Interest Rate Volatility Risk under FRTB

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Abstract

The Fundamental Review of the Trading Book (FRTB) introduces the concept of non-Modellable risk factors to Basel's market-risk framework, which requires extra capital to be held. This new element puts pressure on banks for increasing the modellability of risk factors under FRTB, avoiding or reducing extra capital required to achieve their profitability goals too. Using external data seemed not to be enough and drove banks to explore other methods of reducing the amount of non-modellable risks.

This article focuses on different techniques of modelling approach for Non-Modellable Risk Factors (NMRFs) by means of Modellable Risk Factors (MRFs) and comparing the outcomes of each other. Also, we provide the results from two different perspectives: the P&L at Attribution Test (PLAT), which evaluate the differences between the P&L calculated by the Front Office (F.O) and the P&L calculated from market risk department; and the Internal Model Capital Charge (IMCC).

Keywords: Risk factors, Non-Modelllable Risk Factors (NMRF), Fundamental Review of the Trading Book (FRTB), modellability, PLAT, IMCC, volatility surface, Interpolation approach, Statistical Modelling approach, Parametrization approach.

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1. Introduction

The outbreak of the global financial crisis has highlighted the requirement to take account of the bank's risk management practices. Basel Committee on Banking Supervision (BCBS) has been working with global authorities to strengthen the regulation, supervision and practices of the financial sector with the purpose of improving financial stability.

The Fundamental Review of Trading Book (FRTB), known as Basel IV, is the newest supervision guidelines proposed by the Basel Committee. This overhaul framework for market risk regulatory capital rules focus on: Trading Book and Banking Book boundaries, Internal Model Approach (IMA), Standardized Approach(SA) capital charge calculation and the incorporation of the risk of market illiquidity.

This paper focuses on the second approach in capital requirement of Basel IV: the Internal Model Approach, where banks need to determine an appropriate set of market risk factors to capture an accurate effect on the value of the bank's trading position. In line with banking regulation, a risk factor must fulfil some standards to be classified as modellable, with the result that banks are subject to extra capital requirement for those non-modellable risk factors. Therefore, this new regulatory framework puts pressure on the bank's business goals to maintain profitability at the same time, they are complying with the high capital requirements from Basel IV.

Currently, the challenge for all banks is to meet the modellability criteria and looking for the solution of reductions in NMRF-related capital. Hence, the effort on exploring some possible solutions about that is the aim of this research. We have focused on the use of proxies to determine how a non-modellable risk factor can be explained by a modellable risk factor.

The research has been split into two different parts. The first part is related to *Theoretical Framework* (Section 2), where it is introduced a brief chronological summary to put into perspective the evolution of the market risk framework under Basel regulation. The second part, *Practical Framework (Section 3*) is set out and analysed.

2. Theoretical Framework

2.1. Market Risk regulation review

Market risk is the reason and basis of The Fundamental Review of the Trading Book, since this regulation mainly consists of taking a thorough look at the way a finance entity manages the level of market risk involved in their trading book[1]. The BCBS defines the market risk as to the risk of losses in on and off-balance sheet positions, arising from adverse movements in risk factors that fix their market prices.

Over the past few decades, market risk regulatory capital requirements have changed considerably. Since BCSB's beginnings, it was excluded from Basel I, the first standard published in 1988, being introduced for the first time through an amendment in 1996. By this time, internal models to calculate capital requirements were allowed. The next step was Basel II in 2004, principally focused on self-regulation and market discipline, without making major changes on market risk, fostering greater confidence in bank's models based on Value at Risk (VaR)[2]. This regulatory lasted until 2007-2009 world financial crisis, during which banks incurred significant trading book losses. Subsequent to this, a whole consultation process and amendments emerged, being in the spotlight the need to develop an adequate market risk regulatory framework. Due to this, a stress component was introduced as a market risk measure, Stressed VaR (SVaR)[3].

Subsequently, Basel III was developed to be a remedy and response to address the 2007-2009 crisis. This new regulatory wave involved a source of capital requirements changes, imposing a greater quantity and quality of it. In 2012, The BCBS addressed the recent

[1] *A trading book consists of all instruments comprise financial instruments (primary financial instruments or cash instruments and derivative financial instruments), foreign exchange (FX), and commodities.* (*Minimum capital requirements for market risk -BCBS (2019). Section RB25, paragraphs 25.1 and 25.2)*

[2] *VaR is a measure of the worst expected loss on a portfolio of instruments resulting from current market movements over a given time horizon and pre-defined confidence level. (Minimum capital requirements for market risk -BCBS (2019). Section MAR10, paragraph 10.15)*

[3] SVaR is based on the VaR methodology, but calculations are computed on a stressed historical period in order to give an idea of possible losses given worse market conditions.

persistent wave of regulations, FRTB, with related market risk measures. One proposal was to change the internal model based on VaR and SVaR, introducing the use of Expected Shortfall (ES) for capturing tail risk. Until 2016, the final market risk standards were not published and had not come into effect until January 2019, being a subject of intense debate among the industry.

FRTB, comparing with the last Basel III, has been introduced with new points in the market risk framework. Overall, key-elements are:

- Changes in boundaries of Banking and Trading book to reduce incentives to arbitrage between them.
- Amendments to the internal model approaches, where this research work has been conducted.
- Amendments to standard model approach to make it in better alignment with the bank's actual risk management practices, pulling it to be more granular and risk sensitive.

2.2. Market Risk under FRTB

Under banking regulatory, the measurement and evaluation of the Trading Book Risk, which the bank is exposed, could be done using either the standard approach or internal model approach, respectively submitted for the approval of the designed authorities. Banks are called to manage properly their market risk, in such a way capital charge is being appropriately met. This paper is focused on risk measure techniques by means of the internal model approach, mainly on modellability risk factors requirements.

Quantitative market risk assessment involves defining the probability of occurrence and potential impact of the risk to the organization's business. ES-based metric is currently used to capturing risk, which could be carried out into different alternatives[4]. The historical simulation model is the one we have decided to use along with the research.

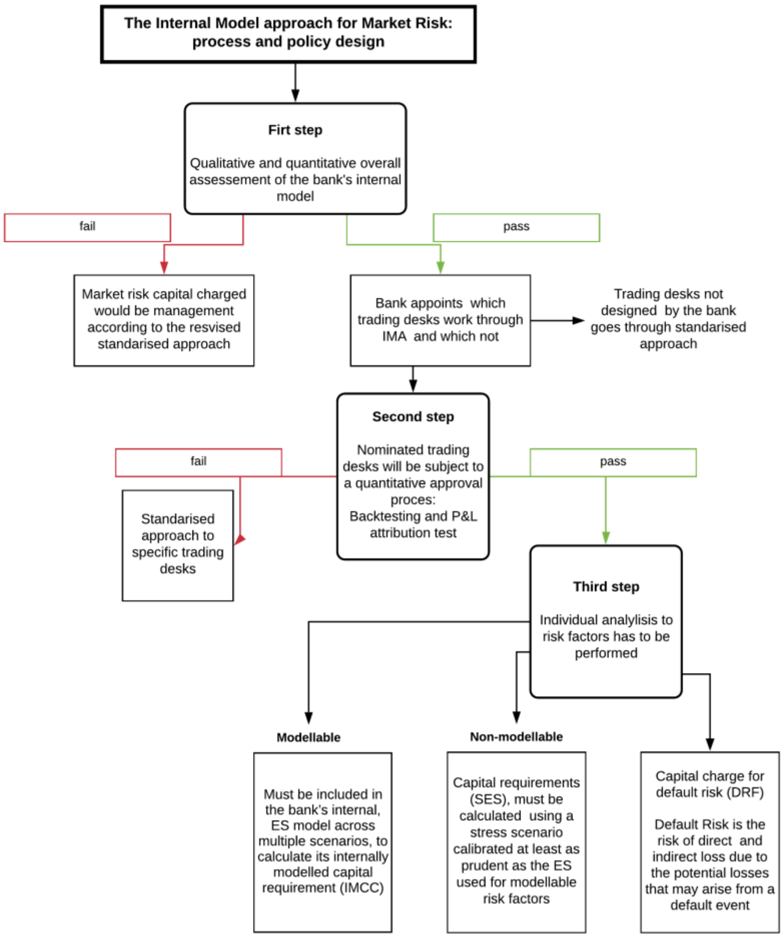
[4] ES metric could be carried out into Model Construction or Variance-Covariance Approach (parametric method), Historical simulation (takes actual past market movements as scenarios) or Monte Carlo simulation (generates random hypothetical scenarios).

This approach is a non-parametric method, that uses empirical distribution of variations time series to generate the related risk metric. ES measure relay on the average of extreme losses in the VaR engine.

The IMA process and design is based on four core checks:

1. Qualitative and quantitative evaluation and endorsement by supervisors of the accuracy and robustness of a bank's risk measure framework.
2. Modellability requirements of underlying risk factors under market prices.
3. Backtesting requirements and PLAT test requirements, which consists of a comparison between daily risk-theoretical P&L (RTPL) signed off by front office and the hypothetical P&L (HPL) used in the ES engine.
4. Calculation of the aggregate capital charge for modellable risk factors (IMCC)

Figure 1 Internal Model approach process for Market Risk



The IMA process illustrated in Figure 1, reveals that trading desks are finally the level where the approval of the bank's internal model is given. Because of the relevance of the quantitative requirements, the following subsections will try to define the ones specified for Backtesting and PLAT, which are determinants if a bank wants to use IMA to determine market risk capital. Moreover, it is introduced the specifications of the calculation of IMCC.

2.2.1. Backtesting Requirements

In respect of backtesting requirements at the trading desk level, FRTB reads as follows:

> *At trading desk level, must be compared the VaR measure calibrated at the 97.5th and the 99th percentile of the actual P&L (APL) and the hypothetical P&L (HPL) using the prior 12-months current observations. The exception threshold is 12 at the 99th percentile and 30 at the 97.5th percentile. If any given trading desk obtains results above those limits, their capital requirements must be determined through the standardised approach. (Minimum capital requirements for market risk - BCBS (2019). Section MAR32, paragraphs 32.18 and 32.19)*

As this requirement is not a subject of this article, for a further analysis please see (Fuentes.N, July 2019).

2.2.2. PLAT Test Requirements

In a nutshell, the rationale of PLAT is to evaluate the differences between P&L computed by Front Office and the one calculated at Risk.

In this regard, PLAT test requirements are worded as follows:

> *Any given trading desk must compare daily risk-theoretical P&L (RTPL) with the daily hypothetical P&L (HPL), which is the same as the HPL used for backtesting. The RTPL is the daily trading desk-level P&L that is produced by the valuation engine of the trading desk's risk management model. There must include modellable, had taken up in the ES model, and non-modellable risk factors, which*

are not included in the ES model. (Minimum capital requirements for market risk -BCBS (2019). Section MAR32, paragraphs 32.22 and 32.24)

PLAT test is based on two different metrics, using the time series of the most recent 250 P&L Vectors:

- **Spearman correlation**: the objective is to find out the correlation between the HPL and RTPL P&L Vectors.

  Assessment process

  As a first step, the bank must rank each P&L vector in ascending order, so that the lowest value receives a rank of 1.

  After ranking, the bank must calculate the Spearman correlation coefficient applying the following formula:

$$r_S = \frac{cov(R_{HPL}, R_{RTPL})}{\sigma_{R_{HPL}} \cdot \sigma_{R_{RTPL}}} \tag{1}$$

  Where, σ_{HPL}, $\sigma_{R_{RTPL}}$ are the standard deviations of each ranked P&L vectors and $cov(R_{HPL,} R_{RRTPL})$ is the covariance between two ranked times series.

  Finally, the output must be compared with the predefined regulatory threshold.

- **Kolmogorov -Smirnov test**: the objective of this test is to measure the similarity of the distribution of HPL and RTPL.

  Assessment process

  Firstly, the bank must calculate the empirical cumulative distribution function (*cdf*) of RTPL and HPL. For any value, *cdf* is the product of 0.004 and the number of each observation that is less than or equal to the specified P&L vector. The KS statistic is the largest absolute difference observed between these two empirical cumulative distributions at any P&L value. This output must be compared with the predefined regulatory threshold.

PLAT test metrics evaluation consists in classifying a trading desk in three zones on the basis of the outcome of the tests described before. Both metrics of PLAT test must meet the thresholds, which are summarised in Table 1.

Table 1 PLAT thresholds

Zone allocation thresholds				**Zone allocation consequences**
Zone	**Spearman Correlation**		**Kolmogorov-Smirnov Statistic**	
🟩	SP > 0.8	and	KS < 0.09	🟩 Trading desks are entitled to go for the IMA
🟧	0.8 < SP < 0.7	or	0.12 < KS < 0.09	🟧 Trading desks are not considered out-of-scope to go for the IMA, but are subject to
🟥	SP < 0.7	or	KS > 0.12	🟥 Trading desks are entitled to go for the IMA

Source: *Minimum capital requirements for market risk -BCBS (2019). Section MAR32, paragraph 32.42*

Any given trading desk allocated in the red PLAT test zone is unfit to apply IMA, having to be used standardised approach to determine market risk capital requirements.

On this point, while PLAT attribution test is being conducted by supervisors, each trading desk must carry out one of the most important parts in risk management: a proper market risk factors selection. According to the regulation, risk factors are the principal determinant of change in the value of an instrument, and they are bound to meet a number of modellability considerations.

Risk Factor eligibility test (RFET) is the name given to the necessary condition that any risk factor in a trading desk has to fulfil. This test calls for a sufficient amount of real[5]

[5] *BSBC establish that a price is considered as real if it must have meet at least one of the following criteria 1. It is a price at which the institution has conducted a transaction. 2. It is a verifiable price for an actual transaction between other arms-length parties. 3. It is a price obtained from a committed quote made by the bank itself or another party.* 4. It is a price that is obtained from a third-party vendor. *(Minimum capital requirements for market risk -BCBS (2019). Section MAR31, paragraph 31.12)*

prices that are representative of each risk factor. Once a price is account as real, it shall serve as an observation of every risk factor for which is representative.

Regarding the modellability, where we focus on, FRTB, in short, sets the following requirements that any risk factor has to satisfy to be considered as modellable:

1) At least, 24 observable real prices per year during the term where ES bank's model has calibrated without more than an observation per day. Moreover, on 12 previous months cannot have passed a 90 days period where less than 4 real price observations. This should be checked monthly.
2) Alternatively, banks can isolate at least 100 observable real prices over the 12 previous months.

The importance of an efficient assessment of modellable and non-modellable risk factors (MRF and NMRF) lies in the impact that NMRFs have in the IMA capital charge. While Modellable Risk capital charge is based on aggregating ES measure across a number of scenarios, Non-modellable Risk Factors will be capitalized individually using stress test scenarios without diversification. No diversification is translated into extra capital to be held. Last year, The International Swaps and Derivatives Association (ISDA), found in a Quantitative Impact Study (QIS) that capital associated with NMRF is excessive, being 4.66 timed the ES component of the FRTB IMA capital (ISDA-GFMA-IIF, June 2018). This links with the following section where it is introduced the IMCC requirements.

2.2.3. IMCC Calculation

IMCC calculation is the subsequent step once Backtesting and PLAT have been successfully passed. As mentioned before, ES-based metric is currently used to capturing market risk, being carried out, in this research, into the historical simulation model, in order to simplify the process. This approach uses the empirical distribution of variations time series to generate the related risk metric.

Under the historical simulation model, past variations are applied to a base scenario, producing stress scenarios and therefore obtaining Profit and Loss (P&L[6]) values.

[6] P&L: profit and losses vector computing from simulating the performance of the current positions held, against historical market movements and compute the difference against base scenario.

Calculating ES implies using the 97.5^{th} percentile one-tailed confidence level of the P&L distribution.

As mentioned before, results are going to be reported from two perspectives: PLAT and IMCC. Thus, it is important to underline the differences in P&L computing for PLAT requirement and P&L for IMCC.

1. In the first case, variations for PLAT metrics are computed for one-day intervals to shock each risk factor. They have to be calculated in a 12-month window and reviewed on a quarterly basis.
2. In the other case, IMCC, base liquidity horizon[7] is established of 10 days, that it means shocking all the risk factors by computing variations for 10 days. They have to be calculated in a 12-month window and on a daily basis.

Following the outline of the document, in the following section, we move into the *Practical Framework* (Section 3) where it is map out the essential parties of the research, the inputs and methodologies needed (Points 3.1 and 3.2) and the final output (Point 3.3). Methodologies needed, refer to this paper aims to set out: different methods to perform the modellability assessment of IR volatility surfaces, in order to reduce, as much as possible, the amount of non-modellable risk factors by determining how a NMRF can be explained by a modellable risk factor.

3. Practical framework

In this section, the main stages that make up the experimental part of the document are described. Firstly, we introduce in point 3.1 an overview of the essential inputs of our empirical research. Continuing in the following point, 3.2, it is displayed and developed

[7] Basel Committee on Banking Supervisions defines Liquidity Horizon as *the time assumed to be required to exit or hedge a risk position without materially affecting market prices in stressed market conditions*. *(Minimum capital requirements for market risk -BCBS(2019).Section MAR10, paragraph 10.20)*

the methodological proxies applied. In the ending point 3.3, we summarized the results in order to compare the different techniques implemented.

3.1. Inputs

This point focusses on the key inputs that have been necessary in order to develop the research properly. First, we described the Stochastic Volatility Model implemented (3.1.1), justified by the emerging of the new negative interest environment and for being one of the models most favoured by the industry in the context of interest rate derivatives. Secondly, volatility market data retrieving through the previous model is explained step by step (3.1.2), pointing out the main features of volatility surfaces that we require for our objective. Finally, volatility data (3.1.3) is characterized in accordance with the purpose of the research, modellability assessment under FRTB.

3.1.1. Stochastic Volatility Model

Before describing the data-sets included in the paper, a brief introduction will be given to explain the new volatility conventions in negative interest environment and the consequences on the models that have been traditionally used by the industry.

Negative interest rates break down any valuation formula that is based on terms that only contemplate positive forward rates, such as logarithm in the Black's model. As a result, current negative interest rates require modified lognormal models, as Shifted SABR, widely used by practitioners, and which has been implemented in this research. Furthermore, this type of model is able to handle another drawback that characterizes Black's model and any traditional one-factor interest rate models too. It is related to the assumption of the independence of implied volatility (σ_B) from the value of the strike (K) and the forward rate (f), which is not consistent with the well-known market volatility smiles or skew. This concept gives the name to the reliance between implied volatility and its strikes. This model was originally proposed by (Hagan et al.,2002), in this original form, Stochastic Alpha Beta Rho model, (SABR). It is a two-factor stochastic volatility model, that provides an approximation formula for the implied volatility. This classic model only allows non-negative rates, due to this, Shift SABR model takes on meaning by replacing the forward rate, f , with a shifted forward rate $f + s$. The shift parameter s, is a positive constant that avoids forward rates and strikes going down to 0. All expression of classic SABR model are the same for the shifted SABR model, by adding the displacement parameter to the forward rate and to the strike.

In the displaced SABR model, the underlying instantaneous forward rate, f_t and its instantaneous volatility σ_t, is modelled as:

$$d(f_T + s) \ = \sigma_T \cdot \ (f_T + s)^{\beta} dW_T \tag{2.a}$$

$$d\sigma_T \ = \nu_T \cdot \alpha_T \cdot \ dZ_T \tag{2.b}$$

Where:

- f_T is the forward of the underlying cap rate for expiry, or fixing at T.
- α may be thought of as the volatility of f_T, related to the at-the-money volatility
- β controls the lognormality of the forward evolution, β =1 represents a stochastic lognormal model, and β = 0 represents a stochastic normal model.
- ν_T is the volatility forward volatility, the volatility of σ_T
- dW_T and dZ_T are standard Brownian motions under the terminal measure Q^T, the drivers for σ_T and f_T, respectively. This also means σ_T and f_T are martingales under Qt.

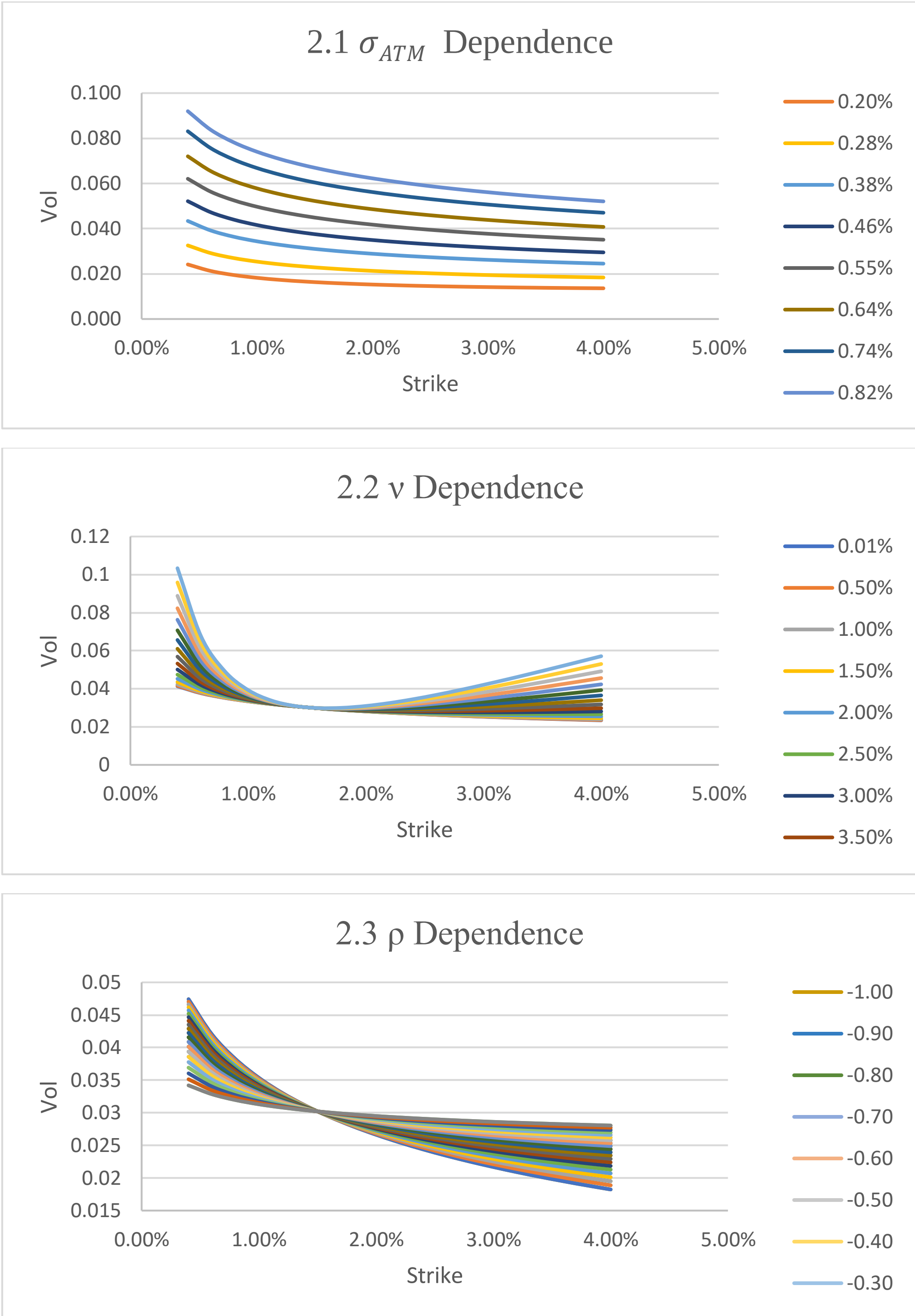
Allowing the relation that prices and market smiles have in reality, moving in the same direction, the forward and the volatility are correlated as follows:

$$E^{Q^T}[dW_t \cdot \ dZ_t] = \rho dt. \tag{3}$$

Where ρ is the correlation between the two above Brownian motions.

Financially speaking, α determines the overall level of at the money forward volatility; ρ also manage the skew shape and ν , volatility of volatility, is a measure of convexity. Figure 2 graphs these effects.

Figure 2 Efects of shifted SABR parameters



For our purposes, the parameter β is considered as a fixed value for simplicity. The displacement s should be selected high enough to prevent values of f and K falling below zero. So, the free parameters of this model are:

• The initial volatility: α

•The leverage: ρ

• The volatility of volatility: ν

The natural bounds of the parameters are: $\alpha > 0,\ -1 \leq \rho \leq 1,\ \nu > 0.$

SABR model provides an estimate of the implied volatility curve which is used as an input in other pricing models such as Black's model. Consequently, giving the implied Black volatility there are many available analytic approximations.

The implied volatility approximation applied in this paper, $\sigma_B(f, K)$, following (Hagan et al.,2002), adding the shift parameter, is given by:

$$\sigma_{B(f,K)} = \frac{\alpha\cdot\left\{1+\left[\frac{(1-\beta)^2}{24}\frac{\alpha^2}{\left((f+s)(K+s)\right)^{1-\beta}}+\frac{1}{4}\cdot\frac{\rho\cdot\beta\cdot\nu\cdot\alpha}{\left((f+s)(K+s)\right)^{\frac{1-\beta}{2}}}+\frac{2-3\rho^2}{24}\nu^2\right]T\right\}}{\left((f+s)(K+s)\right)^{\frac{1-\beta}{2}}\left[1+\frac{(1-\beta)^2}{24}\ln\left(\frac{(f+s)}{(K+s)}\right)^2+\frac{(1-\beta)^4}{1920}\ln\left(\frac{(f+s)}{(K+s)}\right)^4\right]}\cdot\frac{z}{x(z)\}} \tag{4.a}$$

$$z = \frac{\nu}{\alpha}\left((f+s)(K+s)\right)^{\frac{1-\beta}{2}}\ln\frac{(f+s)}{(K+s)} \tag{4.b}$$

$$x(z) = \ln\left(\frac{\sqrt{1-2\rho z + z^2}+z-\rho}{1-\rho}\right) \tag{4.c}$$

In the case of at-the-money (ATM) option, K = f, the formula simplifies to:

$$\sigma_{ATM} = \frac{\alpha}{(f+s)^{1-\beta}}\cdot\left\{1+\left[\frac{(1-\beta)^2}{24}\frac{\alpha^2}{(f+s)^{2-2\beta}}+\frac{1}{4}\frac{\rho\cdot\beta\cdot\nu\cdot\alpha}{\left((f+s)(K+s)\right)^{1-\beta}}+\frac{2-3\rho^2}{24}\nu^2\right]\cdot T\right\} \tag{5}$$

Once the free parameters are calibrated, the implied volatility is a function only of the forward price f and the strike K.

3.1.2. Volatility Market Data

As shifted SABR calibration lies beyond the scope of this paper[8], for the retrieving data that has been implemented along with the practical framework, we have been based on the calibration of SABR parameters obtained from external vendors. These parameters correspond to values of $\rho_t, \nu_t, \sigma_{ATM,t}, \beta$[9]$, f_t$ for one-year historical series, that covers the period from January 31, 2018 to January 31, 2019.

Data-set provided is related to cap[10] volatilities for EURIBOR 6M (at fixed Strikes) along the following range of maturities(years): {0.25, 0.5, 0.9, 1,3,5,10, and 20}.

Parameter α_t has been calibrated using the values of those parameters, applying the following cubic polynomial, obtained as of ATM formula (see Equation (5)).

$$\left(\frac{(1-\beta)^2}{24\cdot(f+s)^{2-2\beta}}T\right)\alpha^3 + \left(\frac{\beta\cdot\nu\cdot\rho}{4\cdot(f+s)^{1-beta}}T\right)\alpha^2 + \left(1+\frac{2-3\cdot\rho^2}{24}\nu^2 T\right)\alpha - \sigma_{ATM}\cdot(f+s)^{1-\beta} = 0 \qquad (6)$$

(West. G,2005) justified along with his research, that this equation may have more than one real root for alpha and his proposal is using the smallest positive one. With this in mind and following (Pineda. A, 2017) and (Tsvetanova. E, 2017), we decided to implement it. Furthermore, the equation may have imaginary solutions, so in this case, we just keep the smallest real part. It should be mentioned that some cases are handled to obtain a more consistent calibration of this parameter.

By computing those parameters as inputs through shifted SABR model, we have built cap volatility surfaces referenced of EURIBOR 6M, replicating implied market volatilities quotes, related to the previously tenors and the following range of strikes:

---

[8] For a further analysis about SABR calibration please see (Pineda. A, 2017) and (Tsvetanova. E, 2017).

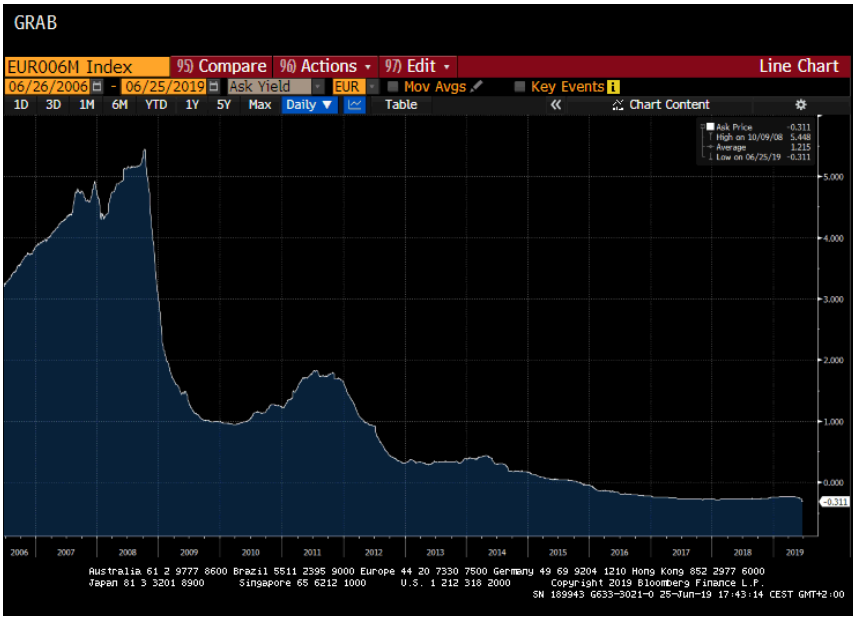
[9] As mentioned before, β is considered as fixed value for simplicity, the same as the parameter s. Them are fixed, respectively, on 0.7, obtained from external vendors, and 0.03, according industry conventions.

[10] The concept of caps through this research, is related to a type of European option which is the sum of caplets. Each caplet is the same type option too, on the 6-month Euribor rate.

{ATM, -1, -0.5, +0.5, +1, +1.5, +2, +2.5, +3, +3.5, +4, 4.5, +5}

In our case, we have built our volatilities surfaces within floating strikes, instead of fixing, in order to provide a more reliable comparison between volatilities at different times. To further understand this reason and strengthen it, Figure 3 displays the temporal evolution of EURIBOR 6M fixings between 2006 and 2019 with maturity 10 years. According to with the figure, not work with floating strikes implies applying historical variations related deep out of the money strike, on today variations related to ATM strike.

Figure 3 EURIBOR6M fixing historical series between 2006 and 2019

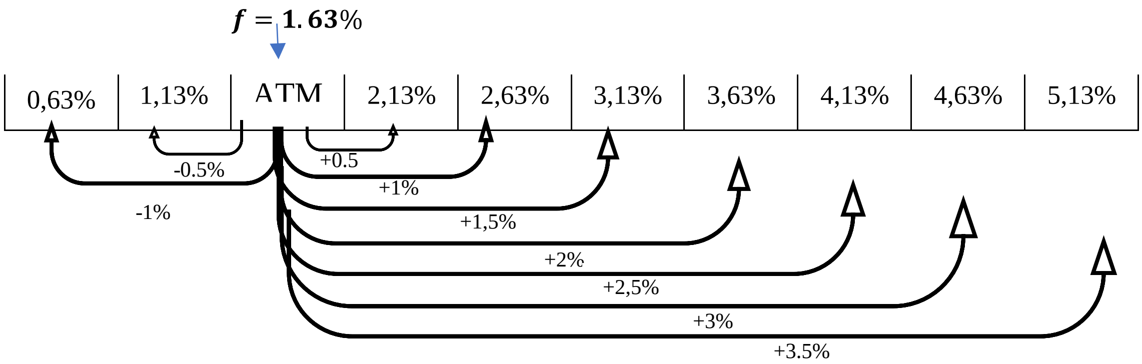


Source: Bloomberg

As it will report later in this section, we are going to compute volatilities variations, due to this, it is important to generalize all the strikes stablishing each ATM strike at the relating forward rate for each time. From this point, it is applied a granularity[11] of (∓ 0.5%) to every ATM strike, finally obtaining the entire volatility surfaces as follows:

[11] It is important to notice that definition of a higher level of granularity in the structure of the volatility smile is restricted to the granularity defined in the modellability map (see 3.1.3).

Figure 4 Generating Floating Strikes



Notes: Forward rate corresponds to the underlying of a cap EURIBOR 6M maturity 10 years

3.1.3. Modellability Map

Recalling the purpose of this research, modellability assessment under FRTB, once the entire volatility smile has been achieved, to address the core of the practical framework, we have had to choose some strikes as modellable risk factors. The selection of these strikes has been made under the premise that the highest trading concentration occurs close to ATM, at the same time we seek to provide results from a conservative view. In this way, a balance is achieved between real and conservative situation. Only three strikes, due to a parametrization proxy, which will be explained later (Point 6 on Section 3.2.2.3), have been set as a modellable risk factor of the full volatility surface computed through SABR model: ATM-0.01, ATM and ATM+0.01.

Figure 5 Modellability implementation

	ATM-0.01	ATM-0.005	ATM	ATM+0.005	ATM+0.01	ATM+0.015	ATM+0.02	ATM+0.025	ATM+0.03	ATM+0.035	ATM+0.04	ATM+0.045	ATM+0.05
MODEBLLABILITY SURFACE													

By this way, we manage to explain complete smiles by means of a conservative scenario, that is, only 3 modellable risk factors, and indeed to be able to make an extrapolation analysis for deep out of the money cap options.

3.2. Methodological Proxies

As mentioned earlier in this research, a proper risk factor selection is one of the most important parts of risk management, in terms of the quantity of capital required and the possibility to compute it going through IMA. According to some modellability specifications (see point 2.2.2), each risk factor selected must be classified as Modellable or Non-modellable with their respective impact on market risk capital requirements. In order to reduce the IMA capital charge of Non-modellable risk factors, banks are focused on looking for ways to decomposite risks into modellable and non-modellable.

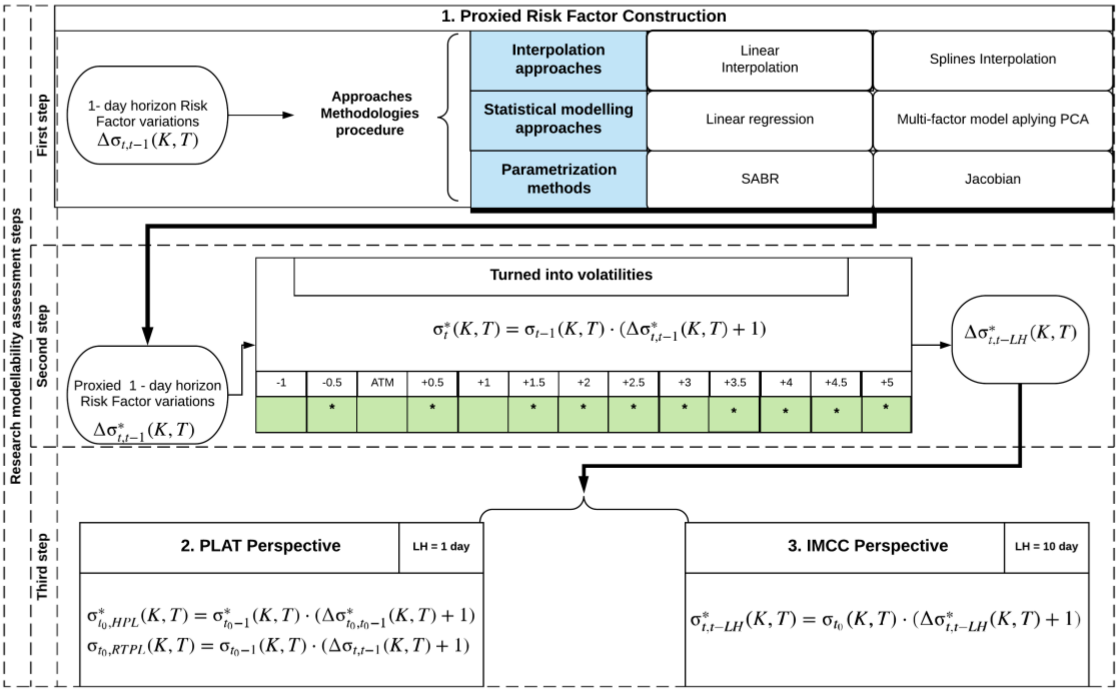
In section 3.2.1, we will describe the common essential steps regarding the modellability assessment. Once the common framework has been established, in section 3.2.2 we will describe different methodologies that shall be considered to perform the modellability assessment of IR volatility surfaces.

3.2.1. General Framework

Before getting to know the common essential steps regarding the modellability assessment, it is important to draw a distinction between the three key parties involved in the whole process: <u>Proxied Risk Factor Construction</u>(1) based on the above section (3.1), <u>PLAT perspective</u> (2) and <u>IMCC perspective</u> (3), both linked to the evaluation of the results of research purpose.

In order to summarize the whole process, the modellability assessment steps followed along this research are summarised in Figure 6:

Figure 6 Modellability steps



Notes: At the second step, squares marked with an asterisk are related to the NMRF partially modelled by approach methods implemented.

1. <u>Proxied Risk Factor Construction</u>:

This party contains **first** and **second steps.** It is the crucial one involved in the process because NMRF approaches in terms of MRF have been made based on one-day historical variations MRF series. In this sense, before starting the process, we should calculate these historical variations of the volatility surface.

As mentioned in the input section 3.1.2, we have built our volatilities surfaces along with floating strikes rates through shifted SABR parameters already calibrated. Working in this line, it has been set out as Risk Factors each original implied volatility: $\sigma_t(K,T)$, where K refers to the floating strike, T refers to the maturity of the option and t is referred to asthe scenario of a given date.

The variation is computed, for each Risk Factor, in a multiplicative way:

$$\Delta\sigma_{t,t-1}(K,T) = \frac{\sigma_t(K,T)}{\sigma_{t-1}(K,T)} - 1,$$
$$i = \{\text{ATM}, -1, -0.5, +0.5, +1, +1.5, +2, +2.5, +3, +3.5, +4, 4.5, +5\} \tag{7}$$

The decision of computing the modellability methodologies as of 1-day variation series has been taken in order to obtain accurate results with lower error rate.

Once the original multiplicative variations of each Risk Factor between scenarios t and $t-1$ have been calculated, the modellability assessment process begins with the **first step.**

The first step consists on to split risk factors into modellable and non-modellable, according to of what has been exposed in Section 3.1.3, (See Figure 5 for the selection of modellable and non-modellable risk factors). Those modellable floats strikes should be set up as modellable risk factor itself, that is, using its own historical series ($\Delta\sigma_{t,t-1}(K,T) = \Delta\sigma_{t,t-1}(K,T)$. For the non-modellable ones, we will use proxy methodologies to make them partially modellable, thus, it is not possible to apply its historical series, leading us to use an approach based on MRF variations, $\Delta\sigma_{t,t-1}(K,T) \cong \Delta\sigma^*_{t,t-1}(K,T)$).

All methods implemented are based on how a non-modellable risk factor can be explained by a modellable risk factor and we have focused on the use of proxies over the historical variations computed previously. As we can see in Section 3.2.2, we have considered the following alternatives options, which will be covered in the later:

- Interpolation approach: Linear Interpolation and Splines Interpolation
- Statistical approach: Simple Linear regression and multi-factor model
- Parametrization approach: Jacobian

The **Second step** is to re-calculate the volatility surface, but now every Risk Factor could be considered as modellable. Risk Factor (original value) in $t-1$ is stressed by shocking it with the Proxied Risk Factor Variations between t and $t-1$ so that variations of risk factors are returned into volatilities, obtaining the Proxied Risk Factor:

$$\sigma_t^*(K,T) = \sigma_{t-1}(K,T) \cdot (\Delta\sigma_{t,t-1}^*(K,T) + 1) \tag{8}$$

This is continued with computing the Stress Risk Factor Variations in order to obtain specific proxied variations requested to IMCC perspective, addressed further in (party 3). They are the proxied multiplicative variations with a liquidity horizon of $LH = 10$ days with respect to the Proxied Risk Factor.

By this way, we situate at the end of the (party 1) in order to get started with the two perspectives: PLAT and IMCC. But first, in order to summarise what is exposed above and make it easier to follow the text, Table 2 sums up the principal variables relating to the party (1):

Table 2 Main variables summary at Proxied Risk Factor Construction

$\sigma_t(K,T)$	**Risk Factor** obtained through the process explained in Section 3.1
$\Delta\sigma_{t,t-1}(K,T)$	**Original Risk Factor Variation,** computed in a multiplicative way between scenarios t and $t-1$
$\Delta\sigma_{t,t-1}^*(K,T)$	**Proxied Risk Factor Variation**: For further information about the methodological proxies, please see section [3.2.2]
$\sigma_t^*(K,T)$	**Proxied Risk Factor**: the variations of risk factors are returned into volatilities.
$\Delta\sigma_{t,t-LH}^*(K,T)$	**Stress Risk Factor Variations:** proxied multiplicative variations with a liquidity horizon of $LH = 10$

The **third step** is divided into two branches that make up the other two parties involved in the modellability assessment process: PLAT perspective (2) and IMCC perspective (3)

2. <u>PLAT Perspective</u>:

There is the essential point to determine the endorsement or not to go through IMA to capital requirement calculation. As has been explained in the theory part (2.2.2), under this view, FRTB states that for PLAT requirements, one-day variation series must be calculated to generate stress scenarios, which are called P&L vectors in this context. P&L vectors would result from:

$$\sigma^*_{t_0,HPL}(K,T) = \sigma^*_{t_0-1}(K,T) \cdot (\Delta\sigma^*_{t_0,t_0-1}(K,T) + 1) \tag{9}$$

$$\sigma_{t_0,RTPL}(K,T) = \sigma_{t_0-1}(K,T) \cdot (\Delta\sigma_{t,t-1}(K,T) + 1) \tag{10}$$

The base scenario always corresponds to volatility from the day before.

Specific features under this perspective among data implemented are gathered in the following notation:

Table 3 Main variables summary in PLAT perspective

$\sigma_{t_0-1}(K,T)$	**PLAT Base Scenario**
$\sigma^*_{t_0,HPL}(K,T)$	**PLAT HPL Scenario:** Volatility surfaces stressed by shocking the PLAT Base Scenario.
$\sigma_{t_0,RTPL}(K,T)$	**PLAT RTPL Scenario:** Original volatility at time t_0

3. IMCC Perspective:

One of the components of capital requirements once a bank has the approval to go through IMA. This component requests the specific features about the manner of computing P&L vectors explained in a previous section (2.2.3).

As FRTB sets for capital purposes, to later ES measurement, base liquidity horizon is established of 10 days. In this sense, once new modellable volatility surface is obtained from the one-day variation series ($\sigma^*_t(K,T)$), it is necessary to calculate variations with 10 days liquidity horizon:

$$\Delta\sigma^*_{t,t-LH}(K,T) = \frac{\sigma^*_t(K,T)}{\sigma^*_{t-10}(K,T)} - 1 \tag{11}$$

These new 10 days variations are used to generate stress scenarios

$$\sigma^{*}_{t,t-LH}(K,T) = \sigma_{t_0}(K,T) \cdot (\Delta\sigma^{*}_{t,t-LH}(K,T) + 1) \tag{12}$$

The base scenario always corresponds to volatility from the current day t. In this case, base scenario matches with the most recent day of the sample:

$$\sigma_{t_0}(K,T) = \sigma_{31/01/\,2019}(K,T)$$

These specific features are summarised with the following notation in Table 4:

Table 4 Main variables summary in IMCC perspective

$\sigma_{t_0}(K,T)$	**IMCC Base Scenario**
$\sigma^{*}_{t,t-LH}(K,T)$	**IMCC Stress Scenario:** Stress volatility surfaces by shocking the IMCC Base Scenario with the Stress Risk Factor Variations.

Even though we have not computed the capital to be held under IMA for reasons of data resources, as we have only worked with one type of Risk Factor, we introduce a brief outline of the ES measure methodology. Under this premise, computing the amount of capital required makes little sense and is not relevant.

Firstly, each P&L vector has been ranked in descending order and comparing with the 97.5th percentile, it should be selected those values larger or equal to that threshold. ES measure is computed as a mean, the sum of those extreme values divided by the numbers of exceptions.

Subsequently, to shock every risk factor to generate stressed scenarios under each perspective, P&L vectors must be calculated. Although the most accurate methodology would be to apply a full revaluation methodology, there exists another alternative that allows us to approximate the results, that is to be the Taylor approach. In order to avoid confusion, both pricing approaches are presented, although only the Taylor approach is the selected one in this document.

1. Full revaluation: involves evaluating the pricing formula to the new stress scenario generated, deriving new stressed cap prices. After that, it is calculated the

differences between cap prices obtained in the stress and in the base scenario. In this way P&L vectors are reached, allowing to observe the change in the value for the total portfolio.

2. Taylor Approximation: it is related to explain price variations of a financial product through the sensitivities of the product and the variations of the risk factors. In our case, we will be interested in finding out how the values of *caps* have changed before variations on the volatility surface. To measure this effect, we have used *vega sensitivity*:

$$vega = \frac{\partial V_{B,cap}(T, K, f, \sigma_B)}{\partial \sigma_B(f, K)} \tag{13}$$

As explained in the previous paragraph, this document is only focused on variations with respect to the volatility parameter and there is no analysis with respect to the variations of any other risk factors involved, such as implied forward rate (sticky strike approach is assumed). Applying Taylor theorem, which is a series expansion to the price function of a cap with respect to the volatility parameter, $f(x) = V_{B,t}\left(T, K, f, \sigma^*_{B,t,t-LH}\right)$, the price of an instrument can be expressed up to first order as follows:

$$f(x) = f(x_0) + f'(x_0 + dx) \cdot dx + O(h)$$

Where x in our case is related to the volatility parameter.

$$x = x_0 + dx \rightarrow \sigma^*_{B,t}(K, T) = \sigma_{B,t_0}(K, T) \cdot \left(\Delta\sigma^*_{B,t,t-LH} + 1\right)$$

To interpret this in our context:

$$V_{B,t}\left(T, K, f, \sigma^*_{B,t}\right) = V_{B,t}(T, K, f, \sigma_B) + \Delta\sigma^*_{B,t,t-LH} \cdot vega_{t_0} \tag{14}$$

Finally, P&L could be approached as follows

$$\begin{aligned} P\&L_t &= V_{B,t}(T, K, f, \sigma_B) - V_{B,t}(T, K, f, \sigma_B) \\ &= \Delta\sigma^*_{B,t,t-LH} \cdot vega_{t_0} \end{aligned} \tag{15}$$

In all the test cases performed in this document, Taylor Approximation has been used for each risk factor instead of Full Revaluation assuming a unitary value of the *vega* sensitivity.

3.2.2. Theoretical Framework

This point develops, in a general way, the several proxy methods that have been implemented, in order to meet modellability requirements under FRTB and to reach accurate approaches of NMRF in terms of the modellable ones too. These methodologies are computed when we are situated at Proxied Risk Factor construction (1), steps one and two of the modellability assessment process. These are applied to Non-Modellable Risk Factor variation ($\Delta\sigma_{t,t-1}(K,T)$), in order to obtain the Proxied Risk Factor Variation **(**$\Delta\sigma^{*}_{t,t-1}(K,T)$). We have implemented a total of 5 methodologies, although there are 6 described below, categorized in three broad groups: Interpolation approach (3.2.2.1), Statistical Modelling approach (3.2.2.2) and Parametrization approach (3.2.2.3). It is worth mentioning that knowing the specific strengths and weak points of each proxy, we decided to sum them up at the end of the results section (See Table 8), in order to not anticipate the final facts.

3.2.2.1 Interpolation approach

We are going to implement two types of interpolations: Linear Interpolation (1) and Splines interpolation (2). In an overview, them consist on interpolating the non-modellable strikes using the modellable ones, applying specific functions related to each interpolation type, which fit the given data. Linear Interpolation matches is a simple method to implement, that use a single polynomial on a closed interval. Splines interpolation, for its parts, divide the approximation interval into a group of subintervals and construct polynomials on each one. It has been applied in order to provided interpolation results from another different alternative and to compare each other.

1. *Linear Interpolation:* curve fitting method using linear polynomials.

Methodology

- We have N intervals from N+1 increasing data points: $x_0 < x_1 < \cdots < x_N$
- Each interval defines a linear polynomial: $f_i(x) = c_i x + d_i \;\; when\; x \in [x_{\{i-1\}}, x_i]$
- Two parameters to be solved in each interval: $y = y_i + (x - x_i)\frac{y_i - y_{\{i-1\}}}{x_i - x_{\{i-1\}}}$

- For those non modellable risk factors lower or higher than the give modellable risk factor, it is setting the nearest one.

2. *Cubic Spline Interpolation*: Splines are usually cubic piecewise polynomial curves

Methodology

- We have N intervals from N+ 1 increasing data points: $x_0 < x_1 < \cdots < x_N$
- Each interval defines a cubic polynomial:
  $f_i(x) = a_i x^3 \ + \ b_i x^2 + c_i x + d_i \ \ when\ x \in [x_{\{i-1\}}, x_i]$.
- 4 N parameters to be solved.

Conditions

1. The first and last splines must pass through its respective first and endpoint:
   $$f(x) \text{ must interpolate the data points}$$
   $$f_i(x_i) = y_i \quad \text{and} \quad f_i(x_{i-1}) = y_{1-1}$$
2. $f'(x)$ must be continuous at each internal knot
3. $f''(x)$ must be continuous at each internal knot
4. Natural spline: $f_o''(x_0) = 0 = \ f_N''(x_N)$ The curvature must be specified at the endpoints.

3.2.2.2 Statistical modelling approach

In this point, we are also viewing two types of this statistical modelling approach: Simple Linear Regression Model (3) and Multi-factor model based on PCA analysis (4). Univariate Linear regression is widely used in social science to describe relationships between variables and provide results in a very intuitive way. The multi-factor model helps us to complete this analysis, in the sense, it allows explaining NMRF by mean of more than one MRF, by contrast with Univariate Linear Regression Model.

3. *Simple Linear Regression Model*

In statistics, this is a mathematical model for assessing the value of one dependent variable from the value of one given independent variable. It is represented by the well-known equation:

$$Y_i = \alpha_i + X_i \cdot \beta_i + u_i, \quad \textit{where } i \textit{ is related to Risk Factors across the volatility surface} \tag{16}$$

The dependent variable, Y_i, match with those Non-Modellable Risk Factors (NMRF) and the selection of the independent variable, X_i, has relied on settle the most correlated modellable risk factor as the best proxy, as will be explained hereunder.

<u>Methodology</u>

The linear regression in our context, in scalar form, would be:

$$NMRF_i \ = \alpha_i + MRF_i \cdot \beta_i + u_i \tag{17}$$

α_i *and* β_i, the level (intercept) and the steepness of the regression line respectively, are unknown fixed parameters, which are estimates applying Ordinary Least Squares (OLS). OLS is based on an objective function $\boldsymbol{S}(\hat{\alpha}_i, \hat{\beta}_i)$ that seeks to minimize the unknown error term u_i as much as possible.

By Ordinary Least Squares (OLS) method:

- u_i is defined by: $u_i = e(\beta) = y_i - \left(\hat{\alpha}_{i_i} + x_i\, \hat{\beta}_i\right) = y_i - \hat{y}_i$, so the sum of the square of the residue is $S\left(\hat{\alpha}_t, \hat{\beta}_i\right) = \sum_{i=1}^{n} (u_i)^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$.
- α_i *and* β_i are estimated through $\hat{\alpha}_t, \widehat{\beta}_\iota$, such that $S\left(\hat{\alpha}_t, \hat{\beta}_i\right) = \min\limits_{\alpha,\beta} S(\hat{\alpha}_t, \hat{\beta}_i)$.

Minimizing the function requires computing the first order derivatives with respect to $\hat{\alpha}_t, \hat{\beta}_i$ and set them equal to 0.

$$\frac{\partial S(\hat{\alpha}_i, \hat{\beta}_i)}{\partial \hat{\alpha}_i} = -2\sum_{i=1}^{n}\left(y_i - (\hat{\alpha}_i + x_i\,\hat{\beta}_i)\right) = 0 \tag{18.a}$$

$$\frac{\partial S(\hat{\alpha}_t, \hat{\beta}_i)}{\partial \hat{\beta}_i} = -2\sum_{i=1}^{n}\left(y_i - (\hat{\alpha}_i + x_i\,\hat{\beta}_i)\right) = 0 \tag{18.b}$$

It is a linear system of two equations with two unknown parameters, which can be solved as follows:

- For alpha: $\hat{\alpha}_i = \sum_{i=1}^{n}(y_i - x_i\,\hat{\beta}_i) \dashrightarrow \hat{\alpha}_i = \bar{y} - \hat{\beta}_i * \bar{x}$ (19.a)
- For beta : $\hat{\beta}_i = \frac{\sum_{i=1}^{n}(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})} = \frac{\sum_{i=1}^{n}(y_i - \bar{y}) \cdot \sum_{i=1}^{n}(x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{Cov(x,y)}{Var(x)}$ (19.b)

The second derivatives of two functions must be positive to ensure that $\hat{\alpha}_i$ and $\hat{\beta}_i$ minimize the sum of squared residuals.

4. *Multi-factor model based on PCA analysis*

In essence, is the same as the statistic model above, but involving more than one explanatory variable. A multi-factor model is represented by the following equation:

$$Y_i = \alpha_i + X_1 \cdot \beta_1 + X_2 \cdot \beta_2 + \cdots + X_N \cdot \beta_N + u_i$$

Where Y_i is related to each NMRF, and X_i are related to the three modellable risk factors.

Using this model could trigger two setbacks:

- Multicollinearity: refers to existing possible linear relationships between model regressors.
- Dimensionality: refers to having many variables but it is possible that not all variables are equally important, not submitting a great deal of information on the variable that we want to explain through the regression model.

Because we have three modellable risk factors to explain each non-modellable, the second problem is not a reality, but to address the first one, we decided to introduce the Principal Components Analysis method (PCA).

Principal Components Analysis method

PCA is a feature extraction technique, where variables are combined in a linear way, explained at a larger stage, allowing for remove variables that provide little information to the model while preserving as much information as possible. Principal components are new variables that are constructed as linear combinations of the initial ones.

The value-added ensuring from PCA is the independence between the new variables.

Methodology

For proper understanding, we sum up the manly concepts and variables involved in this technique.

- The three modellable risk factors are included in a $T \ x \ n$ matrix $\boldsymbol{X}$, where T is the length of the sample used (one-year historical series) and n are the number of independent variables (modellable risk factors).
- V is the covariance or correlation matrix of $\boldsymbol{X}$.
- Let $\boldsymbol{W}$ be the orthogonal matrix of the eigenvectors of $\boldsymbol{V}$, as $\boldsymbol{V}$ is a symmetric matrix.
- The principal components of $\boldsymbol{V}$ are the columns of matrix $\boldsymbol{P}$, the same size as $\boldsymbol{X}$. By this way, $\boldsymbol{X}$ has been transformed into a system of orthogonal variables, which are the volatility variations, $\boldsymbol{P} = \boldsymbol{XW}$.
- Being W an orthogonal matrix implies that $\boldsymbol{W^{-1}} = \boldsymbol{W'}$ (its inverse is equal to its transpose), so $\boldsymbol{X} = \boldsymbol{PW'}$, obtaining a representation of the original variables in terms of the principal components.

Continuing with the methodology. Firstly, normalisation of all input variables so that each one of them contributes equally to the analysis. Computing eigenvalues, by solving the characteristic equation[12], and eigenvectors[13] from correlation or covariance matrix, $\boldsymbol{V}$, inasmuch as both are equivalent because of the normalisation. By this way, it is broken down into eigenvalues and eigenvectors the correlation or covariance matrix of the original variable matrix.

Sorting out the eigenvalues from highest to lowest and choosing the k eigenvectors related with the k highest eigenvalues, we get the Principal Components in order of significance. This means to order the columns of $\boldsymbol{W}$, being the first one which is associated with the largest eigenvalue of $\boldsymbol{V}$.

Each Principal Component[14] is built as linear combinations of the initial variables being the coefficients each eigenvector.

$$CP_{n,i} = w_{n,i} \cdot X_i + w_{n,i} \cdot X_i + \cdots w_{n,i} \cdot X_i$$

Where each $w_{n,i}$ are the corresponding eigenvectors columns of the $n \cdot n$ $\boldsymbol{W}$ matrix. So, the multifactorial model, where principal components are the explanatory variables, would be defined as:

$$Y_i = \alpha_i + CP_1 \cdot \beta_{1i} + CP_2 \cdot \beta_{2i} + \cdots + CP_N \cdot \beta_{Ni} + u_i$$

In our context:

$$NMRF_i = \alpha_i + CP_1 \cdot \beta_{1,i} + CP_2 \cdot \beta_{2,i} + CP_3 \cdot \beta_{3,i} \tag{20}$$

[12] The characteristic equation is defined as follows: $|\boldsymbol{X} - \boldsymbol{\lambda I}| = 0$, where $\boldsymbol{I}$ is the identity matrix.

[13] The eigenvectors are computed by resolving the following system: $(\boldsymbol{X} - \boldsymbol{\lambda I})\boldsymbol{c} = 0$, where $\boldsymbol{c}$ contains the n variables.

[14] The existing PCA literature set the first three principal components as the necessary ones to explain almost the 90% of original variables variability. Within term structure of interest rate framework, these three components are related to the level, slope and curvature of the specified curve.

As applying PCA involves to use only a reduced set of the principal components, the selection has been done by computing the proportion of cumulative variance explained from the eigenvalues until reaching a certain level of explained variance. As a result:

$$\boldsymbol{X} \approx \boldsymbol{P}^{*}\boldsymbol{W}'^{*}$$

Where $\boldsymbol{P}^{*}$ is the chosen principal components matrix and $\boldsymbol{W}'^{*}$ corresponds to the eigenvectors matrix to each elected principal component.

In our research we decided to include the three principal components, since the dimensionality problem is not a reality and using these three components implies that the model is explaining the 100% of the variance of the original variable matrix $\boldsymbol{X}$.
The estimation of the regression above has been reached using OLS too, so what was explained on the matter in the linear regression method applies here.

For further details about this statistical methodology see (Carol Alexander,2008)

3.2.2.3 Parametrization methods

By contrast with previous methodologies, we have decided to implement this parametrization method. It consists on approaches variations of SABR parameters. By this way, we have implemented the Jacobian matrix (6) of SABR and it enables to obtain results based on a coherent volatility surface. Before introducing the Jacobian approach, it is described the SABR method (5), although we have not implemented.

5. SABR

In the case of establishing a reasonable amount of modellable risk factors, this approach shall be interesting. It is related to calibrate all the SABR parameters, using modellable points and obtaining the entire modellable volatility smile.

Having only the three modellable risk factors mentioned before, the calibration of all the SABR parameters cannot be carried out, pulling us to compute a more conservative approach. This approach relies on using the Jacobian matrix of SABR function, approached by means of the three modellable points considered throughout the research. It is explained below.

It is worth mentioning, that if we were in a higher modellable environment, calibration of SABR could be done, but it would also mean that modellability no longer would be a key problem.

6. Jacobian's SABR

This approach consists on using only modellable risks points and recalculates stressed scenarios for SABR parameters[15] to obtain the entire modellable volatility smile. It is worth mentioning that given the different effects[16] on the smile from the three parameters α, ρ and ν , a likeness can be made between PCA analysis described previously. Changing the parameter α influences the level of the smile, ρ impacts directly over the curve's skew and ν controls the curvature.

The aim is to approximate variations on SABR parameters applying the Jacobian matrix. This matrix is made up of the partial derivatives of the three SABR modellable volatilities respect to the parameters: ρ, ν and α. As mentioned in the data section, α is a function of the rest of the parameters, $\alpha = f(\sigma_{ATM}, \rho, \nu, \beta)$, and can be extracted directly from σ_{ATM}. In view of this relation, we decided to approach Jacobian computing first derivatives of the three modellable risk factors with respect ρ, ν and σ_{ATM}, according to the parameterization implemented in the calibration of SABR, explained in the data point, and exploiting that ATM is set as a modellable risk factor.

As mentioned earlier, due to this proxy, the election of the modellable risks across volatility surface strikes has been only three of them. This way, we make sure that the Jacobian matrix is a square matrix, same numbers of parameters and modellable points. If the square matrix has non-zero determinant, then, it is possible to compute its inverse matrix.

[15] Both parameters, β and s, are assumed to be given constants, so derivatives with respect to them are not included.

The Jacobian matrix is a $m \cdot n$ [17] defined and arranged as follows:

$$\boldsymbol{J}_{\boldsymbol{SABR}(\boldsymbol{\rho},\boldsymbol{\nu},\boldsymbol{\sigma}_{\boldsymbol{ATM}})} = \begin{pmatrix} \frac{\partial \sigma_{ATM-0.01}}{\partial \sigma_{ATM}} & \frac{\partial \sigma_{ATM-0.01}}{\partial \rho} & \frac{\partial \sigma_{ATM-0.01}}{\partial \nu} \\ \frac{\partial \sigma_{ATM}}{\partial \sigma_{ATM}} & \frac{\partial \sigma_{ATM}}{\partial \rho} & \frac{\partial \sigma_{ATM}}{\partial \nu} \\ \frac{\partial \sigma_{ATM+0.01}}{\partial \sigma_{ATM}} & \frac{\partial \sigma_{ATM+0.01}}{\partial \rho} & \frac{\partial \sigma_{ATM+0.01}}{\partial \nu} \end{pmatrix} \tag{21}$$

<u>Methodology:</u>
Finite Difference Method, specifically, Forward Difference Method has been applied to determine the entries of the matrix above. It must mention that this approach would be done by the closed-form formulas of each derivative, attaining more favourable results.

A summary of the variables involved in this method is worded:

Being $\boldsymbol{f} : \mathbb{R}^{\boldsymbol{n}} \rightarrow \mathbb{R}^{\boldsymbol{m}}$ a function with as inputs the vector $\boldsymbol{\sigma} \in \mathbb{R}^{\boldsymbol{n}}$ that produces as output $\boldsymbol{f}(\boldsymbol{\sigma}) \in \mathbb{R}^{\boldsymbol{m}}$, carrying it out of our context:

- $\boldsymbol{\sigma} \in \mathbb{R}^{\boldsymbol{n}}$: $\{\Delta\sigma_{ATM,t}, \Delta\sigma_{ATM-1,t}, \Delta\sigma_{\mathrm{ATM+1,t}}\}$
- $\boldsymbol{f}(\boldsymbol{\sigma}) \in \mathbb{R}^{\boldsymbol{m}}$: $\{\Delta\sigma_{ATM,t}\ , \Delta\rho_t, \Delta\nu_t\}$

Where m refers to the original number of parameters, in our case $(\sigma_{ATM}, \rho, \nu)$, and n refers to a number of variables, in our case {ATM, ATM-1, ATM+1}.

Following this notation and given SABR implied volatility function, its derivative can be approximated through finite differences with respect to the base scenario $(\rho = \rho_{t_0}, \nu = \nu_{t_0}, \sigma_{ATM} = \sigma_{ATM_{t_0}})$ as follows:

$$\frac{\partial f}{\partial x}(\mathrm{x}_0) = \frac{f(\mathrm{x}_0 + dx) - f(\mathrm{x}_0)}{dx} + O(dx)$$

[17] In this case, *m* refers to the number of modellable risk factors, which are: ATM-0.001, ATM and ATM+0.01. *n* refers to the number of parameters, they are three too: σ_{ATM}, ρ, ν

In our context:

$$\frac{\partial\sigma_K\left(\sigma_{ATM,t_0},\rho_{t_0},\nu_{t_0}\right)}{\partial\sigma_{ATM}}=\frac{\sigma_K\left(\sigma_{ATM,t_0}+d\sigma_{ATM,t_0},\rho_{t_0},\nu_{t_0}\right)-\sigma_K\left(\sigma_{ATM,t_0},\rho_{t_0},\nu_{t_0}\right)}{d\sigma_{ATM}} \quad (22.a)$$

$$\frac{\partial\sigma_K\left(\sigma_{ATM,t_0},\rho_{t_0},\nu_{t_0}\right)}{\partial\rho}=\frac{\sigma_K\left(\sigma_{ATM,t_0},\rho_{t_0}+d\rho_{t_0},\nu_{t_0}\right)-\sigma_K\left(\sigma_{ATM,t_0},\rho_{t_0},\nu_{t_0}\right)}{d\rho} \quad (22.b)$$

$$\frac{\partial\sigma_K\left(\sigma_{ATM,t_0},\rho_{t_0},\nu_{t_0}\right)}{\partial\nu}=\frac{\sigma_K\left(\sigma_{ATM,t_0},\rho_{t_0},\nu_{t_0}+d\nu_{t_0}\right)-\sigma_K\left(\sigma_{ATM,t_0},\rho_{t_0},\nu_{t_0}\right)}{d\nu} \quad (22.c)$$

Once derivatives have been approximated numerically, to approximate parameters variation, we have considered of the next relation between the Jacobian matrix and the differential of a function:

$$\begin{gathered}\boldsymbol{df}(\boldsymbol{\sigma})=\boldsymbol{J_f}\cdot(\boldsymbol{d\sigma})\\ \boldsymbol{J_f^{-1}}\cdot\boldsymbol{df}(\boldsymbol{\sigma})=(\boldsymbol{d\sigma})\end{gathered} \quad (23)$$

Where $\Delta\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{f_x}\boldsymbol{dx}$ is the differential function of **f** and $\boldsymbol{dx}$ are the vector of the differential variables.

Interpreting it in our context:

$$\begin{pmatrix}\frac{\partial\sigma_{ATM-0.01}}{\partial\sigma_{ATM}} & \frac{\partial\sigma_{ATM-0.01}}{\partial\rho} & \frac{\partial\sigma_{ATM-0.01}}{\partial\nu}\\ \frac{\partial\sigma_{ATM}}{\partial\sigma_{ATM}} & \frac{\partial\sigma_{ATM}}{\partial\rho} & \frac{\partial\sigma_{ATM}}{\partial\nu}\\ \frac{\partial\sigma_{ATM}}{\partial\sigma_{ATM}} & \frac{\partial\sigma_{ATM+0.01}}{\partial\rho} & \frac{\partial\sigma_{ATM+0.01}}{\partial\nu}\end{pmatrix}^{-1}\cdot\begin{pmatrix}d\sigma_{ATM-1\%}\\ d\sigma_{ATM}\\ d\sigma_{ATM+1\%}\end{pmatrix}=\begin{pmatrix}d\sigma_{ATM}^{*}\\ d\rho^{*}\\ d\nu^{*}\end{pmatrix} \quad (24)$$

Computing the inverse matrix of the Jacobian and then multiplying by modellable risk factors, which are the variations of modellable strikes of the volatility surface, it is possible to obtain an approximation of parameters variation.

Applying these variations to the original parameters in *t-1*, we obtain a proxy of parameters in *t*:

$$\sigma_{ATM_{t-1}} \cdot (1 + \; d\sigma^*_{ATM}) = \sigma_{ATM}{}^*_t \tag{25.a}$$

$$\rho_{t-1} \cdot (1 + d\rho^*) = \rho^*_t \tag{25.b}$$

$$\nu_{t-1} \cdot (1 + d\nu^*) = \nu^*_t \tag{25.c}$$

Finally, we are able to obtain the entire volatility surface through SABR model computing these new parameters: $\boldsymbol{\sigma}^*\left(\boldsymbol{\sigma_{ATM_t}} *, \boldsymbol{\rho_t} *, \boldsymbol{\nu_t} *\right) = \; \boldsymbol{\sigma^*_B}(\boldsymbol{f}, \boldsymbol{K})$.

Once it is obtained the volatilities for each strike and tenor, variations across volatility surface must be calculated, as it has been described previously, and compare with the original ones, in order to obtain P&L vector and evaluation PLAT tests.
Recall that one-day variations are related to PLAT and 10-day variations to IMCC perspective.

3.3. Results

The following section has been organized into several sub-parts in order to exhibit the outcomes from different perspectives. The first point (3.3.1) summarizes the main results from each implemented proxy method, presenting them individually and jointly to ensure a proper comparison. The two following sub-parts (3.3.2 and 3.3.3) are focused on evaluating the results from PLAT and IMCC perspective, ending each one with a short balance of which methodology is appropriate under the conditions that FRTB levies to each one. In the end, the main advantages and disadvantages of each approach are summed up in Table 8.

3.3.1. Individual Proxies results

In order to show a thorough report of the results of each proxy, findings have been summarized by tables and graphs to allow comparisons to be made.

Firstly, among non-modellable risk factors, we have selected one enclosed between two modellable risk factors, ATM -0.005, and the other one, ATM+0.035, located in the right corner, away from modellable points. Recalling Figure 5, ATM -0.005 is located between ATM-0.01 and ATM, two of the three modellable risk factors, and ATM +0.035 is surrounded by NMRF, placed five positions away from the modellable risk factor ATM +0.01, counting from its left. Also, three different maturities have been chosen (T=3, 5, 10 years). By this way, it is possible to analyse how proxies perform in different cases.

Along these lines, it is conducted an in-depth analysis of each approach standing by figures. At the last point (3.3.1.2), the first graphs (from Figure 17 to Figure 22) represent the absolute error term between each approach in the two cases of NMRFs chosen. It is followed by the new volatility surfaces resulting after approaches implementation (Figure 23, Figure 24 and Figure 25). In closing, to complete the analysis, it is provided Figure 26, where it is plotted the standard deviation and mean of the error term between all approaches, considering the entire volatility surface, drawing a distinction between how each proxy performs in both MR and NMRF.

3.3.1.1 Comparison of approaches accuracy individually

Along with this sub-section, it goes on to describe the process of each proxy by means of examples and figures, focused on the two NMRF chosen, ending with a comprehensive comparison between all the methodologies across the entire volatility surface. Recall that all approaches have been applied to one-day volatility variations, consequently, all arguments outlined in this point are related to them.

3.3.1.1.1 Linear interpolation

In case of a NMRF is enclosed by two Modellable Risk Factors, linear interpolation is computed as follows:

$$\Delta\sigma^*_{t,t-1}(K_{NMRF},T) \approx \Delta\sigma_{t,t-1}(K_i,T) + (K_{NMRF} - K_i) \cdot \frac{\Delta\sigma_{t,t-1}(K_i,T) - \Delta\sigma_{t,t-1}(K_{i-1},T)}{K_i - K_{i-1}} \quad (26)$$

Where K_i refers to each strike over the volatility surface across MRF.

Example: In the case of ATM - 0.005, which is enclosed by two modellable tenors, linear interpolation is computed as follows:

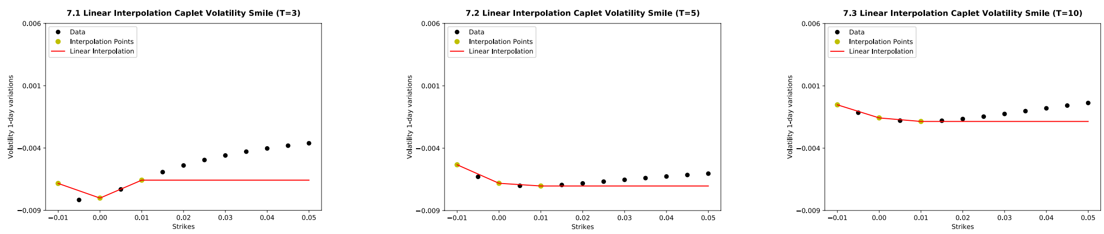
$$\Delta\sigma^{*}_{t,t-1}(K_{ATM-0.005},T) \approx \Delta\sigma_{t,t-1}(K_{ATM},T) + (K_{ATM-0.005} - K_{ATM}) \cdot \frac{\Delta\sigma_{t,t-1}(K_{ATM},T)-\Delta\sigma_{t,t-1}(K_{ATM-0.01},T)}{K_{ATM}-K_{ATM-0.01}}$$

In case of NMRF which are lower or higher than the given MRFs, it is setting the nearest one, that it is to say, we apply variations of the MRF nearest the NMRF.

Example: In the case of ATM + 0.035, a higher point than the modellable settled, it is approached with the nearest one, ATM+0.01.

According to that, Figure 7 displays how linear interpolation performs among the different risk factors in three different maturities

Figure 7 Linear interpolation

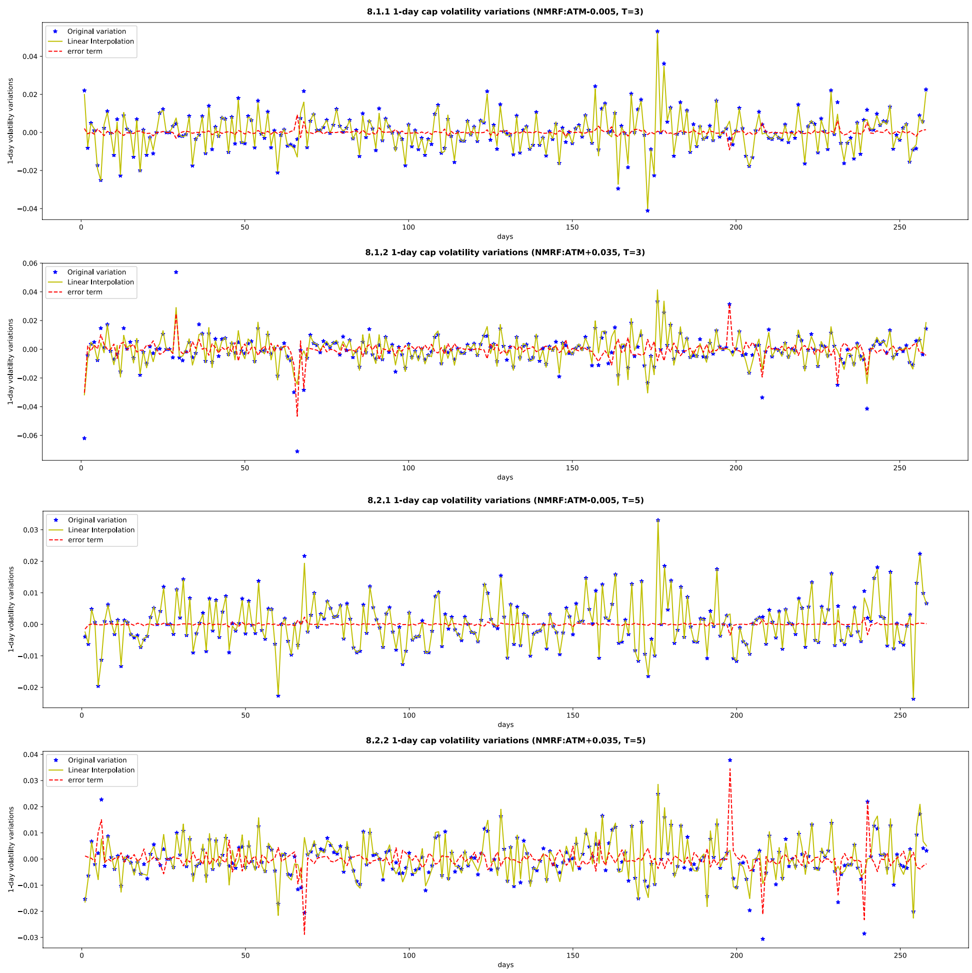


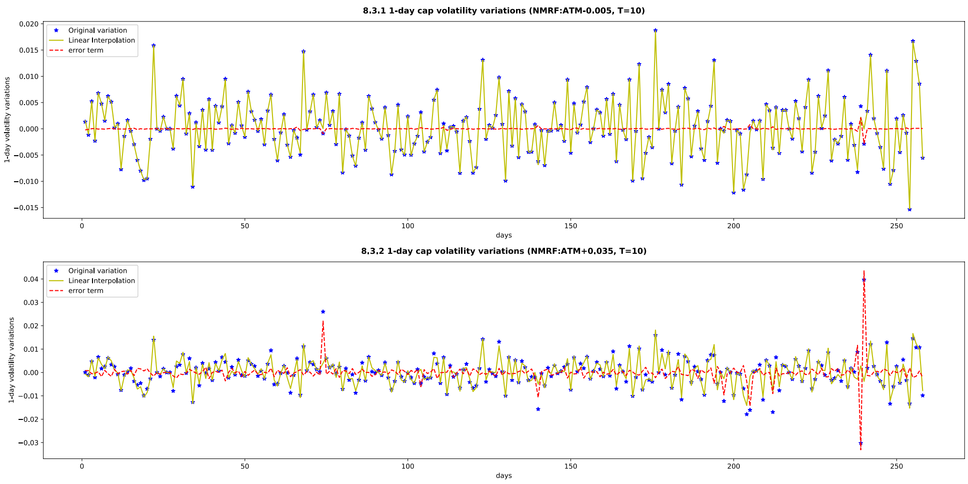
Notes: Data: One-day multiplicative volatility variations of cap EURIBOR6M along each strike of the volatility surface, on 31 January 2019, related to T= 3 years (7.1), 5 years (7.2) and 10 years (7.3). Data is expressed as decimal values.

On the one hand, ATM +0.035, and in general any NMRF as of ATM+0.01 point, is approached by variations of ATM+0.01, its nearest MRF. In these cases, approaches of NMRFs in terms of MRFs is computed by extrapolation technique, that implies using volatility variations of the MRF closer to the NMRF, exemplified above. By this way, linear interpolation is not functioning well enough. On the other hand, this method is very precise at ATM-0.05 point, which is enclosed by two modellable risk factors: ATM-0.01 and ATM. Figure 7 also includes MRF and, of course, linear interpolation performs exactly in these cases.

In addition to this, Figure 8 shows how error term increases in the case of ATM+0.035, the one allocated far away from modellable risk factors, and where it has been computed extrapolation over volatility variations. In the case of ATM-0.005, where interpolation approach over volatility variations is applied, results are highly accurate, the error term is practically zero. Moreover, in this case, this approach appears to be more precise along with maturities 5 and 10, since the error term decreases as maturity increases.

Figure 8 Linear Interpolation cap volatility one-day variations (NMRFs: ATM-0005 and ATM +0.035, t=3,5,10)





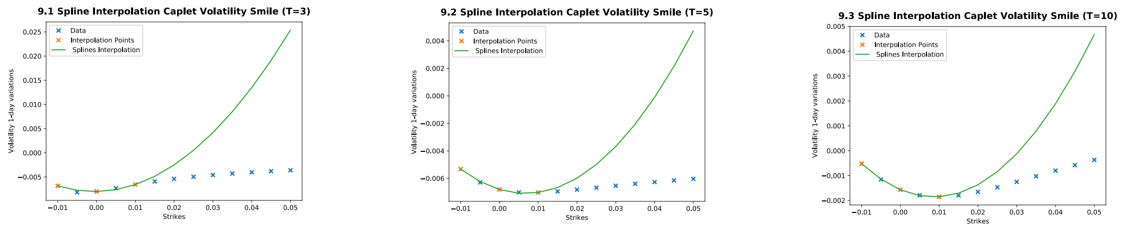
Notes: One-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(8.1.1,8.2.1,8.3.1) and ATM+0.035(8.2.1,8.2.2,8.3.2), t= 3,5,10 years respectively; and error terms Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

3.3.1.1.2 Splines interpolation

According to the previous methodology section (2), as we have three modellable points, cubic polynomials are defined among the two intervals: [ATM-0.01, ATM] and [ATM, ATM+0.01]. ATM -0.005 is located in the first interval and for ATM+0.03 it is used a periodic extrapolation. To further information about Splines Interpolation topic, please see (Burden and Faires,2011).

Figure 9 shows how splines interpolation does not work accurately among outlying Risk Factors. Although among NMRFs enclosed by MRFs, splines interpolation performs accurately, the result, in general, is very distant from reality, given that we have more NMRFs far removed from MRF that enclosed by them.

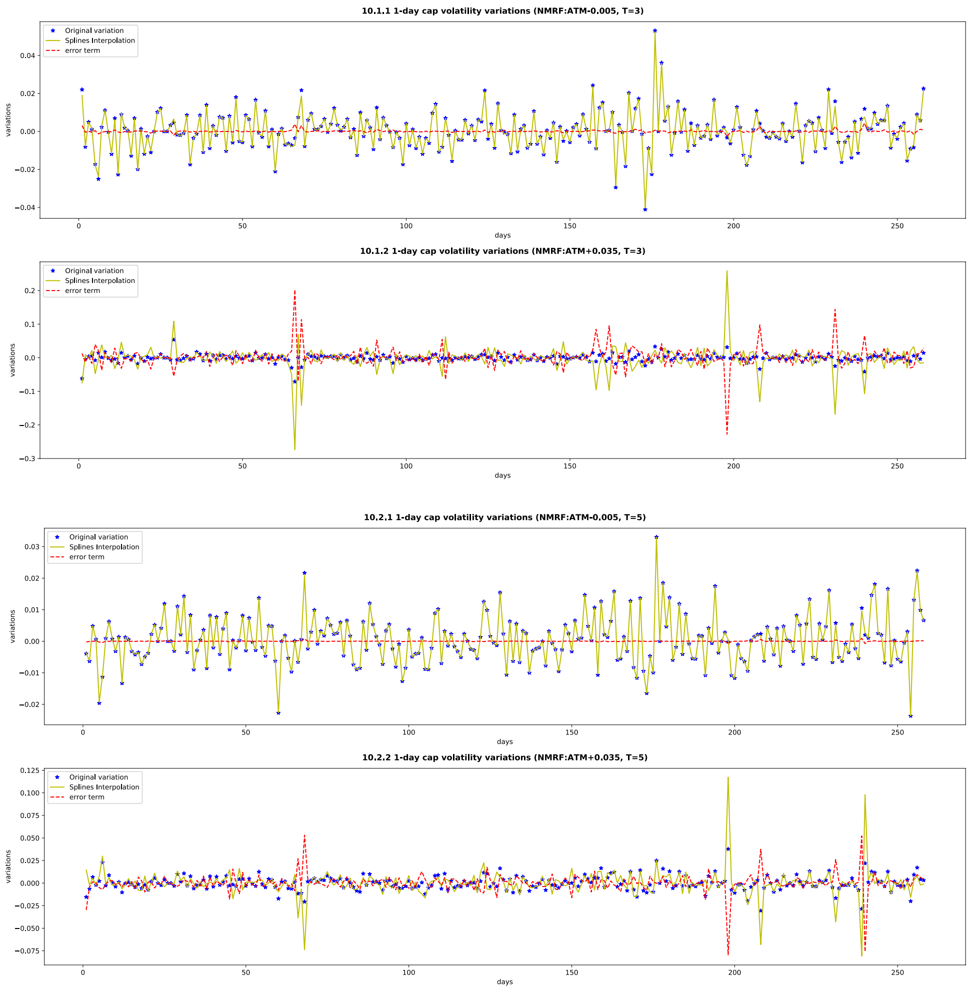
Figure 9 Splines interpolation

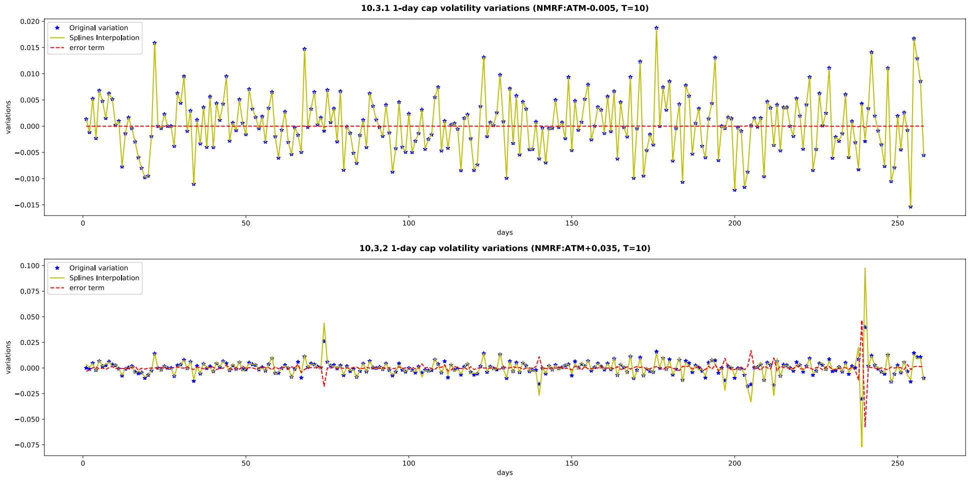


Notes: Data: One-day multiplicative volatility variations of cap EURIBOR6M along each strike of the volatility surface, on 31 January 2019, related to maturities 3 years (9.3), 5 years(9.2) and 10 years(9.3). Data is expressed as decimal values.

Figure 10 evidences one of the constraints of splines interpolations approach since in the ATM-0.005 cases it is reflected that this approach matches almost perfectly, in ATM+0.035 cases, the non-modellable risk factor away from the modellable ones, the error term has increased substantially. Moreover, as occurs with the previous approach, the error term decreases as maturity increases.

Figure 10 Splines Interpolation cap volatility one-day variations (NMRFs: ATM-0.005 and ATM +0.035, t=3,5,10)





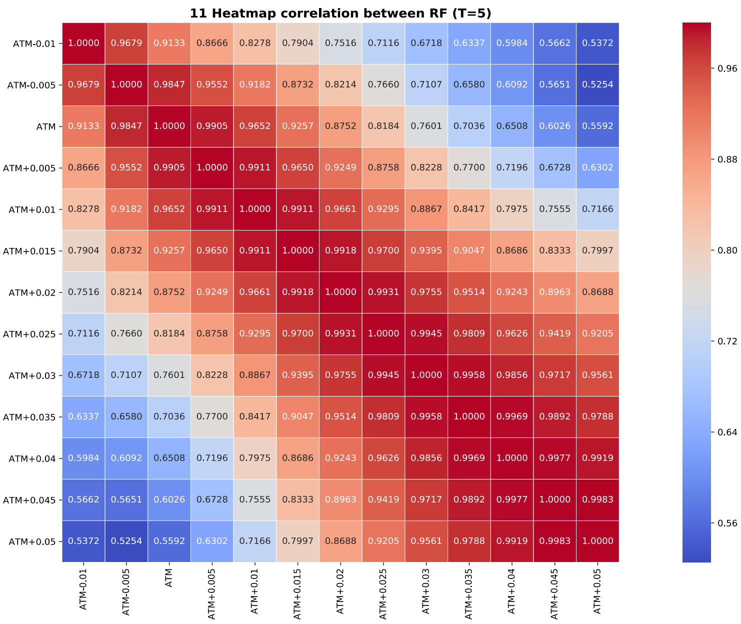
Notes: One-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(10.1.1,10.2.1,10.3.1) and ATM+0.035(10.1.2,10.2.2,10.3.2), maturity 5 years; and error terms. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimals values.

3.3.1.1.3 Simple Linear Regression

Through this approach, it is possible to partially model each NMRF by means of the modellable risk factor with which has a high correlation.

For example, choosing maturity 5 years of implied volatilities of caps reference of EURIBOR6M and computing correlation matrix between risk factors, we obtain the following heatmap in Figure 11.

Figure 11 Heatmap of correlation between Risk Factors



Notes: Correlation between risk factors (RF) of the entire cap volatility surface (T=5). Data is expressed as decimal values.

Figure 11 shows as warm colours higher correlation between two risk factors, being the highest in the main diagonal. Cold colours are related to lower correlations. Because NMRFs only could be explained by modellable risk factors, selection of the independent variable depends on correlations between each non-modellable risk factor and the modellable ones, thus, for example, ATM-0.005 can be explained (as best proxy) by ATM risk factor, with which hold de maximum correlation among MRF. In the case of ATM+0.035, holds a maximum correlation with ATM+0.01.

The approximation to the variation of the partially modelled NMRF chosen, would be as follows:

$$\Delta\sigma^*_{t,t-1}(K_{ATM-0.005}, T) \approx \alpha_{ATM-0.005} + \beta_{ATM-0.005} \cdot \Delta\sigma_{t,t-1}(K_{ATM}, T) \tag{27.a}$$

$$\Delta\sigma^*_{t,t-1}(K_{ATM+0.035}, T) \approx \alpha_{ATM+0.035} + \beta_{ATM+0.035} \cdot \Delta\sigma_{t,t-1}(K_{ATM+0.01}, T) \tag{27.b}$$

A summary of the regression results is listed below (Table 5):

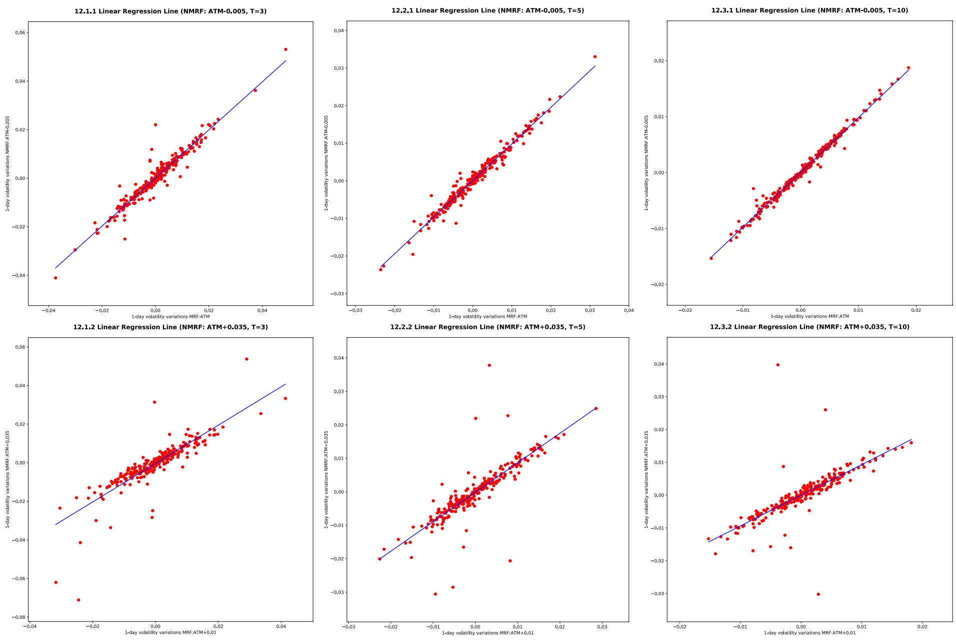
Table 5 Linear Regression OLS estimation results

T= 3	**ATM-0.005**	**ATM+0.035**
	VALUE	VALUE
ALPHA	8.922e-05	-0.0004
BETA	0.9900	0.9945
R^2	0.926	0.729
T=5	**ATM-0.005**	**ATM+0.035**
	VALUE	VALUE
ALPHA	5.792e-05	-0.0001
BETA	0.9740	0.884
R^2	0.970	0.709
T=10	**ATM-0.005**	**ATM+0.035**
	VALUE	VALUE
ALPHA	2.659e-05	-5.197e-05
BETA	0.9796	0.9347
R^2	0.985	0.619

Notes: OLS estimation of each proxy in both cases: ATM -0.005 and ATM +0.035. Data expressed as decimal values.

Linear regression line of each case (ATM -0.005 and ATM+0.035, and T=3,5,10) is represented in Figure 12. Although in graphs, mostly the case ATM+0.035, there are some outliers that the line does not fit, the result adjusted well to the point cloud in both cases

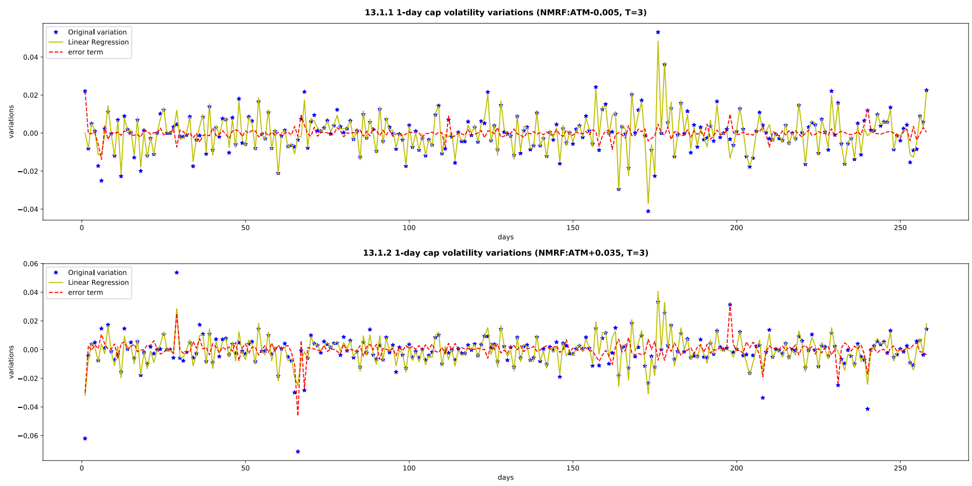
Figure 12 Linear Regression line

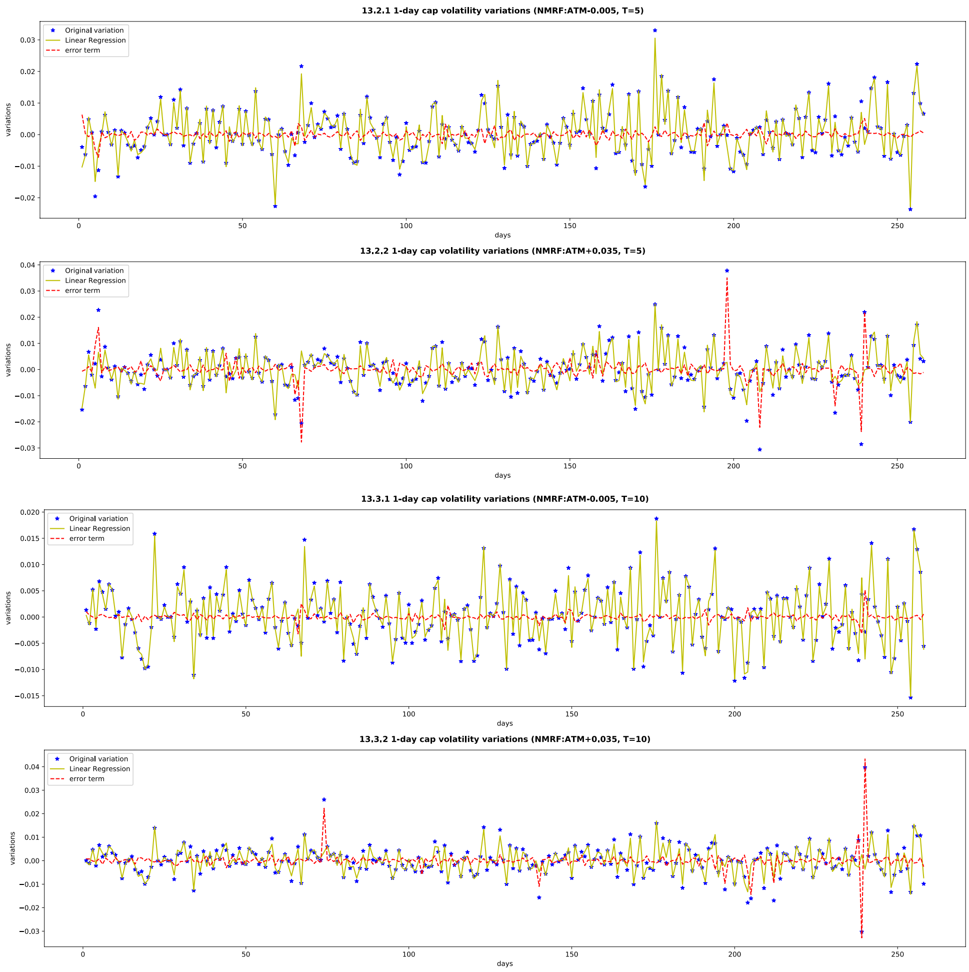


Notes: One-day Multiplicative volatility variations of cap EURIBOR6M, on 31 January 2019,. Abscissa axis: ATM (Figures 12.1.1, 12.2.1, 12.3.1) and ATM+0.01 (Figure 12.1.2, 12.2.2, 12.3.2). Ordinate axis: ATM -0.005 (Figures 12.1.1, 12.2.1, 12.3.1) and ATM+0.035 (Figure 12.1.2, 12.2.2, 12.3.2). Data expressed as decimal values.

Figure 13 yields similar conclusions as previous cases in terms of a worsening on the accuracy of the approach in points allocated far away from MRF. It is worth mentioning that, so far, linear regression is the worst proxy within the first case: ATM-0.005.

Figure 13 Linear Regression cap volatility one-day variations (NMRFs: ATM-0.005 and ATM +0.035, t=3,5,10)



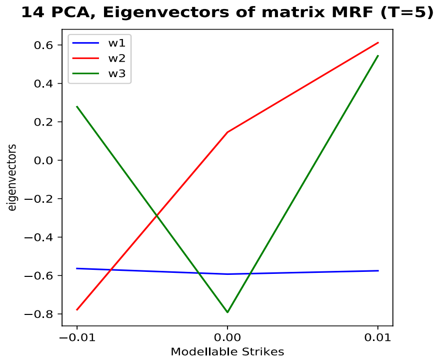


Notes: One-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(13.1.1,13.2.1,13.3.1) and ATM+0.035(13.2.1,13.2.2,13.3.2), maturity 5 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

3.3.1.1.4 Multi-factor model applying PCA

Following methodology explained at previous point (4), Figure 14 shows the eigenvectors related to the correlation matrix of $\boldsymbol{V}$, where is easy to see the interpretation of each one: fthe irst eigenvector refers to level, second to the slope and third to the curvature.

Figure 14 Eigenvectors related to MRF matrix



Notes: Eigenvectors related to the original MRF matrix. Data used: Cap volatilities variations with strikes ATM-0.01, ATM and ATM+0.01. T=5 years. Sample: one year, from 31 January 2019 to 31 January 2018.

Table 6 summarises the cumulative variance explained from each principal component at each tenor. As mentioned in the methodology section (see point 4), we have decided to choose the three CP, because the dimensionality problem is not a reality in our context. Including these three new variables, we avoid the multicollinearity problem, while the model will explain 100% of the variance of the original variable matrix $\boldsymbol{X}$.

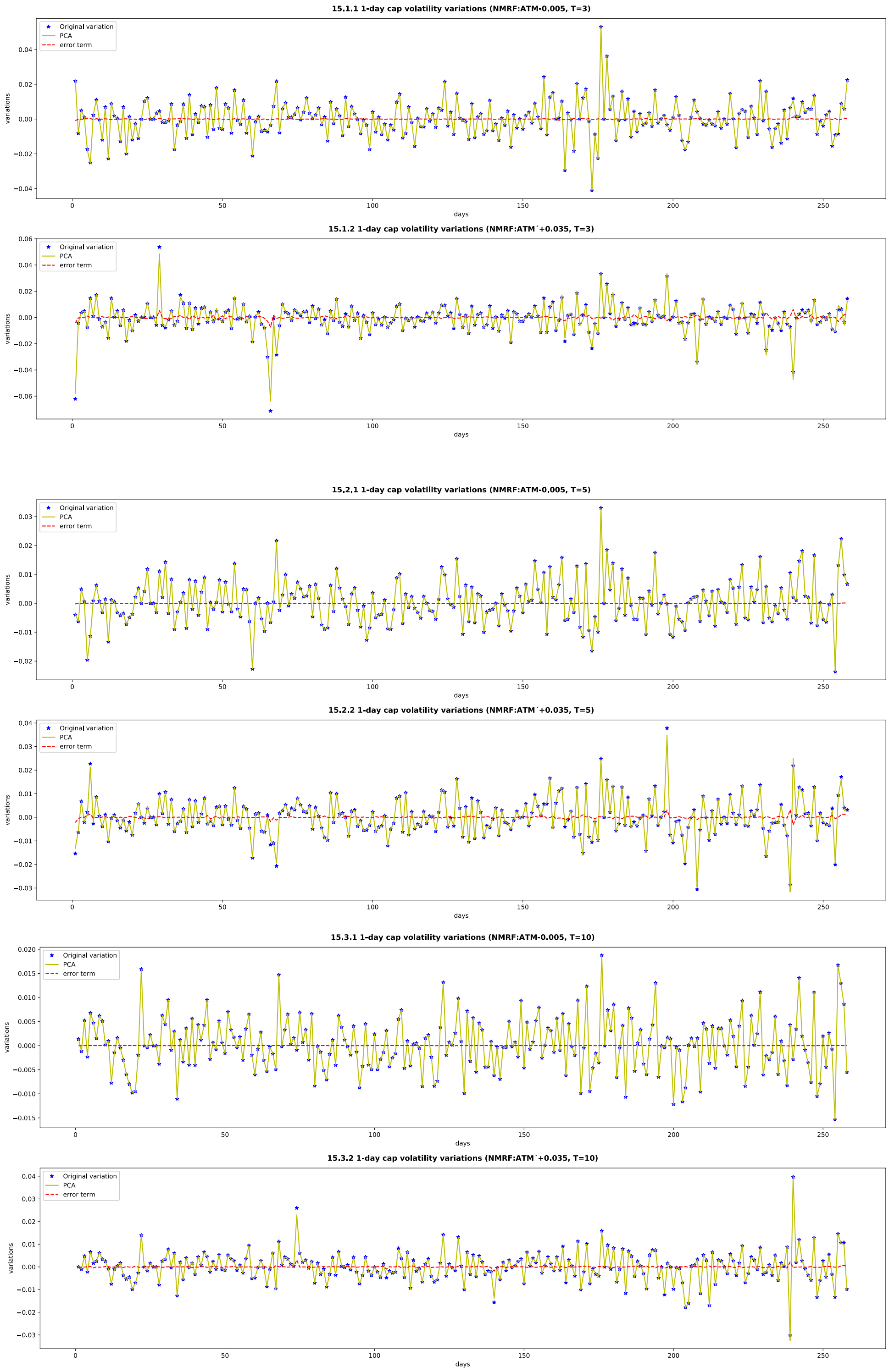
Table 6 Cumulative variance explained from each CP

Cumulative variance explained	**MATURITY 3**	**MATURITY 5**	**MATURITY 10**
CP1	86.3886	93.5105	96.9292
CP2	97.989	99.434	99.5903
CP3	100	100	100

Notes: quantities expressed in percentage. Each cumulative variance computed as:
$Cumulative\ variance = \frac{\lambda_i \cdot 100}{100 \cdot \sum_{i=1}^{3} \lambda_i}, where\ i = eigenvalue\ 1, eigenvalue\ 2, eigenvalue\ 3$

Results of this methodology are exhibit in Figure 15, in accordance with the format that has been taken from previous approaches.

Figure 15 PCA cap volatility one-day variations (NMRFs: ATM-0.005 and ATM +0.035)



Notes: One-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(15.1.1,15.2.1,15.3.1) and ATM+0.035(15.2.1,15.2.2,25.3.2), maturity 5 year; and error terms. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

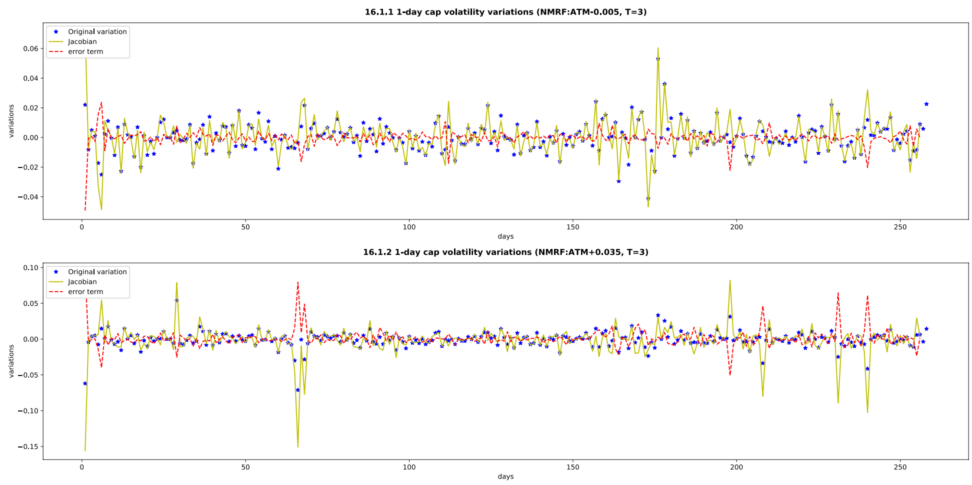
As could be expected, PCA approaches yield the most accurate results among all methods set out above. This model constructed on the basis on PCA, it is using all the information between the three modellable risk factors, besides avoiding multicollinearity problem. For this reason, the error term over ATM-0.05 and ATM+ 0.035 is virtually zero.

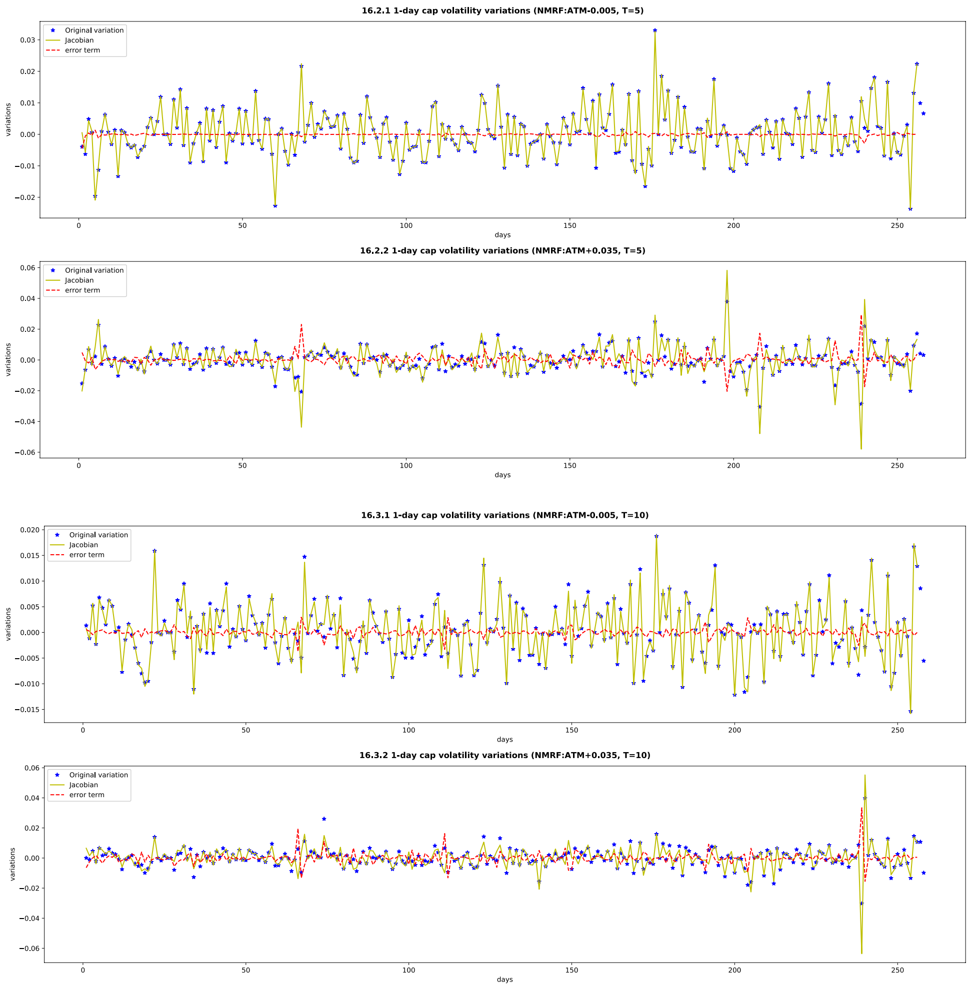
3.3.1.1.5 Jacobian

The methodology implemented is the one explained at the previous methodology point (6). In a nutshell, it has been computed the Jacobian matrix to finally achieve approaching variations of SABR parameters σ_{ATM}, ρ, ν. By calculating the new values at time t of these parameters as it is exhibited before, the new volatilities surfaces have been built. Later, one-day variations have been calculated and they are which have been plotted in figures below.

Figure 16 shows how the Jacobian approach achieves a good fitting of one-day volatility variations in both cases, mostly within ATM-0.005 related to tenors 5 and 10 years. Although both NMRF cases related to maturity 3 year does not really perform well, those results are good news, because, as it is summed up at the end of the section in Table 8, this approach allows us to get a coherent volatility Surface, that means not arbitrability.

Figure 16 Jacobian cap volatility variations (NMRFs: ATM-0.005 and ATM +0.035)





Notes: One-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(16.1.1,16.2.1,16.3.1) and ATM+0.035(16.2.1,16.2.2,16.3.2), t=3,5,10 years respectively) and error terms. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

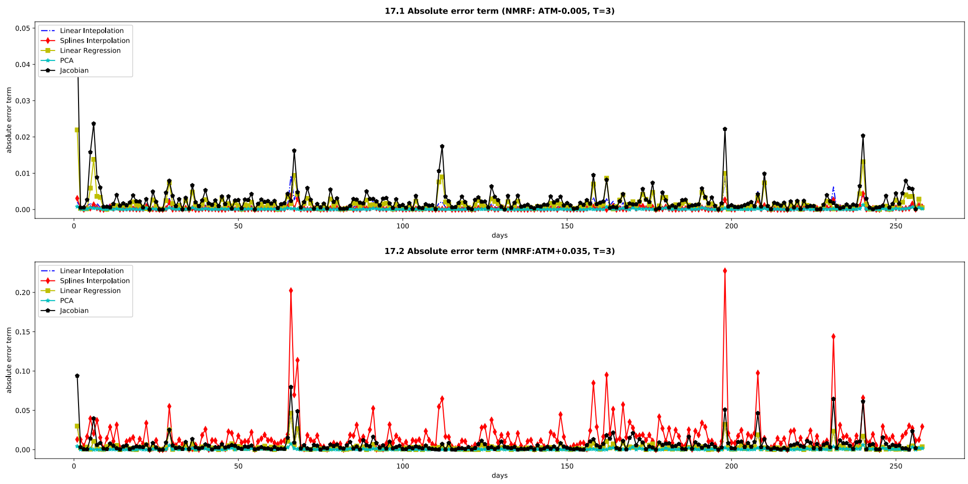
3.3.1.2 Comparison between every approach

Points above have shown steps computed along each proxy to finally get NMRF partially modellable. To complete the analysis, there are provided different figures related to absolute error terms of each proxy among the two cases of NMRFs chosen to tenors 3,5 and 10 years.

Moreover, to illustrate a general view of all the methods, the figures below (Figure 23, Figure 24 and Figure 25) exhibit the new volatility smiles formed on the basis of approached variations. This analysis is complemented by plotting the error term in time of the two NMRF chosen.

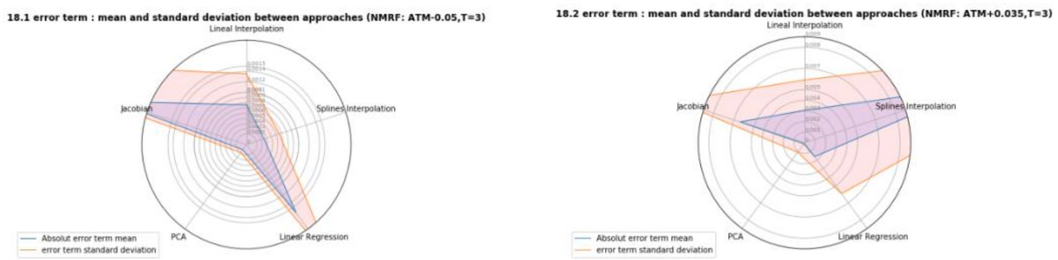
The plots below in Figures from Figure 17 to Figure 22, represent the absolute error term of each approximation to facilitate comparison.

Figure 17 Absolute error term of each proxy in time. (T=3)



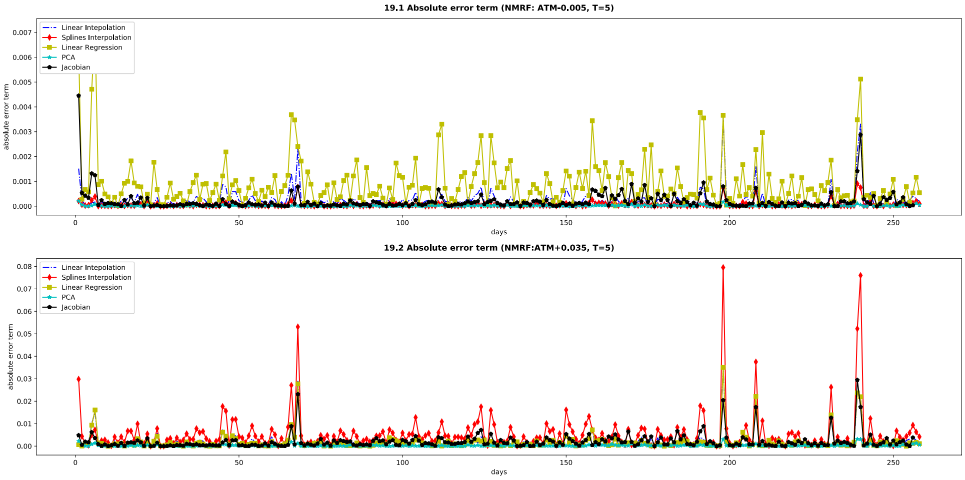
Notes: Absolute error term of one-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(17.1) and ATM+0.035(17.2), maturity 3 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

Figure 18 Absolute error term: mean and standard deviation. (T=3)



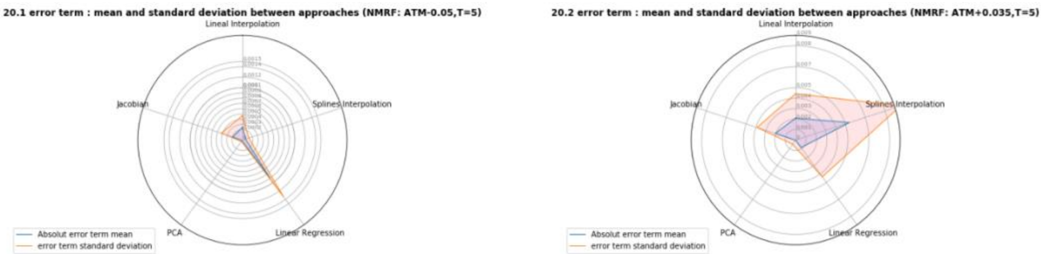
Notes: Mean of absolute error term and standard deviation of error term of one-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(18.1) and ATM+0.035(18.2), maturity 3 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

Figure 19 Absolute error term of each proxy in time. (T=5)



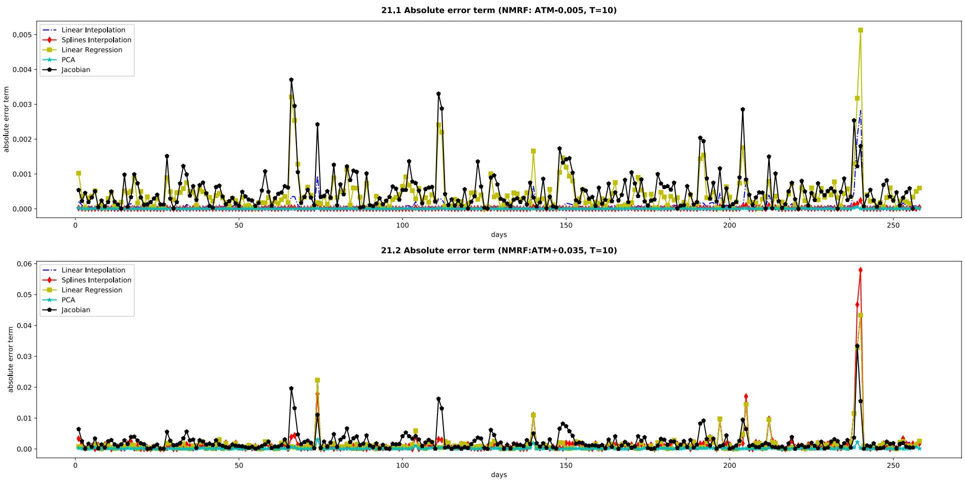
Notes: Absolute error term of one-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(19.1) and ATM+0.035(19.2), maturity 5 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

Figure 20 Absolute error term: mean and standard deviation. (T=5)



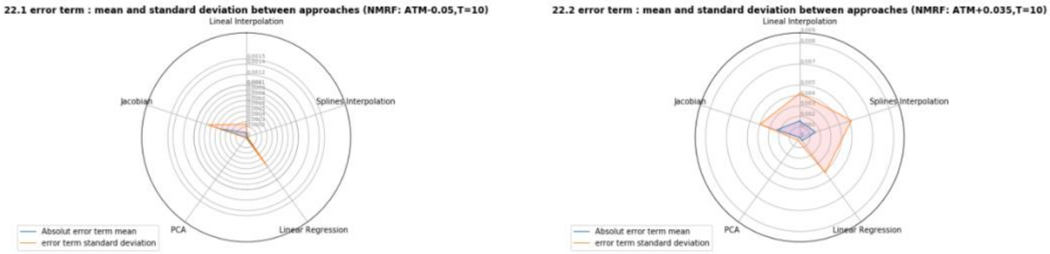
Notes: Mean of absolute error term and standard deviation of error term of one-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(20.1) and ATM+0.035(20.2), T=5 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

Figure 21 Absolute error term of each proxy in time. (T=10)



Notes: Absolute error term of one-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(21.1) and ATM+0.035(21.2), maturity 10 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

Figure 22 Absolute error term: mean and standard deviation. (T=10)



Notes: Mean of absolute error term and standard deviation of error term of one-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(22.1) and ATM+0.035(22.2), T=10 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

From Figure 17 to Figure 22, it is shown the marked contrast between approaching non-modellable risk factors allocated near the modellable ones, and approaching those non-modellable risk factors away from them. The most notable aspect is the evolution of the error with spline interpolation approach. While in the first case, ATM -0.005, performs practically perfect, in the second case, it becomes the worst approach mostly in maturities 5 and 10 years. By contrast, in the case of maturity 3 years, mainly, and 10 years, the Jacobian approach presents a high error rate, positioning at the same level of splines interpolations in terms of accuracy.

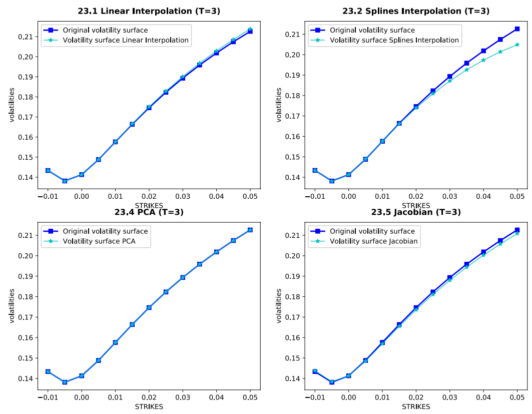
Paying attention to the rest of the proxies, Linear regression approach becomes the worst in the case of ATM-0.005 related to maturity 5 years. In contrast, PCA is, without a doubt, the best approach between all cases.

Seeing pictures temporally, it is remarkable how the magnitude of the error term decreases as maturity increases, reconfirming previous conclusions.

Overall, it is possible to conclude that the error term increases as NMRF move away from the modellable point available.

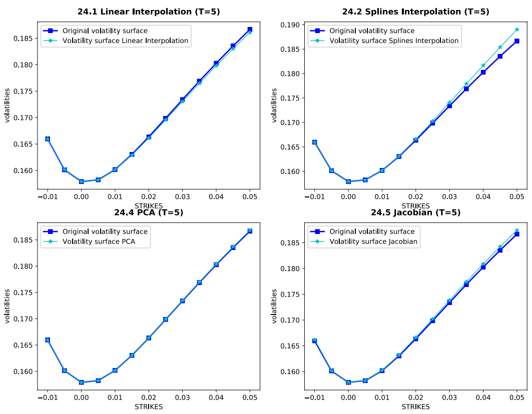
Figure 23, Figure 24, Figure 25 and Figure 26, complete the analysis representing the result of the new volatility surfaces and error term of each proxy, respectively, once one-day variations of each RF have been approached in order to achieve modellability requirements.

Figure 23 Cap volatility surfaces under PLAT (T=3) by comparing all approaches



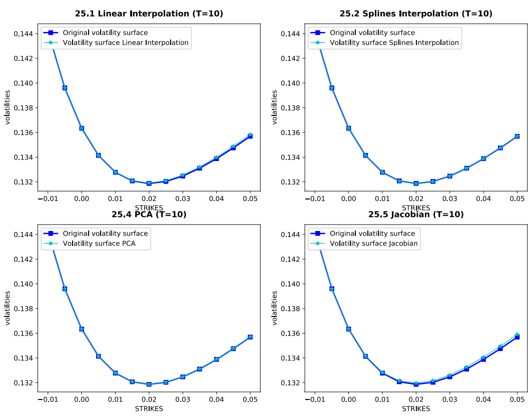
Notes: Data: Cap EURIBOR6M new volatility surfaces on 31 January 2019, related to each proxy. Data is expressed as decimal values.

Figure 24 Cap volatility surfaces under PLAT (T=5) by comparing all approaches



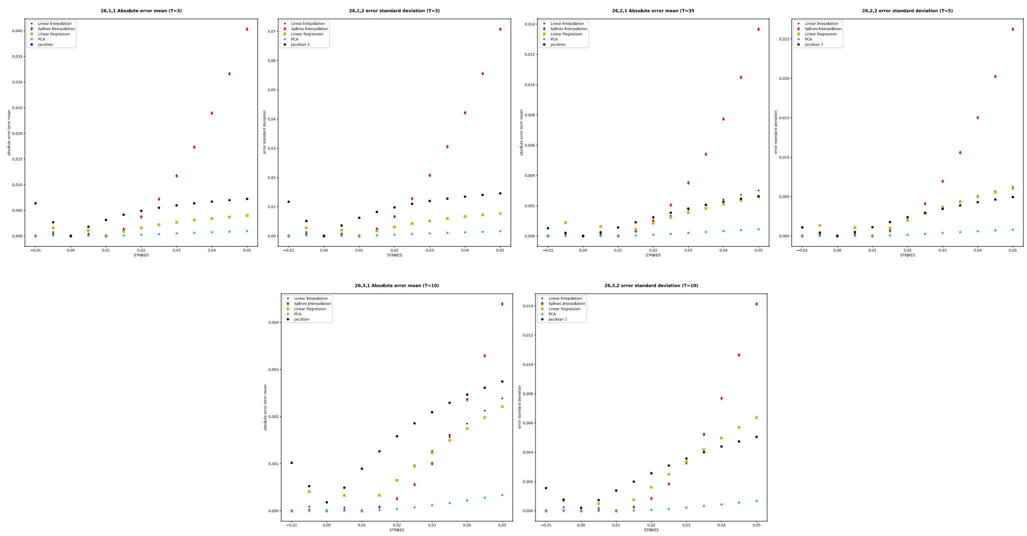
Notes: Data: Cap EURIBOR6M new volatility surfaces on 31 January 2019, related to each proxy. Data is expressed as decimal values.

Figure 25 Cap volatility surfaces under PLAT (T=10) by comparing all approaches



Notes: Data: Cap EURIBOR6M new volatility surfaces on 31 January 2019, related to each proxy. Data is expressed as decimal values.

Figure 26 Mean and standard deviation of error term between all approaches across new volatility surfaces



Notes: Absolute error term mean (26.1.1, 26.2.1, 26.3.1) and error standard deviations (26.1.2 ,26.2.2, 26.3.2) of one-day multiplicative volatility variations approached of cap EURIBOR6M on 31 January 2019, maturity 5 years. Data is expressed as decimal values.

In accordance with the conclusions above, spline interpolation becomes the worst methodology. This is justified by its poor precision among NMRFs not enclosed by MRFs. By contrast, PCA turns into the best approach across the entire volatility surface. Linear interpolation, Linear Regression and Jacobian yields similar outcomes along with maturities 5 and 10 years. As mentioned before, the Jacobian approach is preferable for the reasons about not arbitrability on the new volatility surface, as long as, this approach yields similar accurate results than other approaches.

3.3.2. PLAT metrics results under FRTB

At this point, results related to the fulfilment of the P&L attribution test are summarised to analyse how well each proxy works. As mentioned in the theoretical framework part, PLAT is a crucial point that enables a trading desk to go for the IMA, or not, for calculating capital requirements.

Table 7 exhibit the results of the comparison between HPL and RTPL vectors assessed in the frame of FRTB, that is to say, to applying to HPL and RTPL vectors the two metrics required in PLAT.

Table 7 Spearman correlation and Kolmogorov Smirnov statistic under PLAT test metrics

SPEARMAN CORRELATION	ATM-0.01	ATM-0.005	ATM	ATM+0.005	ATM+0.01	ATM+0.015	ATM+0.02	ATM+0.025	ATM+0.03	ATM+0.035	ATM+0.04	ATM+0.045	ATM+0.05
LINEAR INTERPOLATION	1	0,9888	1	0,9990	1	0,9901	0,9768	0,9642	0,9511	0,9378	0,9243	0,9094	0,8950
SPLINES INTERPOLATION	1	0,9979	1	0,9983	1	0,9824	0,8834	0,6745	0,4337	0,2741	0,1826	0,1391	0,1200
LINEAR REGRESSION	1	0,9525	1	0,9805	1	0,9901	0,9768	0,9642	0,9511	0,9378	0,9243	0,9094	0,8950
PCA	1	0,9997	1	0,9999	1	0,9998	0,9993	0,9986	0,9975	0,9960	0,9944	0,9921	0,9899
JACOBIAN	0,8595	0,949	1	0,960	0,935	0,914	0,887	0,864	0,839	0,821	0,807	0,800	0,794
K-S STATISTIC	**ATM-0.01**	**ATM-0.005**	**ATM**	**ATM+0.005**	**ATM+0.01**	**ATM+0.015**	**ATM+0.02**	**ATM+0.025**	**ATM+0.03**	**ATM+0.035**	**ATM+0.04**	**ATM+0.045**	**ATM+0.05**
LINEAR INTERPOLATION	0	0,027	0	0,019	0	0,031	0,039	0,047	0,047	0,054	0,062	0,070	0,074
SPLINES INTERPOLATION	0	0,023	0	0,027	0	0,035	0,043	0,074	0,112	0,163	0,244	0,291	0,322
LINEAR REGRESSION	0	0,043	0	0,031	0	0,043	0,050	0,058	0,074	0,070	0,074	0,081	0,081
PCA	0	0,023	0	0,019	0	0,019	0,023	0,023	0,027	0,035	0,047	0,043	0,043
JACOBIAN	0,0976	0,047	0,008	0,035	0,051	0,070	0,066	0,074	0,082	0,098	0,098	0,105	0,102

Notes: Maturity 3 years

SPEARMAN CORRELATION	ATM-0.01	ATM-0.005	ATM	ATM+0.005	ATM+0.01	ATM+0.015	ATM+0.02	ATM+0.025	ATM+0.03	ATM+0.035	ATM+0.04	ATM+0.045	ATM+0.05
LINEAR INTERPOLATION	1	0,9966	1	0,9993	1	0,9923	0,9738	0,9558	0,9409	0,9254	0,9102	0,8939	0,8767
SPLINES INTERPOLATION	1	0,9996	1	0,9996	1	0,9973	0,9868	0,9499	0,8514	0,6863	0,5311	0,3967	0,3021
LINEAR REGRESSION	1	0,9815	1	0,9879	1	0,9923	0,9738	0,9558	0,9409	0,9254	0,9102	0,8939	0,8767
PCA	1	0,99997	1	0,9999	1	0,9999	0,9998	0,9996	0,9993	0,9988	0,9981	0,9969	0,9957
JACOBIAN	0,9957	0,9975	1	0,9967	0,99137204	0,9823	0,9801	0,9720	0,9635	0,9544	0,9451	0,9353	0,9270
K-S STATISTIC	**ATM-0.01**	**ATM-0.005**	**ATM**	**ATM+0.005**	**ATM+0.01**	**ATM+0.015**	**ATM+0.02**	**ATM+0.025**	**ATM+0.03**	**ATM+0.035**	**ATM+0.04**	**ATM+0.045**	**ATM+0.05**
LINEAR INTERPOLATION	0	0,0233	0	0,0271	0	0,0233	0,0349	0,0388	0,0543	0,0581	0,0543	0,0581	0,0620
SPLINES INTERPOLATION	0	0,0116	0	0,0116	0	0,0233	0,0349	0,0543	0,0814	0,0504	0,0775	0,1318	0,2093
LINEAR REGRESSION	0	0,0388	0	0,0310	0	0,0310	0,0349	0,0349	0,0426	0,0426	0,0465	0,0504	0,0659
PCA	0	0,0233	0	0,0233	0	0,0233	0,0233	0,0310	0,0194	0,0233	0,0233	0,0349	0,0388
JACOBIAN	0,0390	0,0234	0,0078	0,0273	0,0391	0,0234	0,0352	0,0391	0,0469	0,0508	0,0508	0,0586	0,0625

Notes: Maturity 5 years

SPEARMAN CORRELATION	ATM-0.01	ATM-0.005	ATM	ATM+0.005	ATM+0.01	ATM+0.015	ATM+0.02	ATM+0.025	ATM+0.03	ATM+0.035	ATM+0.04	ATM+0.045	ATM+0.05
LINEAR INTERPOLATION	1	0,9978	1	0,9995	1	0,9848	0,9642	0,9505	0,9384	0,9282	0,9176	0,9066	0,8942
SPLINES INTERPOLATION	1	0,9999	1	0,9990	1	0,9991	0,9986	0,9964	0,9911	0,9780	0,9478	0,8892	0,8239
LINEAR REGRESSION	1	0,9926	1	0,9958	1	0,9848	0,9642	0,9505	0,9384	0,9282	0,9176	0,9066	0,8942
PCA	1	0,9999972	1	0,99999092	1	0,99996646	0,99991964	0,9996604	0,99945007	0,99929774	0,99892809	0,99824958	0,99761021
JACOBIAN	0,9628	0,9889	0,9989	0,9904	0,9631	0,9409	0,9158	0,8856	0,8605	0,8375	0,8155	0,7981	0,7882
K-S STATISTIC	**ATM-0.01**	**ATM-0.005**	**ATM**	**ATM+0.005**	**ATM+0.01**	**ATM+0.015**	**ATM+0.02**	**ATM+0.025**	**ATM+0.03**	**ATM+0.035**	**ATM+0.04**	**ATM+0.045**	**ATM+0.05**
LINEAR INTERPOLATION	0	0,0155	0	0,0194	0	0,0194	0,0271	0,0349	0,0426	0,0349	0,0349	0,0349	0,0426
SPLINES INTERPOLATION	0	0,0116	0	0,0078	0	0,0194	0,0194	0,0233	0,0310	0,0349	0,0465	0,0504	0,0581
LINEAR REGRESSION	0	0,0465	0	0,0310	0	0,0271	0,0310	0,0388	0,0504	0,0504	0,0465	0,0465	0,0581
PCA	0	0,02325581	0	0,0233	0	0,0271	0,0233	0,0155	0,0194	0,0349	0,0233	0,0310	0,0310
JACOBIAN	0,0507	0,0391	0,0234	0,0273	0,0586	0,0508	0,0508	0,0391	0,0352	0,0391	0,0508	0,0508	0,0508

Notes: Maturity 10 years

Colours in Table 7 are in line with the three zones established in this test according to the thresholds related to each measure. By this way, the green cells mean endorsement to apply IMA.

Most of the findings are located in green zone according to PLAT thresholds, but extreme risk factors related to splines interpolation among shorter tenors do not fulfil the requirements.

Consequently, Splines Interpolation is not a good alternative to modellability assessment in this case, due to it is necessary to meet the requirements under PLAT and being allocated in the green zone or at least, in the amber zone with some specifications[18]. By contrast, Linear interpolation, linear regression and multi-factor model provide similar

[18] Please see *(Minimum capital requirements for market risk -BCBS (2019). Section MAR32, paragraphs 32.22 and 32.4)*)

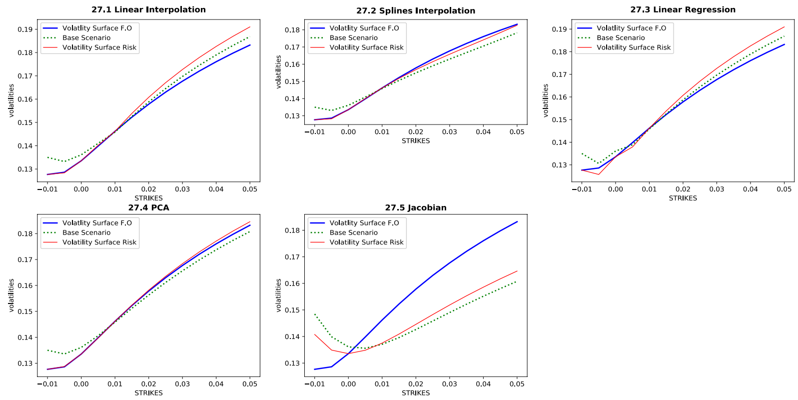
results, and also the Jacobian approach performs accurately with the exception of a few points in the first maturity (3 years).

3.3.3. IMCC Results

To further our analogy between Front Office and Risk, results have been computed the stress scenarios necessary to the ES measure, which is an input in the calculation of capital requirements under the FRTB internal model approach. Despite the fact that FRTB does not establish the need to compare the P&L vectors related to IMCC calculation from F.O and Risk, we decided to provided results from F.O, as every RF was originally modellable, and from Risk, where there are MRF and NMRF, that has been transformed into partially modellable; in order to provide a comparison between each proxy.
FRTB requires to calibrate ES to a period of stress, on a daily basis, using a 97.5th percentile one-tailed confidence level. It is worth mentioning that along with this research, calculations have not been made over a stressed window, we have used the current window. As the base horizon is settled in 10 days, we have computed 10-days variations once we have modelled non-modellable risk factors acquiring the entire surface as modellable. By this way, the following figures show each stressed scenario, the ones computed by FO and the other ones computed by Risk, with each scenario base. Recall that base scenario, under this perspective, is always settled in t_0.

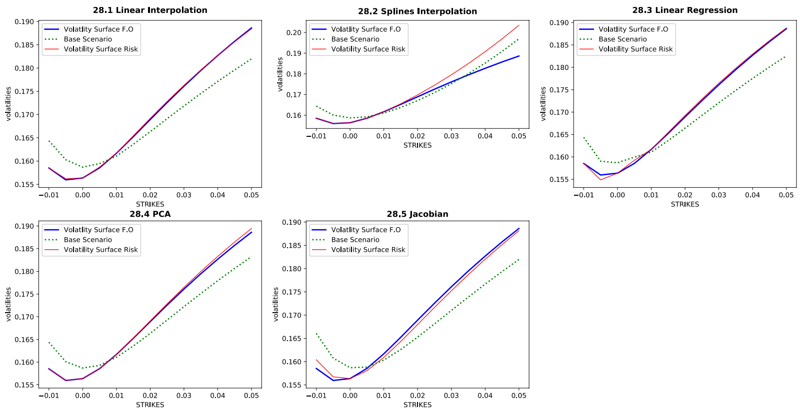
Figure 27, Figure 28 and Figure 29, represent volatilities surfaces stressed by 10-day variations on current window, from FO, considering all RF originally modellable, and form Risk, with MRF and partially modellable NMRF. Moreover, it is plotted the base scenario which allows a clear view of the evolution of volatilities surfaces once have been shocking.

Figure 27 Stressed Scenarios from FO and Risk, maturity 3 years



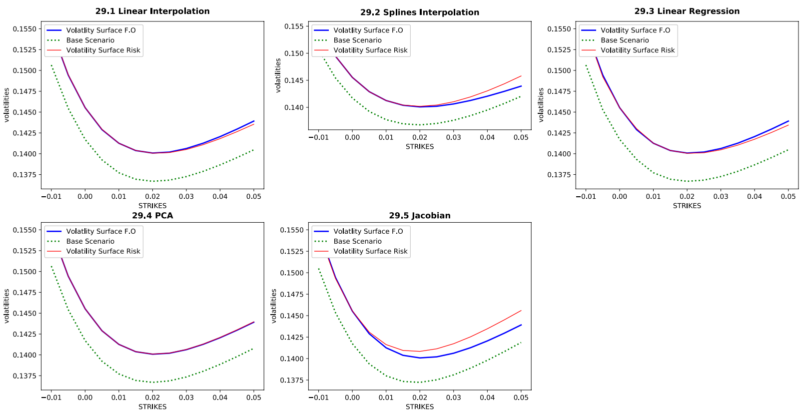
Notes: Data: Cap EURIBOR6M new stressed volatility surfaces on 31 January 2019, related to t= 3 years, among all approaches and under IMCC perspective. Data is expressed as decimal values.

Figure 28 Stressed Scenarios from FO and Risk, maturity 5 years



Notes: Data: Cap EURIBOR6M new stressed volatility surfaces on 31 January 2019, related to t= 5 years, among all approaches and under IMCC perspective. Data is expressed as decimal values.

Figure 29 Stressed Scenarios from FO and Risk, maturity 10 years



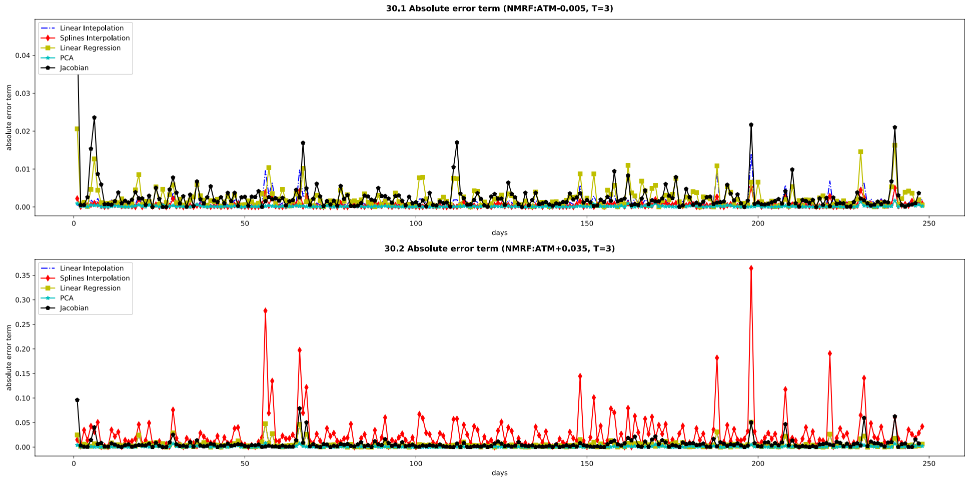
Notes: Data: Cap EURIBOR6M new stressed volatility surfaces on 31 January 2019, related to t= 10 years, among all approaches and under IMCC perspective. Data is expressed as decimal values.

At first glance, Splines interpolation appears to be the worst approach, except on the volatility surfaces related to maturity 3 years, where Jacobian does not yield very good results. Like previous PLAT analysis, these approaches, under IMCC perspective do not perform correctly in these particular cases.

Linear interpolation, linear regression and PCA eclipse the other approaches, having small differences regard to FO results. Considering maturity 5 years, Jacobian also becomes a good alternative to keep in mind.

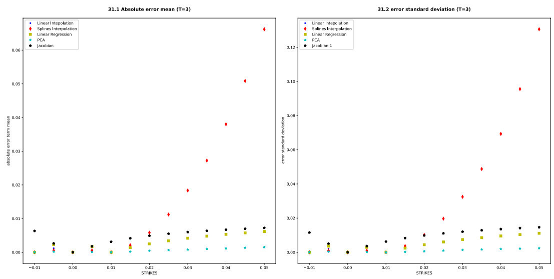
Figure 30, Figure 32, Figure 33, Figure 34 and Figure 35, represent the error term between all approaches under the IMCC perspective, that is, 10-day volatility variations across the new volatilities surfaces once have been approached one-day variations. Results are given to both cases analysed previously (ATM-0.005 and ATM+0.035), and for the entire new stressed volatilities surfaces.

Figure 30 Error absolute term of each proxy under IMCC perspective(T=3)



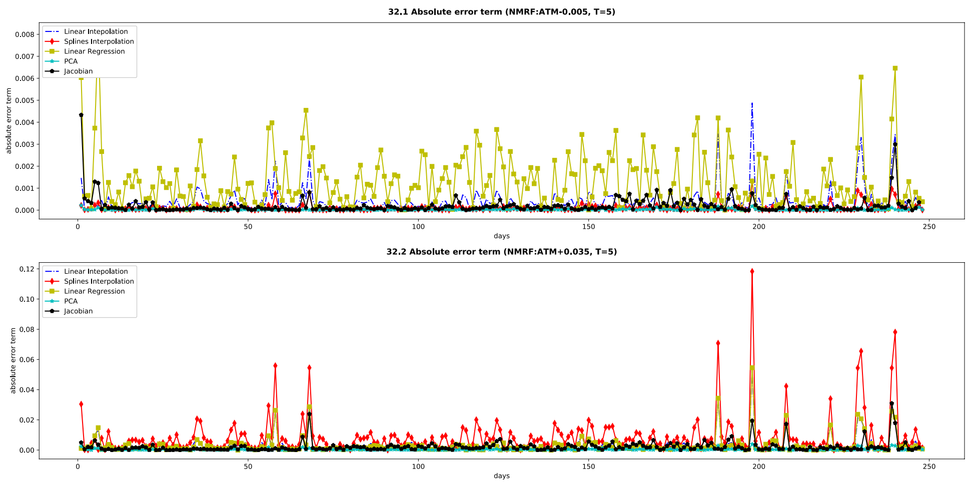
Notes: Absolute error term of 10-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(30.1) and ATM+0.035(30.2), maturity 3 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

Figure 31 Mean and standard deviation of error term between approaches under IMCC: (Entire new volatility surface, T=3)



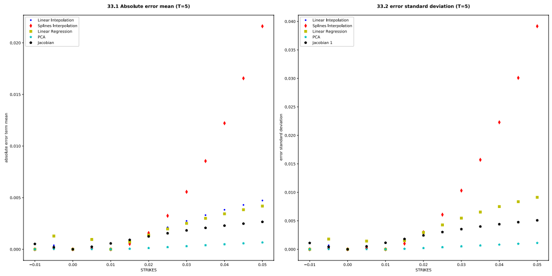
Notes: Absolute error term mean (31.1) and error standard deviations (31.2) of 10-day multiplicative volatility variations approached of cap EURIBOR6M on 31 January 2019, maturity 3 years. Data is expressed as decimal values.

Figure 32 Absolute error term of each proxy under IMCC (T=5)



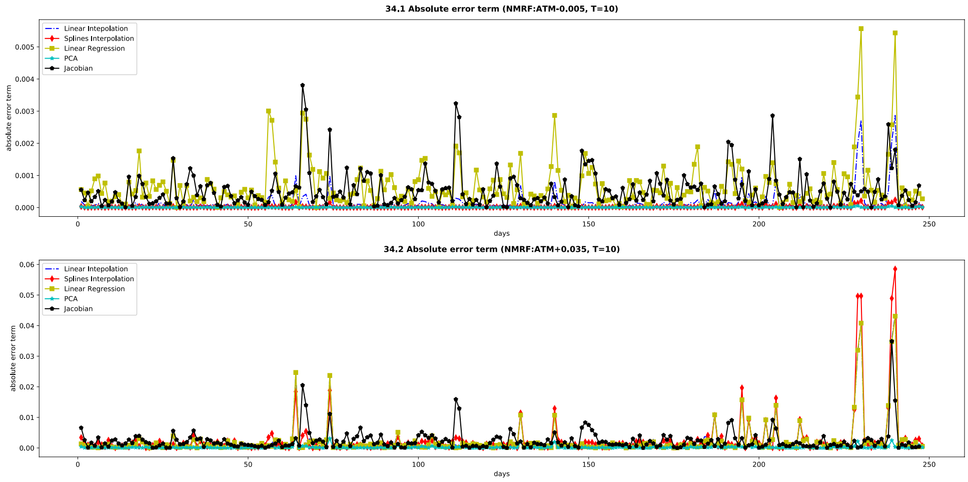
Notes: Absolute error term of 10-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(32.1) and ATM+0.035(32.2), maturity 5 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

Figure 33 Mean and standard deviation of error term between approaches under IMCC: (Entire new volatility surface, T=5)



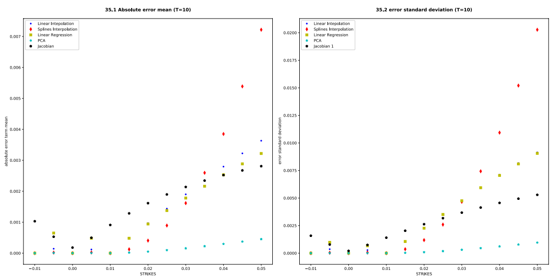
Notes: Absolute error term mean (33.1) and error standard deviations (33.2) of 10-day multiplicative volatility variations approached of cap EURIBOR6M on 31 January 2019, maturity 5 years. Data is expressed as decimal values.

Figure 34 Absolute error term of each proxy under IMCC (T=10)



Notes: Absolute error term of 10-day multiplicative volatility variations of cap EURIBOR6M with strike ATM-0.005(34.1) and ATM+0.035(34.2), maturity 10 years. Sample one year, from 31 January 2019 to 31 January 2018. Data is expressed as decimal values.

Figure 35 Mean and standard deviation of error term between approaches under IMCC: (Entire new volatility surface, T=10)



Notes: Absolute error term mean (35.1) and error standard deviations (35.2) of 10-day multiplicative volatility variations approached of cap EURIBOR6M on 31 January 2019, maturity 10 years. Data is expressed as decimal values.

Figures above report similarities regarding results obtained under PLAT perspective, one-day multiplicative variations context. In general, all proxies have actually got worse, by means of a rise in the error term, but main conclusions between each one are almost the same.

- PCA is preserved with its accurate results.
- Splines interpolation is notable for its weak point between NMRFs far away from MRFs.
- Jacobian, although is the preferred method to provide coherent volatility surfaces, along maturities 3 and 10 years, does not perform accurately, but in the case of maturity 5 years, the error term is not high.
- Linear interpolation and linear regression present similar results.

Table 8 Advantages and Disadvantages of each approach

		Methodology	Advantages	Disadvantages
1	**Interpolation approach**	Linear Interpolation	• Accurate methodology: FO and MR risk factors aligned to both perspectives, PLAT and IMCC • Easy and fast method implementing • Also accurate with not so high correlation between MR and NMRF	• The results do not reflect the proper behaviour of a volatility surface. Admit arbitrability • Depends on the location of the points in the volatility surface, needs to be enclosed by modellable tenors • The bigger of an interval between two modellable points the worst accurate results
1	**Interpolation approach**	Splines Interpolation	• Smooth interpolant between NMRF enclosed by MRF	• The worst approach under the two perspective, particularly where NMRF move away from the modellable ones • results do not reflect the proper behaviour of a volatility surface. Admit arbitrability
2	**Statistical modelling approach**	Linear Regression	• Easy and fast method implementing • This does not depend on the location of the points in the volatility surface, namely, the non-modellable points do not necessarily need to be enclosed by the modellable ones	• Only accurate with very high correlation • Estimation of the β parameter depends on the historical window where is computed: if the window is tiny β not pretty stable, and if the window is wide, β would be less representative • Admit arbitrability
2	**Statistical modelling approach**	Multi-factor model applying PCA	• Has more explicative capability than de linear regression above due to this methodology uses more than one independent variable to explain a dependent one • Does not depend on the location of the tenors in the volatility surface • The best approach in both perspectives	• Admit arbitrability
3	**Parametrization**	Jacobian SABR	• Computing the entire Surface could be done only with a few modellable points. • This approach provides a coherent volatility Surface. No admit arbitrability • Accurate across long maturities	• Most accurate near the modellable risk factors

4. Conclusions

Since the introduction of FRTB, modellability has risen to one of the key concepts for capital risk requirements computation, banks are challenged on exploring reductions in NMRF-related capital by increasing modellability of risk factors. For this reason, through this research, we aim to provide a reference framework for this topic introduced by FRTB. In this respect, this goal has been carried out through different approximation methods based on Interpolation, Statistical Modelling and Parametrization approaches; providing results from two perspectives: PLAT, that allows a trading desk to compute capital requirements through IMA, and IMCC perspective, how capital requirements must be computed.

In our conservative context of interest rate volatility risk, the best approach to partially model those NMRF by mean of modellable risk factors, it has proven to be Multi-factor model base on a previous Principal Components Analysis (PCA), if we focus on accuracy. But if we pay attention to consistency between volatilities surfaces achieved after implementing each approach, Jacobian approach is well regarded, since error terms are not very large and successfully fulfil PLAT requirements.

By contrast, splines interpolation should not be considered as an alternative to modellability assessment. This approach does not yield accurate results in extreme cases, where NMRFs are separate from available MRFs, in addition to not meet PLAT requirements in those cases. Linear interpolation and linear regression exhibit similar perform throughout the research, passing PLAT requirements in all cases considered.

Outcomes under IMCC perspective are similar that the ones obtained under PLAT perspective, reaching the same conclusion about how approaches perform, but in this case, the error term has increased.

5. Further research

In view of the results and bearing in mind that modellability assessment is a topic that has emerged very recently, there are some issues that have been left for further study. There are some that could be considered:

1. Computing the capital charge IMCC over a stressed window to compare results, as FRTB established, between the ones have been obtained in this research.
2. Further analysis of the results obtained with a volatility surface considering floating strikes, by comparing with results on a volatility surface considering fixed strikes, measuring each impact.
3. As we mentioned in Table 8, in the majority of the methodologies applied it is generated volatility surfaces that do not perform coherent, namely, admitting arbitrability. In this sense, the research shall be completed with an analysis of the arbitrability of volatilities surfaces obtained once proxied variations have been computed.
4. Application of smoothing interpolation in order to improve the results obtained. and other parametric approaches (About this last topic see Burden R. and Faires J, 2011).

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