missing data for some asset.

The first stage in the simulation process is transforming the data into logarithm returns. With the aim of getting parameters‘ estimation it is necessary to work with a series of ten years and for some assets, actually equity assets, some data are missing. Specific information about these assets is provided in the following section, where a previous analysis about the sample chosen for the empirical exercise is presented. From the assets quotes, daily returns are calculated as the difference of the logarithm transformation of the price of the assets. Missing values are obtained from an OLS regression where the two highest correlated assets of the sample with the equity with missing data are employed to perform the returns for the period where asset returns are missing. The two highest correlated assets employed are orthogonalized in order to assure that the second employed asset has different information from the first asset about dependent variable. Once the data missing is fulfilled, the returns are added in 22-day intervals in case of employing monthly returns or 66-day interval if quarterly returns are employed for the estimation process.

Next, two types of algorithms in order to simulate the returns are presented depending if the tail dependence between systematic factors is considered or not. The first step for both simulation processes is the same one, generating a series of random uniform distribution between 0 and 1. Depending on how these value are simulated, the method is called MonteCarlo or a Quasi Monte Carlo method. Further information about the process of generating random (or almost random) uniform distribution is provided in the next chapter, where several variance reduction techniques are regarded.

4.3.4.1. Algorithm 1:Case of gaussian marginal distribution and gaussian copula distribution

First step:

For each simulation and each asset, three uniform random variables are generated (u_1, u_2, u_3) in a first step. Then, employing the inverse of the normal cumulative distribution, independent standard normal random variables are obtained$(\tilde{f_1}, \tilde{f_2}, \tilde{\epsilon_i},)$the dash indicates that they are values of a simulation.

Second step:

Then, the composite factor is created combining the weights of the first systematic factor and the second systematic factor. In order to represent an adequate performance of the variability of the factors, the standard normal random variables should be multiply by their standard deviation, and as the composite factor has a standard normal distribution, has to be standardized again.

So a simulated composite factor for the asset I would be like:

$$\tilde{y_i} = \left(\widehat{\omega}_{1,i} * \sigma_1 * \tilde{f_1} + \widehat{\omega}_{2,i} * \sigma_2 * \tilde{f_2}\right) * \frac{1}{\sqrt{\widehat{\omega}_{1,i}^2 * \sigma_1^2 + \widehat{\omega}_{2,i}^2 * \sigma_2^2}} \qquad (16)$$

Equation 16: Simulation of the composite factor

Where n_1 and n_2 are the simulated performance of the systematic factors following a standard normal distribution, $\widehat{\omega}_{1,i}$ and $\widehat{\omega}_{2,i}$ are the estimated parameters for the systematic factors following a quantile regression as in equation (15). An important remark is the fact that σ_1 and σ_2 are the annual standard deviation of the first and second systematic factor respectively.

4.3.4.2. Case of gaussian marginal distribution and hierarchical nested multivariate copula, mixing Clayton copula and Gaussian copula

This issue is rather similar to the previously presented. The main difference arises from the simulation of the values. In other words, the uniform random variables are different, which would imply a calculation of the empirical distribution of the composite factor, due to the fact that the normal random variables are not anymore independent, and as a consequence, the composite factor has a different distribution from normal. Also the empirical distribution of the asset must be estimated, because the combination of the normal distributed idiosyncratic factor and the unknown distributed composite factor lead to a unknown distribution for the asset returns.

In the first place, three uniform distributions are simulated (u_1, u_2, u_3). Then, u_3 that correspond to the idiosyncratic component of equation (2) can be transform into a normal distribution via the inverse normal distribution function ($\tilde{\epsilon_i} = N^{-1}(u_3)$). So it is still independent, while the other normal simulated for factor 1 and 2 are not longer independent. In order to obtain this aim, one factor, for instance F_1, is fixed as it was simulated (so $u_1 = u_1^*$) the distribution of F_2 is obtained given the inverse conditional copula of the Clayton with another uniform distribution simulation and the and the uniform simulated distribution of the first factor. So the conditional copula that must be used is:

$$C_{(2|1)}^{-1} = u_2^* = \left[1 + u_1^{*-\alpha} * \left(u_2^{-\frac{\alpha}{1+\alpha}} - 1\right)\right]^{-\frac{1}{\alpha}} \tag{17}$$

Equation 17: Inverse conditional Clayton copula

The next step is transform the uniform values with asterisk into normal simulated distribution, in order to do that, the inverse normal distribution is employed as before, as a consequence the simulation of factor 1 and factor 2 would be respectively $\widetilde{F_1} = N^{-1}(u_1^*)$and $\widetilde{F_2} = N^{-1}(u_2^*)$, that are dependent though the conditional Clayton copula that was employed.

Marginal inference and canonical maximum likelihood (canonical ML) are equivalent due to the fact that margin normal distribution are not correlated because of the orthogonalization of the variables. In a margin inference, the parameters of the marginal distribution are firstly estimated and once they have been obtained, the parameter for the copula is sought. On the other hand, all the parameters are jointly estimated in the canonical ML. As there are not parameters for estimating on the marginal distribution, the Clayton copula parameter (θ) is estimated via the Clayton copula density in equation (6):

$$\underset{\theta}{Max} \sum_{t=1}^{T} \ln c(F_1(x_1), F_2(x_2); \theta) \tag{18}$$

Equation 18:copula distribution parameters estimation using ML under the supposition of the normal marginal distributions

4.4. LGD model

In this section of the article, LGD model is studied, and could be divided into three parts. First of all, because of the new regulation requirements for Loss Given Default (LGD) are analyzed and the drivers that lead these requirements are studied following a literature review. Then, in a second part different model for LGD are compared in order to choose the more suitable model under the empirical evidence for the loss given default. Finally, the last section of this LGD part proposed a specific model once that all the previous information has been taken into account.

The first step in order to speak correctly about LGD is having a reasonable idea about what is understood by loss given default. LGD is defined as the ratio of losses to exposure at default. The review regulation defines loss given default as 1 – recovery rate[35]. The *recovery rate* for a bond is, in turn, usually defined as the bond's market value after a default as a percent of its face value. In this case the equivalent LGD is known as market LGD. If the recovery rate is based on the observed cash over the course of a workout it is called workout recovery, ultimate recovery or settlement value recovery. However, LGD market is an estimation of workout LGD due to the fact that a buyer of a defaulted bond does not pay more than the expected payoff of the recovery. Moreover, it is not obvious the discount rate to apply to the cash flows. That is the reason why the rating agency recovery studies are based on the market recovery approach. For instance, Moody's estimates defaulted debt recovery rates using market bid prices observed roughly 30 days after the date of default.[36]The main drawback of LGD estimation is the lack of available free information .Due to this reason calibrated values in the literature are going to be used. Concerning the parameters estimation, can be marked that if data from for instance Moody‘s Default and Recovery Database or the Altman-Kuehne/NYU Salomon Center Bond Master Default Database were available, this article could be extended estimating the parameters of the LGD combining Bayesian inference methology and Markov Chain MonteCarlo estimation, as has been showed by Medova (2014) and Luo and Shevchenko (2013). Specific information about the parameters is regarded in data section.

4.4.1. Requirements for the Loss Given Default by the FRTB and empirical evidence

The loss for default must reflect the economic cycle, so at least one of the systematic factors for modeling correlation and default event should be related with the economic situation, obtaining higher defaults in downturns and lower otherwise. This relation with economic cycle must be incorporated also in the dependence of the recovery rate.[37] Losses of portfolio are vastly understated if the lack of correlation between PDs and recovery rates

[35]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (t)

[36]Moody’s Special. 2011. Corporate Default and Recovery Rates, 1920 – 2010. Moody’s Global Corporate Finance, February.

[37]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (l)

(RR) is assumed. If PDs are found to be correlated with RRs, as Alman (2005) pointed out, not only the risk measures based on percentiles, e.g. VaR-type measure, could be critically underestimated, but the amount of expected losses on a given credit portfolio could be misjudged. Altman, Brady, Resti and Sironi (2003) exhibit that when the probability of default is high then the recovery rate tends to be low. Concerning to the most renowned credit models in the industry, recovery rate and the probability of default are treated as two independent variables. While at least *CreditMetrics, CreditPortfolioView* and *CreditPortofolioManager* simulate the recovery rate as a stochastic variable, *CreditRisk* model assumes a constant parameter for the recovery rate.

According to the minimum capital requirements for market risk document published in January 2016, the loss given default (LGD) must carry out a series of standards. As in the case of PD, if an institution has approved LGD estimates as part of the internal rating based (IRB) approach, this data must be used and when such data does not exist or is not enough robust, however LGD provided by external sources may also be used. LGDs must be computed using a methodology consistent with the IRB methodology for credit risk. In our study, LGD parameters are obtained though the previous estimation made in other papers due to the lack of available data about recoveries[38], as was previously stated.

Besides, Committee suggests that the LGD should reflect the type and seniority of the position and cannot be less than zero.[39]Economic literature has reflected the importance of seniority and seniority as drivers of the recovery rate. See, for instance, Acharya, Bharath and Srinivasan (2003) Altman and Kishore (1996), Altman and Eberhart (1994) and Gupton, Gates and Carty (2000)). So this recommendation of the Committee is not baseless.

Moreover, if the seniority driver is included in the simulation of the LGD, the absolute priority rule should be followed. Median recovery for senior corporate bonds is logically higher than lower seniority bonds as was pointed out by Altman and Fanjul (2004).

Another relevant driver for LGD is the issuer‘s industry tangible asset-intensive industries, especially utilities, have higher recovery rates than service sector firms, with some exceptions such as high tech and telecom. Altman and Kishore (1996) and Verde (2003) report a significantly high variance across industrial sectors in terms of recoveries.

[38]However, FDIC provides free data for aggregate time-series data for pools of loans. Unfortunately, this information looks more convenient for consumer credits than for corporate or sovereign bonds, due to that this data do not distinguish enough some features such as credit rating, industry or seniority. More information about the FDIC data is available in: https://www5.fdic.gov/idasp/advSearch_warp_download_all.asp?intTab=4

[39]Minimum capital requirements for market risk-BCBS (2016) Chapter C, Section 8, paragraph 186 (t)

As it was previously stated, recovery rates have to be stochastic and should depend on the systematic factors. That kind of recovery rate is known in financial literature as systematic recovery risk. Frye (2000) has carried out an empirical analysis using recovery data collected by Moody‘s on rated corporate bonds. He found that recovery rates are substantially lower than average in downturn economic periods. The challenge that implies this type of models for recovery rates is the estimation of the dependence of the loss given default on the economic factors.

4.4.2. Most important stochastic distribution for modeling LGD studied in the literature

Several recovery rate distributions have been proposed in the economic literature. For instance Vasicek-type is used by Frye (2000b). This normal distribution for recovery could lead to values outside the range between zero and one, so it is not the better choice.

A similar problem suffer a log-normal distribution for the recovery, that is employed by Bade *et al.* (2011a,2011b) and Wilkens and Predescu (2016) due to the fact that is not a upper-limited at value of one. This issue is the responsible for the fact that the recovery rate has to be scaled using this distribution in order to avoid an ill-fitted model.

A better distribution, which has values between zero and one, can be the logit distribution. This distribution is employed by Wilkens *et al.* (2013), Düllmann and Trapp (2004) and Roesch and Scheule (2005). The main drawback of this distribution is that the structure is not quite flexible in the shapes that allows.

A beta distribution is usually employed to model the recovery rate by the most popular credit risk models. This distribution is quite practical due to the wide range of shapes that can assume. An advantage of this distribution is the fact that following a method-of-moment approach the parameters of the distribution can be obtained based on the historical mean and variance, although this method introduces a loss of efficiency.

However, the histogram of the recovery rate exhibits that the percentage of exposure is either high or low, in other words the histogram of the recovery rate shows a bimodal distribution, while the classic beta distribution is unimodal. Further researches for fitting better the recovery rate have regarded a kernel-beta distribution such as Renault and Scaillet (2004) or a mixture density of beta distribution as Hlawatsch and Ostrowski (2011) have proposed in order to catch this bimodal feature of the recovery rate density function.

4.4.3. Proposed model

The model combined the beta distribution with the effort to catch the relationship between the probability of default and the recovery rate. This relationship is supposed to be a consequence of the fact that both share a common driver, which could be called ~~–g~~lobal factor". This is the linking factor between PD and recovery. The dependence of the PD and the LGD on the same underlying factor is reasonable. Historic data exhibit that recovery rates decrease when default rates increase sharply; see as was pointed out by Altman et al. (2001) and Frye(2000a,2000b). In terms of correlation between them, Hu and Perraudin (2002) estimate that the correlation between recoveries and aggregate default rates for the United States are between 20% and 30%. Moreno and García-Céspedes (2014) also estimate the correlation between the default and the recovery in 22.63% using FDIC historical data, which is the range of Hu and Perraudin.

Following Fry(2000b) an model for the latent variable for the recovery is proposed:

$$L_i = \sqrt{\rho_{RECOVERY}} * F_1 + \sqrt{1 - \rho_{RECOVERY}} * \epsilon_{2,i} \qquad (19)$$

Equation 19: Latent variable of the recovery rate

Where $F_1, and\ \epsilon_2$ are independent and identically distributed standardized normal. The factor F_1 is supposed to be a global factor, which is presented also in the equation (3)

The factor Y_i drives losses in case of default. The LGD driver factors are composed by F_1, a systematic factor of the PD, which is a proxy for global economic situation and an idiosyncratic factor of the recovery rate. However the factor ρ is not calibrated, although Fry(2000b) estimates this parameter of 17%, a different distribution function is chosen to perform the recovery rate.

Then, the inverse of the normal distribution is applied to L_i transforming into a uniform distribution. Finally, using values for the parameters for the beta distribution, the recovery rate is obtained. Different values for the parameters can be use depending on the seniority or the industry. So the assessment of the DRC could be portfolio dependent, depending on industry parameters or seniority parameters are employed.

$$1 - F_{beta}^{-1}\left(N^{-1}(Y_i), \alpha^i, \beta^i\right) \qquad (20)$$

Equation 20: recovery rate approach

Equation (20) represents the Loss Given Default. It is expressed as one minus the recovery rate due to the fact that the values provided by the literature such as Renault and

Scaillet (2004) or Almant and Kalotay (2014) are for the recovery rate. Furthermore, the superscript i stand for the industry or the seniority of the bond.

5. Empirical analysis

In this section an empirical application of the proposed model is exhibited. The current chapter can be organized in four parts.

In the first one, in addition to the data employed, the calibrated parameters are presented, explaining the reasons for choosing them. Moreover, the four principal criteria for the portfolio weighting are presented.

In a second part a general algorithm for the simulation process is showed in order to refresh the most relevant steps in the empirical application of the third and fourth part.

The third part is focused in the sensitivity analysis, mainly analyzing the relevance of the chosen quantile and the parameter that determines the influence of the global factor in the recovery rate. Also, a semi-parametric statistical testing using bootstrapping is regarded for testing if the OLS parameters are significantly different from the quantile parameters, and if the estimated parameters are conditioned by the use of monthly or quarterly data. Apart from that, the result of the DRC measure using a Clayton copula for the systematic factors are showed and compared to the Gaussian copula approach.

Finally, a series of variance reduction techniques are presented in order to get the most accurate value for the DRC given a certain number of simulations.

5.1. Data

In order to represent the third part of this section the Eurostoxx equity returns are employed as a proxy of the corporate assets returns. From the 293 values in the Eurostoxx, they have been filtered and only have been considered those values that have current credit valuation from Standard and Poor‘s according to Reuters. For sovereign debt, eleven bonds with a ten year duration from the Eurozone have been considered. The sovereign debt deemed are from: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain. The period of time on which the data is provided is the period from January 2005 to March 2016.

In relation to the equity data, they are mainly from France Germany, Italy and Spain as can be seen in figure 6, while concerning to their rating credit almost three quarters of the sample are rated between BBB and BB- while higher rated than BBB are less than a quarter of the sample, as it is represented in the below figure.

[Insert Figure 6 here]

[Insert Figure 7 here]

In terms of industry, the sample is quite heterogeneous. Banks, financials companies and insurance companies are around a quarter of the sample, following by chemical and utilities as the next bigger represented sectors, as can be confirmed by the figure 8.

[Insert Figure 8 here]

Regarding the systematic factors, the first one should be a global factor, and as all the assets are European, the Eurostoxx 50 could be a good proxy of such variable. Concerning the second systematic factor, country or industry are employed. For the country systematic factors, the following indexes are considered:IBEX35 for Spain, BEL20 for Belgium, FTSE ATHEX for Greece, CAC40 for France, AEX for Netherlands, DAX30 for Germany, FTSE MIB for Italy, PSI20 for Portugal, ISEQ for Ireland, OMX HELSINKI for Finland, ATX for Austria, LUXX for Luxembourg. If the second systematic factor is an industry factor, index from the Eurostoxx depending of sector are employed: EURO STOXX OIL & GAS, EURO STOXX BASIC MATS, EURO STOXX CHEMICALS, EURO STOXX BASIC RESOURCE, EURO STOXX INDUSTRIALS, EURO STOXX CON & MAT, EURO STOXX INDS GDS & SVS, EURO STOXX CONSUMER GDS, EURO STOXX AUTO & PARTS, EURO STOXX FOOD & BEV, EURO STOXX PERS & H/H GDS, EURO STOXX HEALTH CARE, EURO STOXX CONSUMER SVS, EURO STOXX RETAIL, EURO STOXX MEDIA, EURO STOXX TRAVEL & LEIS, EURO STOXX TELECOM, EURO STOXX UTILITIES, EURO STOXX FINANCIALS, EURO STOXX BANKS, EURO STOXX INSURANCE, EURO STOXX REAL ESTATE, EURO STOXX TECHNOLOGY.

It is necessary to point out that these features about the assets employed in the empirical exercise have been gathered up from Reuters database. These key features are essential in order to determinate their probability of default, the second systematic factor if an industry index or a country index is employed or the recovery if the parameters for alpha and beta are determinate depending on the sector of the company. Regarding to the parameters of the recoveries, the calibrated parameters have been in a first glance obtained from Renault and Scaillet (2004). However, these parameters are previous to the 2008 crisis, so the database could be a bit outdated. That is the reason why the Almant and Kalotay (2014) parameters are employed in order to update the values for alpha and beta of the beta distribution. In the sectors for the recovery rates are fewer groups than in the industry systematic factor. For instance banks, insurance and financial are all of them included in banks group speaking on recovery parameters.

Almant and Kalotay (2014) provide the mean and standard deviation for the main groups of industries and seniorities. In order to obtain the α and the β of the beta distribution, a transformation following the below expression is applied and the results can be observed in the table 1:

$$\alpha = \mu * \left(\frac{\mu * (1-\mu)}{\sigma^2} - 1\right)$$

$$\beta = (1-\mu) * \left(\mu * \frac{(1-\mu)}{\sigma^2} - 1\right)$$

[Insert Table 1 here]

The database from which the parameters are obtained is the Ultimate Recovery Database, which covers three economic cycles and allows linking the recovery with variables like the seniority and the industry.

In relation to the probability of default, the values are obtained from Wilkens and Predescu (2016), due to the fact that a different probability of default is established depending on the rating and distinguishes the type of asset (corporate or sovereign).

[Insert Table 2 here]

A summarized table presents the most remarkable features of the equity and sovereign assets for the empirical exercise, in terms of rating, country and industry.

[Insert Table 3 here]

The goal of using 156 equity assets and 11 sovereign assets, summing a total of 167 assets is regarded the highest number of assets in order to obtain a robust statistical test when the parameters are checked to be different using monthly or quarterly data, or comparing OLS to quantile regression.

When the variance reduction techniques are employed, the number of equity assets is reduced to 39, due to the fact that the algorithm is employed several times in order to obtain a mean and a standard deviation of the DRC measure for the portfolios. Employing 156 assets is not only computing demanding for this propose owing the needs of the virtual memory and the time elapse between each process, but also it is unnecessary for the question of variance is a matter of the loss distribution of the portfolio, the number of assets is not a key feature when this problem is treated.

The 39 equity assets are: ANHEUSER-BUSCH INBEV, TOTAL, SANOFI, BAYER, SAP (XET), LVMH, SIEMENS (XET), DEUTSCHE TELEKOM, ALLIANZ, DAIMLER, UNILEVER, BASF, BNP PARIBAS, BANCO SANTANDER, AXA, ENI, TELEFONICA, AIRBUS GROUP, BMW, DANONE, ORANGE, ENEL, ING GROEP, INTESA

SANPAOLO, IBERDROLA, BBV.ARGENTARIA, NOKIA, SCHNEIDER ELECTRIC SE, MUENCHENER RUCK. , SOCIETE GENERALE, FRESENIUS,CARREFOUR, UNICREDIT, DEUTSCHE BANK,PHILIPS ELTN.KONINKLIJKE, SAINT GOBAIN, UNIBAIL-RODAMCO, VIVENDI, VOLKSWAGEN.

5.1.1. Portfolios

The portfolios constructed to examine the DRC measure are created depending on two features: type of asset and weighting rule for each asset of the portfolio.

Concerning the type of assets that compound the portfolio, those portfolios could be sorted as:

Corporate debt portfolio could be classified depending on the use of country as second factor (portfolio 1.1), or industry (portfolio 1.2). When equity is the main element of the portfolio, this distinction between the systematic factors that is combined with the global factor, that could be a country (portfolio 2.1) or a industry factor (portfolio 2.2) is also developed. A portfolio made of sovereign debt is the fifth alternative in the type of assets (portfolio 3). In relation to portfolio 3, the second factor employed as systematic factor is a country factor. With the aim of showing the advantages of diversification, sovereign and corporate debt are combined creating the portfolio 4. The last considered portfolio combines equity and corporate and sovereign debt setting up the portfolio 5.

On the other hand considering the weights of each assets of the portfolio, four types of portfolio are considered. First an equally-weight portfolio is considered (portfolio A) than is going to be the benchmark in order to considered high or low a DRC measure for a given portfolio. The second portfolio is the minimum variance allowing short positions (portfolio B) and the weights are the consequence of using the following expression from the minimization of the variance of the portfolio:

$$\underset{1xn}{w} = \frac{1_{1xn} * \Sigma^{-1}}{1_{1xn} * \Sigma^{-1} * 1_{nx1}}$$

Where Σ is the matrix of covariance of the assets than compounds the portfolio, 1_{lxk} is a matrix of ones those size has l rows and k columns.

The third portfolio is a similar one to the second, however the long positions are the only considered. This portfolio (portfolio C) try to reflect the fact that short positions are not always allowed in the markets.

The last deemed portfolio (portfolio D) try to minimize the amount of the required DRC. For this propose, the portfolio is mainly composed by the assets of higher rating, that implies that the probability of default of each isolated assets is very low (lower than 1‰) and the weighted are obtained minimizing the default correlation matrix following equation (4), in order to obtain the most independent default possible.

The distribution of pair wise correlation of default between different assets is showed in figure 9 using country as a second systematic factor. Figure 10 and 11 presents a similar image just for corporate assets under a country or an industry second systematic factor. The estimated parameters are obtained under the quantile regression approach using a q=0.2 due to the fact that if the returns of some assets fall into the first quintile they default, like for instance Alpha Bank (see table 2 and 3). The presentation in the three figures is similar, in the left side the cumulative distribution of the correlation with the main statistical values (mean, mode and median), in the right side an histogram of those values is deemed, approximating their density function using a kernel function smoothed by a moving average filter. Also the main statistical values are again showed in the right graph.

As a curiosity, it is remarkable to point out that the highest default correlation between assets is in bank sector. Using country as a second systematic factor, those assets are SANTANDER and BBVA, while if the sector factor is employed the assets are BBVA and SOCIETÉ GENERALE.

As can be observed in Figure 9, mean, mode and median are very close between then, around 0.15. The histogram values near zero correlation are quite high compared to the near correlation values like -0.05 or 0.05. If figure 10 or 11 is observed, one could deduce that the introduction of sovereign debt creates this peak around zero correlation.

[Insert Figure 9 here]

Using an industry factor as a second systematic factor for the corporate debt rises a lower mode and higher mean and median than using a country factor. This could be interpreted as the lower importance of industry in terms of correlation between assets than the country factor. When a country index falls down instead of an industry factor, the sensibility of the assets is higher.

[Insert Figure 10 here]

[Insert Figure 11 here]

In order to confirm this interpretation in Figure 12 shows the number of defaults in a total of 50000 simulation of the corporate assets returns, the minimum required by the

CNMV (Comisión Nacional del Mercado de Valores)[40]. As this figure displays, the number of simulation where none assets defaults is similar under both approaches. However, conditioned to a default, the number of defaults the 60 percentile number of defaults under the industry factor approach is lower than the country approach. Moreover if the 99.9% worst case scenario is regarded concerning the number of defaults, using a country factor almost fifteen firms default while using an industry factor the number is just twelve.

[Insert Figure 12]

5.2. Algorithm

A small algorithm without going into details about the process to generate the Profit &Loss (P&L) distribution of the portfolio is presented in this section.

The process to obtain the DRC of a given portfolio can be divided into five steps. In the first one the data is well-polished, getting the returns and fulfilling the missing data like in the zero step presented in 3.3.5 Simulation process. Additionally to that, the values of the structure parameters of the returns are estimated via maximum likelihood under the assumption of normal distribution for the perturbation of first order autoregressive followed by the assets. These parameters are required in order to acquire the annual mean and standard deviation from the assets and the factors.

In a second step the parameters of the simulating process ($\rho, \omega_{1,i}, \omega_{2,i}$) are estimated following a quantile distribution, the quantile applied for the regression depends on the probability of default of the assets that compound the portfolio. For instance if a high risk assets is included a quantile of 0,2 could be used given the fact that its probability of default is between 5% and 30 %, so if the regression for the 20% lower returns are robustly estimated, a good estimation for the parameters of equation (2) for the aim of measure the default events is obtained. It will be showed in the parameter sensitivity that the chosen quantile for the measure of the DRC is not such important as other parameters that are indefinite like the influence of the systematic factor in the recovery rate.

The third step simulates the assets returns, determinates the default events and it exposure at default. Once the uniform of the variables have been generated, they are inverse

[40]CNMV is the Spanish regulator for investment funds. For pricing OTC instruments, they advise Montecarlo methods using at least 50.000 simulated scenarios http://www.rdmf.es/wp-content/uploads/2015/02/comunicacion.pdf

into normal distribution variables. The assets returns are the result of employing the expression:

$$r_{i,m} = \rho_i * \frac{\omega_{1,i} * \sigma_1 * f_{1,m} + \omega_{2,i} * \sigma_2 * f_{2,m}}{\sqrt{\omega_{1,i}^2 * \sigma_1^2 + \omega_{2,i}^2 * \sigma_2^2}} + \sqrt{1-\rho_i^2} * \epsilon_{i,m}$$

Where: $r_{i,m}$ is the simulated return for asset i in the simulation m,$f_{1,m}$ and $f_{2,m}$are the simulated systematic factors in simulation m, $\epsilon_{i,m}$ is the simulated idiosyncratic factor for the assets i in the m simulation. σ_1 and σ_2 are the annual standard deviation for the systematic factors.

Once the assets are simulated, the default event depends on equation (1), if the value of the function V_i in the m simulation is below zero, the default event occurs, otherwise it does not.

The fourth step is the simulation of the loss given default following equations (19) and (20). It is necessary to point out the importance of the influence of the global factor in the recovery rate. It is a crucial feature as will be showed in the parameters sensitivity section. The Figure 13 could useful to raise awareness about this key parameter for the LGD. In the graph the LGD for a 50000 simulations of the French telecom SFR using different values for the influence of the global factor in the recovery is presented. As can be seen, for higher values for this parameter the LGD has more situations where the loss given default is 100%. A kernel function is employed to see the shape of the LGD depending of the influence of the global factor.

[Insert Figure 13]

Finally the last step consist in combined the default events and the EAD of the third step and the LGD of the fourth step in order to generate a P&L distribution for default risk. Once these information is combined the 99.9 percentile is fixed in order to know the DRC. Note that current capital requirements for the DRC are not just the VaR at 99.9% in a one-year horizon. The DRC capital requirements is the maximum between the average DRC measures over the previous twelve weeks (i.e. three months) and the most recent DRC assessment.

5.2.1. Parallel Computing technique

In order to speed up the assessment of the default event for the assets and the recovery rate simulation, the parallel computing tool available in MATLAB is employed. This tool reduced the elapse time for the default event (step four in the algorithm previously presented) from four hours to an hour and a half or even less (around eighty minutes for some assets), while for recovery rate the time is reduced to half an hour from almost two hours if the script is computing without parallel computing using a cluster with two cores.

The aim of this sub-section is to present the concept of parallel computing and provide some advice about the process in which this tool should be employed.

To begin with, the main idea of parallel computing is to perform many calculations at the same time, dividing a large problem into smaller ones that are solved simultaneously. Formally speaking parallel computing can be defined as the process of programming instructions by dividing them among numerous processors with the goal of saving time in a program running. Parallel computing is widely used in financial industry, not only as a way to use at full capacity the memory resource of the computer, but also as a way of combining a group of computer to work together closely via network , which is known as cluster computing. Each core of the processor computers a portion of the task, and the results are gathered to produce the final results.

Some tips about the parallel computing tool in MATLAB are at this point provided, once that the goal and the concept of this technique have been explained. These tips are based on technical notes written by Mier and Chow (2010). The code matlabpool(‗open',#) inform MATLAB that multiple processors are going to be employed to run the script, specifying in the # the number of cores that are going to be utilized in the parallel computing technique. In the particular case of this article, two cores have been employed to run the script. In order to tell MATLAB to close parallel computing the code matlabpool(‗close') must be introduced. In order to employ the parallel computing in a loop the code for must be changed into parfor, where the loop variable of the parfor must be a vector of consecutive numbers. Moreover, the length of the loop variable must be defined as an integer outside the loop, for instance, the following code will not be run in parallel computing:

```
Load (‗variable.mat')
    parfor i=1:size(variable,2)
        (code (i) )
    end
```

In order to run in parallel computing, the length of the loop variable (i) must be defined outside the parfor:

```
Load (_variable.mat‘)
a=size(variable,2);
        parfor i=1:a
                (code(i))
        end
```

It is important to point out that the parfor code is designed for problems where each iteration is independent of each other iteration. This independence is a crucial idea for using the parallel computing technique. It is also relevant to be aware that a parfor code cannot have inside another parfor.

Finally, a last tip is provided in order to obtain a lower elapse time in the script performance. The fact of creating the matrices before enter into a loop code is a way of allocate memory that speed up the assessments. The memory problems have been an important issue when the code has been run. In order to fix the out-of-memory errors, the virtual memory has been expanded[41] and the workspace have been saved each time the quantile or the influence of the global factor in the recovery rate have been changed using either pack function or saving manually when the workspace has a size bigger than 2 GB.

This brief section has tried to explain the main features of the parallel computing in order to ease the implant of this useful technique. Moreover the last consideration about the memory problems is an issue that the programmer has to be aware of in a computing task so demanding and which so many iterations as assessing a high quantile credit value at risk.

5.3. Parameter sensitivity

This section a parameter sensitivity approach is carried out in a wide sense. To begin with, a series of three dimension graphs are showed in order to discern if the influence of the global factor in the recovery is more important than the quantile employed in the regression, which affect the correlation of default between assets. As a consequence of so many iterations (a grill of six different values for the quantile and six different values of the influence of the global factor in the recovery rate are carried out, making a total of 36 different values at risk for each portfolio) lack of virtual memory problems arise. The recommendations suggested in the previous section about memory saving tips have been

[41] http://es.mathworks.com/help/matlab/matlab_prog/resolving-out-of-memory-errors.html#brh72ex-54

employed for dealing this issue. The number of simulation for this analysis is 50000. However, in the probability of default and the recovery rate sensibility the simulation is fixed in 100000 in order to obtain more robust results.

Then, a change in the probability of default is also considered. The probability of default is changed a basic point, developing a value at risk with higher and lowers PDs, but always having in mind the lower bound of three basic points of the PDs established by the Committee. Given the fact that the lower assets have a probability of default around 20-30% (see table 2), in order to have robust estimations a quantile of 20 is employed in this subsection. Also, the influence of the global factor in the recovery is fixed ad hoc at 50%

In a third subsection a more precise analysis of the change in the influence of the global factor in the recovery rate is regarded.

Note that for each subsection, in order to make a reasonable comparison between the VaR values if a parameter is changed, the same seed has to be employed, otherwise the comparison is not feasible.

Finally, in the last subsection of the parameter sensitivity the estimation method is assayed. It should be questioned if the quantile regression employing a value of 20 is statistically different form the values employing an OLS regression. Lastly, the data time interval (monthly or quarterly) is compared in order to discern if for the analyzed sample it is an important feature. In this last subsection a semiparametic contrast is deemed using a boostraping technique.

5.3.1. Different levels of quantiles and the influence of the global factor in the recovery rate

A grill of values for the quantile in the regression is chosen for zero to the median regression. Additionally, the influence of the global factor in the recovery is also grilled from zero to one into intervals of 0.2, where zero means independence of the recovery rate from the probability of default and one is the maximum dependence on the recovery rate of the probability of default via a global factor.

As a result, three-dimensional graphs are showed, where one can distinguish the importance in DRC on the dependence of the recovery rate of the probability of default. The chosen quantile keeps in a secondary level compared to the influence of the global factor in the recovery for all the analyzed debt portfolios. The change on the quantile level does not modify considerably the DRC assessment. The difference in a percentage on the nominal value of the portfolio between the quantile zero to the quantile 50 is lower than a hundred basis points.

[Insert Figure 14 here]

Considering the equally-weighted portfolios, the difference between the zero influence from the global factor and the dependence from the global factor is conditional of the type of assets. For corporate bonds its minimum value is a three per cent while the maximum around eight per cent. For sovereign debt the range of 300 basis points between the minimum (five per cent) and the maximum (eight per cent) values of DRC.

[Insert Figure 15 here]

Concerning the minimum variance portfolios, the corporate portfolio DRC is between 14 % and 24%, a thousand basis points between both values. The highest range between DRCs conditioned to the influence on the recovery rate of the global factor. If only long positions are available, the minimum is a three per cent and the maximum only a six per cent. However the lower values for the corporate debt portfolios is the weighted by rating portfolio that is compounded minimizing the default correlation. Those values are between less than one per cent and two percent.

5.3.2. Changes in the probabilities of default

In this section the change in the DRC when the probabilities are changed in a basis point. Table 4 exhibit the growth of the DRC when the probabilities of default are modify a basis point, always having in mind the lower bound of three basis points established by the Committee. This is the reason why the DRCs of the sovereign debt do not decrease if the PDs fall a basis point. The fact that having a PD of three basis points means that no decrease of PD can be done.

[Insert Table 4 here]

The minimum default correlation weighted by rating portfolio is the most sensitive portfolio to a basis point change in the PDs, followed by the minimum variance allowing only long positions portfolio. The equally weighted and the minimum variance portfolios are the less sensitive to a change of the PD.

[Insert Table 5 here]

The huge variability of the DRC in some portfolios when the probability of default changes in a basis point shows the problem of the cliff effect that was mentioned in the subsection ―Challenges" of the section ―Default Risk Charge as a risk measure". Obviously, a common seed has been employed to measure the change in the DRC. The values of the

DRC for the portfolios with the original PDs are exhibited in Table 10. For instance, using country as a second systematic factor, the increase of the DRC is lower in the equity case than in the debt case. This does not implies that the DRC of the corporate debt is higher than the DRC of the corporate equity portfolio if the probability of default increases a basis point, because of the different initial value of the DRC with the original probability of default.

[Insert Table 10 here]

5.3.3. Changes in the influence of the global factor in the recovery rate

A change in the influence of the global factor in the recovery rate is considered in this section. Taking in account that the influence is fixed ad hoc in 0.5, the change of this parameter is a key issue due to the lack of information about the possible value that could achieve.

The most remarkable feature about the Table 6 is the fact that with a lower influence of the global factor in the recovery the sovereign debt DRC increases and if the influence is higher, the DRC for the sovereign debt decreases. This is a consequence of the relationship between the sovereign debt and a global factor such as a stock index as the Eurostoxx 50. When the economy growth is high, the stock index is increasing and the demand of sovereign debt falls due to the low profitability that this kind of asset offers. As a consequence, the price falls down and the returns also decrease. So the lower returns for the sovereign debt occur when the probability of default is lower and “the things are going well”. On the other hand, if the economy collapses, the stock index falls down and the investors run for shelter in the sovereign debt, and then the price of the sovereign debt increases and the same happens with the returns. Basically, the main idea is the countercyclical behavior of the sovereign debt. This feature makes that the increase of the influence of the global factor in the recovery supposes a decrease of the sovereign debt DRC, while in a corporate debt DRC generates the opposite effect. A wise choice between corporate and sovereign debt in the composition of the portfolio could achieve a hedge against a change of the influence of the global factor in the recovery rate. As an example, the debt portfolio choosing the assets following the lower default correlation weighted by rating arise that if the influence changes from 0.5 to 0.25 or 0.75 the DRC is modified in less than a one per cent.

[Insert Table 6 here]

The best performance of the portfolios in terms of being more stable to the changes of the influence of the global factor in the recovery rate is achieved by the lower default correlation portfolio weighted by rating, where the variation is very low. The highest variation on this type of portfolio arises in the sovereign portfolio, that it is modified a 36 per cent if the influence of the global factor is 1. However it is important to take in account the initial value for the DRC of this portfolio in Table 10, which is the lowest value in the entire table, with only 0.36 for a nominal of 100 euros. Concerning that in absolute terms, a variation of 36 per cent is not quite relevant.

The most stable portfolios in terms of modifying the influence of the global factor in the recovery seem to be the lower default correlation portfolio weighted by rating followed by the minimum variance portfolio.

5.3.4. Study about the different estimation methods

In this section the estimation method is evaluated. For this purpose a semiparametric test for finding significantly differences between the estimated values employing different interval time data or a OLS regression over a quantile regression with q=0.2. Additionally, the DRC changing the estimated parameters in order to determinate if changing the regression method or the interval time data has important effects in the assessment.

The semiparametric test is based on the boostraping method. The model for the returns that is in equation (2) is not questioned. Also the independence of the idiosyncratic factor and its distribution as a normal variable are considered axiom on this test. Returns are simulated based on the estimated values for the parameters (in the first case using OLS regression or quantile regression with q=0.2, or in the second factor using monthly data or quarterly data in order and employing a quantile regression with q=0.2) and the systematic factor data. The simulation is created when a normal variable vector of the same length than the data serie is generated to fulfill the part of the idiosyncratic factor. There were generated a hundred vectors of idiosyncratic factor for each asset. The each one of the hundred simulated returns series for each asset is again estimated. The fact of introducing noise in the idiosyncratic factor allows generating an empirical distribution of the estimated parameter, based on the previous axioms.

The null hypothesis and the alternative one are in the first case:

$$\begin{cases} H_o: \widehat{\omega}_{1,i}^{Q} - \widehat{\omega}_{1,i}^{OLS} = 0 \\ H_1: \widehat{\omega}_{1,i}^{Q} - \widehat{\omega}_{1,i}^{OLS} \neq 0 \end{cases}$$

$$\begin{cases} H_o: \widehat{\omega}_{2,i}^{Q} - \widehat{\omega}_{2,i}^{OLS} = 0 \\ H_1: \widehat{\omega}_{2,i}^{Q} - \widehat{\omega}_{2,i}^{OLS} \neq 0 \end{cases}$$

$$\begin{cases} H_o: \hat{\rho}_i^{Q} - \hat{\rho}_i^{OLS} = 0 \\ H_1: \hat{\rho}_i^{Q} - \hat{\rho}_i^{OLS} \neq 0 \end{cases}$$

On the other hand, in the second case the null hypothesis and the alternative one are:

$$\begin{cases} H_o: \hat{\omega}_{1,i}^{Q,mont\,hly} - \hat{\omega}_{1,i}^{Q,quarterly} = 0 \\ H_1: \hat{\omega}_{1,i}^{Q,mont\,hly} - \hat{\omega}_{1,i}^{Q,quarterly} \neq 0 \end{cases}$$

$$\begin{cases} H_o: \hat{\omega}_{2,i}^{Q,mont\,hly} - \hat{\omega}_{2,i}^{Q,quarterly} = 0 \\ H_1: \hat{\omega}_{2,i}^{Q,mont\,hly} - \hat{\omega}_{2,i}^{Q,quarterly} \neq 0 \end{cases}$$

$$\begin{cases} H_o: \hat{\rho}_i^{Q,mont\,hly} - \hat{\rho}_i^{Q,quarterly} = 0 \\ H_1: \hat{\rho}_i^{Q,mont\,hly} - \hat{\rho}_i^{Q,quarterly} \neq 0 \end{cases}$$

Then, the sorted estimated values for each asset using a different technique or a different interval time data are subtracted. The difference between the estimated parameters for asset I would be these three vectors in the first case:

$$\left(\hat{\omega}_{1,i}^{1,Q}, \hat{\omega}_{1,i}^{2,Q}, \dots, \hat{\omega}_{1,i}^{100,Q}\right) - \left(\hat{\omega}_{1,i}^{1,OLS}, \hat{\omega}_{1,i}^{2,OLS}, \dots, \hat{\omega}_{1,i}^{100,OLS}\right) = \left(\hat{\omega}_{1,i}^{1,Q-OLS}, \hat{\omega}_{1,i}^{2,Q-OLS}, \dots, \hat{\omega}_{1,i}^{100,Q-OLS}\right)$$

$$\left(\hat{\omega}_{2,i}^{1,Q}, \hat{\omega}_{2,i}^{2,Q}, \dots, \hat{\omega}_{2,i}^{100,Q}\right) - \left(\hat{\omega}_{2,i}^{1,OLS}, \hat{\omega}_{2,i}^{2,OLS}, \dots, \hat{\omega}_{2,i}^{100,OLS}\right) = \left(\hat{\omega}_{2,i}^{1,Q-OLS}, \hat{\omega}_{2,i}^{2,Q-OLS}, \dots, \hat{\omega}_{2,i}^{100,Q-OLS}\right)$$

$$\left(\hat{\rho}_i^{1,Q}, \hat{\rho}_i^{2,Q}, \dots, \hat{\rho}_i^{100,Q}\right) - \left(\hat{\rho}_i^{1,OLS}, \hat{\rho}_i^{2,OLS}, \dots, \hat{\rho}_i^{100,OLS}\right) = \left(\hat{\rho}_i^{1,Q-OLS}, \hat{\rho}_i^{2,Q-OLS}, \dots, \hat{\rho}_i^{100,Q-OLS}\right)$$

Analogously, for the second case would be:

$$\left(\hat{\omega}_{1,i}^{1,Q,mont\,hly}, \hat{\omega}_{1,i}^{2,Q,mont\,hly}, \dots, \hat{\omega}_{1,i}^{100,Q,mont\,hly}\right) - \left(\hat{\omega}_{1,i}^{1,Q,quarterly}, \hat{\omega}_{1,i}^{2,Q,quarterly}, \dots, \hat{\omega}_{1,i}^{100,Q,quarterly}\right) = \left(\hat{\omega}_{1,i}^{1,mont\,hly-quarterly}, \hat{\omega}_{1,i}^{2,mont\,hly-quarterly}, \dots, \hat{\omega}_{1,i}^{100,mont\,hly-quarterly}\right)$$

$$\left(\hat{\omega}_{2,i}^{1,Q,mont\,hly}, \hat{\omega}_{2,i}^{2,Q,mont\,hly}, \dots, \hat{\omega}_{2,i}^{100,Q,mont\,hly}\right) - \left(\hat{\omega}_{2,i}^{1,Q,quarterly}, \hat{\omega}_{2,i}^{2,Q,quarterly}, \dots, \hat{\omega}_{2,i}^{100,Q,quarterly}\right) = \left(\hat{\omega}_{2,i}^{1,mont\,hly-quarterly}, \hat{\omega}_{2,i}^{2,mont\,hly-quarterly}, \dots, \hat{\omega}_{2,i}^{100,mont\,hly-quarterly}\right)$$

$$,\left(\hat{\rho}_i^{1,Q,mont\,hly}, \hat{\rho}_i^{2,Q,mont\,hly}, \dots, \hat{\rho}_i^{100,Q,mont\,hly}\right) - \left(\hat{\rho}_i^{1,Q,quarterly}, \hat{\rho}_i^{2,Q,quarterly}, \dots, \hat{\rho}_i^{100,Q,quarterly}\right) = \left(\hat{\rho}_i^{1,mont\,hly-quarterly}, \hat{\rho}_i^{2,mont\,hly-quarterly}, \dots, \hat{\rho}_i^{100,mont\,hly-quarterly}\right)$$

Hence, a histogram of differences between estimated values can be exhibited, and looking at the 5, 25, 75 and 95 percentile, the test is performed. If the zero is outside the range between the 5 and the 95 percentile, then the null hypothesis is rejected in a strict sense, giving the fact that just a 10% of the empirical distribution is obviated. While if the zero value is inside the 5-95 percentile range but outside the 25-75 percentile range the null hypothesis could be rejected in a weak sense but not in a stronger one. If zero is inside the 25-75 percentile range, the null hypothesis is not rejected.

As an example, the distributions of the subtraction of the estimated parameters are exhibited for the BBVA. In Figure 18 the first case is analyzed and in the figure 19 the second one.

[Insert Figure 18 here]

[Insert Figure 19 here]

The following subsection performs a brief resume of the test and the change of the DRC considering the estimated parameters. Due to the huge number of contrasts carried out the values for the percentiles are not showed in this article, only the most relevant conclusions are presented.

5.3.4.1. OLS against quantile regression

Establishing country as second factor, a total of 26 corporate assets over 156 reject the null hypothesis for at least one of their estimated parameters. The number of assets which reject the null hypothesis depending on the contrast is: 21 for the first factor, 11 for the second one and 18 for the rho estimated parameter. Within the 26 assets, the 69,23% of them were speculative assets.

Establishing sector as second factor, a total of 20 corporate assets over 156 reject the null hypothesis for at least one of their estimated parameters. The number of assets which reject the null hypothesis depending on the contrast is: zero for the first factor, 13 for the second one and 16 for the rho estimated parameter. Within the 20 assets, the 70% of them were speculative assets.

Finally, for the sovereign assets, a total of 5 assets over 11 reject the null hypothesis for at least one of their estimated parameters. The number of assets which reject the null hypothesis depending on the contrast is: 2 for the first factor, 5 for the second factor and zero for the rho estimated parameter. Within the 5 assets, the 60% of them were speculative assets.

5.3.4.2. Monthly data against quarterly data

Establishing country as second factor, a total of 74 corporate assets over 156 reject the null hypothesis for at least one of their estimated parameters. The number of assets which reject the null hypothesis depending on the contrast is: 47 for the first factor, 25 for the second one and 62 for the rho estimated parameter. Within the 74 assets, the 70,27% of them were speculative assets.

Establishing sector as second factor, a total of 54 corporate assets over 156 reject the null hypothesis for at least one of their estimated parameters. The number of assets which reject the null hypothesis depending on the contrast is: zero for the first factor, 45 for the second one and 42 for the rho estimated parameter. Within the 54 assets, the 68,51% of them were speculative assets.

Finally, for the sovereign assets, a total of 6 assets over 11 reject the null hypothesis for at least one of their estimated parameters. The number of assets which reject the null hypothesis depending on the contrast is: zero for the first factor, 5 for the second factor and 3 for the rho estimated parameter. Within the 5 assets, the 33,33% of them were speculative assets.

Taking everything into account final considerations and comments about the results of the contrasts are now considerer. First of all, for both types of contrast, either the OLS against quantile regression or the monthly against quarterly data, the mayor type of asset that reject at least one of the null hypothesis were speculative assets, defined as those assets which rating is BBB or below that. Moreover all the contrasts were made using the interquantile range. If the percentile 5 and 95 were employed most of the assets would have not rejected the null hypothesis. Finally, it is always important to keep in mind the assumptions on which these contrasts rest such as the assumption that the model is the right one.

The variation of the DRC if the OLS estimated parameters are employed could lead a variation of the measure lower than the 20%, while if the quarterly data is employed to obtain the DRC measure, the new DRC could be twice the value obtained with monthly data. This should be make us aware of the variability that the DRC has and the importance of having a more precise specification of the estimation process by the Committee, otherwise the variability of the capital requirements could suppose a serious drawback of this measure.

[Insert Table 7 and 8 here]

5.4. Variance reduction techniques

In this section, the goal is reducing the variance of the DRC assessment using different types of techniques. There were simulated a hundred times the DRC for each portfolio in order to obtain consistent means and standard deviation. The sort in each subsection is similar. First the technique is presented, showing the advantages and disadvantages of using it. Once the idea of employing that technique is clear, an empirical performance is commented in the following table. A hundred thousand simulated returns would be employed in order to get the empirical distribution of the credit losses. In addition to this, the computation elapse time is also showed.

[Insert Table 11 here]

[Insert Table 12 here]

This section tries to have a general idea about variance reduction techniques for the DRC measure, however there are more variance reduction techniques that are not showed in this part such as Importance Sampling MC or Quasi-Montecarlo methods. Other methods as the latin hypercube presents some drawback as the fact that in large portfolios the application is not feasible. Moreover, the matching moments methods does not assured the reduction of the variance, so in this section is not employed. The main goal of the article is not focused on the variance reduction methods but the model process for the DRC, so this section is only preview of possible methods in order to obtain a more accurate VaR value for this risk measure. Using the following method, the half of the considered portfolios does not reduce its variance, while for the others the variance is reduced a 40% for some portfolios, and between a 10% and 4% for most portfolios. These conclusions are deemed given the fact that the variance of the portfolios under the antithetic technique should be multiplied by two due to the reasons that are explained in the next section.

5.4.1. Antithetic Variates

The antithetic variates technique tries to reduce the variance of the estimation by introducing negative dependence between pairs of replications. Considering a vector of uniform random variables for the simulated factor k $(U_1^k, \dots U_M^k)$ that are going to be employed in order to simulate each component of the asset returns, the symmetry distribution $(1 - U_1^k, \dots, 1 - U_M^k)$ is also taking into account under the antithetic variates technique, where M is the number of simulation, in our case fifty thousand. The $N^{-1}(U_1^k)$ *and* $N^{-1}(1 - U_1^k)$ have the same magnitude but the opposite sign given the fact that the normal distribution is employed for simulated the marginal distribution of the factors. This suggests that an outstanding output computed by the first vector may be balance by the values computed by the antithetic path. Those vectors are going to be generated a hundred times in order to have an estimation of the mean and the standard deviation. From now on, the $(U_1^k, \dots U_M^k)$ is going to be known as the positive seed and $(1 - U_1^k, \dots, 1 - U_M^k)$ as a negative seed.

An empirical P&L distribution of the credit risk is then computed using both seeds.

The percentile 99.9% is observed once that the empirical distribution of the credit losses is computed. There are then two value-at risk-measures, one for each seed (DRC_n^+, DRC_n^-), the average value-at-risk is the mean of both values$(\frac{DRC_n^+ + DRC_n^-}{2} = DRC_n^{AV})$. The subscript n represents the original employed seed, n rages from one until a hundred. So the mean of the DRC and the variance of the DRC would be without antithetic variates:

$$DRC_{PLAINMC}{}^{MEAN} = \sum_{i=1}^{N=100} \frac{DRC_n^+}{100}$$

$$DRC_{PLAINMC}{}^{VARIANCE} = Var(DRC^+)$$

On the other hand, DRC_n^{AV} can be deemed a sample mean of n independents observations of the value-at-risk measure:

$$(DRC_1^{AV}, \dots, DRC_n^{AV}, \dots, DRC_{100}^{AV}) = (\frac{DRC_1^+ + DRC_1^-}{2}, \dots \frac{DRC_n^+ + DRC_n^-}{2}, \dots, \frac{DRC_{100}^+ + DRC_{100}^-}{2})$$

The antithetic variate estimation is preferred to an ordinary Monte Carlo estimator based on independent replications when:

$$2 * DRC_{ANTITHETICVARIATE}{}^{VARIANCE} < DRC_{PLAINMC}{}^{VARIANCE}$$

Note that the variance of the plain MC is multiplied by two due to the fact that in antithetic variate technique the number of simulations is twice the number of simulations in the plain MC.

The variance of the DRC under the antithetic technique is:

$$DRC_{ANTITHETICVARIATE}{}^{VARIANCE} = Var\left(\frac{DRC^+ + DRC^-}{2}\right) = \frac{Var(DRC^+ + DRC^-)}{4}$$

$$= \frac{Var(DRC^+) + Var(DRC^-) + 2 * Cov(DRC^+, DRC^-)}{4}$$

Where if $DRC^{+} and DRC^{-}$ are independently and identically distributed then $Var(DRC^{+}) = Var(DRC^{-})$and the variance would be $DRC_{ANTITHETICVARIATE}{}^{VARIANCE} = \frac{DRC_{PLAINMC}{}^{VARIANCE}}{2} = \frac{Var\,(DRC^{+})}{2}$.

Therefore, the condition for antithetic sample to reduce variance arises from the fact that the value-at-risk calculated with the positive and negative seed are not lower independent, and the covariance $Cov(DRC^{+}, DRC^{-})$ is negative.

5.5. Further consideration: copulas

Following the subsection of ~~"~~3.3.3.2 Rank correlation issue: another copula family is possible" and the proposed model section ~~"~~3.3.4.2 ~~"~~Case of Gaussian marginal distribution and hierarchical nested multivariate copula, mixing Clayton copula and Gaussian copula" a different value of DRC is performed using a Clayton copula between the systematic factors. As a consequence, the simulated returns are no longer normal distributed, so the empirical distribution should be computed. Consequently, the value of λ instead of being obtained for the inverse distribution of the normal is obtained from the inverse of the returns empirical distribution.

The value of the alpha parameter of the Clayton copula is estimated following the relation between the alpha and the Kendall's tau. Then, the DRC is calculated and is showed in the following table as the growth of the DRC without rank correlation.

[Insert Table 9 here]

The use of the Clayton copula supposed an increase in the DRC for all the portfolios considered. The lower increase is in the minimum variance portfolio followed by the lower default correlation weighted by rating. This change on the multivariate distribution of the components of the returns supposes an increase between a 10% and a 125% depending on the type of portfolio and its composition. The fact of employing a copula with the industry factor instead of country factor looks that arise higher relative increase for the portfolios. Moreover the sovereign debt portfolio seems the most stable DRC portfolio if a Clayton copula is employed between the systematic factors, due to the fact that the increase is just between 10% and 15% depending on the weights for each sovereign debt.

[Insert Figure 18 here]

In figure 18 the DRC of the different portfolios is deemed. The red section of the column corresponds to the increase of the DRC due to the fact of employing a Clayton copula in the systematic factors.

6. Conclusion

The present thesis looks for a model approach to the Basel 4 measure known as Default Risk Charge. In order to achieve this goal, the history of the Basel commitments seems to be a previous key step to understand the dynamics that roll the financial regulation process. Though the drawbacks and the disadvantages of the previous measure, the DRC could be understood as a enhance measure to catch tail events concerning credit risk for trading book positions.

Taking in account the recommendations of the Committee and using as starting point the article of Wilkens and Predescu (2016), a bifactorial model for the default event is proposed following Pykhtin (2004) and a recovery rate related to the probability of default is also conceived using a beta distribution and a link between the recovery rate and the probability of default though a global factor. The novelty resides in the estimation using a quantile regression, an approach understudies to date. The main advantage of this choice is the robust estimation for the left tail values of the distribution of the returns, where the default occurs.

With the aim of determinate a period of stress for the data a DECO approach is employed, which has better properties than DCC due to the fact that DECO approach drawing on broader information set when formulating the correlation process of each pair. The drawback of DCC compared to DECO is it failure in capturing the information pooling aspect.

In the sensibility analysis, a broader set of portfolios are considered standing out a portfolio made of investment grade assets where the weights are choosing in order to minimize the default correlation between them. This portfolio seems to have a good behavior, robust values than do not suffer from the cliff-effect when a change in the probability of default or in the influence of the global factor in the recovery rate is considered. Relating to the DRC and the consequences of the relation between the PD and the recovery on the measure, the article concludes that a wise choice of corporate and sovereign debt in the portfolio could lead to a hedge against changes in the parameter of the influence of the global factor in the recovery, which has so much lack of information about it. Also it is the portfolio that has the lower increase when a copula approach is employed.

Indeed, a Clayton copula is employed in the systematic factors in order to obtain a more realistic simulation of the returns. This copula has implied 10% higher values of the DRC, which could trouble us due to possible insufficient capital requirements for DRC using a model without taking in account rank correlation. This is an important result that should be taking into account when the effectiveness of measure is analyzed.

Stands of work that could go deeper in some features that are analyzed in this article could be related to more variance reduction techniques or quantile regression. Actually, studying a specific quantile for the regression in order to get the most accurate estimation of the parameters seems an interesting field.

Also the quantile regression could be combined with a regression discontinuity design (RDD), where depending on the value of the simulation of a discriminating variable, a sort of estimated parameters or another are employed to simulate returns, and the feature of a different correlation of default depending on the economic cycle could be reflected in the simulation. The discriminating variable should be one that performs the state of the economy such as a global factor.

Moreover, extending the analysis to global portfolios instead of the present Euro zone portfolios is an interesting line of work.

Furthermore it is necessary to point out the lack of information about the recovery rate that is present in this article. An estimation of the parameters of the recovery should be employed in order to have a deeper knowledge about the data that is being employed.

Finally, it is necessary to point out that the process of building a model approach of the DRC under the internal approach must be aware of the Committee steps on IRB approach, in order to unify and build a consistent framework that could prevent from inconsistencies.

To sum up, this thesis starts from the article of Wilkens and Predescu (2016) in order to build a consistent framework for the DRC model and, through a series of innovations such as quantile regression, DECO and copulas concludes that the two most important features are the influence of the global factor in the recovery and the use of copulas in the simulation of the systematic factors.

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8. Appendix

8.1. Appendix of tables

	INDUSTRY			
	mean	*std. Deviation*	*alpha*	*beta*
FOOD	**0,692**	**0,400**	**0,230**	**0,102**
MINING	**0,623**	**0,346**	**0,599**	**0,363**
OIL	**0,545**	**0,369**	**0,448**	**0,374**
CLOTHES	**0,625**	**0,345**	**0,606**	**0,363**
CONSUMER DURABLES	**0,605**	**0,396**	**0,317**	**0,207**
CHEMICAL	**0,698**	**0,373**	**0,360**	**0,156**
DRUGS	**0,594**	**0,422**	**0,210**	**0,144**
CONSTRUCTION	**0,584**	**0,399**	**0,307**	**0,219**
STEEL	**0,551**	**0,410**	**0,260**	**0,212**
FABRICATED PRODUCTS	**0,709**	**0,376**	**0,326**	**0,134**
MACHINERY	**0,624**	**0,375**	**0,417**	**0,251**
AUTOMOTIVE	**0,657**	**0,385**	**0,342**	**0,178**
TRANSPORT	**0,517**	**0,362**	**0,468**	**0,437**
UTILITIES	**0,864**	**0,259**	**0,649**	**0,102**
RETAIL	**0,540**	**0,403**	**0,286**	**0,244**
FINANCIAL	**0,564**	**0,417**	**0,234**	**0,181**
OTHER	**0,561**	**0,397**	**0,316**	**0,247**

	SENIORITY			
	mean	*std. Deviation*	*alpha*	*beta*
SENIOR SECURED	**0,635**	**0,340**	**0,638**	**0,367**
SENIOR SUBORDINATED	**0,294**	**0,335**	**0,250**	**0,600**

SENIOR UNSECURED	**0,486**	**0,375**	**0,377**	**0,399**
JUNIOR OR SUBORDINATED	**0,274**	**0,343**	**0,189**	**0,502**

	SOVEREIGN			
	mean	***std. Deviation***	***alpha***	***beta***
BANKRUPCY	**0,909**	**0,132**	**3,406**	**0,341**

Table 1: Values of alpha and beta for the beta distribution

	Corporate	***Sovereign***
AAA	**0,03%**	**0,03%**
AA+	**0,03%**	**0,03%**
AA	**0,03%**	**0,03%**
AA-	**0,03%**	**0,03%**
A+	**0,06%**	**0,03%**
A	**0,07%**	**0,03%**
A-	**0,07%**	**0,03%**
BBB+	**0,14%**	**0,03%**
BBB	**0,20%**	**0,03%**
BBB-	**0,35%**	**0,03%**
BB+	**0,47%**	**0,10%**
BB	**0,71%**	**0,41%**
BB-	**2,10%**	**0,70%**
B+	**2,40%**	**2,06%**
B	**5,10%**	**2,50%**
B-	**8,17%**	**6,30%**
SD	**26,85%**	**34,00%**

Table 2: Probability of default obtained from Wilkens and Predescu (2016)

CODE	*NAM*	**S&P**	**Country**	**Sector Recovery**	**Sector factor**
ABI.BR	AB INBEV ORD	A-	BELGIUM	FOOD	FOOD
ABE.MC	ABERTIS INFRAESTRUCTURAS ORD	BBB	SPAIN	CONSTRUCTION	CON
ACCP.PA	ACCOR ORD	BBB-	FRANCE	OTHER	CONS. SERV.
AEGN.AS	AEGON ORD	A-	NETHERLANDS	FINANCIAL	INSURANCE
AGES.BR	AGEAS ORD	BBB	BELGIUM	FINANCIAL	INSURANCE
AIRP.PA	AIR LIQUIDE ORD	A+	FRANCE	UTILITIES	UTILITIES
AIR.PA	AIRBUS GROUP ORD	A	FRANCE	OTHER	IND.GOODS

CODE	NAM	S&P	Country	Sector Recovery	Sector factor
ALVG.DE	ALLIANZ ORD	AA	GERMANY	FINANCIAL	INSURANCE
ACBr.AT	ALPHA BANK R ORD	SD	GREECE	FINANCIAL	BANKS
ALSO.PA	ALSTOM ORD	BBB-	FRANCE	TRANSPORT	TRAVEL
AMA.MC	AMADEUS IT HOLDING ORD	BBB	SPAIN	OTHER	TECH
ISPA.AS	ARCELORMITTAL ORD	BB	LUXEMBOURG	STEEL	BASIC RESOURCE
AKE.PA	ARKEMA ORD	BBB	FRANCE	CHEMICAL	CHEMICAL
ASMI.AS	ASM INTL ORD	BB+	NETHERLANDS	FABRICATED PR.	BASIC RESOURCE
ATL.MI	ATLANTIA ORD	BBB+	ITALY	CONSTRUCTION	CON
AXAF.PA	AXA ORD	A-	FRANCE	FINANCIAL	INSURANCE
EMII.MI	BANCA POPOLARE DELL EMILIA ROMA. ORD	BB-	ITALY	FINANCIAL	BANKS
BBVA.MC	BANCO BILBAO VIZCAYA ARGENTARIA ORD	BBB+	SPAIN	FINANCIAL	BANKS
BCP.LS	BANCO COM ORD	B+	PORTUGAL	FINANCIAL	BANKS
SABE.MC	BANCO DE SABADELL ORD	BB+	SPAIN	FINANCIAL	BANKS
POP.MC	BANCO POPULAR ESPANOL ORD	B+	SPAIN	FINANCIAL	BANKS
SAN.MC	BANCO SANTANDER ORD	A-	SPAIN	FINANCIAL	BANKS
BKIR.I	BANK OF IRELAND ORD	BBB-	IRELAND	FINANCIAL	BANKS
BKIA.MC	BANKIA ORD	BB+	SPAIN	FINANCIAL	BANKS
BKT.MC	BANKINTER ORD	BBB-	SPAIN	FINANCIAL	BANKS
BASFn.DE	BASF N ORD	A	GERMANY	CHEMICAL	CHEMICAL
BAYGn.DE	BAYER N ORD	A-	GERMANY	DRUGS	CONS. GOODS
GBFG.DE	BILFINGER ORD	BB+	GERMANY	CONSTRUCTION	CON
BMWG.DE	BMW ORD	A+	GERMANY	AUTOMOTIVE	AUTO
BNPP.PA	BNP PARIBAS ORD	A	FRANCE	FINANCIAL	BANKS
BOUY.PA	BOUYGUES ORD	BBB	FRANCE	CONSTRUCTION	CON
BNRGn.DE	BRENNTAG N ORD	BBB	FRANCE	CHEMICAL	CHEMICAL
CABK.MC	CAIXABANK ORD	BBB	SPAIN	FINANCIAL	BANKS
CAPP.PA	CAP GEMINI ORD	BBB	FRANCE	OTHER	TECH
CARR.PA	CARREFOUR ORD	BBB+	FRANCE	RETAIL	RETAIL
CASP.PA	CASINO GUICHARD PERRACHON ORD	BB+	FRANCE	RETAIL	RETAIL
CLNX.MC	CELLNEX TELECOM ORD	BB+	SPAIN	OTHER	TELECOM
CNHI.MI	CNH INDUSTRIAL ORD	BB+	NETHERLANDS	MACHINERY	INDUSTRIALS
COFB.BR	COFINIMMO ORD	BBB	BELGIUM	OTHER	REAL STATE
CBKG.DE	COMMERZBANK ORD	BBB+	GERMANY	FINANCIAL	BANKS
SGOB.PA	COMPAGNIE DE SAINT GOBAIN ORD	BBB	FRANCE	FABRICATED PR.	BASIC RESOURCE
CAGR.PA	CREDIT AGRICOLE ORD	A	FRANCE	FINANCIAL	BANKS
CRH.I	CRH ORD	BBB+	IRELAND	FABRICATED PR.	BASIC

CODE	NAM	S&P	Country	Sector Recovery	Sector factor
					RESOURCE
DAIGn.DE	DAIMLER N ORD	A-	GERMANY	AUTOMOTIVE	AUTO
DANO.PA	DANONE ORD	A-	FRANCE	FOOD	FOOD
DELB.BR	DELHAIZE ORD	BBB	BELGIUM	FOOD	FOOD
DBKGn.DE	DEUTSCHE BANK N ORD	BBB+	GERMANY	FINANCIAL	BANKS
DB1Gn.DE	DEUTSCHE BOERSE N ORD	AA	GERMANY	FINANCIAL	FINANCIALS
LHAG.DE	DEUTSCHE LUFTHANSA ORD	BBB-	GERMANY	TRANSPORT	TRAVEL
DTEGn.DE	DEUTSCHE TELEKOM N ORD	BBB+	GERMANY	OTHER	TELECOM
DWNG.DE	DEUTSCHE WOHNEN ORD	A-	GERMANY	OTHER	REAL STATE
DIDA.MC	DISTRIBUIDORA INTERN.DE ALIMEN. ORD	BBB-	SPAIN	FOOD	FOOD
EONGn.DE	E.ON N ORD	BBB+	GERMANY	UTILITIES	UTILITIES
EDEN.PA	EDENRED ORD	BBB+	FRANCE	FINANCIAL	FINANCIALS
EDF.PA	EDF ORD	A+	FRANCE	UTILITIES	UTILITIES
ELI1V.HE	ELISA ORD	BBB+	FINLAND	OTHER	TECH
ELE.MC	ENDESA ORD	BBB	SPAIN	UTILITIES	UTILITIES
ENEI.MI	ENEL ORD	BBB	ITALY	UTILITIES	UTILITIES
ENGIE.PA	ENGIE ORD	A	FRANCE	UTILITIES	UTILITIES
ENI.MI	ENI ORD	BBB+	ITALY	OIL	OIL
ERST.VI	ERSTE GROUP BANK ORD	BBB+	AUSTRIA	FINANCIAL	BANKS
EURBr.AT	EUROBANK ERGASIAS/R ORD	SD	GREECE	FINANCIAL	BANKS
ETL.PA	EUTELSAT COMMUNICATIONS ORD	BBB	FRANCE	FABRICATED PR.	TELECOM
EVKn.DE	EVONIK INDUSTRIES ORD	BBB+	GERMANY	CHEMICAL	CHEMICAL
EXOR.MI	EXOR ORD	BBB+	ITALY	FINANCIAL	FINANCIALS
FER.MC	FERROVIAL ORD	BBB	SPAIN	CONSTRUCTION	CON
FCHA.MI	FIAT CHRYSLER AUTOMOBILES ORD	BB	ITALY	AUTOMOTIVE	AUTO
SIFI.MI	FINMECCANICA ORD	BB+	ITALY	OTHER	TECH
FDR.PA	FONCIERE DES REGIONS REIT	BBB	FRANCE	FINANCIAL	FINANCIALS
FUM1V.HE	FORTUM ORD	BBB+	FINLAND	UTILITIES	UTILITIES
FMEG.DE	FRESENIUS MEDICAL CARE ORD	BBB-	GERMANY	OTHER	HEALTH CARE
FREG.DE	FRESENIUS ORD	BBB-	GERMANY	OTHER	HEALTH CARE
GAS.MC	GAS NATURAL ORD	BBB	SPAIN	UTILITIES	UTILITIES
GFCP.PA	GECINA REIT	BBB+	FRANCE	OTHER	IND.GOODS
GRLS.MC	GRIFOLS ORD CL A	BB	SPAIN	OTHER	HEALTH CARE
HNRGn.DE	HANNOVER RUCKVERSICHERUNG N	AA-	GERMANY	FINANCIAL	INSURANCE

CODE	NAM	S&P	Country	Sector Recovery	Sector factor
	ORD				
OTEr.AT	HELLENIC TELECOM ORG R ORD	B+	GREECE	OTHER	TELECOM
HNKG_p.DE	HENKEL& KGAA PRF	A	GERMANY	CHEMICAL	CHEMICAL
IBE.MC	IBERDROLA ORD	BBB+	SPAIN	UTILITIES	UTILITIES
ICAD.PA	ICADE REIT	BBB+	FRANCE	OTHER	REAL STATE
IFXGn.DE	INFINEON TECHNOLOGIES N ORD	BBB	GERMANY	FABRICATED PR.	TECH
ING.AS	ING GROEP GDR	A-	NETHERLANDS	FINANCIAL	BANKS
ISP.MI	INTESA SANPAOLO ORD	BBB-	ITALY	FINANCIAL	BANKS
ITAI.MI	ITALCEMENTI FABBRICHE RIUNITE ORD	BB	ITALY	FABRICATED PR.	BASIC MATS
JCDX.PA	JCDECAUX ORD	BBB	FRANCE	OTHER	MEDIA
SDFGn.DE	K+S N ORD	BBB-	GERMANY	CHEMICAL	CHEMICAL
KBC.BR	KBC GROEP ORD	BBB+	BELGIUM	FINANCIAL	BANKS
PRTP.PA	KERING ORD	BBB	FRANCE	CLOTHES	CONS. GOODS
KYGa.I	KERRY GROUP ORD	BBB+	IRELAND	FOOD	FOOD
KGX.DE	KION GROUP ORD	BB+	GERMANY	OTHER	BASIC RESOURCE
LOIM.PA	KLEPIERRE REIT	A-	FRANCE	FOOD	FOOD
AHLN.AS	KONINKLIJKE AHOLD ORD	BBB	NETHERLANDS	RETAIL	RETAIL
PHG.AS	KONINKLIJKE PHILIPS ORD	BBB+	NETHERLANDS	OTHER	TECH
KPN.AS	KPN KON ORD	BBB-	NETHERLANDS	OTHER	TELECOM
LXSG.DE	LANXESS ORD	BBB-	GERMANY	CHEMICAL	CHEMICAL
LEGD.PA	LEGRAND ORD	A-	FRANCE	FABRICATED PR.	BASIC RESOURCE
LING.DE	LINDE ORD	A+	GERMANY	UTILITIES	UTILITIES
LUX.MI	LUXOTTICA GROUP ORD	A-	ITALY	CLOTHES	CONS. GOODS
LVMH.PA	LVMH MOET HENNESSY LOUIS VUITTON SE ORD	A+	FRANCE	CLOTHES	CONS. GOODS
MRCG.DE	MERCK ORD	A	GERMANY	DRUGS	CHEMICAL
MRL.MC	MERLIN PROPERTIES REIT	BBB	SPAIN	OTHER	REAL STATE
MEO1V.HE	METSO ORD	BBB	FINLAND	MINING	BASIC MATS
MUVGN.DE	MUENCHENER RUECKVER N ORD	AA-	GERMANY	FINANCIAL	INSURANCE
CNAT.PA	NATIXIS ORD	A	FRANCE	FINANCIAL	BANKS
NOKIA.HE	NOKIA ORD	BB+	FINLAND	CON. DURABLES	TECH
NUME.PA	NUMERICABLE SFR ORD	B+	FRANCE	OTHER	TELECOM
ONTEX.BR	ONTEX GROUP ORD	BB	BELGIUM	MACHINERY	INDUSTRIALS
ORAN.PA	ORANGE ORD	BBB+	FRANCE	OTHER	TELECOM
PERP.PA	PERNOD RICARD ORD	BBB-	FRANCE	FOOD	FOOD

CODE	NAM	S&P	Country	Sector Recovery	Sector factor
PST.MI	POSTE ITALIANE ORD	BBB-	ITALY	OTHER	CONS. SERV.
PTNL.AS	POSTNL ORD	BBB-	NETHERLANDS	RETAIL	RETAIL
PROX.BR	PROXIMUS ORD	A	BELGIUM	OTHER	TELECOM
PUBP.PA	PUBLICIS GROUPE ORD	BBB+	FRANCE	OTHER	MEDIA
RBIV.VI	RAIFFEISEN BANK INTERNATIONAL ORD	BBB	AUSTRIA	FINANCIAL	BANKS
REE.MC	RED ELECTRICA CORPORACION ORD	A-	SPAIN	UTILITIES	UTILITIES
RENA.PA	RENAULT PAR	BBB-	FRANCE	AUTOMOTIVE	AUTO
REP.MC	REPSOL ORD	BBB-	SPAIN	OIL	OIL
RXL.PA	REXEL ORD	BB	FRANCE	FABRICATED PR.	BASIC RESOURCE
RRTL.DE	RTL GROUP ORD	BBB+	GERMANY	OTHER	MEDIA
RWEG.DE	RWE ORD	BBB	GERMANY	UTILITIES	UTILITIES
RYA.I	RYANAIR HOLDINGS ORD	BBB+	IRELAND	TRANSPORT	TRAVEL
SAMAS.HE	SAMPO A ORD	A-	FINLAND	FINANCIAL	FINANCIALS
SASY.PA	SANOFI ORD	AA	FRANCE	OTHER	HEALTH CARE
SAPG.DE	SAP ORD	A	GERMANY	OTHER	TECH
SCHN.PA	SCHNEIDER ELECTRIC SE ORD	A-	FRANCE	UTILITIES	TECH
SCOR.PA	SCOR ORD	AA-	FRANCE	FINANCIAL	INSURANCE
SESFd.PA	SES FDR	BBB	LUXEMBOURG	OTHER	TELECOM
SIEGN.DE	SIEMENS N ORD	A+	GERMANY	CON. DURABLES	CONS. GOODS
SKG.I	SMURFIT KAPPA GROUP ORD	BB+	IRELAND	FABRICATED PR.	BASIC RESOURCE
SRG.MI	SNAM ORD	BBB	ITALY	OIL	OIL
SOGN.PA	SOCIETE GENERALE ORD	A	FRANCE	FINANCIAL	BANKS
EXHO.PA	SODEXO ORD	A-	GERMANY	OTHER	CONS. SERV.
SOLB.BR	SOLVAY ORD	BBB-	BELGIUM	CHEMICAL	CHEMICAL
STM.PA	STMICROELECTRONICS ORD	BBB-	FRANCE	OTHER	TECH
STERV.HE	STORA ENSO R ORD	BB	FINLAND	FABRICATED PR.	BASIC RESOURCE
TCH.PA	TECHNICOLOR ORD	B+	FRANCE	OTHER	MEDIA
TECF.PA	TECHNIP ORD	BBB+	FRANCE	CONSTRUCTION	CON
TLIT.MI	TELECOM ITALIA ORD	BB+	ITALY	OTHER	TELECOM
TEF.MC	TELEFONICA ORD	BBB	SPAIN	OTHER	TELECOM
TNET.BR	TELENET GROUP HOLDING ORD	B+	BELGIUM	OTHER	TELECOM
TRN.MI	TERNA RETE ELETTRICA NAZIONALE ORD	BBB	ITALY	UTILITIES	UTILITIES
TKAG.DE	THYSSENKRUPP ORD	BB	GERMANY	STEEL	BASIC MATS
TNTE.AS	TNT EXPRESS ORD	BBB	NETHERLANDS	RETAIL	RETAIL

CODE	NAM	S&P	Country	Sector Recovery	Sector factor
TOTF.PA	TOTAL ORD	A+	FRANCE	UTILITIES	UTILITIES
UNBP.AS	UNIBAIL RODAMCO REIT	A	FRANCE	FINANCIAL	FINANCIALS
CRDI.MI	UNICREDIT ORD	BBB-	ITALY	FINANCIAL	BANKS
ULVR.L	UNILEVER ORD	A+	LUXEMBOURG	CHEMICAL	CHEMICAL
UBI.MI	UNIONE DI BANCHE ITALIANE ORD	BBB-	ITALY	FINANCIAL	BANKS
UPM1V.HE	UPM KYMMENE ORD	BB+	FINLAND	FABRICATED PR.	BASIC RESOURCE
VLOF.PA	VALEO ORD	BBB	FRANCE	AUTOMOTIVE	AUTO
VIE.PA	VEOLIA ENVIRONNEMENT VE ORD	BBB	FRANCE	UTILITIES	UTILITIES
VIV.PA	VIVENDI ORD	BBB	FRANCE	OTHER	MEDIA
VOWG_p.DE	VOLKSWAGEN NV PRF	BBB+	GERMANY	AUTOMOTIVE	AUTO
VNAn.DE	VONOVIA ORD	BBB+	GERMANY	OTHER	REAL STATE
MWDP.PA	WENDEL ORD	BBB-	FRANCE	FINANCIAL	FINANCIALS
WLSNc.AS	WOLTERS KLUWER C ORD	BBB+	NETHERLANDS	OTHER	TECH
95740@orgid	AUSTRIA	AA+			
95744@orgid	BELGIUM	AA			
95765@orgid	FINLAND	AA+			
95766@orgid	FRANCE	AA			
96805@orgid	GERMANY	AAA			
95769@orgid	GREECE	B-			
69201@orgid	IRELAND	A+			
95779@orgid	ITALY	BBB-			
95794@orgid	NETHERLANDS	AAA			
95804@orgid	PORTUGAL	BB+			
162600@orgid	SPAIN	BBB+			

Table 3: Assets employed in the empirical exercise

Note for the tables 4, 5, 6,7, 8, 9, 10, 11 and 12:

The composition of each portfolio is the following:

1. **Corporate debt**
   - 1.1 **Second factor: country**
   - 1.2 **Second factor: industry**
2. **Equity**
   - 2.1 **Second factor: country**
   - 2.2 **Second factor: industry**

3 **Sovereign debt**
4 **Corporate and sovereign debt**
5 **Debt and equity**

The criteria for building the portfolios are the following:

A. **Equally-weighted**
B. **Minimum variance**
C. **Minimum variance (allowing only long positions)**
D. **Lower default correlation weighted by rating**

COMPOSITION		1		2		3	4	5
PORTFOLIOS	MINUS ONE BP	1.1	1.1	2.1	2.2			
	A	-3,16%	-1,78%	0,00%	0,00%	0,00%	-3,16%	-2,89%
	B	-0,58%	-2,62%	-2,63%	-3,56%	0,00%	-1,63%	-4,55%
	C	-3,35%	-2,77%	-2,36%	-1,57%	-0,34%	-2,82%	-2,80%
	D	-0,92%	-13,59%	-16,67%	-3,98%	0,00%	-0,98%	-11,32%

Table 4: Change in the DRC (%) if the probabilities of default decrease a basis point

COMPOSITION		1		2		3	4	5
PORTFOLIOS	PLUS ONE BP	1.1	1.1	2.1	2.2			
	A	2,86%	2,80%	4,17%	7,69%	0,80%	2,86%	2,22%
	B	0,80%	2,99%	2,14%	0,66%	4,71%	4,12%	3,02%
	C	2,49%	1,15%	2,06%	1,35%	6,20%	4,52%	4,74%
	D	7,36%	13,08%	0,01%	11,60%	109,54%	14,93%	7,09%

Table 5: Change in the DRC (%) if the probabilities of default increase **a basis point**

COMPOSITION		1		3	4	5
PORTFOLIOS	RECOVERY	1.1	1.1			
	R=0					
	A	-51,62%	-51,40%	17,75%	-51,62%	-26,38%
	B	-17,15%	-30,41%	13,28%	-29,52%	-18,08%
	C	-29,93%	-37,35%	17,41%	-50,92%	-25,42%
	D	-0,58%	-27,88%	23,03%	-0,23%	6,77%
PORTFOLIOS	R=0.25					
	A	-26,37%	-25,03%	9,76%	-26,37%	-13,56%
	B	-7,54%	-17,32%	4,00%	-13,46%	-10,28%
	C	-20,16%	-22,86%	10,25%	-25,60%	-13,23%
	D	-0,59%	-16,15%	12,71%	-0,37%	3,92%
PORTFOLIOS	R=0.75					
	A	10,79%	14,96%	-14,75%	10,58%	7,22%
	B	12,09%	10,38%	-10,06%	12,25%	6,50%
	C	10,38%	11,05%	-32,79%	9,44%	-3,40%
	D	1,91%	0,91%	-11,31%	0,13%	0,07%
PORTFOLIOS	R=1					
	A	10,94%	13,75%	-21,08%	10,94%	4,54%
	B	16,43%	20,95%	-23,94%	16,76%	10,97%
	C	11,81%	12,50%	-24,03%	16,11%	9,63%
	D	3,26%	18,94%	-36,01%	10,77%	-0,71%

Table 6: Change in the DRC (%) if the influence of the global factor in the reocvery is different from 0.5

COMPOSITION		1		2		3	4	5
PORTFOLIOS	OLS	1.1	1.1	2.1	2.2			
	A	4,37%	0,74%	7,69%	7,69%	0,43%	4,37%	5,25%
	B	19,51%	-1,49%	9,88%	1,50%	-8,41%	4,65%	0,05%
	C	15,56%	-8,47%	10,93%	-2,58%	-9,38%	4,96%	5,71%
	D	-6,66%	15,44%	-9,99%	-11,20%	-47,07%	-2,61%	-5,87%

Table 7 : Change in the DRC (%) if the OLS values of the parameters are employed

COMPOSITION		1		2		3	4	5
PORTFOLIOS	QUARTERLY	1.1	1.1	2.1	2.2			
	A	-2,82%	9,81%	7,69%	15,38%	-0,55%	-2,82%	-3,20%
	B	50,36%	0,46%	38,33%	19,88%	10,04%	-25,38%	-27,86%
	C	0,74%	134,48%	170,98%	149,84%	-23,15%	-26,83%	-27,31%
	D	23,16%	10,80%	-18,66%	-5,53%	47,16%	50,49%	-9,41%

Table 8: Change in the DRC (%) if the quarterly data is employed

COMPOSITION		1		2		3	4	5
PORTFOLIOS	COPULA	1.1	1.1	2.1	2.2			
	A	90,66%	115,35%	84,62%	100,00%	12,37%	90,13%	94,33%
	B	32,93%	29,84%	19,46%	22,84%	15,42%	14,16%	11,36%
	C	77,64%	110,24%	74,03%	103,42%	10,90%	80,70%	82,57%
	D	18,67%	65,68%	20,56%	36,02%	10,79%	14,74%	21,23%

Table 9: Change in the DRC (%) a Clayton copula is employed in the systematic factors

COMPOSITION		1		2		3	4	5
NOMINAL:100 €	INITIAL VALUES	1.1	1.1	2.1	2.2			
PORTFOLIOS	A	6,93	7,33	7,69	8,33	4,23	3,47	4,91
	B	17,32	21,35	22,53	26,18	1,08	11,20	11,89
	C	5,80	6,59	6,94	7,45	0,88	3,56	3,77
	D	4,08	5,95	10,01	8,97	0,35	2,74	4,90

Table 10: Initial DRC values for the portfolios

NOMINAL: a million	PLAIN MONTECARLO						
	1		2				
MEAN	1.1	1.2	2.1	2.2	3	4	5
A	50962,04	49511,23	51282,05	76923,08	41216,28	25490,16	34114,53
B	126884,10	127461,09	215169,39	226470,14	10534,85	79882,33	84544,99

C	**78957,68**	**73913,37**	**113993,83**	**114783,19**	**8821,83**	**17949,19**	**18977,46**
D	**54231,82**	**60578,99**	**101628,08**	**109204,99**	**3072,96**	**39709,63**	**44698,81**
NOMINAL: a million	*ANTITHETIC VARIANTES MONTECARLO*						
MEAN	*1*		*2*		*3*	*4*	*5*
	1.1	*1.2*	*2.1*	*2.2*			
A	**50949,19**	**49426,59**	**51282,05**	**76923,08**	**41411,62**	**25482,86**	**34110,03**
B	**126960,41**	**127432,89**	**215413,28**	**225607,79**	**10526,81**	**79680,21**	**84611,88**
C	**79285,22**	**73847,23**	**113993,83**	**115267,20**	**8821,16**	**17769,19**	**18821,23**
D	**54159,96**	**59847,17**	**101464,46**	**109536,86**	**2980,06**	**39563,45**	**44617,75**

Table 11: Mean of the values of VaR using plain MC and antithetic MC

NOMINAL: a million	*PLAIN MONTECARLO*						
VARIANCE	*1*		*2*		*3*	*4*	*5*
	1.1	*1.2*	*2.1*	*2.2*			
A	**84513,042**	**1187980,04**	**8,56E-22**	**3,42E-21**	**1199024,32**	**20412,3564**	**6175,05319**
B	**12013816,8**	**7672854,81**	**199724509**	**13596419,1**	**178007,626**	**5386021,86**	**2406490,54**
C	**10676049,6**	**12553324**	**7,70E-21**	**4639290,23**	**143187,452**	**762502,147**	**676632,747**
D	**3583995,1**	**9301576,25**	**5876518,24**	**122563954**	**1174558,98**	**4882379,49**	**6740081,31**
NOMINAL: a million	*ANTITHETIC VARIANTES MONTECARLO*						
VARIANCE	*1*		*2*		*3*	*4*	*5*
	1.1	*1.2*	*2.1*	*2.2*			
A	**40383,6889**	**585852,419**	**8,5559E-22**	**3,4224E-21**	**578669,339**	**9718,58452**	**2930,06151**
B	**5428711,07**	**3304426,89**	**113109304**	**17811248,9**	**87209,7088**	**3126272,79**	**1390247,68**
C	**2989810,52**	**9282759,93**	**7,7003E-21**	**3411591,68**	**71736,6875**	**411034,659**	**380788,867**
D	**2196621,33**	**4784478,21**	**1773391,37**	**56080330,2**	**685460,003**	**2495714,88**	**3662342,24**

Table 12: Variance of the values of VaR using plain MC and antithetic MC

8.2. Appendix of figures

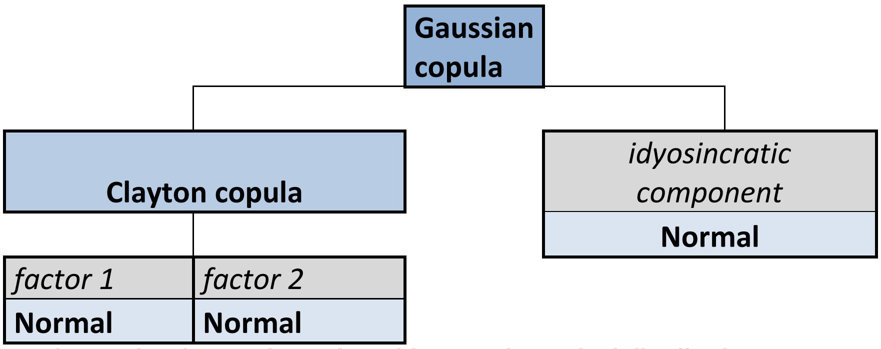

Figure 1: Structure of a two-level nested copulas with normal marginal distribution

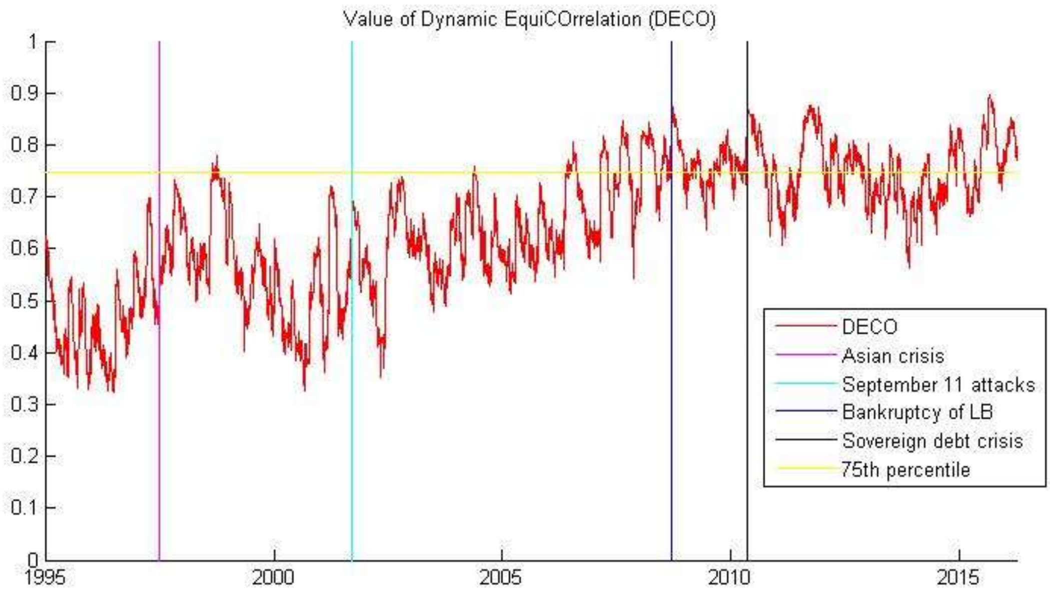

Figure 2: DECO correlation from 1995-2005 across the main stock market indexes.

Note: the indexes employed for this goal are the followings: AEX index for Netherlands, BEL20 for Belgium, DAX30 for Germany, CAC40 for France, IBEX35 for Spain, ATX for Austria, ISEQ for Ireland and PSI20 for Portugal.

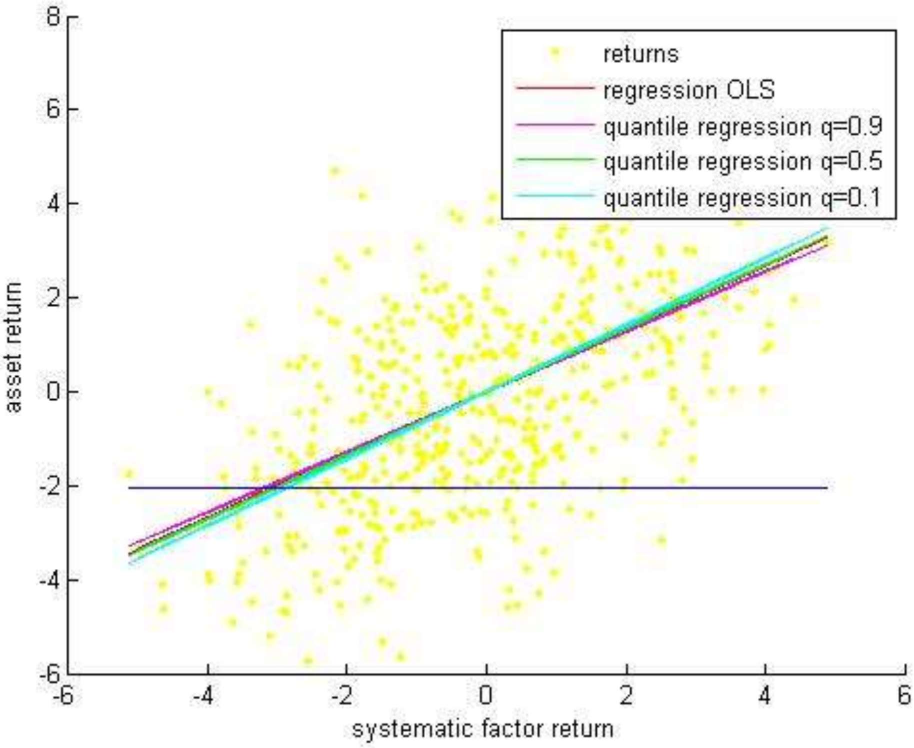

Figure 3: Regression simulated under OLS and different quantile regressions

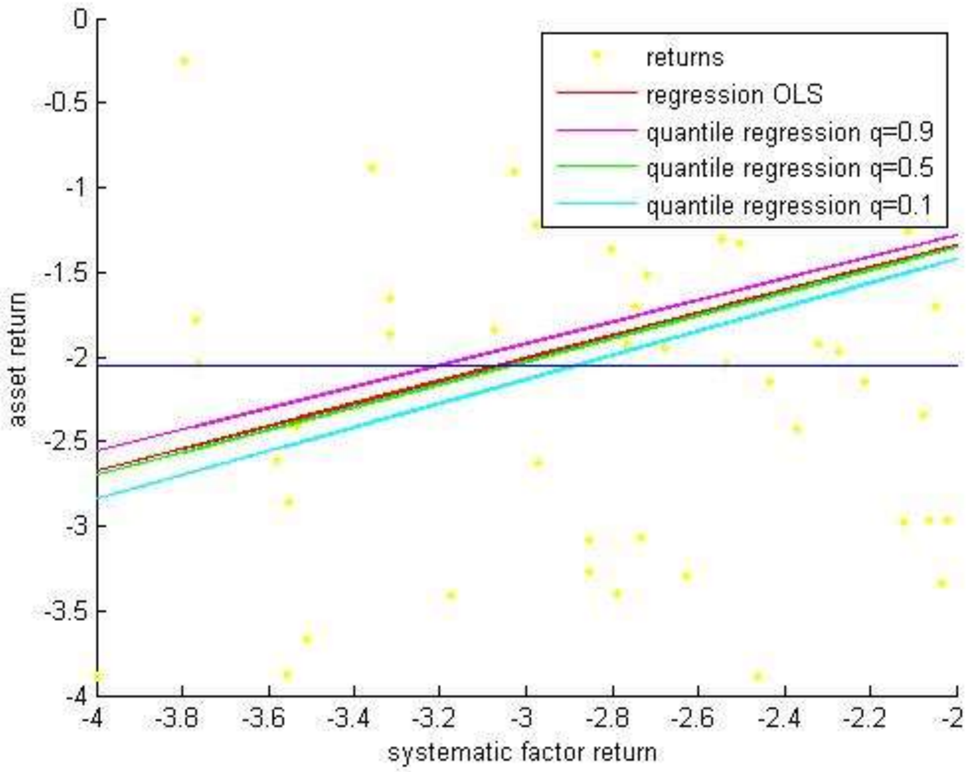

Figure 4: Regression simulated under OLS and different quantile regressions (detail)

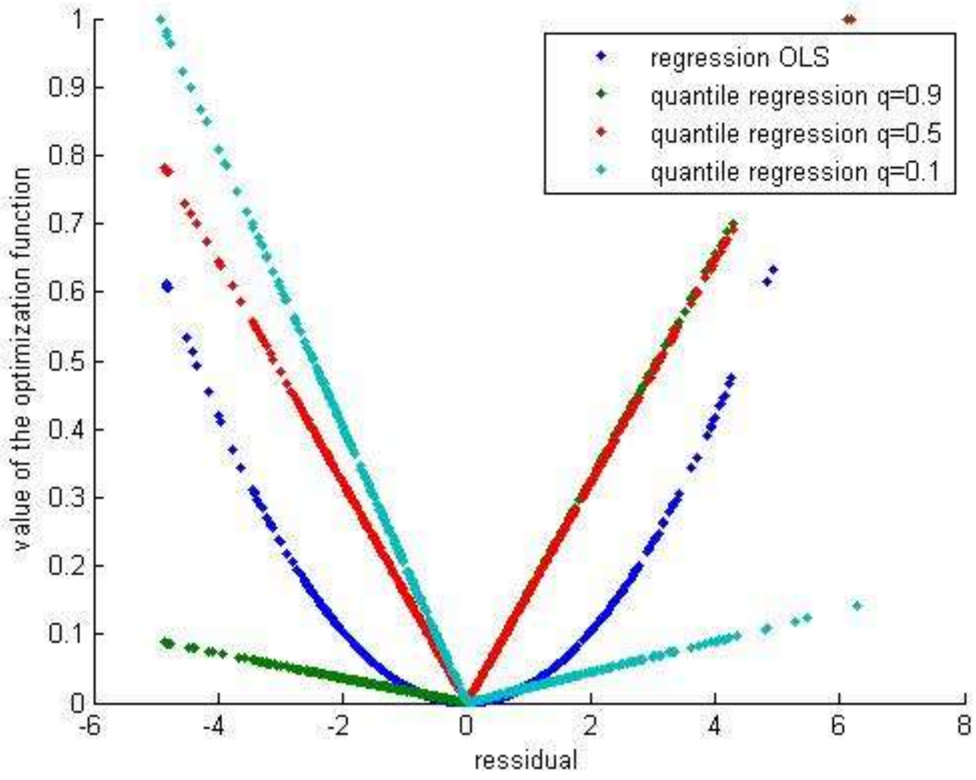

Figure 5: Residuals and weight for the residual under each different regression employed
Note that the values of the optimization function have been scaled in order to compare sensibly.

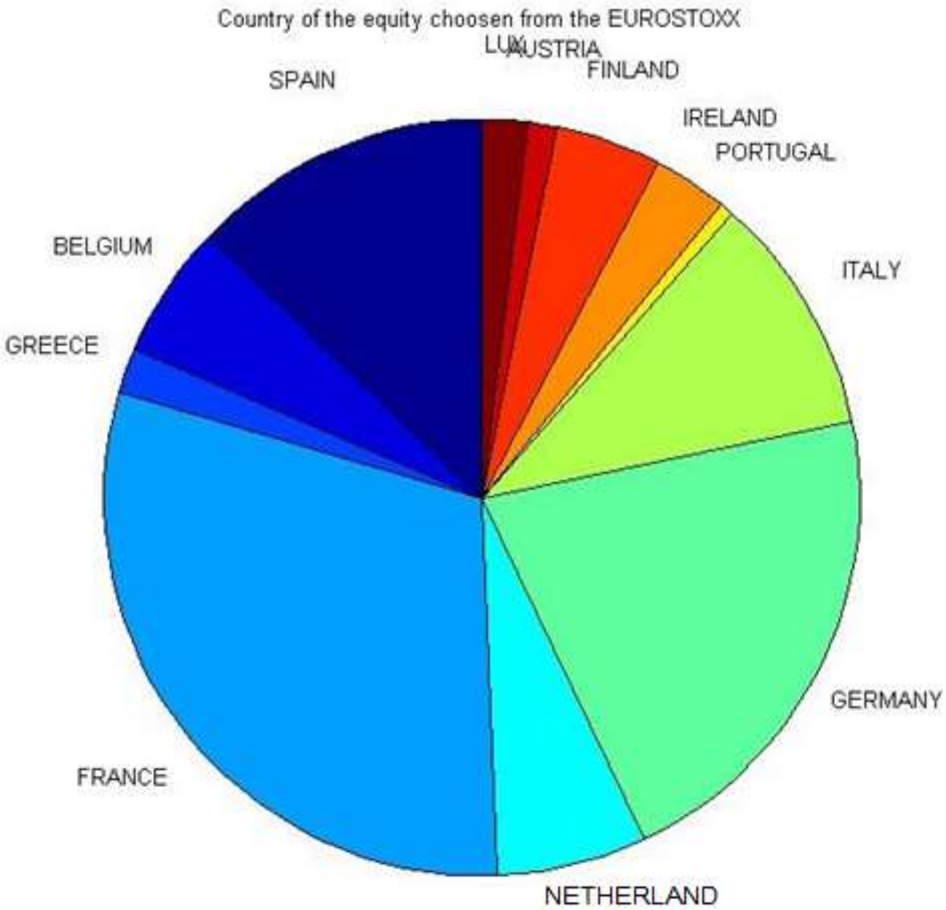

Figure 6: Country of the equity from Eurostoxx employed in the sensitivity analysis

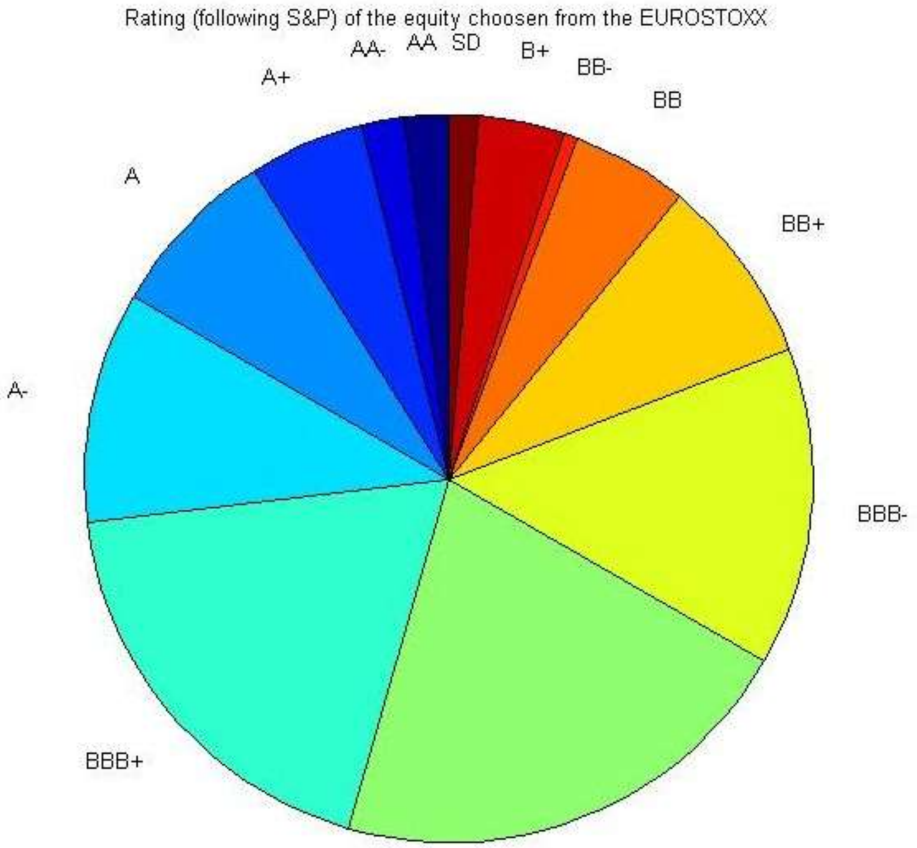

Figure 7: Rating of the equity from Eurostoxx employed in the sensitivity analysis

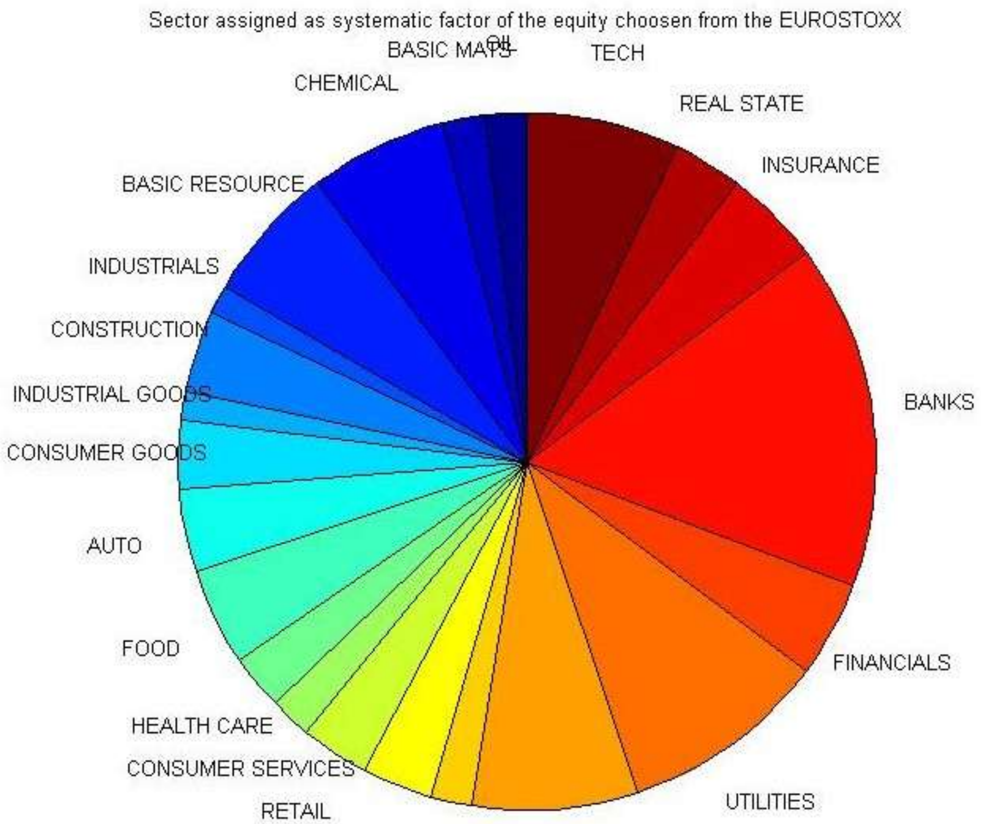

Figure 8: Sector of the equity from Eurostoxx employed in the sensitivity analysis

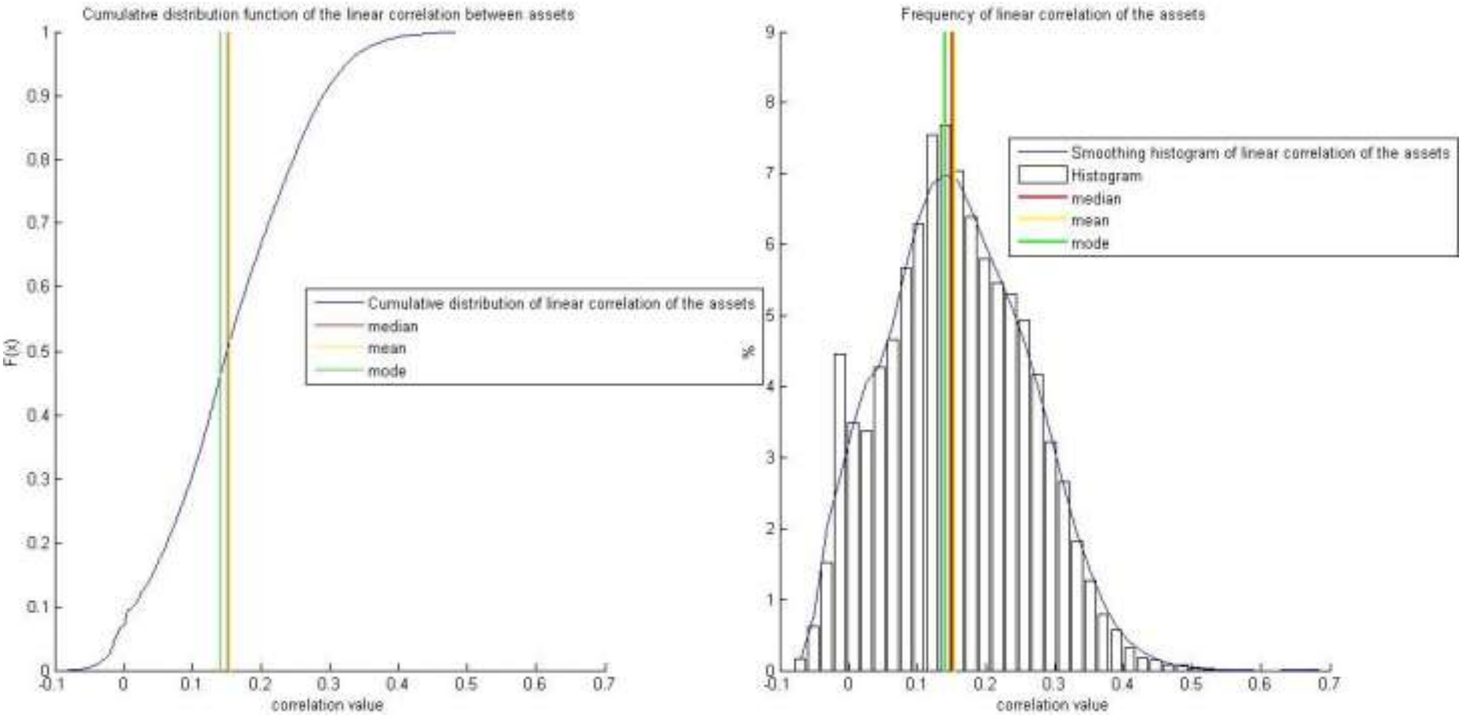

Figure 9: Correlation default distribution of corporate and sovereign assets

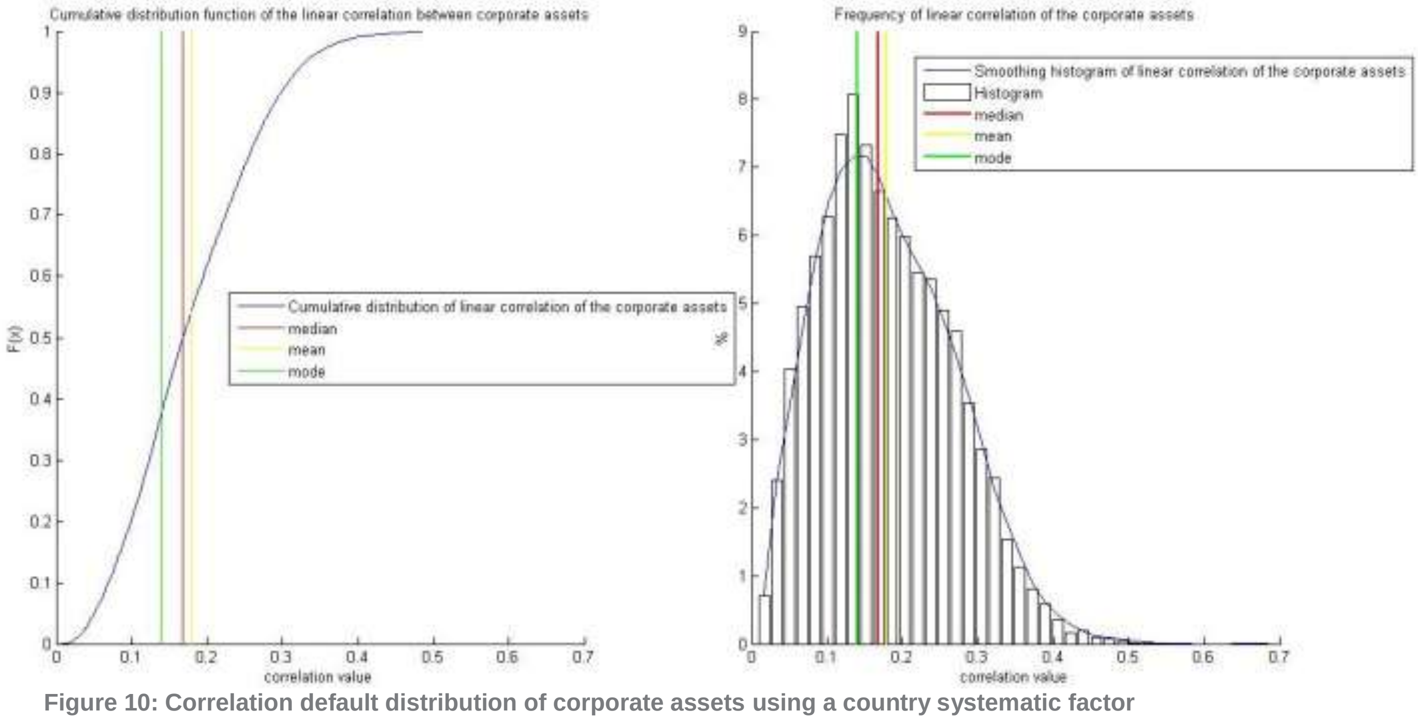

Figure 10: Correlation default distribution of corporate assets using a country systematic factor

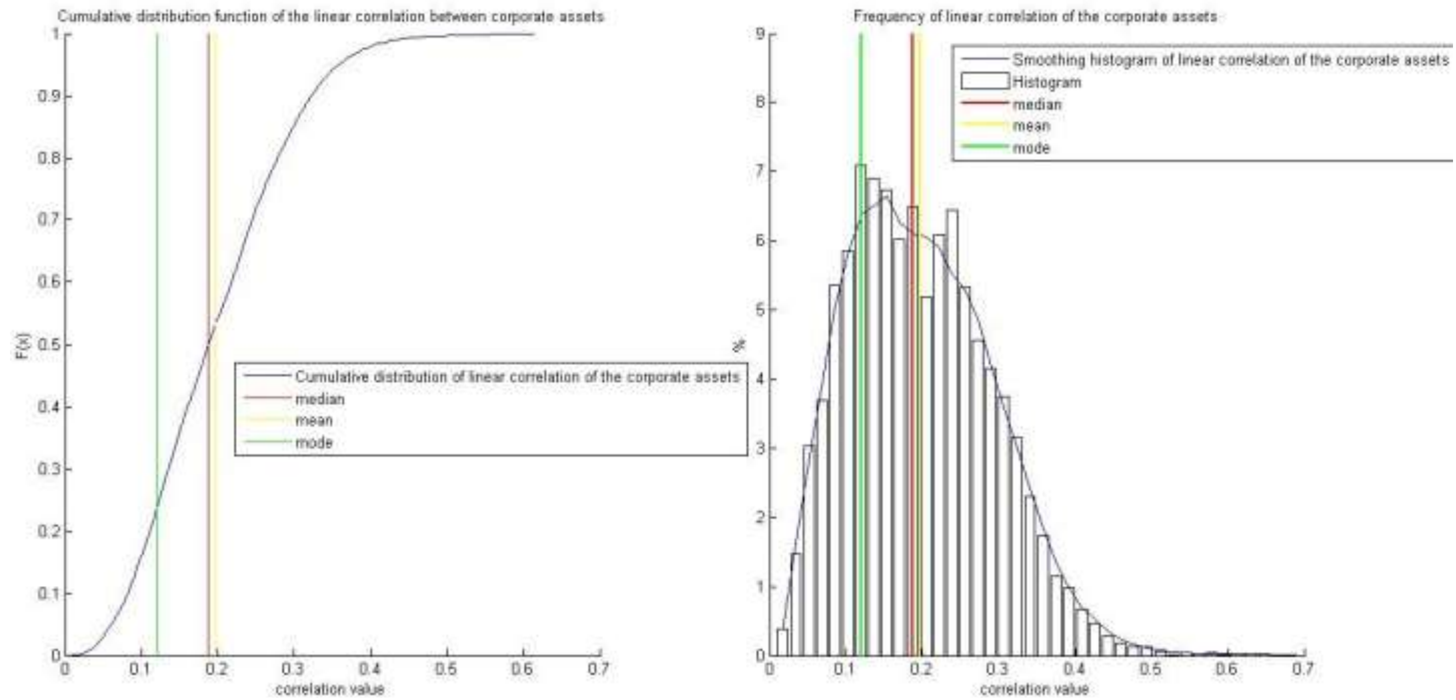

Figure 11: Correlation default distribution of corporate assets using a industry systematic factor

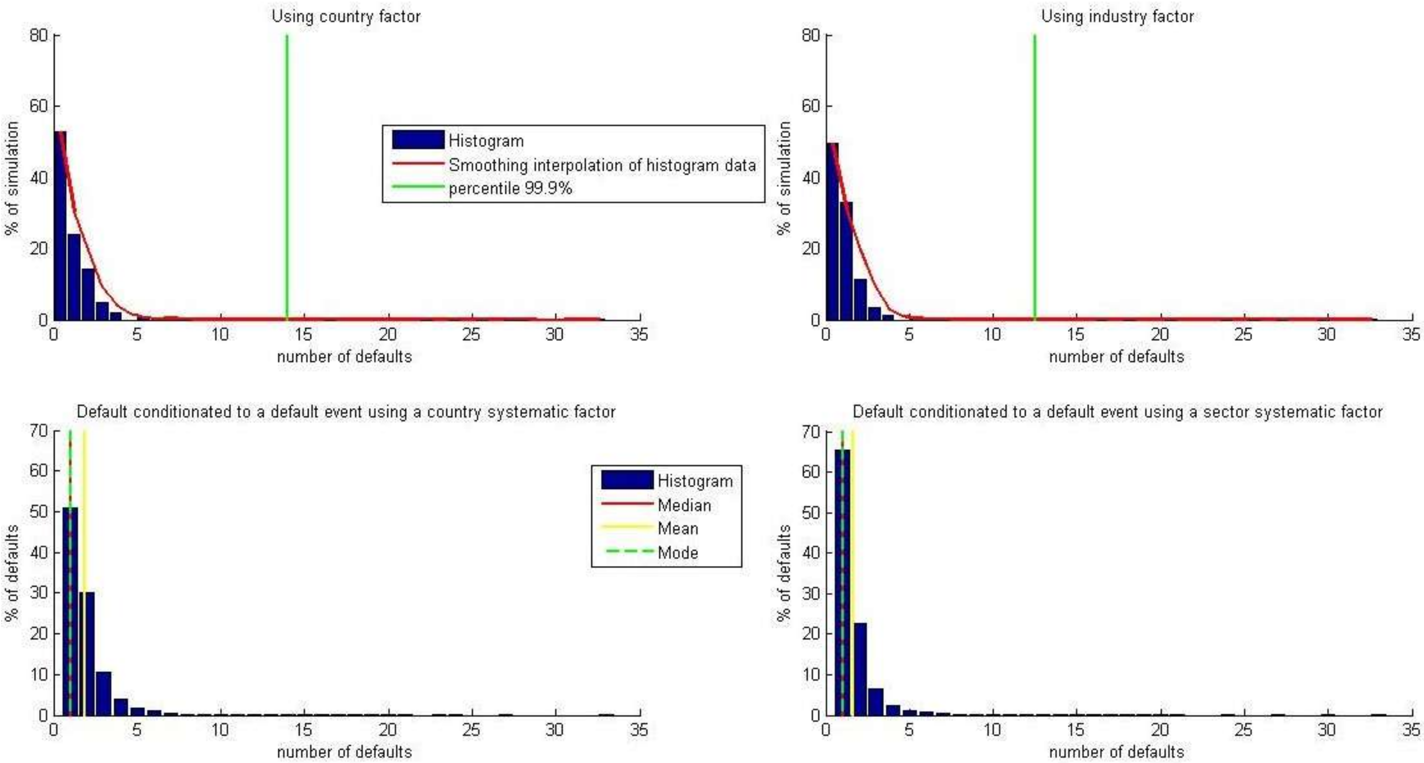

Figure 12: Number of defaults depending on the chosen second systematic factor

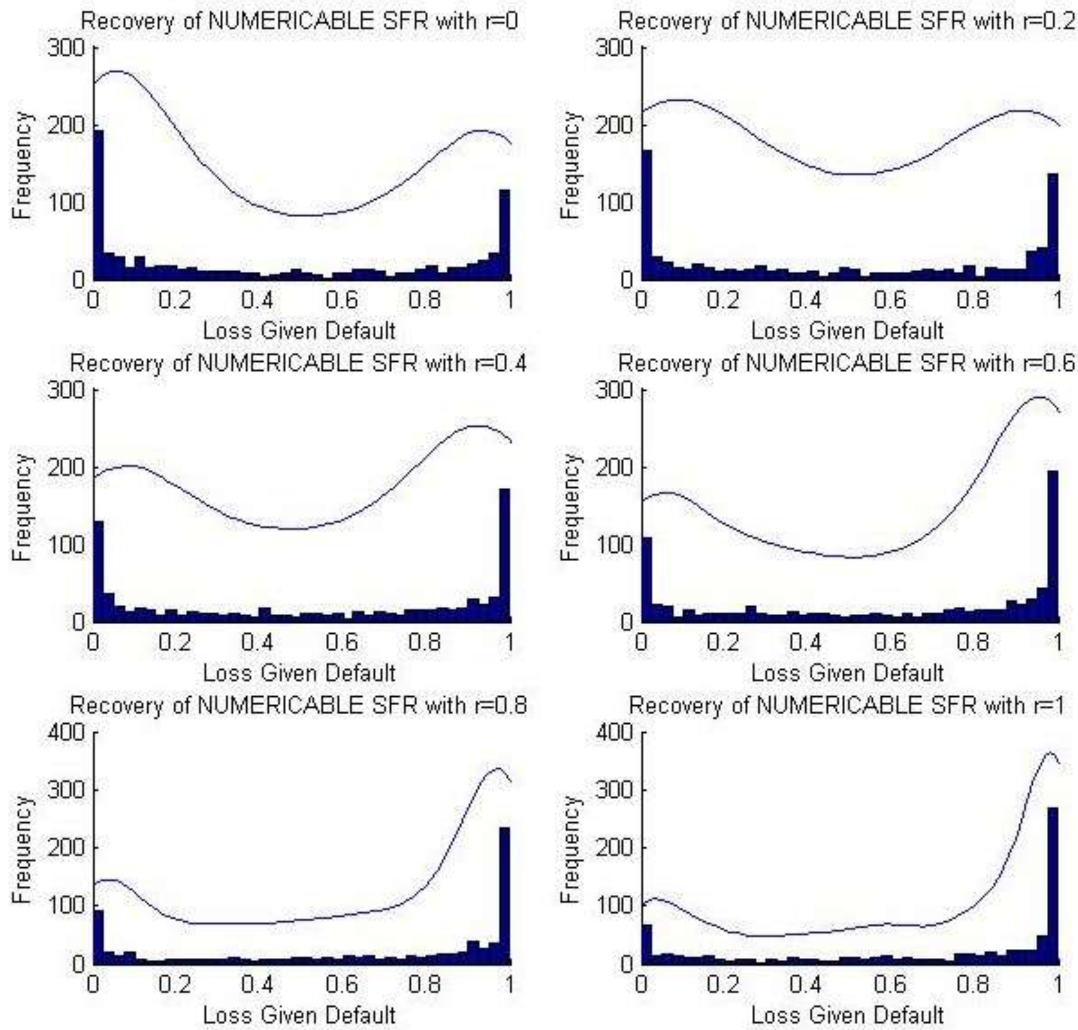

Figure 13: LGD for NUMERICABLE SFR using different values for the parameter of influence of the global factor in equation (19)

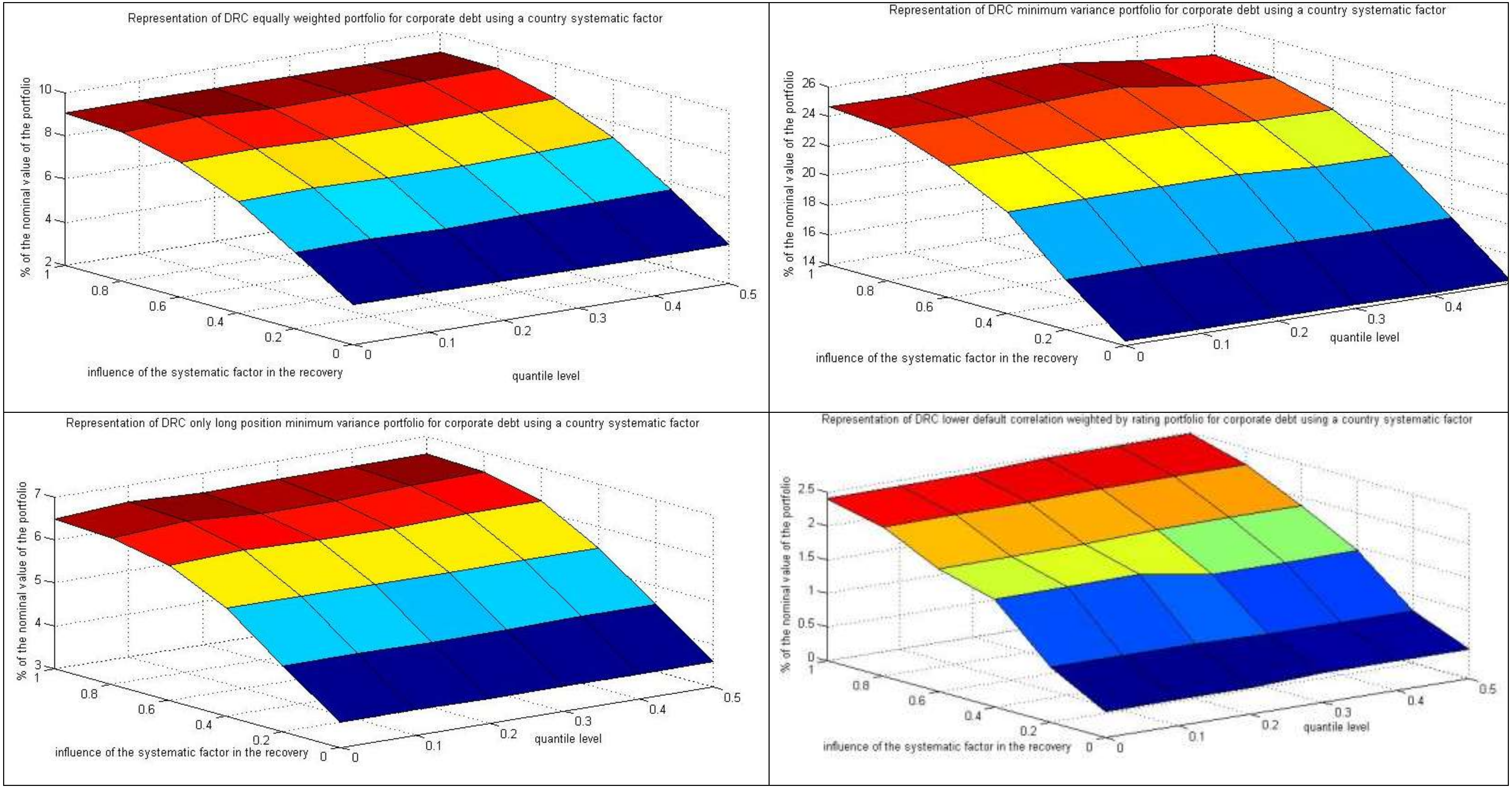

Figure 14: Different VaR depending on the quantile of the regresion and the influence of the global factor in the recovery rate for a portfolio made of corporate debt using country as a second systematic factor.

Note: the upper left graph represents the equally-weighted portfolio, while the upper right graph represents minimum variance portfolio. The lower left graph represents the minimum variance portfolio allowing only long positions, while the lower right graph represents the lower default correlation weighted by rating.

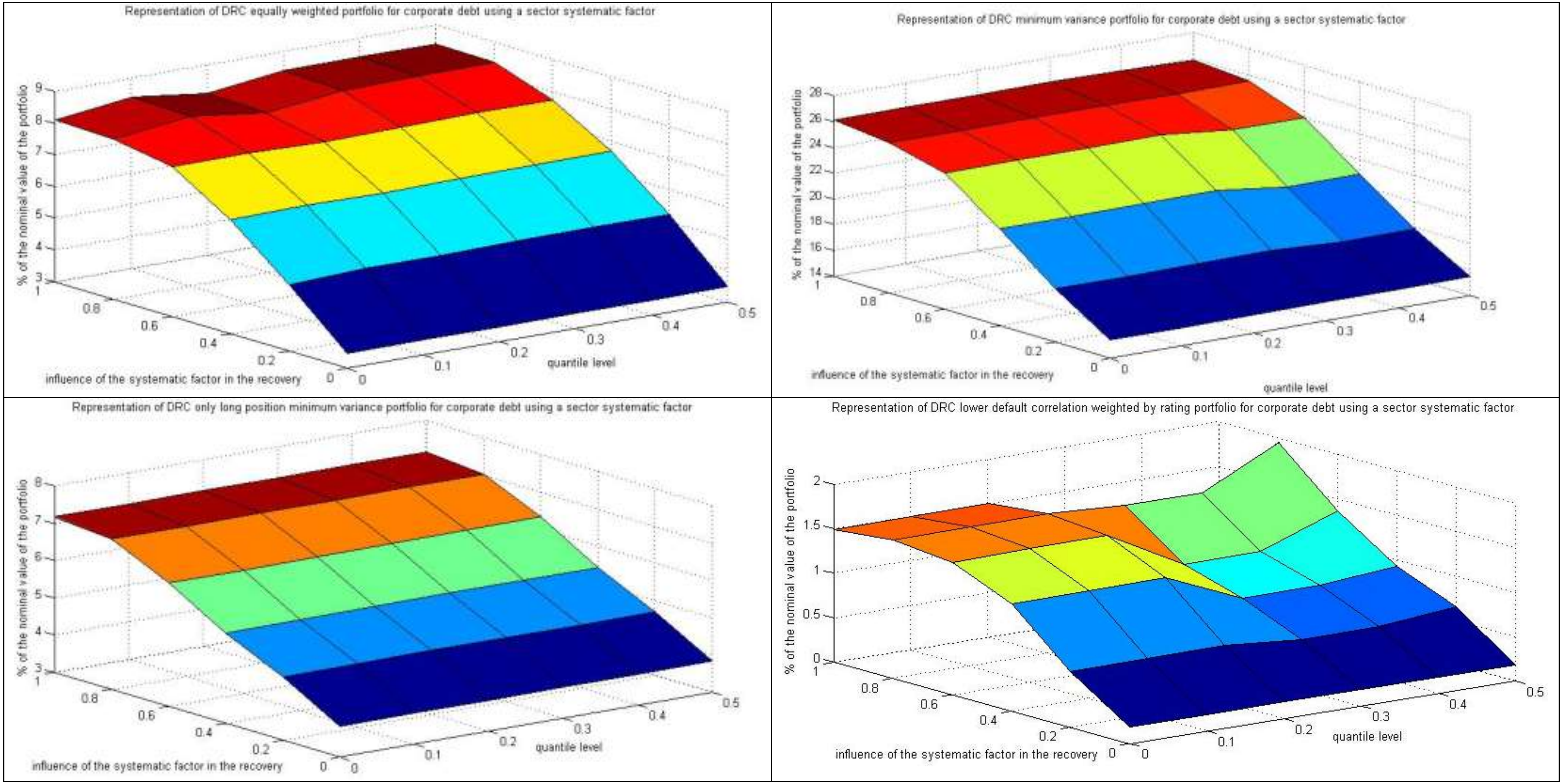

Figure 15: Different VaR depending on the quantile of the regresion and the influence of the global factor in the recovery rate for a portfolio made of corporate debt using industry as a second systematic factor.

Note: the upper left graph represents a portfolio made of corporate debt using country as a second systematic factor, while the upper right graph represents the same portfolio using industry as a second systematic factor. The lower left graph represents a sovereign debt portfolio, while the lower right graph represents a portfolio of corporate and sovereign debt.

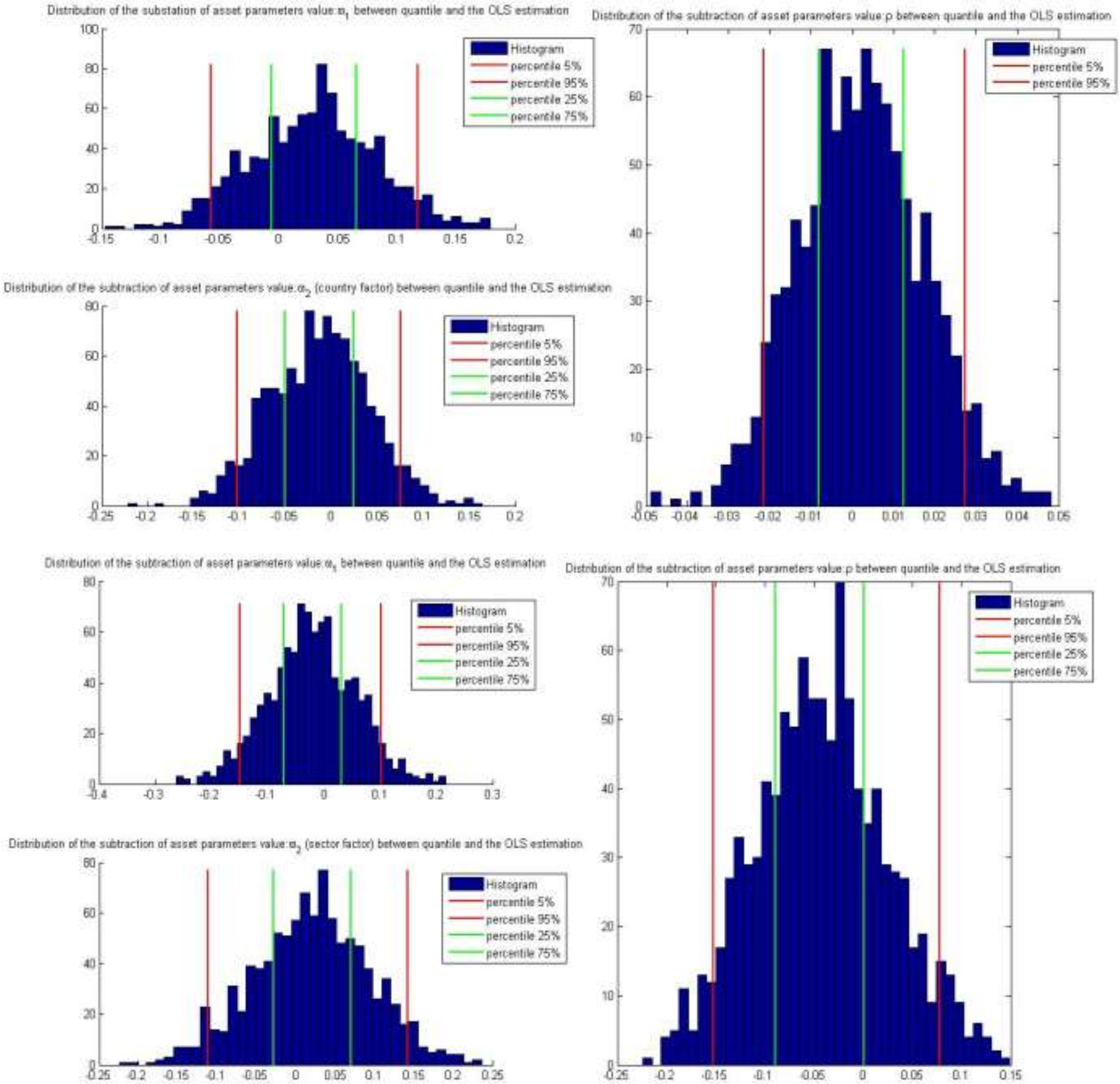

Figure 16: Estimated parameters test (q=0.2 regression against OLS regression) for BBVA returns using country (upper)or sector (lower) as a second factor

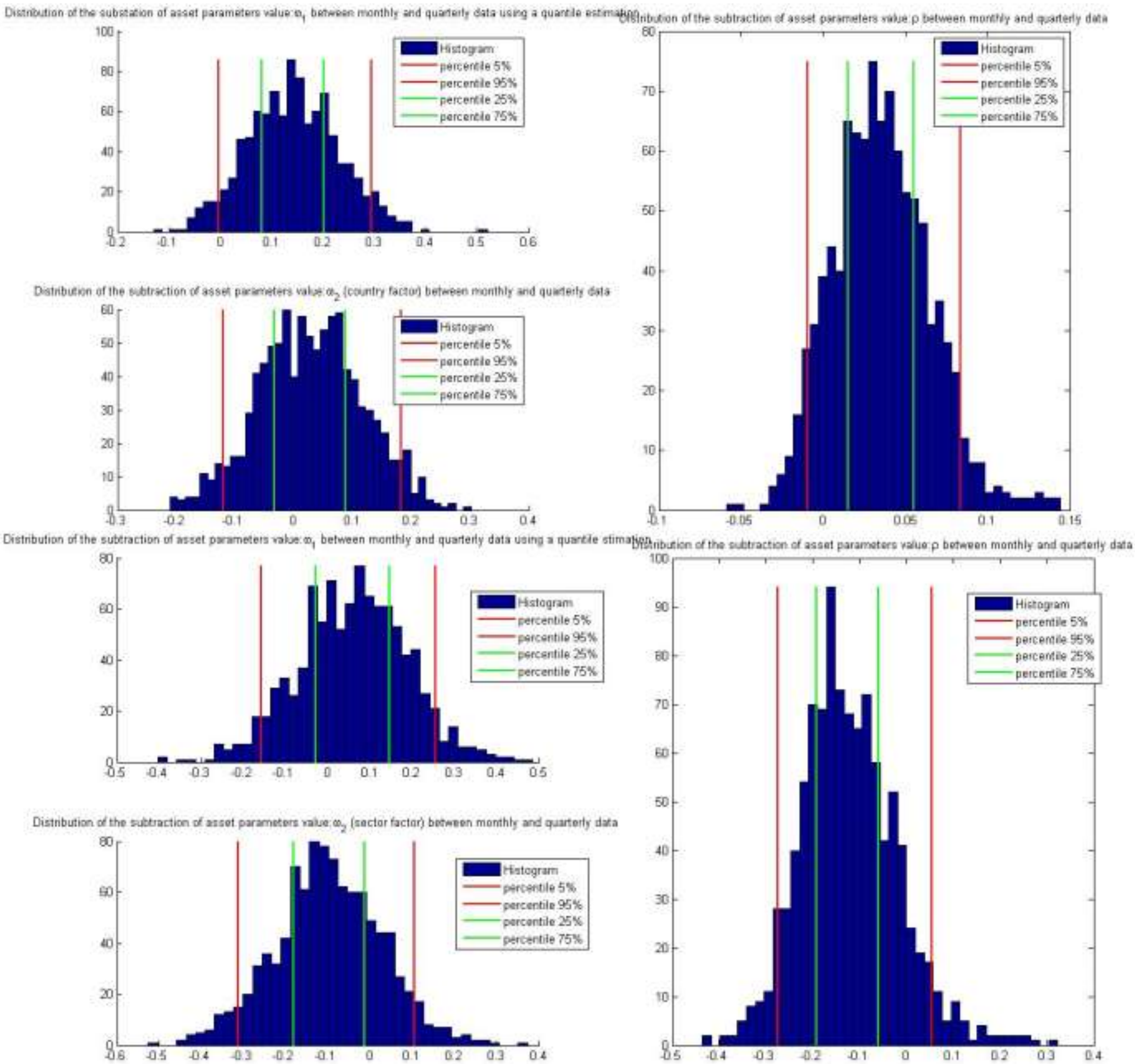

Figure 17:Estimated parameters test (monthly against quarterly data employed to a q=0.2 quantile regression) for BBVA returns using country (upper)or sector (lower)as a second factor

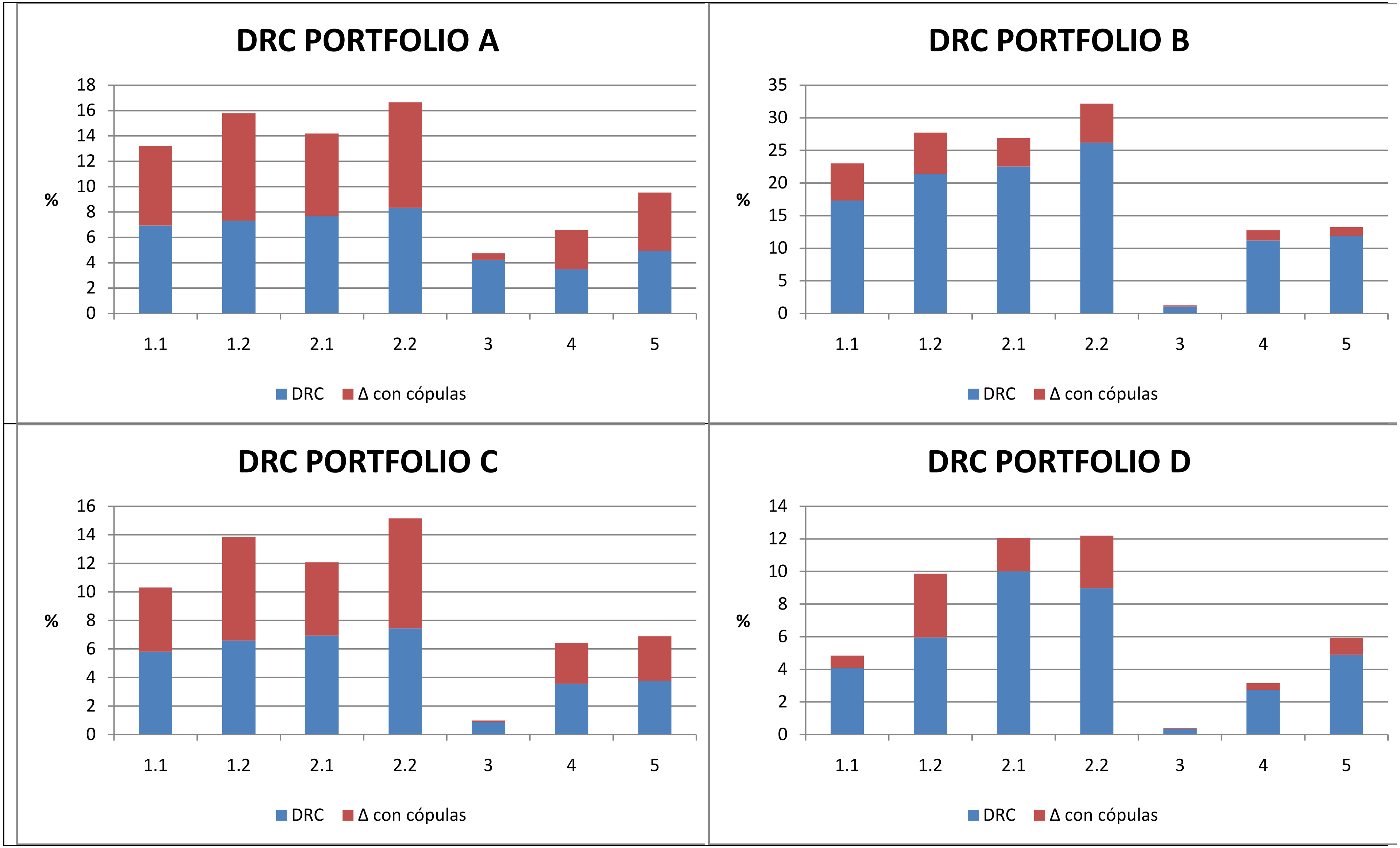

Figure 18: DRC of the different portfolios and the increase if a Clayton copula is employed in the systematic factors