AN UPDATED MODEL IMPLEMENTATION FOR INCREMENTAL RISK CHARGE

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Abstract

The unsecured positions and the biased allocation between banking and trading book derived in huge losses due to credit risk that the banking framework of the time of the previous financial crisis could not contain. Since then and still in force, *Basel II.5* implemented the Incremental Risk Charge adding the information that credit risk entails on a market risk measurement in order to hold capital against default and credit migration. Based on Creditmetrics methodology, and taking Vasicek's Gaussian model as a starting point, this master's thesis aims to go deeper in the credit risk modelling. It will compare an estimated Student-t copula with the last ECB's requirement to outdo the flaws of the basic Gaussian model of the ASRF of Vasicek. Additionally, we will develop a Clayton copula model to catch up with the empirical evidence drew by the default and migration risk correlation among issuers causing left-asymmetry.

Keywords: Basel, Incremental Risk Charge, market risk, credit risk, default, credit migration, factor copula.

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Introduction

Basel framework

The Basel Committee on Banking Supervision (the Committee or BCBS) have been proposing the regulatory international banking frame since its inception in 1974 from the quest of closing gaps in banking matters as the financial risks that it involves. The main financial risks adressed by the regulation are credit risk, operational risk and market risk.

As Bill Maris said, "the reality is regulation often lags behind innovation" so that the lack of scope will be shown in each agreement of banking regulation due to the performance that the financial sector does both in terms of side-stepping the regulation and the empirical evidence drew. The breakdown in 2007 resulted a severe proof of that since it meant an inflection point within the financial sector and its consequent regulation. It supposed an enormous source of losses mainly in the trading book[1] due to its increasingly exposures to unsecuritized credit products. There was an important factor that did not contain this problem: the market risk framework of *Basel II*, based on the 1996's amendment. This regulatory framework computed the necessary capital using VaR based on long smooth period prior to the crisis. The situation concluded that the models used clearly underestimated the losses and risks and that the regulatory capital for trading was much lower than the banking book.

Even before the financial crisis of 2008, the weakness of *Basel II* had severely manifested by itself so as stop-gap response, the Committee introduced a revision to the *Basel II* market risk framework known as *Basel II.5* —BCBS (2009)[8]. Both in terms of standarised and internal model approach (SA and IMA, respectively), it involved some key points to surpass the shortcomings of the VaR itself such an incremental charge for credit risk, an stressed VaR and other additional ones.

Considering that the main source of losses were emerging from the credit risk, the Committee led up in 2009 to a revised framework for computing the charge for this risk –see BCBS(2009)[9]– under the IMA range. It brought a novel framework's risk coverage in certain areas with a particular focus on the trading instruments exposed to credit risk.

[1] The trading book is held to speculate or to hedge as well as marked to market daily while the banking book is composed by the banking products held until its maturity and carried at amortized cost.

The IRC interest lies into manifest quantitatively a risk measure that it may keep capital in case of financial disturbance which can severely affect the certainty of the ordinary activity of the bank.

Aim and motivation

This study deals with the challenge of the manner that the credit risk can be modelled in order to add it on a market risk measurement under the current framework named Incremental Risk Charge (IRC) that will be valid until the end of 2022[2]. From that year on, it is estimated that the Fundamental Review of Trading Book set basis on the new market risk regulation framework replacing the IRC by the Default Risk Charge (DRC).

The credit risk models proposed by the regulator are based on a factor copula approach. In such context, the Vasicek's model is applied in our work as the basis model. Despite this, the approach manifested several flaws, being the contagion the one addressed by this master's thesis. It uses a Gaussian copula approach not allowing dependence beyond the middle percentiles of the variables. Additionally, to give more realism to the structures of dependence within the factor model, we will fit the empirical distribution of the variables by which such dependence is obtained. It will allow to put aside the normality with the exception of the basis model.

The main objective of this study, hence, falls to make certain assumptions in the modelling of credit risk with the purpose of capturing more accurately the previously stated empirical evidence that the financial performance extracts: increasingly left-tail dependence in periods of market turmoil. It will take advantage of the last requirement made by the European Central Bank of including a Student-t copula within the implementation model to finally extending it through other copulas. Henceforth, the ECB's requirement stated in the Targeted Review of Internal Models[3] (TRIM) – ECB (2017)[19]–, will be mentioned as the ECB's proposal.

Finally, due to the lack of default data to estimate the needed dependence parameters, we will make use of stock data for corporate issuers and the yield of 1-year sovereign bond for sovereign. The proposed IRC models will be implemented through six portfolios with different risk composition.

The IRC consists on model specific risk and hold a capital buffer against default risk that escape from the VaR scope. IRC not only captures the credit risks due to default but also credit migrations combined with widening of credit spreads and the loss of liquidity. Preventing capital arbitrage between banking and trading book is another goal sought by the IRC, setting consistent charges for similar positions held in both mentioned books. The Committee assessed a quantitative impact study –BCBS (2009)[10]– to measure the impact the IRC charge on market risk changes

[2]See BCBS (2019)[6] page 1, first paragraph of "Introduction".
[3]See page 119, paragraph 152.

which concluded that new requirements will increase the trading book market risk capital requirements by two or three times on average.

The background literature for the quantitative modelling of the credit risk form a group of three discernible models: structural (Moody's KMV model and Credit-Metrics), macro-factors (Econometric model) and actuarial models (CreditRisk+). This master's thesis is focused on the structural models which are based on the Merton's model –Merton (1974)[41]– and in the Credit-Metrics model –Gupton and Finger (2007)[31]– in particular, since it takes into account not only the default probability but also the rating migration.

Structure definition

This research is organized in three parts. *Base line* the firt one, provides an explanation of the regulation's framework starting by giving a brief glance of the agreement periods and their coverings. After that, from the bottom to the top, the correspondent Basel accord at which this master's thesis is based (Basel II and its enhancements) will be described to finally end up with the exposition of the basis model of Vasicek.

The second part named *Methodology* is devotedly focused on describing the assumptions and methods whereby the model is based on. To capture default and migration risk with increasing robustness several statistical techniques are explained. It is divided in four parts where the definition, other considerations in terms of regulation procedures, a model explanation and a different background theory with the use of copulas are addressed.

The last one, *Empirical analysis*, presents different model applications. Firstly the data sources are described, following with the methodology carried out to finish with the results obtained through the model implementation.

1 Base line

1.1 Market risk framework

The first permission of using internal models –or recognised as Value-at-Risk models– arose with little importance in *Basel I* as foundation for measuring the buffer capital for market risk. These internal models developed by banks since then were subject to strict quantitative and qualitative standards.

Under this approach, the equation for computing the total capital into the *Basel II.5* market risk framework is the sum of the five components as follows Brunac –(2012)[13]–:

$$\text{Market risk capital} = \text{VaR } + \text{Stressed VaR} + \text{IRC} + \text{CRM} + \text{SC} \tag{1.1}$$

where

VaR is the standard value-at-risk daily measured at 99.9% of confidence level over an horizon of 10 days. It is the result of the higher VaR between the previous calculation or the average of the value-at-risk measures on each preceding sixty business days multiplied by a factor $m_c \geq 3$ set on the basis of the quality of the bank's risk management system. Basicaly, it will depend on the ex-post performance of the model that will be check by the so-called backtesting[1].

$$\text{VaR} = max\left(VaR_{t-1}, m_c \cdot \sum_{i=1}^{60} \frac{VaR_{t-i}}{60}\right)$$

Stressed VaR is a similar to the previous value-at-risk measure but at least weekly calculated and with model inputs calibrated to historical data from a continuous 12-month period of a notable financial stress. It is intended to replicate a calculation that would be presented in a bank's portfolio if market risk factors had such a stressed influence as well as to ameliorate the procyclicality of the minimum capital requirements.

$$\text{Stressed VaR} = max\left(SVaR_{t-1}, m_s \cdot \sum_{i=1}^{60} \frac{SVaR_{t-i}}{60}\right)$$

[1] The multiplicative factors (m_c and m_s) are between 0 and 1. They will depend on the performance of the model during the last 250 business days in accordance with EUR-lex(2013)[21] (See the Table 3.17 by which they are selected).

IRC is an incremental charge to the market risk due to credit risk. It is similar to the VaR charge as has herein-above been stated but in this case the average built on the previous twelve business weeks and weekly calculated. It will be vastly outlined in the section 2.2.

$$\text{IRC} = max\left(IRC_{t-1}, m_c \cdot \sum_{i=1}^{12} \frac{IRC_{t-i}}{12}\right)$$

CRM is another incremental charge for correlating and securitised trading activities (at least weekly computation).

SC is sandardised charge on securitisation exposures not covered by CRM and comparable to the banking book.

1.2 IRC and the Committee's proposal

According to the proposals made by the Committee and once the European Banking Authority (EBA)[2] was established, a guidelines on the IRC modelling were provided to set guidance. It is not a batch of rules but a break new ground as a high level principles to develope models for calculating the IRC by those banks with a wide extensive trading activities.

Consequently, to construct our model proposal to the IRC, the most relevant aspects have to be expounded[3].

Scope of application

The calculation will be applied to unsecuritised credit products held in the trading book. Unsecuritised positions liable to IRC calculation should include long and short positions subject to:

- Specific interest rate risk

  Within this item are included sovereign bonds, other structured bonds, money market loans and covered bonds simply collateralised but not asset-backed.

- A listed equity and derivatives positions based on such listed equity

  Compulsorily these related positions have to be jointly managed by a predetermined trading unit.

[2] The EBA took responsability of all existing and ongoing tasks and responsabilities from the predecessor Committee of European Banking Supervisors (CEBS) since 1th January 2011. The EBA is in regard to monitor and to draw up the guidelines to a level playing field in the EU. Furthermore, transposes the BCBS amendments through their Directives to enhance alignment among the national banking authorities.

[3] See guidance in EBA (2012)[18] and the proposals BCBS (2009)[9] by which this section is taken.

Needless to say that securitised products and n-th-to-default credit derivatives [4] are excluded over the IRC scope.

Individual Modelling

In view of the fact that reducing arbitrage to allocate credit sensitive instrument into the trading book rather than in the banking book was stated as the main principle for introducing the IRC, there must be consistency in both capital charges for similar book's positions (once adjusted for illiquidity) and sources of model parameters:

- Comparability to the internal ratings based approach (IRB)[5] soundness, the IRC must measure losses from default or migration events at 99.9% confidence interval over a capital horizon of one year.
- When using internal sources in order to rate obligor's positions or to estimate probabilities of loss given default(LGDs) and default(PDs), it should be coherent with the IRB approach:
  - If there are LGDs and PDs internally estimated as part of IRB, this may be used as source for obtaining LGDs and PDs for IRC purposes.
  - If not, LGDs and PDs should be computed using a consistent technique subject to mandatory approval for IRC application.

  Our model proposal will consider a process to calculate the LGDs (= 1 - Recovery Rate) that is described in the upcoming lines. Similarly, the PDs' consideration are addressed in the next part so are included within the migration matrices.

Interdependence

The interdependence is the key element in the risk management field due to the multivariate nature of risks. We cannot focus on a single risk but on an aggregate risks so as was quoted by 1998 in *Bloomberg Businessweek* "[...] synchronized rises and falls in financial markets occur infrequently but they do occur, [...] in which many things go wrong at the same time —the perfect storm scenario."

From this high-dimensional scenario, it can be distinguished interdependence among obligors and among underlying risk factors:

- Correlations between default and migration events

  This type is in regard to the purpose of the IRC, which is to capture the financial and economic dependence that causes clustering of default and migration events.

[4] N-th-to-default credit derivative is a credit default swap whose reference is a basket of underlying credits.

[5] The IRB is the internal methodology adopted under Basel II with regard to credit risk in the banking book.

The approach should be adequate enough to capture the inderdependence among the risk drivers (obligors) of credit risk events.

- Underlying risk factors

  The EBA put a combination of risk factors forward to trace the process of any firm's asset value through an idiosyncratic (i.e. individual of each obligor) and one or multiple systemic factor. In such a way, the authority draws forth the use of dimension-reductions models to gather high-dimensional nature of risk into a smaller subsets of essential risk factors. A one-factor model is used in our follow-up model proposal.

Since this dependence may not share the same structure among all the distribution quantiles, the institution may select possible copula candidates according to its faculty to explain default or migration clusters for historical tail events.

Lastly, to model the interdependence structure (i.e. the rating migration process) must be mentioned the EBA's proposal with regard to migration matrices:

- Migration matrices

  Containing the probabilities of all tranches to migrate from the initial credit rating to any other (possible downgrade, upgrade and even default-PDs) at a given time horizon. It is essential to simulate the rating migration process.

  – Either internal or external, i.e. rating agencies where internal historical data is sparse or less than 5 years of observation period.

  – Separated migration matrices may be applied depending on (i) portfolio composition, (ii) possible differences in migration characteristics across issuers and geographical areas and (iii) availability. To sovereign obligors, specific transition matrix should be applied as well.

Constant level of risk over one-year capital horizon

In accordance with the Basel framework, an IRC model should be modelled based on the assumption of a constant level of risk over a one-year capital horizon. In this manner, it reflects an appropriate capital to buffer the risk assumed in the trading book as well as to provide capital capacity to keep fluid the liquidity to the financial markets in turmoil scenarios. To meet with this constant level risk assumption, a bank should rebalance, or roll over, those positions in the trading book whose credit rating have afflicted or ascended to maintain the initial level over one-year horizon. The liquidity horizon of a given position will condition the frequency of rebalancing.

Liquidity horizon

The liquidity horizon indicates the time required to sell a position or to hedge all risks covered by the IRC in a stressed market scenario. Several aspects have to be exposed:

- It may be defined by position or on an aggregated basis ("buckets") depending on the portfolio composition.
- A floor of three months is set.
- Banks should be based on past experience, market structure, the quality of the product or its complexity in order to select the liquidity horizon.

In our model proposal, the liquidity horizon will be one-year based. So any instrument within our portfolio must have at least one year of maturity from the valuation date.

P&L valuation

Once the rating migrations have been simulated (i.e. through simulated asset's returns and ranked by migration matrices thresholds), it may be converted into price variations according to market conditions. Three assumption can be made:

- The obligor company presents no credit rating change

  No change in the portfolio value is understood.

- The case in which the company's credit rating migrates

  Full recalculation is needed in order to get a vector of prices obtained according to their new credit quality. It requires the application of the riskless zero coupon yield plus the interest rate term structure (forward curve) correspondent to each rating class.

- The default state is triggered for the obligor

  Recovery rate (RRs) data should be gathered additionally. Following Creditmetrics, our model will consider stochastic RRs yet uncorrelated with PDs –see Altman *et al(2001)*[3]. Consequently, the LGD = 1 - RR.

Each simulated scenario will extract a portfolio valuation so that a future P&L distribution is obtained available for quantitative analysis.

1.3 Model foundations

The certainty that came up with the financial crisis of 2008 is that the most of the losses were mainly caused by downgrades combined with widening of credit spreads and the loss of liquidity instead of defaults itself. In short, most of the losses were located in the trading book due to banks have often chosen to hold products dependent to credit risk within its trading book instead of in the banking book considering that the previous regulation framework did allow that arbitrage. The instruments allocated in the banking book were required to hold credit risk capital while those in the trading book were subject to market risk capital, resulting less regulatory capital as a whole in this second book

1.3.1 Creditmetrics and asset value models

Most of the credit risk models are based on the fact of obtaining the probability of the issuers' default within a loan portfolio, but to calculate the Value-at-Risk founded on the impact that credit risk's changes entails in a trading portfolio it is necessary a methodology that takes into account credit migration besides default. Riskmetrics Group (A JPMorgan division) primarily developed the technique named Creditmetrics –Gupton and Finger (2007)[31]– that better fits with the credit migration and the default risks mentioned. It is recognised as one of the structural models and it is a Merton-type model.

The Merton asset-value model (AVM) –Merton (1974)[41] is a latent variable approach, often interpreted as the asset's value of the firm because of its dynamic are not observable. It defines that the value of a firm follows a log-normal process distribution and the defaults occurs when the asset's value falls below the nominal value of their debts (B^i) on the date of maturity.

Following Merton's approach, let V^i be the i-th obligor assets' value, with the process of a geometric brownian motion (GBM):

$$\frac{dV_t^i}{V_t^i} = \mu^i dt + \sigma^i dW_t \tag{1.2}$$

The asset value V_t^i can be obtained by integrating:

$$V_t^i = V_0^i \exp\left[\left(\mu^i - \frac{\sigma^{2,i}}{2}\right)t + \sigma^i\sqrt{t}X^i\right] \tag{1.3}$$

with $X_i \sim N(0,1)$, μ^i and $\sigma^{2,i}$ being the mean and variance, respectively, of the instantaneous rate of return on the firm's assets dV_t^i/V_t^i.

The probability of default of the i-th obligor is given by:

$$p_{DEF} = P[V_t^i < B^i] = P[X^i < \zeta^i] = N(\zeta^i)$$

where N(·) is the cumulative normal distribution, B^i is the debt value and

$$\zeta^i = \frac{\ln(B^i) - ln(V_t^i) - (\mu^i - \frac{1}{2}\sigma^{2,i})t}{\sigma^i\sqrt{t}}$$

The term firm refers to the obligor which issue the asset position held in the trading book (issuer). Since the issuer may signal the likelihood of non-dealing with the obligation's payment taken, the credit risk is inherent in the portfolio and the credit quality of the firm can vary. In that case, the probability of default or that the credit rating changes (migrates) entails credit migrations producing a market risk fluctuation.

According to that, the credit quality categories $\{CR_j\}_{j=1,2,\ldots,8}$ that Creditmetrics considers are given by seven credit ratings besides the default state which the external credit agency Standard & Poors establishes:

AAA - AA - A - BBB - BB - B - CCC - DEF (default)

where AAA refers to extremely reliable with regard to financial obligations therefore the best credit rating, CCC indicates the worst credit rating and currently vulnerable to non-payment before DEF rated, when default has actually occured meaning also an absorbing state.

1.3.2 Factor model. Application of Vasicek

The high-dimensional field of risk management commented on in section 1.2 is addressed with techniques of dimension reduction such as factor modelling and principal components. In Creditmetrics methodology, the factor model is applied.

Factor models are techniques from multivariate statistics that allows to tackle the randomness of a countless components that explain a d-dimensional vector by reducing these into a set of common factors. In our Merton's context, the d-dimensional vector may be a butch of firm's asset values which processes are dependent on underlying common factors such as industrial, regional influences likewise general economic situation with regard to drive the financial future of the firms.

Retrieving the Merton's model jointly with the factor model application and after standardization of the asset value log-returns, it admits a linear representation according to Bluhm *et al* (2003)[11]:

$$ln\left(\frac{V_t^i}{V_0^i}\right) \equiv r_i = R_i F + \varepsilon_i \qquad (1.4)$$

The asset value log-returns are normally distributed, so due to that standardization we have

$$r_i \sim N(0,1), \qquad F \sim N(0,1), \qquad \text{and } \varepsilon_i \sim iid\; N(0,1)$$

where R_i^2 quantifies the volatility of r_i that can be explained by the volatility of F and can be written as

$$R_i^2 = \frac{\beta_i^2 \, \mathrm{Var(F)}}{Var(\varepsilon_i)} = \beta_i^2 = \left(\rho \frac{\sigma_F \sigma_{r_i}}{\sigma_F^2}\right)^2 = \rho^2$$

Among the Merton-type factor models in the context of Creditmetrics is located the Aymptotic Single Risk Factor (ASRF) model –Vasicek (2002)[54]–, with a wide range of applications in the Basel regulatory framework. It is a factor copula model since it relies on linear structure dependence among the obligors by describing the asset's returns with two principal elements. It has a market-dependent common factor and an idiosyncratic element (or non-systematic) driving the i-th return, hence, the main property of the model is its implicit Gaussian copula.

Described by the specializing equation 1.4 and following the derivation in the Appendix B, then it can be written:

$$ln\left(\frac{V_t^i}{V_0^i}\right) \equiv r_i = \sqrt{\rho} \cdot F + \sqrt{1-\rho} \cdot \xi_i \tag{1.5}$$

where

$\{r_i\}_{i=1,2,\ldots,n}$ are jointly standard normal firm's asset returns

ρ is the uniform asset's pair-wise correlation among firms

F is the portfolio systematic factor usually represented by a market factor

$\{\xi_i\}_{i=1,2,\ldots,n}$ are independent and identically standard normal distributed, is the idiosyncratic component inherent at each asset firm's return

F $\perp$ ξ_i are uncorrelated and independent random variables

In addition, ρ measures the sensivity of the systematic risk so that $\sqrt{\rho} \cdot F$ can be the firm's exposure to the common factor and $\sqrt{1-\rho} \cdot \xi_i$ represents the firm's specific risk.

1.3.3 IRC basis model proposal

Once the described methodology is taken together, the model proposal allow to derive an analytical formula for credit IRC calculation based on the following assumptions:

- default-mode (Merton-type) model extended to a migration-mode based on Creditmetrics
- unique systematic factor considered (single factor model)
- an infinitely granular trading portfolio (i.e. composed by a large amount of positions of differents products to diversify the idiosyncratic risk)
- dependence structure among firms described by the Gaussian copula (Vasicek basis model)
- returns on the firm's stocks as principal variable (i.e taken as a consistent variable that describes the credit structure among asset value of the firms)

For the purpose of avoiding committing calculus that extracts complex numbers in case of negative correlation as well as using a more pragmatical approach, the basis model equation turns into as follows:

$$r_i = \rho \cdot F + \sqrt{1-\rho^2} \cdot \varepsilon_i \tag{1.6}$$

Summing up, giving a process for the latent variables as Merton's model does, permits to approximate the firm's asset value with firm's stock value, and moreover, the derivative ASRF (Vasicek) of factor model approaches accordingly the dependence among the latent variables as an approximation to model the credit risk dependence among issuers (i.e. dependence of default or credit downgrades).

The crucial point of the factor model is that the entire dependence structure among issuers is given by the common factor, see next Figure 1.1. For the simplified example of two firms, A and B, the dependence structure between them is expressed uniquely by means of their correlation with the shared factor, having each firm a direct dependence of the common factor while creating through an indirect way the dependence structure between A and B (i.e. gaussian in this basis point).

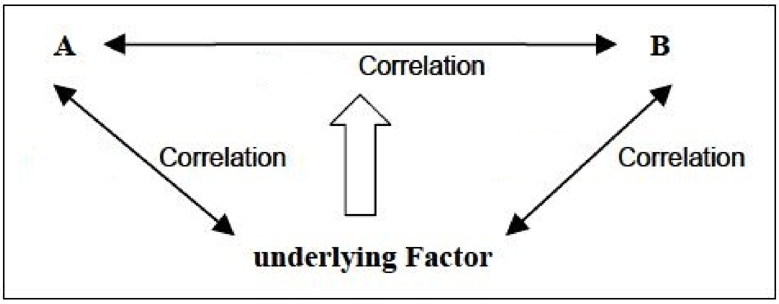

Figure 1.1: Factor model performance

Since Merton approach is a two-stated model (defaul/no-default) besides it could be easily generalised to more-state models with the credit classes above mentioned, Creditmetrics is extended to include them by slicing the distribution of the asset return into eight bands according to the S&P qualifications. If we draw randomly from the distribution, it will get reproduced the migration frequencies shown in a transition matrix.

Given that the migration and default probabilities are pre-determined for any given issuer-rated by the migration matrices, the model interest lies into delimiting the thresholds for each credit rating. To do that, it turns into simply establishing $\{V_{CR_j}\}_{j=1,2,...,8}$ as the firm value at any eight of the credit categories reachable, $\{Z_{CR_j}\}_{j=1,2,...,7}$ as their corresponding thresholds, and then using the inverse of the probability of ending up in one of them allows to extract the thresholds as follows:

$$\begin{aligned} pr_{CR_j \to DEF} &= pr_{CR_j < DEF} = pr_{DEF} \\ &= P[V_t < V_{DEF}] = P[r_i < N^{-1}(pr_{DEF})] \\ &= P[r_i < Z_{DEF}] \\ \mathbf{Z_{DEF}} &= N^{-1}(pr_{DEF}) \end{aligned}$$

$$\begin{aligned} pr_{CR_j \to CCC} &= pr_{DEF < CR_j < B} \\ &= P[V_{DEF} < V_t < V_{CCC}] = P[Z_{DEF} < r_i < Z_{CCC}|F] \\ &= P[r_i < Z_{CCC}] - P[r_i < Z_{DEF}] \\ &= pr_{CCC} - pr_{DEF} = \\ \mathbf{Z_{CCC}} &= N^{-1}(pr_{DEF} + pr_{DEF < CR_j < B}) \end{aligned}$$

and similarly

$$\begin{aligned}
\mathbf{Z_B} &= N^{-1}(pr_{CCC} + pr_{CCC<CR_j<BB}) \\
\mathbf{Z_{BB}} &= N^{-1}(pr_B + pr_{B<CR_j<BBB}) \\
\mathbf{Z_{BBB}} &= N^{-1}(pr_{BB} + pr_{BB<CR_j<A}) \\
\mathbf{Z_A} &= N^{-1}(pr_{BBB} + pr_{BBB<CR_j<AA}) \\
\mathbf{Z_{AA}} &= N^{-1}(pr_A + pr_{A<CR_j<AAA})
\end{aligned}$$

The following Figure 1.2 shows an example of the normalized asset's return distribution for a given BB-rated issuer delimited by its corresponding thresholds (Z_{CR_j}). These credit ratings thresholds correspond to the transition probabilities for a BB-rated issuer and are the same for any issuer within the same rating qualification.

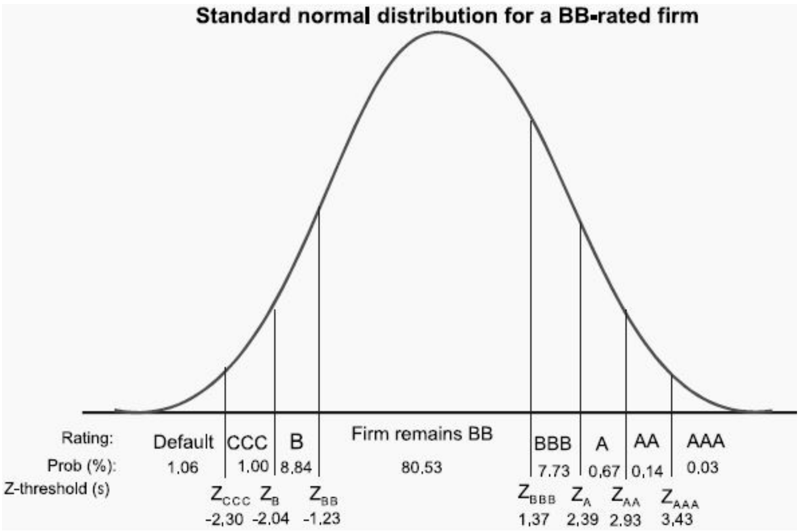

Figure 1.2: Example of the thresholds and the return's classification following Creditmetrics methodology.

Model extension

The Vasicek's model of ASRF has shown some weaknesses on its hypothesis so that our model proposal based on it can be a starting point in credit risk modelling that may be extended to outdo the following shortcomings:

- Name concentration

  It makes reference to the imperfect diversification of the idiosyncratic risk, i.e. the above-mentioned infinite granularity in the trading portfolio does not hold in possible cases.

- Sector concentration

  It refers to the imperfect diversification across systematic components of risk.

- Contagion

  Exposures to independent issuers have increasingly exhibited default or downgrades dependencies in turmoil periods so that the losses also exceeds the ones expected in those cases. This is well known as the asymmetric (or skewed) distribution of instruments subjet to credit risk.

Pykhtin (2004)[50] introduced the Multi-Factor Merton model in order to address both name and sector concentration. In his work is employed a combination of independent factors such as industry, geography or economic drivers creating a composite factor that may affect obligor's defaults in a systematic manner.

The third item, contagion, is the shortcoming addressed by this master's thesis. It can be tackled by extending the Gaussian-copula-based model proposal to other factor copula model that better fits with the fatter tails in the distribution of credit risk as a Student-t or Clayton copula could do.

2 Methodology

This chapter describes the entire process of obtaining the IRC through our model proposal from the first step of getting the model dependence structure to producing the trading portfolio loss distribution. It also goes deeper in several considerations explaining their academical background which has been used to generate the results.

2.1 Dependence structure

As it has been exposed in section 1.2 (interdependence), in risk management field, the multivariate dimension and its (inter)dependence is a decisive facet. For that very reason it has became the first step and the most important one in the IRC process (and also in every risk management model). This section is developed through three parts considered to completely understand the process of generating correlated migration events.

2.1.1 Driving factor

High dimensional financial risk applications often requires strong simplifications in order to keep the aim tractable. The use of factor modelling is one of such techniques of dimension reduction that results a essential tool to explain the multi-dimensional randomness in the components of a trading portfolio. The factor methods tries to run with a reduced set of risk sources or just with a single common factor a wide range of assets. It works well where the goal is to explain the equity returns or to predict them in order to simulate future dependent structure scenarios. That allows us to describe the firm's asset value process as a risk driver of the default or credit downgrades in a portfolio.

Factor models are not only computationally simpler but also they make it possible to derive scenarios to the wide range of portfolio assets besides keeping their statistical properties. Particularly, the computational gain is due to all calculus can be made by considering the covariance matrix of the factor model, instead of the high-dimensional matrix covariance of the original assets. In case of interest in such an original matrix, it can be effortlessly extracted.

In the factorial methods when modelling equity returns, is habitual the use of macroeconomic factors such as inflation rates to incorporate inflation risk, industrial production index or the employment rates to incorporate cyclical drivers. Additionally, it may be added other financial indicators.

As the purpose of our IRC model is to simulate equity returns (i.e. a proxy for the latent firm's asset value processes) as driver of the default or migration events, our model will use a financial-indicator-type factor as driver of systematic risk. They are the followings:

- An European equity index leading the global behaviour of the european economies (i.e. the risk inherent in all of them).
- A principal component derived from the PCA (Principal Component Analysis) of the twelve eurozone national equity indices plus two European bond's indices.

  The PCA is the second method of dimension reduction of a system of variables. In this case, the system is a butch of the twelve national market indices plus two European bond's indices where our issuer are listed. In short, the method is based on certain linear combinations capable of explaining the most variability of considered variables. The variables' variability runs into the variances-covariances matrix so that the PCA are constructed based on such matrix while keeping the same statistical information as the principal variables contain. The output it provides is a matrix with as many observations and series (principal components) as the original variables have. With a reduced selection of this principal components it may be explained the most of their variances.

Once the factors are specified, and reminding that our interest rely on the prediction or simulation of the future equity returns, it is first needed to predict the future process (i.e. fit the empirical distribution) of the financial indicators. In the macroeconomic factor model, the factor data is observable so it does not mean a problem in order to fit a distribution for then simulate them.

At this point, it can be made two assumptions concerning different levels of realism:

1. Standard normal distribution

   The simplest consideration so the most unrealistic one at the same time is fitting a standard normal distribution for the factor[1]. Even so, this is the primarily assumption made by the Gaussian factor model (see Vasicek on its ASRF (2002)[54], page 105 of McNeil *et al* (2005)[40] or page 78 of Bluhm *et al* (2003)[11]). Thus, it will be our first fitted factor distribution in our basis IRC model.

[1] There is no need to estimate the parameters in this case due to both factor will have zero mean and unit variance

2. Fitting a distribution to the factor

   Going further in the robustness of the model and according to the figures annexed (see QQ-plot Figure 3.18 for Stoxx factor and 3.19 for PCA factor in the Appendix C), a Student-t distribution may be accepted where the degrees of freedom of the factor are estimated by maximum likelihood estimation (MLE).

2.1.2 Copula assumption

Let us say that we are studying a trading portfolio with n counterparties. From that basis and over a year, every n-th rated-obligor may have changed its creditworthiness meaning a market impact (i.e. a loss in case of long position) ending up at default or other ratings considered. Since it is tried to model the credit dependence structure among all the issuers in the portfolio (i.e. trading book), such dependence is caused by the latent variable of the obligor's assets value. It is explained as previously stated through a common factor and an idiosyncratic element.

Following Merton's approach it can be written that:

$$\begin{gathered} r_n = f(F, \varepsilon_n), \quad n = 1, 2, ..., N \\ r_n \sim G_{r_n}, \quad F \sim G_F, \quad \varepsilon_n \sim iid\ G_\varepsilon \text{ and } F \perp \varepsilon_n \forall n \\ [r_1, r_2, ..., r_n]' \equiv \mathbf{R} \sim \mathbf{F}_R = \mathbf{C}(G_1, ..., G_n) \\ \text{implicitly from } F_{bivariate} = C(G_{r_n}(r_n), G_F(F)) \end{gathered} \tag{2.1}$$

Signifying that any individual issuer's stock return is a function of a common factor and an idiosyncratic element, where from now on G_{r_i}, G_F and G_ε are the marginal distribution, respectively, of the i-th return distribution, the common factor (systematic risk) and the idiosyncratic one, respectively.

Since all individual issuer's returns are described by the same common risk, a copula will be fit for them but with the indirect manner of a bivariate copula between the factor and the return. F_R is the supposed multivariate joint distribution in general terms and $F_{bivariate}$ the bivariate copula running such implicit dependence of the trading portfolio.

The most commonly and simplest technique to study dependence among variables is to calculate the Spearman's linear correlation but this approach is problematic and limited. It cannot establish the quantile-dependence structure beyond giving a scalar measure of the general dependence so the asymmetry besides different dependencies over quantiles are not managed. The reason by which copulas are used is that luckily they can overcome such these limitations.

Definition and basic properties

Copulas have been used as an statistical tool for constructing multivariate distributions and they have been increasingly applying since their discovery as a valuable technique in risk management. It is the most natural way in static distributional context of treating dependence in multivariate risk models and help to overcome the pitfalls of dependence that only focus on correlation allowing alternative measures ones. Copulas express dependence on a quantile scale being very much useful paying attention at the extremes outcomes that credit risk entails. Even more where Value-at-Risk or Expected-Shortfall measures has led us to think of risk in terms of extreme quantiles.

The multivariate joint distribution implicitly contains both the characterization of the marginal behaviour of individual obligor's credit process and a description of their dependence structure. The copula, therefore, provides a manner of isolating the description of that structure dependence among the variables $r_1, ..., r_n$ within our portfolio.

In the academic field of copulas it is found the well-kwon Sklar's Theorem which states that all multivariate distribution functions contain copulas and that copulas may be used alongside with univariate distribution functions (i.e. marginal distribution) to construct multivariate distribution functions.

Sklar's Theorem. Considering our model, let F_R be the portfolio joint df with continuous marginals $G_1, ..., G_n$. Then there exist a unique copula $C : [0,1]^n \to [0,1]$ if considered in R^n (e.g. n obligors) such that

$$F_R(r_1, ..., r_n) = C\left(G_1(r_1), ..., G_n(r_n)\right) \quad \text{where } r_1, ..., r_n \in \mathbb{R} \tag{2.2}$$

and conversely, if C is a copula and $G_1, ..., G_n$ are univariate marginal dfs, then equation (2.2) defines the multivariate df F_R with margins $G_1, ..., G_n$.

If we denote the uniformly distributed variables as $r_1 = G_1^{-1}(u_1), ..., r_n = G_n^{-1}(u_n)$, given the marginals G_i, every multivariate distribution defines an implicit copula:

$$C\left(u_1, ..., u_n\right) = F_R\left(G_1^{-1}(u_1), ..., G_n^{-1}(u_n)\right) \tag{2.3}$$

so that there is an implicit copula at any multivariate df. This is the very used method on the copulas construction.

Before listing the copulas used in this master's thesis it is necessary to point out two of the groups that we will consider in this master's thesis: elliptical and archimedean[2]. The elliptical copulas (Gaussian and Student-t) are the ones that share a linear dependence among the marginals dfs., they are copulas implicitly taken from their corresponding multivariate distributions due to there is no closed-form for them. These distributions have elliptical equi-probability lines (i.e. symmetric) and it is only needed to apply the Cholesky decomposition in order to simulate them. On the other hand, archimedean copulas share a non-linear dependence meaning asymmetry besides that the linear correlation coefficient does not make sense in these cases. They do have closed-form named as generating function to simulate them. Clayton, Gumbell or Frank's copula are a few of this group.

Elliptical copulas. Following the Creditmetrics method based on Gaussian copula, the aim is to apply that copula to our basis model before extending it to the Student-t copula as the EBA have proposed.

If we consider that $[r_1, r_2, ..., r_N]' \equiv \mathbf{R} \sim N_R(0, \Gamma)$[3], following equation (2.3) the copula will be

$$C\left(u_1, ..., u_n; \Gamma\right) = P\left(\Phi_1(r_1) \leq u_1, ..., \Phi_n(r_n) \leq u_n\right) = \Phi_R\left(\Phi_1^{-1}(u_1), ..., \Phi_n^{-1}(u_n)\right) \tag{2.4}$$

where $\Phi_R(\cdot)$ is the multivariate standard normal distribution and $\Phi_n(\cdot), n = 1, ..., N$ are likewise the univariate standard normal marginals.

Otherwise, if we face $[r_1, r_2, ..., r_N]' \equiv \mathbf{R} \sim T_{R,\nu}(0, \Gamma)$[4] and again with (2.3) we have

$$C\left(u_1, ..., u_n; \Gamma\right) = P\left(T_{1,\nu}(r_1) \leq u_1, ..., T_{n,\nu}(r_n) \leq u_n\right) = T_{R,\nu}\left(T_{1,\nu}^{-1}(u_1), ..., T_{n,\nu}^{-1}(u_n)\right) \tag{2.5}$$

respectively as in the gaussian copula case, but here $T_{R,\nu}(\cdot)$ is the multivariate Student-t distribution and $T_{n,\nu}(\cdot), n = 1, ..., N$ the correspondent Student-t univariate distribution.

Before concluding this theoretical subsection, it is important to note that in order to build multivariate elliptical distributions there is the possibility to assume other marginal distribution than its respectively same univariate creating meta-distributions.

[2] There exists other copulas distinctions as Extreme Value Theory copulas (ETV).

[3] Variables jointly standard normally distributed where Γ is the correlation matrix due to standardization.

[4] Standardized variables jointly Student-t distributed.

Archimedean copulas. Unlike elliptical case with its inversion-obtention mode, archimedian copulas do have an alternative method using a generating function $\Psi(u) : [0,1] \rightarrow [0,\infty]$. It is defined then an archimedean copula as

$$C\left(u_1,...,u_n;\theta\right) = \Psi_R^{-1}\left(\Psi(u_1) + ... + \Psi(u_n)\right)\right) \tag{2.6}$$

Considering $[r_1, r_2, ..., r_N]' \equiv \mathbf{R} \sim$ with a Clayton copula, the expression (2.6) takes the particular form of

$$C\left(u_1,...,u_n;\alpha\right) = \left(1 + \alpha\alpha^{-1}\left(\sum_{n=1}^{N} u_n^{-\alpha} - N\right)\right)^{-1/\alpha} = \left(u_1^{-\alpha},...,u_N^{-\alpha} - N + 1\right)^{-1/\alpha} \tag{2.7}$$

Meta-distributions. Taking advantage of the Sklar's Theorem (2.2) through its useful contribution to obtain probability multivariate distributions from whatsoever marginals with the use of a copula, one can assume that to define a multivariate df there is only needed a copula and any type of marginals. In terms of density function is as follows

$$f_R(r_1,...,r_n) = c\left(G_1(r_1),...,G_n(r_n)\right) \cdot g_1(r_1),...,\cdot g_n(r_n) \tag{2.8}$$

resulting very flexible when one desires to make a better empirical fit of its model without making any unrealistic assumption.

Conditional copulas and factor models

Gathering the reduction-dimension technique of factor model with its characteristic of creating the interrelated relation of the high-dimensional scheme mainly depending on that common factor, we locate the factor copula models.

By using copula models conditional on the common factors, it may be established the dependence relationship between the stock returns. It allows us to use the marginals distribution of the common factor (besides the marginal distribution of the idiosyncratic risk) instead of directly applying the marginals of every stock return.

To focus solely on the interactions among the returns, the factor copula correlations conditional on the common factors tackles more accurately the dependence structure among multiple variables. With such bivariate copula it may be derived two conditional distribution fuctions but only resulting in our use the second one:

$$\begin{aligned} C_{F|r_i}(G_F(F)|G_{r_i}(r_i)) = C_{F|r_i}(u_F|u_{r_i}) = P(U_F < u_F|U_{r_i} < u_{r_i}) \\ C_{r_i|F}(G_{r_i}(r_i)|G_F(F)) = C_{r_i|F}(u_{r_i}|u_F) = P(U_{r_i} < u_{r_i}|U_F < u_F) \end{aligned} \tag{2.9}$$

From that consideration it is found that there is an implicit copula through the specification of the factor model. Considering again the linear factor model, $f(F, \varepsilon_i)$, this master's thesis firstly develops the IRC model having chosen to work with the elliptical copulas implicitly inherent in the kind of multivariate distribution that may be considered as a natural alternative models for asset values. It would then be also possible to work with the archimedian family copulas in order to catch up with the asymmetry and the non-linearity as Clayton copula does –see Nadaraja *et al* (2017)[43] or Novales (2017)[44].

Gaussian factor copula model. The less reasonable one due to in the academic word it is known that the probability dfs of financial series tend not to be normally distributed as the data frequency increases. Even worse when that data is related to credit risk returns extracting fat-tailed and non-symetrical distributions that gaussian copula does not consider because of its zero tail dependence. Anyhow, Creditmetrics and KMV model are based on that copula and for that reason it will be our basis model.

In this Gaussian bivariate case with $\mu_F = \mu_{r_i}$ and unit variance by construction (i.e. after an standardization of the data), we know that the univariate distribution of r_i conditional to F = F, is univariate standard normal with:

$$E(r_i|F = F) = \rho \cdot F, \quad Var(r_i|F = F) = 1 - \rho^2$$

so that, conditioned on F=F, the random variable $Z = \dfrac{r_i - \rho F}{\sqrt{1 - \rho^2}} \sim N(0, 1)$

After some trivial calculus, it can be expressed from the quantile curve already in returns terms:

1. With standard normal marginals as showed before in equation (1.6) so r_i, F and $\varepsilon_i \sim N(0, 1)$

$$r_i^{gauss} = \rho \cdot F + \sqrt{1 - \rho^2} \cdot \varepsilon_i$$

2. Student-t marginals, then a meta-distribution with r_i and $F \sim T(\nu)$, while $\varepsilon_i \sim N(0, 1)$

$$r_i^{stud-t} = T_{\nu_{r_i}}^{-1} \left[\Phi \left(\rho \cdot \Phi^{-1}[T_{\nu_F}(F)] + \sqrt{1 - \rho^2} \cdot \varepsilon_i \right) \right] \tag{2.10}$$

   where $\Phi(\cdot)$ is the general univariate standard normal distribution, $T_{\nu_{r_i}}$ the univariate Student-t distribution of the i-th return and T_{ν_F} their correspondent univariate of the common factor.

   See both previous representations in the appendix C (Figure 3.15).

The study will consider two assumptions of the marginals distribution increasing the model realism from normality to contemplate Student-t distribution fitting the degrees of freedom to the equity and factor data. Nevertheless, as explained before, in the case of simulating a meta-distribution copula (Clayton with Student-t marginals), the dependence structure does not vary but consistency will improve.

Student-t factor copula model. The financial crisis of late 2007 came up with the surprise of the most assets that had previously behaved mostly independently suddenly moved together with crashes being more correlated than booms. Only left-tailed dependence could better fit with that, but in order to start the following modification of Creditmetrics by systematically moving away from normality, it will be employed the Student-t copula by decreasing the degrees of freedom with its symmetrical imposition of tail dependence.

Considering now that (r_i, F) follows a Student-t bivariate distribution with ν degrees of freedom, then, conditioned on F=F, the random variable $T = \sqrt{\frac{\nu+1}{\nu+F^2}}\frac{r_i - \rho F}{\sqrt{1-\rho^2}} \sim T(\nu+1)$.

Again with few steps we have our return's simulation process:

1. With Student-t marginals so that r_i and $F \sim T(\nu)$, while $\varepsilon_i \sim T(\nu+1)$

$$r_i^{stud-t} = \rho \cdot F + \sqrt{(1-\rho^2)\frac{\nu+F^2}{\nu+1}}\varepsilon_i \tag{2.11}$$

2. Standard normal marginals, then a meta-distribution with r_i, and $F \sim N(0,1)$, while ε_i still $\sim T(\nu+1)$

$$r_i^{gauss} = \Phi_{r_i}^{-1}\left[T_\nu\left(\rho \cdot T_\nu^{-1}[\Phi_F(F)] + \sqrt{(1-\rho^2)\frac{\nu+T_\nu^{-1}[\Phi_F(F)]^2}{\nu+1}}\varepsilon_i\right)\right] \tag{2.12}$$

where $T_\nu(\cdot)$ is the general univariate Student-t distribution, $\Phi_{r_i}(\cdot)$ and $\Phi_F(\cdot)$ the univariate standard normal of the i-th return and the common factor, respectively.

See the representation in the appendix C (Figure 3.16). Note than we present the Gaussian and Student-t meta-distributions but this work will not use it.

Clayton copula factor model. Additionally, the use of IRC model is intended to estimate measures of tail risk as Value-at-Risk and Expected-Shortfall, then we should be concerned with the Clayton copula that properly estimate the left tail of the portfolio loss distribution. Otherwise, they might underestimate these measures. Furthermore, this copula seems to capture the left-asymmetric behaviour of the credit risk's returns as next figure shows.

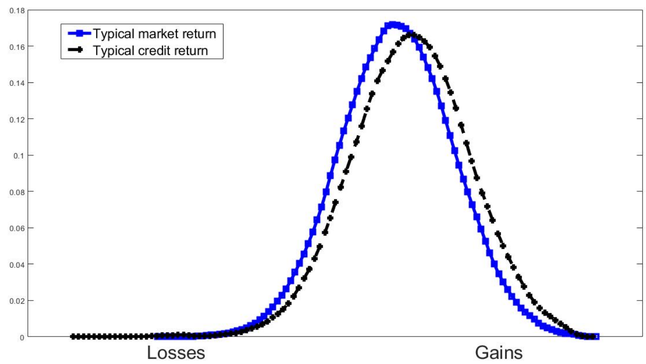

Figure 2.1: Comparison of typical market and credit returns.

The conditional copula of this case with any marginal distribution (i.e. Gaussian or Student-t) for the common factor $F \sim G_F$ or for the i-th return $r_i \sim G_{r_i}$ results

$$r_i = G_{r_i}^{-1}\left[\left(1 + G_F(F)^{-\alpha}(q^{-\alpha/(1+\alpha)} - 1)\right)^{-1/\alpha}\right] \tag{2.13}$$

where α is the driving parameter of the left tail's positive dependence and q is the q-quantile of any distribution, in this case, $q = G_\varepsilon(\varepsilon_i) \equiv U(0,1)$.

See the representation in the appendix C (Figure 3.17).

Making use of these copulas conditioned on the common factor with yet no meta-distribution considered, for any random-pair of returns we have the next example:

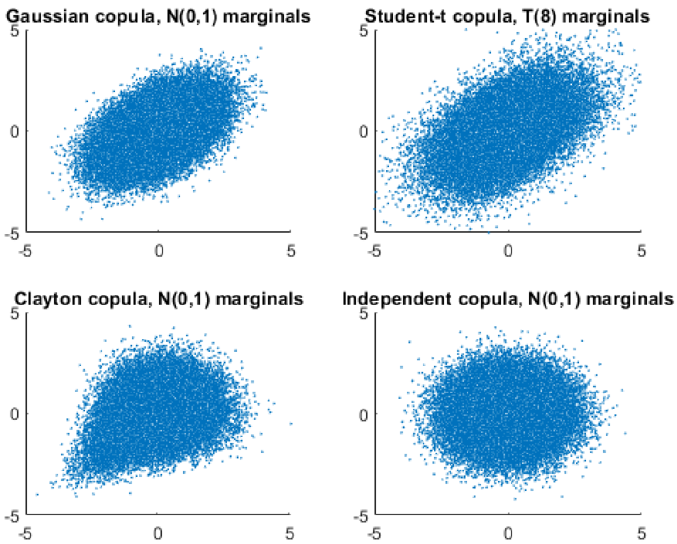

Figure 2.2: Conditional copula comparison, $M = 100000$, $\rho = 0.7$, $\nu = 8$ and $\alpha = 0.87$.

2.1.3 Issuer correlation

The issuer correlation is also a key point in the model since it is an input parameter necessary to the copula in order to simulate the creditworthiness' process of each obligor. As it has already been stated before, the conditional driving factor traces the general economic situation which may affect the issuer's credit behaviour. Therefore, it is needed an exposure parameter quantifying that effect (i.e. correlation coefficient).

Purely, it should be calibrated from default data due to that way the factor copula model will describe straightforwardly the credit process imputing the default correlation. Unfortunately, there exist a lack of such data so insted of that we will use the next one described in the upcoming paragraphs.

Even so, Merton's model exhibits the first flaw assuming the credit behaviour ran through the asset value, and as a consequence of these latent variables nature, the model ends up having as a proxy of default correlation the equity correlation making finally the flaws double.

The literature in this regard is varied, but it seems to agree with Frye (2008)[28] and Düllmann *et al* (2008)[17] as they state the vast difference when using the equity or default correlation besides the higher values obtained by the first type of data. Qi *et al* (2010)[51] find the equity correlation as a proxy for unobservable asset correlation not valid, while on the other hand Düllmann *et al* (2008)[17] justify the more effectiveness of using equity finding, moreover, the differences caused by substantial downward bias characteristic of estimates based on default data. Other investigations carried out by Hull and White (2001)[34], Overbeck and Schmidt (2001)[48] or Cedeno and Jansson (2018)[14] uses joint default probabilities[5] in order to obtain default correlations. However, that approaching does not hold since it is a pair-based default correlation among issuers and our need claims for pair-based correlation between the driving factor and the equity issuer's data according to the factor model structure.

Finally, our model proposal will fit the default correlations assuming:

1. Estimates from asset correlation as Zhou (2001)[59] or Frey *et al*[25] do, while taking advantage of this last work by using a factor model to describe asset correlations.

2. Following Creditmetrics —Gupton and Finger (2007)[31]— with its narrowed assumption of asset correlation equal to equity return correlation if leverage levels (i.e. debt ratio over capital share) are low and horizons are short[6].

Conversely for sovereign issuers, there is also the necessity for a dynamic's credit quality descriptor but the equity national indices no longer hold in such case. The reason is countered by Aslanidis *et al*(2018)[5] so the two most traded in investment markets –bond and stock market– have evidenced opposite perfomances in a macroeconomic sphere creating a trade-off: there are simultaneous episodes of large negative bond returns and large positive stock return. At this point, it doesn't exist valid driver of credit worthiness dynamic through stock market but in the corporate case.

For that very reason and given availability, the sovereign correlation is deduced from constructed series deriving prices through daily 1-year[7] sovereign bond yields:

$$price = \frac{100}{1 + y_{daily}} \tag{2.14}$$

where y_{daily} is the daily yield of the sovereign bond issued to a 1-year maturity.

The data considered in this subsection is shown in the Table 3.1 (Appendix C).

[5] To calculate the default-paired correlation ϱ_{jk} between obligor j and k, with $p_{jk} = P(\tau_j < t_0, \tau_k < t_0)$ as the joint default probability between times 0 and t_0, and p_j,p_k the univariate probability of default, they use $\frac{p_{jk} - p_j \cdot p_k}{\sqrt{p_j(1-p_j)p_k(1-p_k)}}$

[6] Mainly the reason why Creditmetrics is used by the common industry models (i.e. our master thesis proposal) over a one-year horizon. Otherwise, longer horizons based on Merton's approach encounter inconsistency (Zhou, 2001)[59].

[7] According to the capital horizon made by our model. It will not be any rebalanced position in a one-year horizon so that the default's event may take place once reached that space of time.

2.2 Model inputs

In accordance with Creditmetrics methodology, several inputs have to be gathered to our IRC model such as a rating system, transition matrices and PDs, interest rates term structures for each rating class or the mean and standard deviation of RRs if they are stochastic. They are cleared up in this section through the following four parts.

2.2.1 Migration matrices

The migration matrices or also named transition probability matrix (TPM) serves as a primarily input in the IRC model. The approach made by Creditmetrics set basis of the market risk due to credit risk descending not only from default but also from changes in value due to downgrades. Thus, the credit migration matrix is the specification of magnitude that any individual credit product in the portfolio may default or migrate its creditworthiness in a pre-specified time horizon.

From a given initial rating which classifies the credit quality of a counterpary into seven states, the entries in the matrix denote what is the likelihood to migration to another credit rating or the default. Since our assumption lies into one-year capital horizon, the TPMs are selected consistently with the likelihood of having a rating migration for one-year based.

As already mentioned herein-above (check the guidelines concerning migration matrices section 1.2, there exist various methodologies in the estimation of the TPM drawing estimates that can vary substantially. According to the EBA's guidelines it may be considered either internal or external sources of such data.

External ratings

Due to the narrow scope of data availability, our model proposal consider both corporate and sovereign TPM from external data sources. Concretely besides in accordance with Creditmetrics, from Standard & Poors Global Ratings –see Vazza and Kraemer (2017)[55] for corporate and Witte (2017)[57] for sovereign data–, even though several other credit rating agencies are gathering that data as Moody's Investors Service or Fitch Ratings.

The migration data has been adjusted from the one disclosed by S&P in its annual report of default study by assembling the credit final ratings of (+) and (-) into the middle-rated. For example, the probabilities of AAA+, AAA and AAA- have been uniquely gathered to AAA. The matrix for both corporate and sovereign issuers can be seen in the Appendix c (Table 3.7 and 3.8, respectively) where additional tables and figures are attached.

Internal estimation

When institutions meet the requirements, an internal estimation of the TPM may be carried out. There are two main different methodologies estimations based on Markov chains –see Christensen *et al* (2004)[15] for further detail– that could have been tackled in this master's thesis if we had had acces to historical migration data. Nonetheless, we give a dub explanation following Gunnvald (2014)[30] and Van Der Stel (2010)[53].

Applying Markov chain model. A Markov chaing model theory can be used to build up a theorical framework around credit migration estimates. Let N be the number of states for the credit rating framework (i.e. the number of possibly ratings), M the TPM and the corresponding P in the Markov chain theory where entries p_{ij} denotes the probability of transition from initial rating i to rating j during a given time horizon[8].

Also let G be the generator matrix, corresponding to Q in the continuous time Markov chain (CTMC) framework. Composed by its analogous entries q_{ij}, where it have an initial state i and a final one j. The default state D is assumed as an absorbing state, therefore, once reached it cannot leave.

$$M = \begin{pmatrix} p_{11} & \cdots & p_{1(N-1)} & p_{1N} \\ \vdots & \ddots & \vdots & \vdots \\ p_{(N-1)1} & \cdots & p_{(N-1)(N-1)} & p_{(N-1)N} \\ 0 & \cdots & 0 & 1 \end{pmatrix}$$

and

$$G = \begin{pmatrix} q_{11} & \cdots & q_{1(N-1)} & q_{1N} \\ \vdots & \ddots & \vdots & \vdots \\ q_{(N-1)1} & \cdots & q_{(N-1)(N-1)} & q_{(N-1)N} \\ 0 & \cdots & 0 & 1 \end{pmatrix}$$

The two most commonly methods of estimating the entries of M referred to are the so called *cohort* and *duration*, in discrete and continuous time, respectively.

Cohort method. The first approach and the most widely used by the industry due to its simplicity. It employs a discrete-time setting by letting $t_0, t_1, ..., t_n$ be the discrete-time points with an arbitrary and constant time interval $t_{k+1} - t_k = \Delta t_k$.

The estimator of p_{ij} over one period is

$$\hat{p}_{ij}(t_k) = \frac{n_{ij}(\Delta t_k)}{n_i(t_k)} \tag{2.15}$$

with $n_{ij}(\Delta t_k)$ the number of counterparties that have migrate from state i to j between t_k and t_{k+1}, and $n_i(t_k$ the number of companies in the initial state i at initial time t_k.

[8] The sum by rows has to be the unit.

Duration method. In accordance with Lando and Skødeberg (2002)[35], with the assumption of time-homogeneity and in a continuous time range, the matrix M can be obtaining by first applying the ML estimation to the generator matrix G.

The ML estimates the elements q_{ij} in a time horizon between t and T following

$$\hat{q}_{ij}(t,T) = \frac{n_{ij}(t,T)}{\int_t^T Y_i(s)ds} \text{ for } i \neq j \tag{2.16}$$

where $n_{ij}(t,T)$ is the number of companies migrated from state i to j during the period of [t,T] and $Y_i(s)$ the number of issuers remaining in the initial rating i at time s.

Then, having the entries estimates of the generator G, the matrix M can be calculated as follows for all t ≥ 0

$$M(t) = e^{tG} = \sum_{k=0}^{\infty} \frac{(tG)^k}{k!} \tag{2.17}$$

Inherently, the entries of the generator matrix G describe the probabilistic behaviour of the remain time in state i as an exponential distribution with parameter $\lambda = q_{ii}$ and the probability of migrate from rating state i to j is given by $\dfrac{q_{ij}}{q_{ii}} = \dfrac{\lambda_{ij}}{\lambda_{ii}}$ for $i \neq$j.

This duration method ameliorate the shortcomings of the cohort method but, on the other hand, it induces to a hard estimation of the bands of confidence and the standard errors for the migration probabilities.

2.2.2 Credit curves

The present value of a financial instrument is nothing else but its future payment updated to valuation date. Specially from the last financial crisis, every instrument has to be valued according its creditworthiness depending on its ensuing counterparty credit quality. From that basis, there exist a different interest rate term structure or the so called forward zero curves for each rating class that uniquely discernible by applying different (credit) spreads.

This zero curves are a riskless yield curve plus a credit spread according to the quality, following

$$f_{t,k}^{j} = f_{t,k}^{risk-free} + s^{CR_m^j} \tag{2.18}$$

where t is the valuation time, k is the future cash flow payment date and j the credit rating $\forall j = 1, ..., 8$ of the m simulated scenario.

As we want an updated capital measurement (IRC) for the date on which the model is calculated[9], there is no need to extract forward curves.

[9]We recall the weekly frequency of the IRC calculation.

Summing up, each future cash flow (FCF) of the bond (i.e. coupon or coupon plus nominal) have to be valued according to its correspondent issuer's credit quality. Following the financial literature of the money markets, the lower rating, the higher spread applied –see Galliani *et al* (2014)[29] or Livingston *et al* (2018)[37]– in order to premium the increase in risk, otherwise the instrument will not be negociated.

2.2.3 Recovery rates

If the credit risk move to default, the RRs are applied to the face value of the bond. At this point, and following the approach that was stated in the section 1.2, we will use a stochastic process for dynamic of the RRs. In accordance with Fisher *et al* (2016)[23], Frye (2000) and Altman *et al*(2002), the RRs values may be obtained from the beta distribution. This distribution function allows to extract values between the range of zero and one, besides, through its parameters alpha and beta permits us to give the shape that better fits with the seniority or industry of the bond.

In relation to this, there exist several works which have carried out investigations to measure the distribution of the RRs. The papers of Renault and Scaillet (2004)[52], Bruche and Gonzalez-Aguado (2009)[12] and Altman and Kalotay (2014)[1] provide the data concerning the RRs.

As Renault and Bruche data gathering is prior to the financial crisis of 2007, we choose the RRs data presented by Altman and Kalotay in order to simulate the process of the RR accordingly with the most current market situation.

To do that, the beta distribution paramaters have to be derived. It may be done by applying the mean and the standard deviation to the next formulas:

$$\begin{aligned} \alpha &= (\frac{1-\mu}{\sigma^2} - \frac{1}{\mu})\mu^2 \\ \beta &= \alpha(\frac{1}{\mu} - 1) \end{aligned} \tag{2.19}$$

The RRs simulated depending on the industry of the issuer are attached in the Appendix C (Figure 3.26).

Based on Frye (2000)[28], Ojea (2016)[22] also stated the link that stochastic RRs have with PDs by correlating them through another factor copula model [10]. It goes beyond our data availability so its correlation parameter has to be estimated strictly with default data. Moreover, Creditmetrics does not consider such dependence so we only apply to our models stochastic RRs uncorrelated with the PDs.

[10] $RR_i = \sqrt{\rho_{recovery}}F + \sqrt{1-\rho_{recovery}}\varepsilon_i$

2.2.4 Portfolio and model selection

IRC models

The models developed in this master's thesis are succinctly attached in the next table.

	Copula	**Marginal distributions (r,F)**	**Factor**	**Copula d.o.f.**
Model 1	Gaussian	Standard normal	European Index	–
	Gaussian	Standard normal	PCA	–
Model 2	Student-t	Student-t standardized	European Index	Estimated
	Student-t	Student-t standardized	PCA	Estimated
Model 3	Student-t	Student-t standardized	European Index	Imposed 8
	Student-t	Student-t standardized	PCA	Imposed 8
Model 4	Clayton	Student-t standardized	European Index	–
	Clayton	Student-t standardized	PCA	–

Table 2.1: Model proposals

In this master's thesis, we have tested a group of 4 model proposal for modelling the credit risk. They all set basis in the Model 1 that is just the Vasicek's approaching presented in the base line section –1.3.3 IRC basis model proposal–.

To continue adding soundness gradually to our framework, we have developed next an IRC model based on Student-t copula. It aims to better catch up with the extreme-values dependence that the general returns on financial markets performances. It is the Model 3 where the copula parameters have been estimated from the market data in compliance with our research.

Model 4 starts up the herein-above cited ECB's proposal of imposing eight degrees of freedom to the factor Student-t copula model. In the forward analysis is proved the capital requirement of this proposal compared to the other models.

Lastly, in our seek for presenting a model that better agrees with the left-skewed distribution of credit instruments as was displayed in past Figure 2.1, we formulate a factor Clayton copula approaching.

In order to distinguish which factor is being used by any model, it will be defined as *Stoxx* the European equity index and as *PCA* the factor derived from the PCA of the mentioned indices.

Bonds portfolios composition

In order to test our IRC measurement, several hypothetical trading portfolios should be built. We have build six different portfolios with only long positions taken.

The first two portfolios are very basic and only incorporate corporate and sovereign bonds positions meeting with the minimum denomination requirement on each position. With pie charts we present the credit rating percentage in each portfolio. Portfolio 1 and 2 are presented in the Figure 3.20 and 3.21., both with a vast accumulation in the BBB rating.

The next four portfolios have been formed in order to make an study of the IRC demand when the risk differs. Therefore, the Portfolio 3 conforms a high investment grade portfolio (is attached in the Figure 3.22). The next one, Portfolio 4, decreases to lower-medium investment grade (check Figure 3.23). Finally, to show how a highly risk concentration could rise the capital requirement, we form a non-investment grade/speculative and an extremely speculative portfolios (Portfolio 5 and 6, respectively in Figure 3.24 and 3.25).

2.3 Marked-to-market valuation

Once stated the likelihood of credit migration for any rated issuer, the simulation process take place following Merton's approach. It is accomplish through the factor copula model which will determine possible upgrades, downgrades or even default's triggered given the TPM. Accordingly, if the credit quality move is to another credit rating rather than default, the exposure should be revaluated as follows:

$$B_t^i(CR_m^j) = \sum_k^M \frac{FCF_k^i}{(1+f_{t,k}^j)^{k-t}} = \sum_k^M \frac{FCF_k^i}{(1+f_{t,k}^j)^{DCF}} \quad \forall k \geq 1\text{year}, \forall j = 1, ..., 7 \tag{2.20}$$

where $B_t^i(CR_m^j)$ is the bond value of the issuer i, valued at time t (in our case t = 0 = IRC calculation day) and given the credit rating j in the m future 1-year scenario.

Otherwise, if default state is triggered, the marked-to-market valuation will result simply by:

$$B_t(CR_m^8) = B_t \cdot (1 - LGD^{ind}) = B_t \cdot RR^{ind} \tag{2.21}$$

where ind is the company's industry, therefore RR^{ind} is subject to the industry to which the defaulted issuer belongs.

Therefore, each bond will present a valuation table like the next. Only one value will coincide with its initial, apart from the defaulted valuation because of such state is an absorbing one meaning that any defaulted bond can be initially considered.

Year-end rating	Value
AAA	$B_t(CR_m^1)$
AA	$B_t(CR_m^2)$
A	$B_t(CR_m^3)$
BBB	$B_t(CR_m^4)$
BB	$B_t(CR_m^5)$
B	$B_t(CR_m^6)$
CCC	$B_t(CR_m^7)$
DEF	$B_t(CR_m^8)$

Table 2.2: Example of the bond valuation

where here the m simulated scenario does not matter.

2.4 Portfolio loss distribution and IRC

Simulating new asset returns will end up generating new creditworthiness for each obligor once classified by thresholds. After that, we can map the into new bond prices that will cause portfolio value movements.

Let us now introduce the portfolio value as the PFV acronym and make a simple example to show how the IRC is calculated given 3 issuers with one bond each.

$$\begin{aligned} PFV_{t,m} &= \sum_{i=1}^{3} B_t^i(CR_m^j) \\ \Delta PFV_{t,m} &= PFV_{t,m}(CR_m^j) - PFV_{t,0}(CR_0^j) = P\&L_{t,m} \end{aligned} \tag{2.22}$$

Resulting $PFV_{t,m}$ and $\Delta PFV_{t,m}$, respectively, the portfolio value and the change presented in the portfolio in relation to its initial value at time t and in the m scenario.

To show an IRC calculation, let us also simplify it to $B(CR^j)$ and take this portfolio example with three bonds position and five scenarios simulated:

Scenarios	Issuer 1	Issuer 2	Issuer 3	PFV	P&L
0 (Initial)	$B(CR^3)$	$B(CR^5)$	$B(CR^1)$	$PFV_{t,0}$	–
1	$B(CR^1)$	$B(CR^3)$	$B(CR^6)$	$PFV_{t,1}$	$\Delta PFV_{t,1}$
2	$B(CR^4)$	$B(CR^2)$	$B(CR^3)$	$PFV_{t,2}$	$\Delta PFV_{t,2}$
3	$B(CR^5)$	$B(CR^2)$	$B(CR^1)$	$PFV_{t,3}$	$\Delta PFV_{t,3}$
4	$B(CR^2)$	$B(CR^2)$	$B(CR^7)$	$PFV_{t,4}$	$\Delta PFV_{t,4}$
5	$B(CR^8)$	$B(CR^4)$	$B(CR^1)$	$PFV_{t,5}$	$\Delta PFV_{t,5}$

As it can be seen, the asset value's simulation primarily mapped with its new issuer's credit rating turns out to generate possibly different market-to-marked bonds positions $B(CR^j)$ with j=1,...,8 (from AAA to DEF). At any scenario the PFV and its respective ΔPFV are calculated so that it will get a couple of vectors with five different scenarios where the distribution of the values could therefore be analysed.

Recalling the IRC measure defined in one of the initials sections –1.1 Market risk framework–, it concerns the Value-at-Risk of the P&L distribution at 99.9% confidence level (i.e. the 0.1% percentile).

We would like also to propose the IRC measurement based on the Expected Shortfall (ES) following Artzner *et al* (1999)[4] instead of only the VaR. The Committee already shifted it in its consultative document – BCBS (2013)[7]– due to the flaws of the VaR: "A number of weaknesses have been identified with using VaR for determining regulatory capital requirements, including its inability to capture tail risk".

In agreement with its proposal for internal models, we derived a renovated IRC measurement under the ES at same confidence level besides the stated VaR. Taking both measures of the portfolio P&L and in terms of losses (- P&L):

$$\begin{aligned} VaR_{99.9\%} &= inf\{P\&L : P(Loss > P\&L) \leq (100-99.9)\%\} \\ ES_{99.9\%} &= E[P\&L | P\&L \geq VaR_{99.9\%}] \end{aligned} \tag{2.23}$$

at a given time horizon, in our case one-year capital horizon.

Hence, the IRC will be based on the Value-at-Risk measure unless otherwise stated.

3 Empirical analysis

The empirical analysis has required different steps to obtain the final IRC value. At this section, it is first justified the data sources used through the three main steps carried out: first the valuation of the each bond's position at any credit rating possible, second the estimation of the parameters which are utilized in the factor copula model, till third make use of them to simulate an accordingly migration structure following the actual market performance. Lastly, the results of the model proposal are presented respectively.

3.1 Data sources

In accordance with the generation's steps of the IRC models, several types of data have been used:

- General model inputs
  - TPM. As before commented, this data is extracted from S&P report updated to 2017 from estimates between 1981-2017 for global corporates.
  - Credit curves. Following subsection 2.2.2, the riskless yield curve plus the credit spread are obtaining from Reuters Eikon Database.
  - RR. From the given data of Altman and Kalotay (2014)[1] for either industry or seniority. Although in our model we simulate the RRs according to the industry of the issuer.
- Dependence structure

  The dependence structure (section 2.1) is an important aspect in the IRC modelling. In order to catch up with the market features[1], the estimates may be extracted according to such data. For our model proposal, the selected sample data comprises 251 business days prices from our valuation date on 26th April backwards. We have gathered data from Thomson Reuters Eikon for the factor copula model need. It means data for the factor besides for each issuer's credit driver in order to obtain their dependence.

[1] Although it is more interesting to estimate the model from turmoil periods as was the last financial crisis that started in 2007, due to limits of historical data extraction, we finally considered another more recent data period.

Two factors are minded:

– Stoxx Europe 50 EUR Price Index (Europe, .STOXX50)

– The first component of the PCA from the following national equity indices: Amsterdam Exchanges Index (Netherlands, .AEX), Athex Composite Share Price Index (Greece, .ATG), Austrian Traded Index (Austria, .ATX), BEL 20 Index (Belgium, .BFX), CAC 40 Index (France, .FCHI), FTSE Italia All-Share Index (Italy, .FTITLMS), Deutsche Boerse DAX Index (Germany, .GDAXI), IBEX 35 Index (Spain, .IBEX), ISEQ Overall Price Index (Ireland, .ISEQ), OMX Helsinki 25 Index (Finland, .OMXH25), OMX Stockholm 30 Index (Sweden, .OMXS30) and Euronext Lisbon PSI 20 Index (Portugal, .PSI20); and from the European Debt indices of FTSE MTS Eurozone Government Bond Index (EXEG5=) and EURO STOXX 50 Corporate Bond (.SX5BPI).

- Marked-to-market valuation

  Bond features from Reuters Eikon Database are noted down as it is shown from Table 3.2 to 3.5. The complete range of bonds start by considering an unique bond for each corporate issuer. In the sovereign case, from twelve different countries where the corporate issuers are headquartered, we only have eight country issuers for data reasons. We finally have 117 corporate and 97 sovereign bonds making a sum of 214 valued instruments and 125 counterparties credit processes' analysed.

3.2 Valuation

The valuation step has been performed following the section 2.3 formulas. As it is necessary the bond price for all eight possible final credit class, the formula (2.20) is taken to the valuation among AAA - CCC simply by updating the future cash flows of the instrument with the correspondent interest rates term structure. In the eighth state –defaulted–, the formula followed is (2.21) applying the RR to the bond notional amount.[2].

3.3 Estimation

The estimation comprehends two encompassing parts or levels. It mainly lies into first fit a distribution (i.e. estimate its parameters) to the factors and to the issuer's credit driver. The next level is to estimate the factor copula model. In other words, first is estimated the marginals of the copula distribution and then, the copula itself as a

[2] It has to be pointed out that in the valuation procedure it was taken into account the +2 business days proceedings, real coupon frequencies and the correspondent DCF conventions of each bond.

multivariate distribution function.

- Driving factor

  Following subsection 2.1.1, two factors will describe the credit process of the companies. When normality assumption is outdone by the model 2, 3 and 4, there is a distribution estimation process.

  - Stoxx Europe 50 EUR Price Index

    As it may be checked in the quantile-quantile plot (QQ-plot) with a normal theoretical distribution (see Figure 3.18), this index data no longer seems to fit empirically with normality –both upper and lower values extracts longer probabilities (i.e. fatter tails than the normal distribution).

    A Student-t univariate distribution is fitted for this serie. The degrees of freedom estimated are 13.183.

  - First principal component derived from PCA

    Once the study of the principal components are done, the serie of the first component is selected as our second factor. It inherently have the higher variance explanation resulting in about 40% of the total original variables' variance (See Figure 3.19).

    After the selection of this factor, the analysis of its quantiles behaviour in a QQ-plot guides by itself to a Student-t copula as well. Its estimation of the degrees of is 12.526.

- Issuer's credit driver

  Already justified in the subsection 2.1.3, our IRC model uses the issuer's stock prices as creditworthiness' driver. Analogously as in the driving factor case, with the exception of Model 1 following Vasicek's approach using normal marginals, the fitted distribution for each of the issuer's series are Student-t univariate distributions with the degrees of freedom estimated.

  It makes sense since all data sources are financial series, hence, when increasing the data frequency which distances it from normality in most of the cases – see Figure 3.27 (corporate issuers) and Figure 3.28 (sovereign). When the degrees of freedom increases above 30-40 it may be supposed as normality (also delimited in the figures mentioned).

- Bivariate copula

  Once the marginals distributions are estimated, next step is the copula estimation. Since our credit risk approach is modelled by a factor copula model, we use the bivariate copula. More precisely, the conditional bivariate copula as shown in subsection 2.1.2 ,conditional copulas and factor models.

  At this point, it has to be brought up that following our model concerns, the bivariate copula estimation changes over the model proposals (Gaussian, Student-t and Clayton). Therefore, the estimates of the copulas will output the maximum-likelihood value reached by the optimization plus the correspondent parameters.

Even in the Gaussian copula where there is no need to estimate the parameter (correlation's parameter), it will be used the estimator.

Maximum-likelihood estimator. According to the estimation process of the copulas, we use the second strategy based on two-stage estimation method instead of the first one, canonical maximum likelihood (ML). This strategy is named inference function for margins (IFM) and consists of two steps[3] :

1. Estimating the parameters of each univariate density function (marginal density) using the maximum likelihood method

$$\underset{\alpha_i}{Max} \sum_{t=1}^{T} ln\ f_i(x_{i,t}; \alpha_i) \tag{3.1}$$

   where α_i are the degrees of freedom estimate of the Student-t univariate distribution case.

2. Estimating the parameters of the copula solving the optimization problem, conditioned to the univariate parameter's estimates

$$\underset{\theta}{Max} \sum_{t=1}^{T} ln\ c(F_1(x_{1,t}; \hat{\alpha}_1), F_2(x_{2,t}; \hat{\alpha}_2); \theta) \tag{3.2}$$

   where θ contains a scalar or vector of copula parameters and $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are the estimates in the previous step.

3.4 Simulation and thresholds

In the aftermath of the factor copula estimation it follows the simulation process of the asset values returns. In this regard, it should take into account as many different simulation processes and thresholds sorting as models defined in subsection 2.2.4. 100,000 simulations have been carried for every bond scenario to consequently obtain a vector of 100,000 P&L.

Again, we recall the factor models (see 1.3.2 Factor model. Application of Vasicek) together with the conditional copulas presented in subsection 2.1.2. Taking both approaches, we could construct factor copula models by conditioning the simulated return to the explainer factor. As a result, it forms a bivariate copula where the simulation draws returns conditioned to the factor at the same time. Both marginals, the issuer's asset returns and the driving factor are required, as well as the one of the idiosyncratic risk that is not a marginal for the copula input per se.

[3]As the maximum likelihood is applied to the density function of the bivariate copula (2.8), once taken the logarithm it is shown the possible maximization through the two addend.

Model 1

Formula used:

$$r_i^{gauss} = \rho_i \cdot F + \sqrt{1-\rho_i^2} \cdot \varepsilon_i$$

where in this particular case $r_i \sim N(0,1)$, $F = N^{-1}(seed_1)$, $\epsilon_i = N^{-1}(seed_2)$[4] and ρ_i is the pair-wise dependence between the factor and the issuer.

The thresholds obtaining may be derived from two methods: considering the probabilities in the TPMs as unconditionally or transform them to conditionally independent.

Unconditionally. If it is considered the approach applied by Gupton and Fingers (2007)[31], Van der Stel (2010)[53] and Forsman (2012)[24] the thresholds are calculated by simply applying the univariate inverse distribution to the probabilities in the entries of the TPMs. It should be the respective through which the asset's returns of every issuer are distributed.

Since this Model 1 is the basis model of Vasicek's approach, the thresholds have already been expressed in subsection 1.3.3.

Conditionally. On the other hand, several research works have utilized the conditionally independent probabilities when considering the common factor fixed. See Vasicek(2002)[54], Mosconi (2015)[42], Martin *et al*(2011)[39] or Zhang and Jiao(2012)[58]. The main purpose of this master's thesis does not rely on take the common factor as fixed. Even though, we will develop an analysis taking advantage of such derivation in order to differentiate how the IRC value could vary depending on the economic cycle.

In such context, the conditional thresholds are obtained as it follows[5]:

$$\begin{aligned}
pr_{CR_j \to DEF|F} &= pr_{CR_j < DEF|F} = pr_{DEF|F} \\
&= P[V_t < V_{DEF}] = P[r_i < N^{-1}(pr_{DEF})|F] \\
&= P[\rho_i \cdot F + \sqrt{1-\rho_i^2} \cdot \varepsilon_i < Z_{DEF}|F] \\
&= P\left[\varepsilon_i < \frac{Z_{DEF} - \rho_i \cdot F}{\sqrt{1-\rho_i^2}} |F\right] \\
&= N\left(\frac{Z_{DEF} - \rho_i \cdot F}{\sqrt{1-\rho_i^2}}\right) \\
&= N\left(Z_{DEF|F}\right) \\
\mathbf{Z_{DEF|F}} &= N^{-1}(pr_{DEF|F})
\end{aligned}$$

[4] To reduce the variability of the Monte Carlo simulation, we use the same seed in all the models. One vector of uniform variables for the factor –$seed_1$– and a matrix of them –$seed_2$– for the idiosyncratic risk.

[5] It is only given an example in the default's case. By accumulating the probabilities from DEF to AAA can be derived all seven thresholds.

It can be simply and equally as:

$$\mathbf{Z_{DEF|F}} \quad = \frac{N^{-1}(pr_{DEF}) - \rho \cdot F}{\sqrt{1-\rho^2}} \tag{3.3}$$

Model 2

The general formula results

$$r_i^{stud-t} = \rho_i \cdot F + \sqrt{(1-\rho_i^2)\frac{\nu_i + F^2}{\nu_i + 1}}\varepsilon_i$$

but in order to get the simulated values according to the univariate Student-t marginals estimated, we have made used of the next formula which is the same as the equation (2.12), but in this case with the marginals sought according to the univariate estimation

$$r_i^{stud-t} = T_{\nu_{r_i}}^{-1}\left[T_{\nu_i}\left(\rho_i T_{\nu_i}^{-1}(T_{\nu_F}(F)) + \sqrt{(1-\rho_i^2)\frac{\nu_i + T_{v_i}^{-1}(T_{\nu_F}(F))^2}{\nu_i+1}}\varepsilon_i\right)\right] \tag{3.4}$$

where

$F \sim \mathrm{T}(\nu_F)$ estimated

$r_i \sim T(\nu_{r_i})$ fitted and estimated for each issuer's returns,

ν_i the degrees of freedom estimates from the i-*th* Student-t bivariate copula [6], and

$\varepsilon_i = T_{\nu_i+1}^{-1}(seed_2)$.

Consistently, the thresholds are derived as follows for those either unconditionally or conditionally independent.

Unconditionally. From the point that r_i is Student-t distributed with ν_{r_i} degrees of freedom, we obtain the thresholds by applying the respective Student-t distribution with its correspondent degrees of freedom to the given probabilities.

For instance, in the case of default:

$$\begin{aligned} pr_{CR_j \to DEF} &= pr_{CR_j < DEF} = pr_{DEF} \\ &= P[V_t < V_{DEF}] = P[r_i < T_{\nu_{r_i}}^{-1}(pr_{DEF})] \\ &= P[r_i < Z_{DEF}] \\ &= T_{\nu_{r_i}}(Z_{DEF}) \\ \mathbf{Z_{DEF}} &= T_{\nu_{r_i}}^{-1}(pr_{DEF}) \end{aligned}$$

[6]Point out that $\nu_{r_i} \neq \nu_i$, since the first one is the univariate serie estimation of the issuer's return and the second is the degrees of freedom estimated in each Student-t bivariate copula.

Conditionally. Following the explanation of the Model 1, it can be obtained the pertinent conditional thresholds given a fixed factor.

The derivation in this case and model is such as:

$$\begin{aligned}
pr_{CR_j \to DEF|F} &= pr_{CR_j < DEF|F} = pr_{DEF|F} \\
&= P[V_t < V_{DEF}] = P[r_i < T^{-1}_{\nu_{r_i}}(pr_{DEF})|F] \\
&= P\left[\rho_i \cdot F + \sqrt{(1-\rho_i^2)\frac{\nu_i + F^2}{\nu_i + 1}}\varepsilon_i < Z_{DEF}|F\right] \\
&= P\left[\varepsilon_i < \frac{Z_{DEF} - \rho_i \cdot F}{\sqrt{(1-\rho_i^2)\frac{\nu_i + F^2}{\nu_i + 1}}}|F\right] \\
&= T_{\nu_i+1}\left(\frac{Z_{DEF} - \rho_i \cdot F}{\sqrt{(1-\rho_i^2)\frac{\nu_i + F^2}{\nu_i + 1}}}\right) \\
&= T_{\nu_i+1}\left(Z_{DEF|F}\right)
\end{aligned} \tag{3.5}$$

$$\mathbf{Z_{DEF|F}} = T^{-1}_{\nu_i+1}(pr_{DEF|F}) = \frac{Z_{DEF} - \rho_i \cdot F}{\sqrt{(1-\rho_i^2)\frac{\nu_i + F^2}{\nu_i + 1}}} \tag{3.6}$$

Model 3

This model implements the proposal made by the ECB of imposing eight degrees of freedom in the Student-t copula. As the simulation and thresholds are analogously to the previous model, only one consideration has to be metioned according to the bivariate copula parameter.

Thus, we will follow the previous formulas yet with $\nu_i = 8$ for every copula simulated, hence, $\varepsilon_i = T^{-1}_{\nu_i+1}(seed_2)$ will also change.

Model 4

$$r_i^{clayton} = T^{-1}_{\nu_{r_i}}\left[\left(1 + T_{\nu_F}(F)^{-\alpha_i}(q^{-\alpha_i/(1+\alpha_i)} - 1)\right)^{-1/\alpha_i}\right] \tag{3.7}$$

where $q = T_{\nu_i+1}(\varepsilon_i)$ and α_i is the only parameter estimate in any bivariate Clayton copula.

In this instance, the non-linear dependence between the common factor and each issuer's returns is held by that parameter. Indeed, from the point that the Clayton copula catches up with the left-asymmetry among variables, α_i is the decisive parameter in the left-tailed dependency that equals to $\lambda = 2^{-1/\alpha_i}$ when it is greater than zero.

Thresholds derivation.

Unconditionally. Although this model put into practice a bivariate Clayton copula, we have still imposing the marginals distribution that we had estimated in such section. Due to from the Model 3 onwards the marginals distribution are univariate Student-t, the unconditional thresholds case follow accurately the formula (3.6).

Conditionally. The Clayton's copula thresholds derivation are carefully calculated following the next steps: first the conditional probabilities are generated, and then, its respective thresholds. As any of the previous demonstrations.

$$\begin{aligned}
pr_{CR_j \to DEF|F} &= pr_{CR_j < DEF|F} = pr_{DEF|F} \\
&= P[V_t < V_{DEF}] = P[r_i < T^{-1}_{\nu_{r_i}}(pr_{DEF})|F] \\
&= P\left[T^{-1}_{r_i}\left[\left(1 + T_{\nu_F}(F)^{-\alpha_i}(q^{-\alpha_i/(1+\alpha_i)} - 1)\right)^{-1/\alpha_i}\right] < Z_{DEF}|F\right] \\
&= P\left[\left(1 + T_{\nu_F}(F)^{-\alpha_i}\left(q^{-\alpha_i/(1+\alpha_i)} - 1\right)\right)^{-1/\alpha_i} < T_{\nu_{r_i}}(Z_{DEF})|F\right] \\
&= P\left[1 + T_{\nu_F}(F)^{-\alpha_i}\left(q^{-\alpha_i/(1+\alpha_i)} - 1\right) < T_{\nu_{r_i}}\left(Z_{DEF}\right)^{-\alpha_i}|F\right] \\
&= P\left[T_{\nu_F}(F)^{-\alpha_i}\left(q^{-\alpha_i/(1+\alpha_i)} - 1\right) < T_{\nu_{r_i}}\left(Z_{DEF}\right)^{-\alpha_i} - 1|F\right] \\
&= P\left[q^{-\alpha_i/(1+\alpha_i)} - 1 < \left[T_{\nu_{r_i}}\left(Z_{DEF}\right)^{-\alpha_i} - 1\right] \cdot T_{\nu_F}\left(F\right)^{\alpha_i}|F\right] \\
&= P\left[q^{-\alpha_i/(1+\alpha_i)} < \left[T_{\nu_{r_i}}\left(Z_{DEF}\right)^{-\alpha_i} - 1\right] \cdot T_{\nu_F}\left(F\right)^{\alpha_i} + 1|F\right] \\
&= P\left[T_{\nu_i+1}(\varepsilon_i) < \left(\left[T_{\nu_{r_i}}\left(Z_{DEF}\right)^{-\alpha_i} - 1\right] \cdot T_{\nu_F}\left(F\right)^{\alpha_i} + 1\right)^{-(1+\alpha_i)/\alpha_i}|F\right] \\
&= P\left[\varepsilon_i < T^{-1}_{\nu_i+1}\left(\left(\left[T_{\nu_{r_i}}\left(Z_{DEF}\right)^{-\alpha_i} - 1\right] \cdot T_{\nu_F}\left(F\right)^{\alpha_i} + 1\right)^{-(1+\alpha_i)/\alpha_i}\right)|F\right] \\
&= T_{\nu_i+1}\left(T^{-1}_{\nu_i+1}\left(\left(\left[T_{\nu_{r_i}}\left(Z_{DEF}\right)^{-\alpha_i} - 1\right] \cdot T_{\nu_F}\left(F\right)^{\alpha_i} + 1\right)^{-(1+\alpha_i)/\alpha_i}\right)\right) \\
&= T_{\nu_i+1}(Z_{DEF|F})
\end{aligned} \tag{3.8}$$

$$\mathbf{Z_{DEF|F}} = T^{-1}_{\nu_i+1}(pr_{DEF|F}) = T^{-1}_{\nu_i+1}\left(\left(\left[T_{\nu_{r_i}}\left(Z_{DEF}\right)^{-\alpha_i} - 1\right] \cdot T_{\nu_F}\left(F\right)^{\alpha_i} + 1\right)^{-(1+\alpha_i)/\alpha_i}\right) \tag{3.9}$$

3.5 Results

3.5.1 Statistical adequacy to data

Prior to show the results of our model proposals, we want to analyse the statistical adequacy to our data ir order to ascertain which best fits. Though, on the back pages are attached all the results with no discrimination of adequacy. It is simply to make a study on which features could explain better the considered data.

Driving factor choice

Given that in our model proposals there is a unique driving factor, the importance falls on its selection. To measure the ability to explain the complete range of variables selected (i.e. the credit process of corporate and sovereign issuers) we have calculated the pair-wise dependence between the respective factor (recall from subsection 2.1.1 our two considerations) and each of the issuer's data series driving the credit process.

Hence, the interest lies into compare the dependency parameters that both Gaussian and Student-t bivariate copula extracts while maximizing the log-likelihood. In the Copula Clayton cases, the parameter only focus on the left-tailed positive dependence in accordance to its properties. Such left-dependence parameter is obtained through the unique alpha parameter that the algorithm optimizes[7].

In the tables below, we have displayed through a stacked bar graphs the dependency parameters both in relation to the Euro Stoxx and PCA factor, in orange and blue, respectively. Moreover, the mean among all pair-wise dependencies in order to show which factor better fits with the data on average.

[7] $\lambda_{r_i} = 2^{-1/\alpha_i}$ if $\alpha_i > 0$ and $\lambda_{r_i} = 0$ otherwise

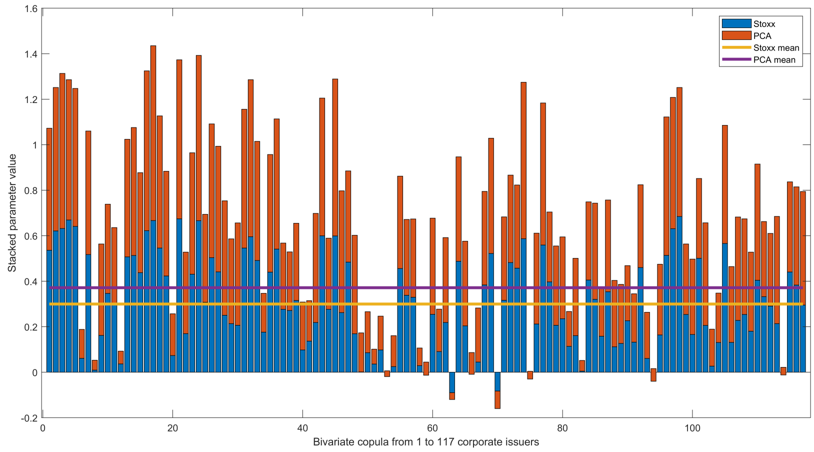

Figure 3.1: Dependency parameter from Gaussian copula between factors and corporate issuer's data

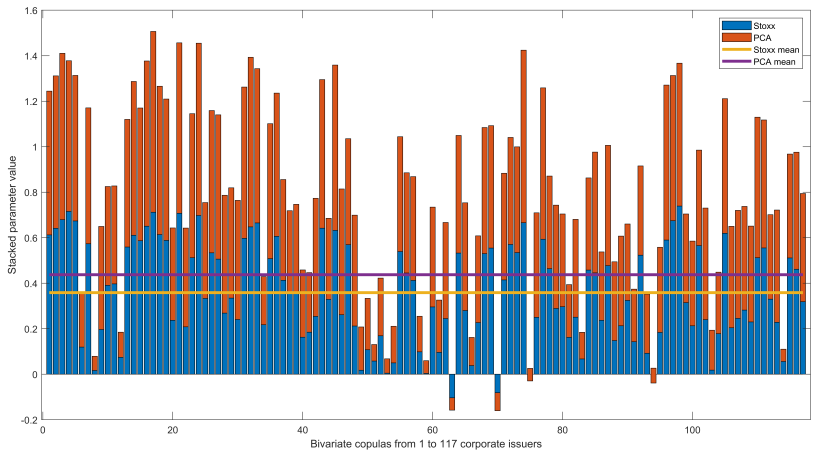

Figure 3.2: Dependency parameter from Student-t copula between factors and corporate issuer's data

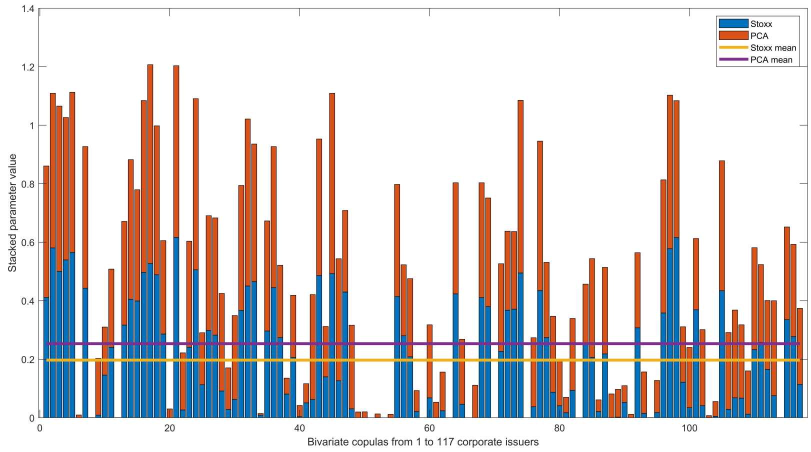

Figure 3.3: Left-tailed dependency parameter from Clayton copula between factors and corporate issuer's data

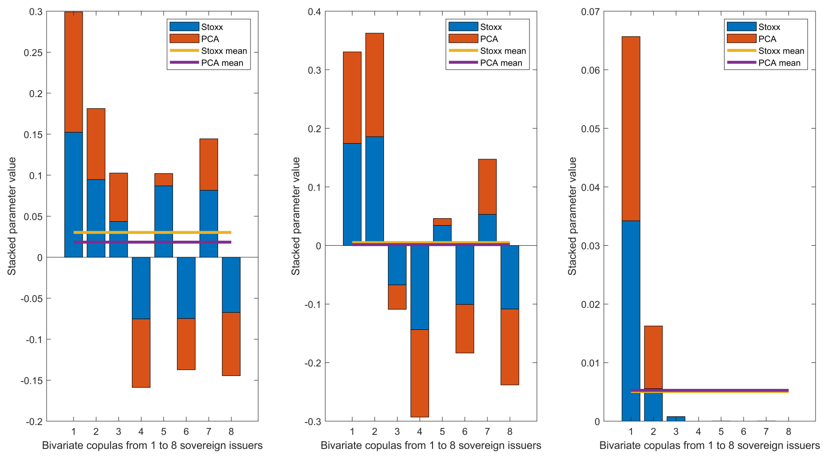

Figure 3.4: Dependency parameter from Gaussian, Student-t and Clayton between factors and sovereign issuer's data

As it can be seen, the PCA from the equities national indices plus the two European debt indices could explain better the corporate credit process. On the other hand, when we compare the dependency of the two factors in relation to the sovereign data, the capacity to better explain such processes is not that clear. Even though, when we put aside the Gaussian copula to move towards the Clayton copula, again the PCA gains explanatory ability in detriment of the Euro Stoxx factor.

Therefore, we believe that an IRC model proposal based on one-factor approach has more accuracy if it is employed a PCA for the factor.

Copula selection

It is important to remember that the simulation process (section 3.4) is a crucial step within the IRC model. In accordance, an estimation process of the data has to be carried out as explained in section 3.3 (bivariate copula). A natural way of comparing copulas is through the maximum value reached by the maximum-likelihood estimator. Taking also the number of parameters input, there exist two possibilities to make a comparison among each pair-wise data: the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC).

$$\begin{aligned} AIC &= 2k - 2\widehat{lnL} \\ BIC &= T^{-1}(k \cdot lnT - \widehat{lnL}) \end{aligned} \tag{3.10}$$

where k are the number of parameters and T the number of observations.

As stated in Novales (2017)[44], the lower value of the information criteria, the higher copula adequacy.

Following the higher adequacy values of the PCA factor, the copula comparison is based on the information criteria of the bivariate copula between issuer's data and the PCA factor uniquely. In the next tables, we attached three stacked bar graphs with the AIC and BIC for each bivariate copula. The first two for corporate data while the third one for sovereign. The x-axis indicates the

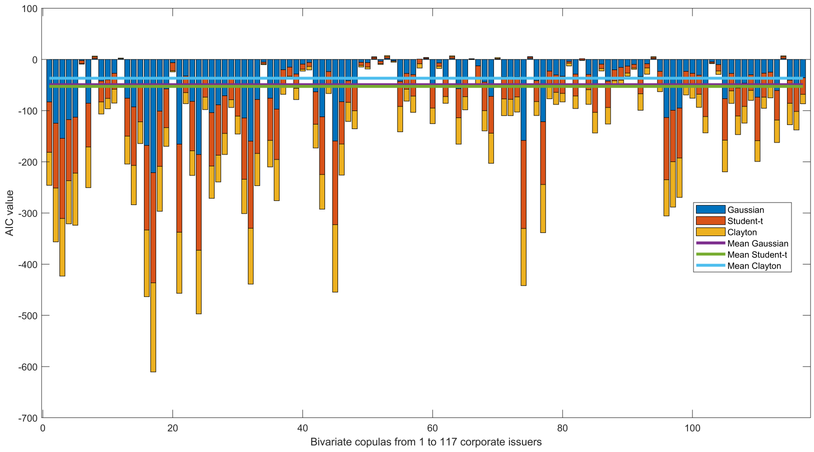

Figure 3.5: AIC of the bivariate corporate-PCA copulas

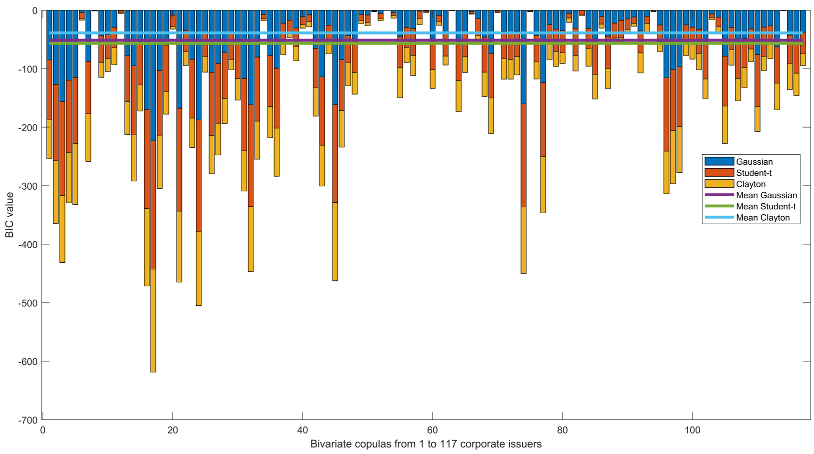

Figure 3.6: BIC of the bivariate corporate-PCA copulas

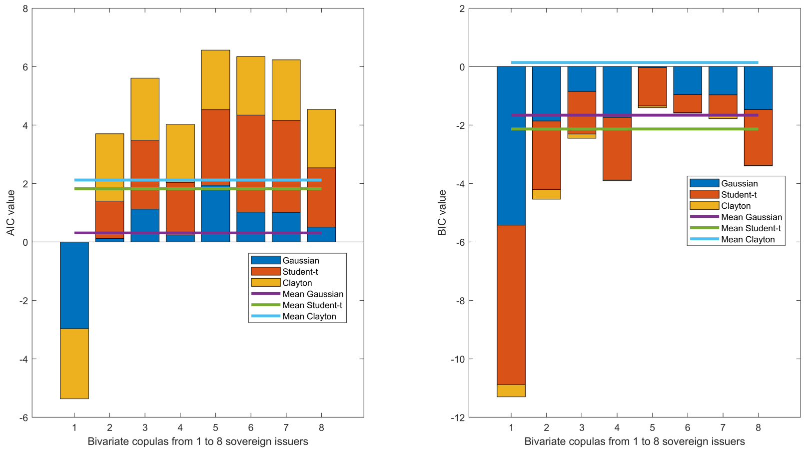

Figure 3.7: AIC and BIC of the bivariate sovereign-PCA copulas

In accordance with the results, several considerations can be made. While in the corporate-PCA copulas the conclusion is quite clear, on the sovereign side there is controversy once again.

From the corporate-PCA it seems obvious that the Clayton copula engages the worst. If we look through the figure 3.5 (AIC), the Student-t copula fits better in 78 cases out 117 over the three others. In the case of the bayesian criteria (figure 3.6) the fraction increases to 106 for the Student-t copula again. So within this pair-wise relation, it seems clear that the Student-t bivariate copula better explains the relation between corporate data and the factor.

On the sovereign-PCA side, the clearest conclusion is that the Clayton copula does not explain well its relation either. The AIC and BIC extract conflicting values: the Gaussian copula gets the lower AIC value in all pair-wise relations, while the Student-t in 6 out 8 according to the BIC. Only for the first issuer conditioned to the PCA copula (i.e. Germany) seems to fit well a conditional copula to the PCA. If we only consider the BIC, the values would be closer to the corporate-PCA case and the conclusion will be that the Student-t copula explain better their dependence than the others, followed by the Gaussian and finally the Clayton copula.

The last consideration is in relation to the Clayton copula. It has the worst ability to explain the pair-wise relation in all the cases contra prognosis (considering we are within a credit risk model). This is due to the type of data by which the model is based on to drive the credit issuer's process and the factor. Such data is mainly equity prices (subsection 2.1.3) because of the lack of default data. Besides that reason, the limit of historical data extraction forced us to select the data sample of an stable period.

If it had been data availability since 2007, the Clayton copula might have explained better the dependence. Moreover, we also could have observed this fit if we had used credit data in our model, specially because our portfolios are based on long positions only. It would mean that it does not take advantage of the opportunity to partially offset losses due to downgrades or default by gains from upgrades on other bonds. Such tail events may be avoided through semi-active approach whereby large positions are combined with short positions. Even though, the probability of a sudden default or downgrade remains and the left-skewed distribution as well –see ECB (2007)[20]–.

Without this two limitations, and as it was displayed through figure 2.1, we had demonstrated the higher left-tailed dependence due to a credit instrument returns and the estimation of the model within a turmoil period.

Otherwise, the estimation is one step among several other. It does not mean that Clayton copula is not valid to simulate the factor copula model in order to correlate the default events among issuers within our portfolios.

3.5.2 Factor copula model analysis

The main objective of this master's thesis is to propose several factor copula models and their implementation according to the IRC obtaining as was described in subsection 2.2.4. Consequently, the comparison among such factor copula models is the most important part.

Within this subsection, we show the most remarkable aspects to take into account derived from the use of different copulas among the factor models. It is also needed to point out that the following results concerns the factor copula models based on PCA factor, stochastic RRs and unconditional thresholds. Moreover, in order to simplify the results, the following figures are from the portfolio 1 (Figure 3.8, 3.9 and 3.10), the portfolio 2 (Figure 3.11) and both of them (Figure 3.12). To check the entire range of results see the annexed tables at the appendix C (Table 3.12).

In the following two graphs it can be demonstrate the use of all copula range, since the basis factor model of Vasicek (Model 1) until the factor model using a Clayton copula (Model 4). To display the key aspects of these models, we show the histogram of the distribution of the $P\&L$ scenarios (Portfolio 1) through two graphs, the first one focused on the left tail (scenarios lower than the percentile 0.1%) and the second one on the right tail (scenarios upper than the percetile 50%).

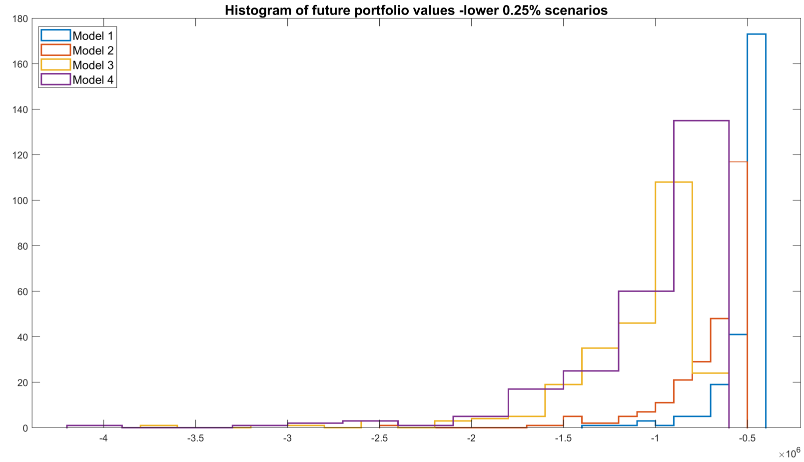

Figure 3.8: Histogram of the $P\&L$ scenarios lower than the 0.25% percentile

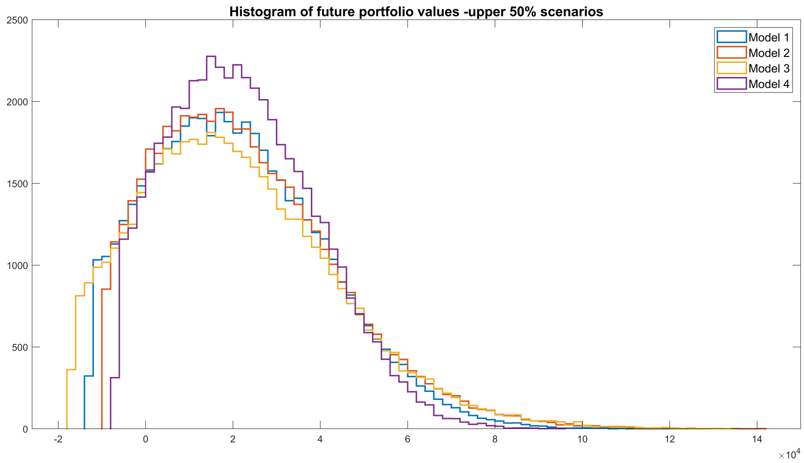

Figure 3.9: Histogram of the $P\&L$ scenarios upper than the 50% percentile

In the previous figure 3.8 can be demonstrate that the Clayton-based model (Model 4) has the most lower values of the future portfolio changes but the least higher as well (observed in figure 3.9). It would be in accordance with the

The Student-t-based model with the parameters proposed by the ECB (Model 3) would result the next in extracting lower values of the future portfolio P&L (both in the left and right tail) then followed by our estimated Student-t-based model (Model 2). Finally, the basis model (Model 1) would perform the most P&L values grouped around the middle percentiles.

Through the next bar graph (figure 3.10) we would also demonstrate the effect of the different factor copula models selection. Based on Portfolio 1 results, it can be checked that Model 1 requires the lowest IRC from the 4.88% of the portfolio value until the highest 10.61%, more than doubled.

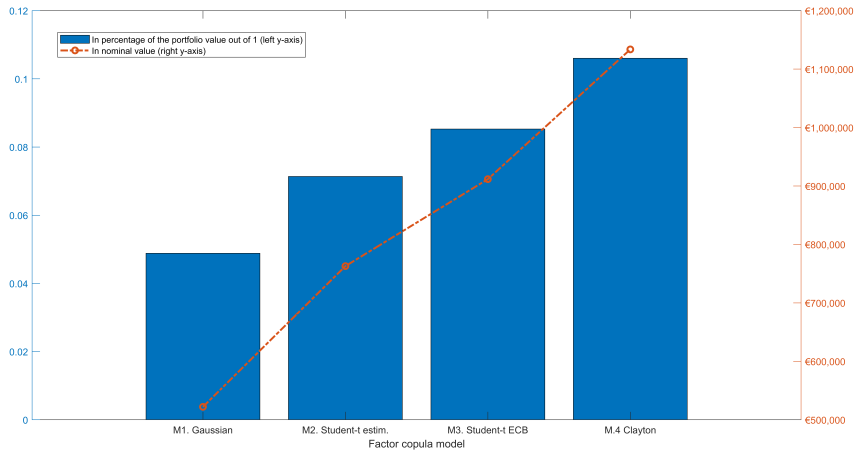

Figure 3.10: IRC values depending on the factor copula model (Portfolio 1 - PCA factor)

The next figure 3.11 is similar than the previous one, but here the results are obtained from the Portfolio 2 where all the positions held are sovereign issuers. It calls the attention that almost in every factor copula model the IRC percentage is the same, in between the 3.62% (Model 1) and the 4.07% (Model 3). So the copula assumption at this point is not such relevant.

It is due to sovereign issuers are unlikely to default with the exception of an initial BB-rated or lower –see the table 3.6 of the TPMs of sovereign issuers–. Besides, the considered initial rating of sovereign issuers are BBB or above (i.e. zero probability of default). By not having a probability of default, it means that although there are extreme scenarios simulated, these will not be classified as such. Therefore, the factor copula models do not vary their IRC requirements to a large extent.

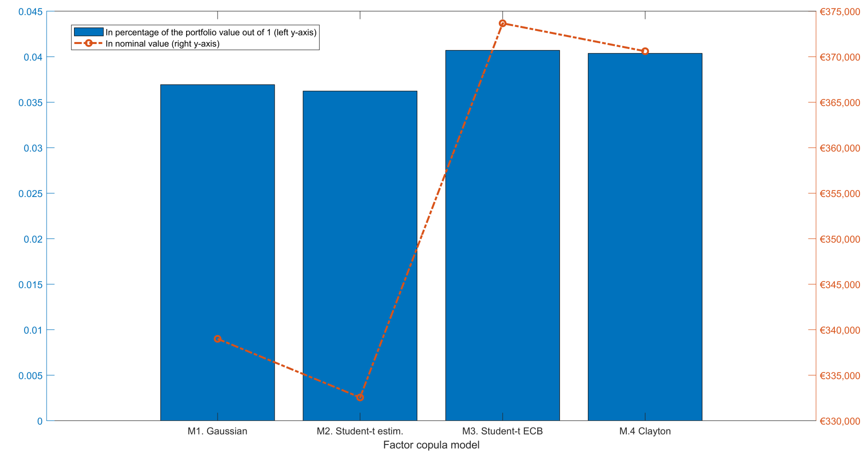

Figure 3.11: IRC values depending on the factor copula model (Portfolio 2 - PCA factor)

To finish with this section, we display in the next figure a graphical measurement of the distance performed by the IRC based on a Expected Shortfall over the IRC based on a Value-at-Risk. The middle dots crossed by the sticks indicate the IRC (VaR) while the sticks' endpoint note the IRC (VaR) plus and less the difference from the IRC (ES). Even though we are also interested in the upper endpoint, this MATLAB graph help us out to show relatively how is increasing the distance between both measures while putting the Gaussian model aside.

In other words, it is just demonstrating us the effectiveness of the factor copula model when going further than the basis model of Vasicek (M1. Gaussian). The distance rises from 173 basis points (bp), 262 bp, 356 bp till 608 bp in the M4. Clayton (Portfolio 1 + PCA). Once again and in accordance with the above-mentioned, the sovereign-based portfolio (Portfolio 2) does not show any remarkable difference so the distance does not surpass 62bp in any factor copula model (nor in any factor used).

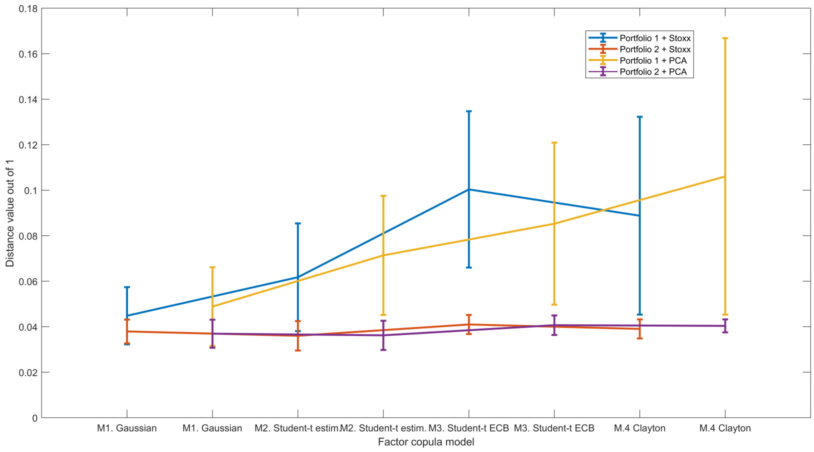

Figure 3.12: Distance of ES measure over VaR

3.5.3 Portfolio creditworthiness comparison

The next study we have carried out is how the IRC requirement could vary considering different concentration of credit ratings within the portfolio. In the subsection 2.2.4. we presented a group of different formed portfolios. The reason lies in implement to them our four model proposals in order to verify if the capital required comply with the credit risks (i.e. rating concentration within the portfolio).

Hence, we test the IRC models to four different portfolios with different ratings concentration. If we set apart the Portfolio 1 and 2 (they only have one type of issuer each), Portfolio 3 (see figure 3.22) is the least likely to default since its 79% is composed by AAA, AA and A issuer's rated. Portfolio 4 –figure 3.23– increases the previous probabilities of default so here a 78% is concentrated in BBB debt. Finally, Portfolio 5 and 6 (see figure 3.24 and 3.25) are much likely to default since they have reached the non-investment grade (75% of BB and B bonds) and extremely speculative (74% in CCC) consideration, respectively.

To do that, we present in the next figure the comparison among portfolios but all of the values extracted from our estimated factor copula model (Model 2 Student-t), based on PCA factor with stochastic RRs and unconditional thresholds.

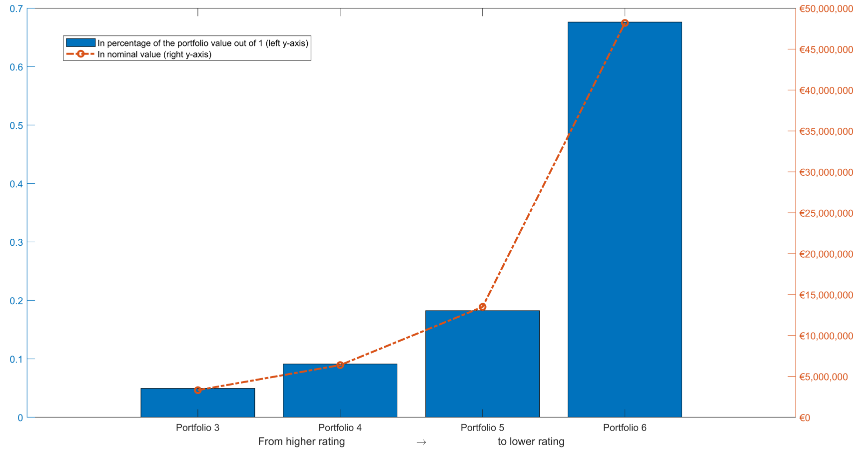

Figure 3.13: IRC comparison among portfolios (Model 3 - PCA factor)

Figure 3.13 justifies that the variation of the predominant credit rating in the portfolio leads to different capital requirements. We can see that the more risk the portfolio has, the more IRC requirement demands the model. In fact, such capital requirement increases exponentially till an approximately 64% of IRC when the 74% of the portfolio has high likelihood of default (27% for CCC).

3.5.4 Deterministic Recovery Rate assumption

There are three different manners of assuming the RRs in the credit risk models. The first one lets the RRs be fixed so it accepts a deterministic value for them as CreditRisk+ (Actuarial model) takes. Another assumes an stochastic process for them, yet uncorrelated with PDs (Creditmetrics), or also correlating the RRs with PDs as the last one does.

Rejecting the last one as explained in subsection 2.2.3, and taking the second as our basis RRs process, we have implemented a 40% deterministic RR following Brunac (2012)[13] in order to measure such impact in the IRC.

The IRC change is attached in the Table 3.13 of appendix C. Additional tables and figures. The results have shown that by applying a deterministic RR, as we leave aside the Gaussian model but still within a portfolio rated as investment grade (Portfolio 1 to 4), the minority of the defaulted scenarios affect increasing the IRC capital required with regard to stochastic set.

If we have a look at the two speculative portfolios (5 and 6), with a much higher percentage of scenarios defaulting, although on average the stochastic process of the RRs simulation will extract higher values than a 40% (check Table 3.11), the impact that those lower scenarios make is also higher. It means that as a result of the high standard deviation of the stochastic RRs, a default scenario extracts much lower P&L which finally ends up demanding more capital than in the deterministic case (i.e. negative percentages in the Table 3.13)

3.5.5 Basis point impact

One of the processes carried out by the ECB in order to do the validation of the internals models is to apply an up and down shift in the PDs. It quantitative impact on the IRC may, among others processes, determine if they are well specified or not.

To do that, we have implement such proposal in order to get a sensitivity analysis therefore a measurement of the model itself. Hence, a basis point (up and down) shift have been implemented in our corporate TPM (only in the default entry).

The sensitivity of our model to an increase of a basis point is attached in the Table 3.14. From 48 IRC calculations (summing all different portfolios and models), almost in all of them it has derived in a rise in the capital demand. The ones in the Model 4 (Clayton copula) that most reaching a 2.88% shift. In four or five of them, such impact is not so clear observed.

On the other hand, when shifting down a basis point (Table 3.15), the impact is almost the same as the previous.

3.5.6 Conditional thresholds

Following the approach that several researchers had also carried out, we implement the thresholds conditional to the factor state as described in the section 3.4. Only in this subsection has been developed the conditional thresholds to apply into our models. The portfolio which we have selected to show the comparison is the Portfolio 4, so it seems quite close to the one that a bank institution could held. The IRC values of applying conditional thresholds are shown in table 3.14 (Appendix C).

The conditionally thresholds obtaining that we have put into effect is carried out by conditioning the thresholds derivation to the factor in two opposite cases. The first one, when our factor is at its low percentile (i.e. 10th percentile) and the second, at its high percentile (i.e. 90th). The reason of doing this analysis is simply to state how the IRC could change depending on the economic situation, accordingly, through a period of turmoil or growth (low and high percentiles, respectively).

Taking advantage of the statistical information of the RRs given by Altman and Kalotay (2014)[1] we have applied the stochastic RRs values in its 10% and 90% percentile to our model as well (it can be checked in the Table 3.9).

In order to demonstrate the differences, the next bar graph displays the values of the conditional IRC taking the unconditional IRC as 100. Only the models conditioned to the PCA factor are analysed.

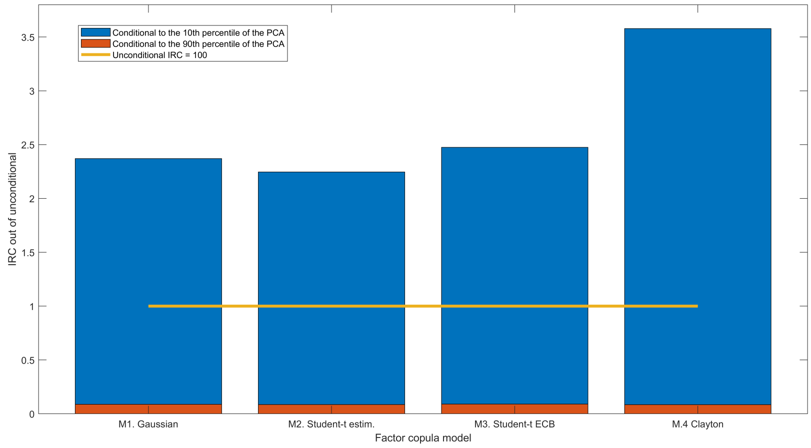

Figure 3.14: IRC values obtained from the conditional thresholds (Portfolio 4 - PCA factor)

The IRC values conditional to a turmoil economic situation are on average 2.5 times over the unconditional values appart from in the Model 4 where it reaches up to 3.5. On the other hand, when the situation is within a growth period, the conditional IRC decreases until an approximately tenth of the unconditional values.

Far from being applicable an IRC requirement as the obtained when the PCA factor is at its 90th percentile (so it results hugely short), we believe that one interest in this approach could rely on imposing a minimum IRC requirement when the factor is at its lower percentiles. For instance, imposing a minimum of IRC capital condional to a factor at its lower percentiles calibrated in the prior crisis of 2007 in order to avoid the losses that took place at that time.

Conclusion

One of the main reason why the Committee led to the incorporation of the IRC through the market risk framework was that *Basel II* had manifested a weakness by utilizing only a VaR based measurement. As it was described, in 2009 finally took effect its demand and the IRC began to be applied. But, it is quite likely that such models were based on a basis model with a Gaussian copula approach since two years ago was the time when the ECB disclosed its proposal of computing a Student-t copula. Through this research, it is demonstrated that such approach based in a Gaussian copula could hardly keep capital to cope the losses occasioned even when most of the economic players were being affect by the turmoil at the same time and with much difficulties to pay their financial obligations.

By implementing the ECB proposal, it may be noticed that the institution have made both a wise and a conservative suggestion so it has resulted the most stable capital demand through all our portfolio testing. Even though, our Student-t estimation model results a sensible proposal. It is principally estimate from stock data and it could also lead to a higher requirements than from default data, meaning a successful outcome in terms of the regulation interests. It meets with the regulation's requirements besides it results a reduction of the needed capital by a bank institution if compared with the ECB proposal.

The further application of a Clayton copula is the outcome of the robustness sought by the credit risk evidence. In accordance with the rationality of the economic players and their risk-aversion, they tend to go together in extreme economic situation. The peculiarity is that in turmoil periods, such dependence that we have tried to model is enormously high, even more demonstrated in credit instruments than in others. It is the main reason of our Model 4 proposal that in concordance demands more capital.

As a result, it is quite proved that the capital requirement increases in order when a basis model of a Gaussian, our Student-t estimate, then the Student-t ECB proposal and finally a Clayton is implemented. It is also true that this ordering does not always hold so when the model is implemented in a highly speculative portfolio, its IRC stands stable around a 66-67% of its value with whatever model used. On the other investment side, we can test that in a highly conservative portfolio with only sovereign debt, the IRC does not discriminate with any copula.

With our simple but useful validation process through the basis point shift application, it may be seen that the model does not run as expected in the entire range of portfolios and factor copula models, but it is also a vast minority. We impute this to the only one factor that we have considered to our model. Such flaw was assumed since other researchers as Pykhtin tackled it in order to cover the sector concentration (imperfect diversification across systematic components of risk). Nevertheless, we focused this work on the side of the contagion risk across obligors.

To finish, we want to expose some further work which still present in the master's thesis if it is interested to apply more accurately the model or to a wider range of financial instruments. In accordance with the dependence estimation, as it has already been stated, it may be obtained from default data if there is availability. Moreover, if such availability stands, the TPMs can also be obtained internally by following the methods presented in the subsection 2.2.1. In relation to the liquidity horizon, it could be reduced to a quarter of year so it would be needed to consider the rebalancing of the position in order to hold constant the risk in the portfolio during the capital horizon. This assumption only has to meet with the minimum of three months required.

In addition to that, if a portfolio of derivatives is formed meeting also the scope of application (section 1.2), the IRC process calculation will require a further study concerning a stochastic process which defines the "moneyness" of the instrument and its correspondent credit risk inherent within the capital horizon.

Through the histograms of the distribution of the P&L, it have been noticed that there are few mode values under certain circumstances. We believe that such statistical properties are nothing but the result of the high percentages of standing in its correspondent initial credit rating (with the exception of the CCC where it decreases) for an issuer[8]. Moreover, it is specially exhibited in the implementation of the Model 1 and 2 (See both histograms attached in the Figure 3.29 with the use of the Portfolio 4). It would mean that in both factor copula models, the scenarios's simulation gathers around its correspondent initial thresholds because of the Gaussian copula has not been left out too far yet strictly in all the Student-t estimated bivariate copulas (check the degrees of freedom in Figure 3.27 and 3.28). It could set basis of a deeper study of the IRC through the mixture of normals methodology.

The IRC will keep being implemented till the DRC (*Basel IV*) takes effect. Although this master's thesis is focused on the IRC, the upcoming DRC methodology also shares a factor copula approach. Hence, this work could result profitable in order to apply other factor copula models beyond the Gaussian approach that fits better with the reality of the markets.

[8]see TPMs in Table 3.7 and 3.8.

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APPENDIX

A. Vasicek derivation

We can express the equity return of the i-*th* obligor r_i with jointly standard normal varibles (F, ξ_i)

$$r_i = aF + b_i\xi_i \tag{3.11}$$

where F and $\xi_i, i = 1, ..., n,$ are mutually independent variables following standard normal distribution.

The derivation of coefficient a begins by examining the correlation of r_i and r_j:

$$\begin{aligned} corr(r_i, r_j) = \frac{cov(r_i, r_j)}{\sigma_F \sigma_{r_i}} &= \frac{E\left[(r_i - E[r_i])(r_j - E(r_j))\right]}{\sigma_F \sigma_{r_i}} \\ &= \frac{E[r_i r_j]}{1 \cdot 1} \\ &= E\left[(aF + b_i\xi_i)(aF + b_j\xi_j)\right] \\ &= a^2E[F^2] + ab_iE[F]E[r_i] \\ &+ ab_jE[F]E[r_j] + b_ib_jE[\xi_i]E[\xi_j] \\ &= a^2E[F^2] \\ &= a^2(Var[F] + E[F]^2) \\ &= a^2 \\ \Rightarrow a = \sqrt{corr(r_i, r_j)} &= \sqrt{\rho} \end{aligned} \tag{3.12}$$

The values for b_i may likewise be derived using the variance of r_i:

$$\begin{aligned} Var[r_i] &= E[r_i^2] - E[r_i]^2 \\ &= a^2E[F^2] + b_i^2E[\xi_i^2] + 2ab_iE[F]E[\xi_i] \\ &= a^2 + b_i^2 \\ \Rightarrow b_i &= \sqrt{Var[r_i - a^2]} = \sqrt{1-\rho} \end{aligned} \tag{3.13}$$

So variables r_i can be expressed in the form

$$r_i = \sqrt{\rho}F + \sqrt{1-\rho}\xi_i$$

B. Copulas extension

Gaussian copula

$$c(u_1, u_2; \rho) = \frac{1}{\sqrt{1-\rho}} exp\left\{ -\frac{\rho^2\Phi^{-1}(u_1)^2 - 2\rho\Phi^{-1}(u_1)\Phi^{-1}(u_2) + \rho^2\Phi^{-1}(u_2)^2}{2(1-\rho^2)} \right\} \tag{3.14}$$

Student-t copula

$$\begin{aligned} c(u_1, u_2; \nu, \rho) = {} & K \frac{1}{\sqrt{1-\rho^2}} \\ & \left[1 + \frac{T_\nu^{-1}(u_1)^2 - 2\rho T_\nu^{-1}(u_1) T_\nu^{-1}(u_2) + T_\nu^{-1}(u_2)^2}{2(1-\rho^2)} \right]^{-\frac{\nu+2}{2}} \\ & \left[(1 + \nu^{-1} T_\nu^{-1}(u_1)^2)(1 + \nu^{-1} T_\nu^{-1}(u_2)^2) \right]^{\frac{\nu+1}{2}} \end{aligned} \tag{3.15}$$

where

$$K = \Gamma\left(\frac{\nu}{2}\right) \ \Gamma\left(\frac{\nu+1}{2}\right)^{-2} \ \Gamma\left(\frac{\nu+2}{2}\right)$$

Clayton copula

$$c(u_1, u_2; \alpha) = (\alpha+1)\left(u_1^{-\alpha} + u_2^{-\alpha} - 1\right)^{-2-\frac{1}{\alpha}} (u_1 u_2)^{-\alpha-1} \tag{3.16}$$

C. Additional tables and figures

	Corporate													Sovereign
	Amsterdam Exchanges	Athex Composite Share Price	Austrian Traded	BEL 20	CAC 40 Index	FTSE Italia All-Share	Deutsche Boerse DAX	IBEX 35	ISEQ Overall Price	OMX Helsinki 25	OMX Stockholm 30	Euronext Lisbon PSI 20	All	1-year Bond Yield
	.AEX	.ATG	.ATX	.BFX	.FCHI	.FTITLMS	.GDAXI	.IBEX	.ISEQ	.OMXH25	.OMXS30	.PSI20		
Country														
AT - Austria	-	-	4	-	-	-	-	-	-	-	-	-	4	YES
BE - Belgium	-	-	-	4	-	-	-	-	-	-	-	-	4	YES
DE - Germany	-	-	-	-	-	-	17	-	-	-	-	-	17	YES
ES - Spain	-	-	-	-	-	-	-	21	-	-	-	-	21	YES
FI - Finland	-	-	-	-	-	-	-	-	-	8	1	-	9	NO
FR - France	-	-	-	-	22	-	-	-	-	-	-	-	22	YES
GR - Greece	-	4	-	-	-	-	-	-	-	-	-	-	4	NO
IE - Ireland	-	-	-	-	-	-	-	-	3	-	-	-	3	YES
IT - Italy	-	-	-	-	-	13	-	-	-	-	-	-	13	YES
LU - Luxemburg	1	-	-	-	-	-	-	-	-	-	-	-	1	NO
NL - Netherlands	14	-	-	-	-	1	-	-	-	-	-	-	15	NO
PT - Portugal	-	-	-	-	-	-	-	-	-	-	4	-	4	YES
TOTAL CORPORATE													**117**	
TOTAL SOVEREIGN														**8**
Industry														
Food	1	-	-	1	2	-	-	-	-	-	-	-	4	
Mining	-	-	-	-	-	-	-	-	-	1	-	-	1	
Oil	1	-	1	-	-	2	-	1	-	-	-	1	6	
Consumer durables	1	-	-	-	-	-	2	-	-	1	-	-	4	
Chemicals	1	-	-	-	1	-	1	-	-	-	-	-	3	
Drugs, soap	1	-	-	-	1	-	1	2	-	-	-	-	5	
Construction	-	-	1	-	1	-	-	3	-	-	-	-	5	
Steel	-	-	-	-	1	-	1	1	-	-	-	-	3	
Fabricated products	-	-	-	-	2	-	-	-	-	-	-	1	3	
Automotive	-	-	-	-	2	2	4	-	-	-	-	-	8	
Transport	-	-	-	-	-	1	-	-	1	-	-	-	2	
Utilities	-	-	-	1	2	1	1	2	-	1	-	1	9	
Retail	-	-	-	-	2	-	1	1	-	1	-	-	5	
Financial	5	3	2	1	5	6	4	6	2	2	1	-	37	
Other	2	1	-	1	4	2	4	5	-	2	-	1	22	
TOTAL													**117**	

Table 3.1: Corporate and sovereign issuer’s data

CODE	ISSUER	DOMICILE	CPN	FREQ	INDUSTRY	RATING	MAT YEAR
FR0012386688	ACCOR SA	FR (France)	1.679	Annually	Other	BBB	2022
XS1960353388	ACS	ES (Spain)	1.875	Annually	Construction and materials	BBB	2026
XS1529854793	AEGON NV	NL (Netherlands)	1	Annually	Financial	A	2023
FR0011344076	AIR LIQUIDE FINANCE SA	FR (France)	2.125	Annually	Utilities	A	2021
XS1128224703	AIRBUS SE	NL (Netherlands)	2.125	Annually	Other	A	2029
FI4000167176	AKTIA BANK ABP	FI (Finland)	2.5	Annually	Financial	A	2020
XS0809847667	AKZO NOBEL NV	NL (Netherlands)	2.625	Annually	Chemicals	BBB	2022
FI4000375241	ALANDSBANKEN ABP	FI (Finland)	0.125	Annually	Financial	AAA	2024
XS1921451040	ALLIANZ SE	DE (Germany)	1.413	Annually	Financial	AA	2028
XS1799975765	ALLIED IRISH BANKS PLC	IE (Ireland)	1.5	Annually	Financial	BBB	2023
XS1919894813	ALMIRALL SA	ES (Spain)	0.25	Semiannually	Drugs	BB	2021
XS1762980065	ALPHA BANK SA	GR (Greece)	2.5	Annually	Financial	B	2023
XS1501162876	AMADEUS IT GROUP SA	ES (Spain)	0.125	Annually	Other	BBB	2020
XS1244060486	AMRO BANK NV	NL (Netherlands)	0.75	Annually	Financial	A	2020
BE6285454482	AB INBEV NV	BE (Belgium)	1.5	Annually	Food	A	2025
XS1167308128	ARCELORMITTAL SA	ES (Spain)	3.125	Annually	Steel	BBB	2022
FR0011651389	ARKEMA SA	FR (France)	3.125	Annually	Chemicals	BBB	2023
XS1405774990	ASML HOLDING NV	NL (Netherlands)	0.625	Annually	Consumer Durables	A	2022
FR0012830685	ATOS SE	FR (France)	2.375	Annually	Other	BBB	2020
IT0005108490	AUTOSTRADE PER L'ITALIA SPA	IT (Italy)	1.625	Annually	Transport	BBB	2023
FR0011655612	AXA SA	FR (France)	2.625	Annually	Financial	A	2022
XS1533918584	AZIMUT HOLDING SPA	IT (Italy)	2	Annually	Financial	BBB	2022
XS1876076040	BANCO DE SABADELL SA	ES (Spain)	1.625	Annually	Financial	BBB	2024
ES0413900475	BANCO SANTANDER SA	ES (Spain)	0.13	Quarterly	Financial	BBB	2022
XS0867469305	BANK OF IRELAND	IE (Ireland)	10	Annually	Financial	BBB	2022
ES0413307119	BANKIA SA	ES (Spain)	0.875	Annually	Financial	BBB	2021
ES0413679350	BANKINTER SA	ES (Spain)	0.625	Annually	Financial	BBB	2020
XS0883560715	BASF SE	DE (Germany)	1.875	Annually	Chemicals	A	2021
DE000A2E4GF6	BAYER AG	DE (Germany)	0.05	Annually	Drugs	BBB	2020
XS1105276759	BMW AG	NL (Netherlands)	1.25	Annually	Automotive	A	2022
ES0413211121	BBVA	ES (Spain)	3.5	Annually	Financial	A	2021
FR0013078748	BNP PARIBAS SA	FR (France)	0.67	Annually	Financial	AAA	2023
FR0010379255	BOUYGUES SA	FR (France)	5.5	Annually	Fabricated products	BBB	2026
FR0124147135	CAISSE REG CREDIT AGRIC AQUITAINE SC	FR (France)	0.36	Annually	Financial	A	2021
XS1936805776	CAIXABANK SA	ES (Spain)	2	Annually	Financial	BBB	2024
FR0012821932	CAPGEMINI SE	FR (France)	1.75	Annually	Other	BBB	2020
XS0529414319	CARREFOUR SA	FR (France)	3.875	Annually	Retail	BBB	2021
FR0013260379	CASINO GUICHARD PERRACHON SA	FR (France)	1.865	Annually	Retail	BB	2022
XS1468525057	CELLNEX TELECOM SA	ES (Spain)	2.375	Annually	Other	BB	2024
XS0946179529	CITYCON OYJ	FI (Finland)	3.75	Annually	Retail	BBB	2020
XS1513765922	CODERE SA	ES (Spain)	6.75	Semiannually	Other	B	2021
DE000CZ40NS9	COMMERZBANK AG	DE (Germany)	1	Annually	Financial	A	2026
XS1881593971	COMPAGNIE DE S. G. SA	FR (France)	1.875	Annually	Fabricated products	BBB	2028
FR0011442979	COVIVIO SA	FR (France)	3.3	Annually	Construction and materials	BBB	2020
FR0010905133	CREDIT AGRICOLE SA	FR (France)	4.5	Quarterly	Financial	A	2020
DE000A169G07	DAIMLER AG	DE (Germany)	0.875	Annually	Automotive	A	2021
FR0010967216	DANONE SA	FR (France)	3.6	Annually	Food	BBB	2020
DE000DB2GTD0	DEUTSCHE BANK AG	DE (Germany)	0.65	Annually	Financial	BBB	2021
DE000A1RE1W1	DEUTSCHE BOERSE AG	DE (Germany)	2.375	Annually	Financial	AA	2022
XS0503603267	DEUTSCHE TELEKOM AG	DE (Germany)	4.875	Annually	Other	BBB	2025
XS1250867642	DEUTSCHE WOHNEN SE	DE (Germany)	1.375	Annually	Other	A	2020
XS1400342587	DIA SA	ES (Spain)	1	Annually	Retail	CCC	2021
FI4000312095	DNA OYJ	FI (Finland)	1.375	Annually	Other	BBB	2025
XS0388366097	E.ON SE	NL (Netherlands)	5.684	Annually	Utilities	BBB	2022
FR0011244367	EDENRED SA	FR (France)	3.75	Annually	Financial	BBB	2022
XS1222590488	EDP ENER. DE POR. SA	PT (Portugal)	2	Annually	Utilities	BBB	2025
FR0013213303	ELECTRICITE DE FR. SA	FR (France)	1.875	Annually	Utilities	A	2036
BE0002629104	ELIA SYSTEM OPERATOR SA	BE (Belgium)	1.375	Annually	Utilities	BBB	2026
XS1578886258	ELISA OYJ	FI (Finland)	0.875	Annually	Other	BBB	2024
XS0521000975	ENI SPA	IT (Italy)	4	Annually	Oil	A	2020
XS1647857264	ERG SPA	IT (Italy)	2.175	Annually	Utilities	BBB	2023
AT0000A1GMA0	ERSTE GROUP BANK AG	AT (Austria)	1.99	Annually	Financial	A	2025
XS1709545641	EUROBANK ERGASIAS SA	GR (Greece)	2.75	Annually	Financial	B	2020
XS0940284937	FERROVIAL SA	ES (Spain)	3.375	Annually	Construction and materials	BBB	2021
XS0124085951	FIAT CHRYSLER AUTO. NV	LU (Luxembourg)	4	Annually	Automotive	BB	2021
XS0629937409	FORTUM OYJ	FI (Finland)	4	Annually	Utilities	BBB	2021
XS1554373248	FRESENIUS SE & CO KGAA	DE (Germany)	1.5	Annually	Other	BBB	2024
XS1071419524	GALAPAGOS SA	LU (Luxembourg)	5.375	Semiannually	Drugs	CCC	2021
PTGGDAOE0001	GALP GAS NAT. DISTRIB. SA	PT (Portugal)	1.375	Annually	Oil	BBB	2023
GRC4191173B0	GREEK ORG. OF FOOTBALL PROG. SA	GR (Greece)	3.5	Semiannually	Other	BB	2022
XS1598757760	GRIFOLS SA	ES (Spain)	3.2	Semiannually	Drugs	BB	2025
XS0811554962	HEINEKEN NV	NL (Netherlands)	2.125	Annually	Food	BBB	2020
XS0350850177	IBERDROLA SA	ES (Spain)	5.808	Annually	Utilities	BBB	2023
XS1053594385	ING BANK NV	NL (Netherlands)	2	Annually	Financial	A	2024
XS1539873437	INNOGY SE	DE (Germany)	3.5	Annually	Consumer Durables	BBB	2037

Table 3.2: Corporate bonds considered

CODE	ISSUER	DOMICILE	CPN	FREQ	INDUSTRY	RATING	MAT YEAR
XS0753480457	INTESA SANPAOLO SPA	IT (Italy)	5.53	Annually	Financial	BBB	2020
BE0002272418	KBC GROEP NV	BE (Belgium)	0.75	Annually	Financial	A	2022
XS1531060025	KNORR BREMSE AG	DE (Germany)	0.5	Annually	Automotive	A	2021
XS0811124790	KONINKLIJKE KPN NV	NL (Netherlands)	3.25	Annually	Other	BBB	2021
XS0999654873	LEONARDO SPA	IT (Italy)	4.5	Annually	Other	BB	2021
XS1346762641	MEDIOBANCA BANCA DI CRED. FINAN. SPA	IT (Italy)	1.625	Annually	Financial	BBB	2021
DE000A13R8M3	METRO AG	DE (Germany)	1.375	Annually	Retail	BBB	2021
XS0795500437	METSO OYJ	FI (Finland)	3.8	Annually	Mining	BBB	2022
XS1698932925	NATIONAL BANK OF GREECE SA	GR (Greece)	2.75	Semiannually	Financial	B	2020
XS0981438582	NATURGY ENERGY GROUP SA	ES (Spain)	3.5	Annually	Utilities	BBB	2021
PTPTIUOE0006	NAVIGATOR COMPANY SA	PT (Portugal)	1.575	Semiannually	Fabricated products	BB	2021
XS1497527736	NH HOTEL GROUP SA	ES (Spain)	3.75	Semiannually	Other	B	2023
XS1222431097	NIBC BANK NV	NL (Netherlands)	0.25	Annually	Financial	BBB	2022
XS1960685383	NOKIA OYJ	FI (Finland)	2	Annually	Consumer Durables	BB	2026
XS0569852717	NORDEA BANK ABP	FI (Finland)	4.16	Annually	Financial	AA	2022
XS1206510569	OHL SA	ES (Spain)	5.5	Semiannually	Construction and materials	CCC	2023
XS1734689620	OMV AG	AT (Austria)	1	Annually	Oil	A	2026
FR0011798115	PERNOD RICARD SA	FR (France)	2	Annually	Food	BBB	2020
XS1808984501	PIAGGIO & C SPA	IT (Italy)	3.625	Semiannually	Automotive	BB	2025
BE6252911977	PROXIMUS NV	BE (Belgium)	2.256	Annually	Other	A	2023
XS1510547810	RAIFFEISEN BANK INT. AG	AT (Austria)	0.695	Annually	Financial	BBB	2021
FR0011769090	RENAULT SA	FR (France)	3.125	Annually	Automotive	BBB	2021
XS1334225361	REPSOL SA	ES (Spain)	2.125	Annually	Oil	BBB	2020
XS1476654238	ROYAL DUTCH SHELL PLC	NL (Netherlands)	0.375	Annually	Oil	AA	2025
XS1077584024	RYANAIR DAC	IE (Ireland)	1.875	Annually	Transport	BBB	2021
XS1487498922	SAIPEM SPA	IT (Italy)	3	Annually	Oil	BB	2021
FR0012146777	SANOFI SA	FR (France)	1.125	Annually	Drugs	AA	2022
XS1874128033	SIEMENS AG	DE (Germany)	1	Annually	Consumer Durables	A	2027
AT0000A1C741	STRABAG SE	AT (Austria)	1.625	Annually	Construction and materials	BBB	2022
XS0486101024	TELECOM ITALIA SPA	IT (Italy)	5.25	Annually	Other	BB	2022
XS0907289978	TELEFONICA SAU	ES (Spain)	3.961	Annually	Other	BBB	2021
XS0765448757	THYSSENKRUPP AG	DE (Germany)	5	Annually	Steel	BB	2022
XS0661287507	UNICREDIT SPA	IT (Italy)	0.5	Annually	Financial	BBB	2022
IT0005347346	UNIONE DI BANCHE ITALIANE SPA	IT (Italy)	1.3	Semiannually	Financial	BBB	2020
XS1784311703	UNIPOLSAI ASSICURAZIONI SPA	IT (Italy)	3.875	Annually	Financial	BBB	2028
FR0011689033	VALEO SA	FR (France)	3.25	Annually	Automotive	BBB	2024
XS1700480160	VALLOUREC SA	FR (France)	6.625	Semiannually	Steel	B	2022
XS1260665895	VAN LANSCHOT KEMPEN NV	NL (Netherlands)	7.3	Annually	Financial	BBB	2020
XS1167644407	VOLKSWAGEN AG	DE (Germany)	0.875	Annually	Automotive	BBB	2023
DE000A19UR61	VONOVIA SE	DE (Germany)	0.75	Annually	Other	BBB	2024
XS0907301260	WOLTERS KLUWER NV	NL (Netherlands)	2.875	Annually	Other	BBB	2023
PTNOSFOM0000	ZOPT SGPS SA	PT (Portugal)	1.125	Annually	Other	BBB	2023

Table 3.3: Corporate bonds considered (cont.)

CODE	ISSUER	CPN	FREQ	RATING	MAT. YEAR
DE 2Y BUND	DE (Germany)	0	Annually	AAA	2021
DE 3Y BUND	DE (Germany)	0	Annually	AAA	2022
DE 4Y BUND	DE (Germany)	0	Annually	AAA	2023
DE 5Y BUND	DE (Germany)	0	Annually	AAA	2024
DE 6Y BUND	DE (Germany)	0.5	Annually	AAA	2025
DE 7Y BUND	DE (Germany)	0.5	Annually	AAA	2026
DE 8Y BUND	DE (Germany)	6.5	Annually	AAA	2027
DE 9Y BUND	DE (Germany)	4.75	Annually	AAA	2028
DE 10Y BUND	DE (Germany)	0.25	Annually	AAA	2029
DE 15Y BUND	DE (Germany)	4.75	Annually	AAA	2034
DE 20Y BUND	DE (Germany)	4.25	Annually	AAA	2039
DE 25Y BUND	DE (Germany)	2.5	Annually	AAA	2044
DE 30Y BUND	DE (Germany)	1.25	Annually	AAA	2048
PT 2Y T-BOND	PT (Portugal)	3.85	Annually	BBB	2021
PT 3Y T-BOND	PT (Portugal)	2.2	Annually	BBB	2022
PT 4Y T-BOND	PT (Portugal)	4.95	Annually	BBB	2023
PT 5Y T-BOND	PT (Portugal)	5.65	Annually	BBB	2024
PT 6Y T-BOND	PT (Portugal)	2.875	Annually	BBB	2025
PT 7Y T-BOND	PT (Portugal)	2.875	Annually	BBB	2026
PT 8Y T-BOND	PT (Portugal)	4.125	Annually	BBB	2027

Table 3.4: Sovereign bonds considered

CODE	ISSUER	CPN	FREQ	RATING	MAT. YEAR
PT 9Y T-BOND	PT (Portugal)	2.125	Annually	BBB	2028
PT 10Y T-BOND	PT (Portugal)	1.95	Annually	BBB	2029
PT 15Y T-BOND	PT (Portugal)	2.25	Annually	BBB	2034
PT 20Y T-BOND	PT (Portugal)	4.1	Annually	BBB	2037
PT 30Y T-BOND	PT (Portugal)	4.1	Annually	BBB	2045
ES 2Y T-BOND	ES (Spain)	0.05	Annually	A	2021
ES 3Y T-BOND	ES (Spain)	5.85	Annually	A	2022
ES 4Y T-BOND	ES (Spain)	5.4	Annually	A	2023
ES 5Y T-BOND	ES (Spain)	0.35	Annually	A	2023
ES 6Y T-BOND	ES (Spain)	1.6	Annually	A	2025
ES 7Y T-BOND	ES (Spain)	1.95	Annually	A	2026
ES 8Y T-BOND	ES (Spain)	1.5	Annually	A	2027
ES 9Y T-BOND	ES (Spain)	1.4	Annually	A	2028
ES 10Y T-BOND	ES (Spain)	1.45	Annually	A	2029
ES 15Y T-BOND	ES (Spain)	2.35	Annually	A	2033
ES 20Y T-BOND	ES (Spain)	4.2	Annually	A	2037
ES 25Y T-BOND	ES (Spain)	5.15	Annually	A	2044
ES 30Y T-BOND	ES (Spain)	2.7	Annually	A	2048
BE 3Y OLO	BE (Belgium)	4	Annually	AA	2022
BE 4Y OLO	BE (Belgium)	2.25	Annually	AA	2023
BE 5Y OLO	BE (Belgium)	0.2	Annually	AA	2023
BE 6Y OLO	BE (Belgium)	0.5	Annually	AA	2024
BE 7Y OLO	BE (Belgium)	1	Annually	AA	2026
BE 8Y OLO	BE (Belgium)	0.8	Annually	AA	2027
BE 9Y OLO	BE (Belgium)	5.5	Annually	AA	2028
BE 10Y OLO	BE (Belgium)	0.9	Annually	AA	2029
BE 15Y OLO	BE (Belgium)	1.25	Annually	AA	2033
BE 20Y OLO	BE (Belgium)	1.9	Annually	AA	2038
FR 2Y OAT	FR (France)	0	Annually	AA	2021
FR 3Y OAT	FR (France)	0	Annually	AA	2022
FR 4Y OAT	FR (France)	0	Annually	AA	2023
FR 5Y OAT	FR (France)	0	Annually	AA	2024
FR 6Y OAT	FR (France)	0.5	Annually	AA	2025
FR 7Y OAT	FR (France)	3.5	Annually	AA	2026
FR 8Y OAT	FR (France)	1	Annually	AA	2027
FR 9Y OAT	FR (France)	0.75	Annually	AA	2028
FR 10Y OAT	FR (France)	0.5	Annually	AA	2029
FR 15Y OAT	FR (France)	1.25	Annually	AA	2034
FR 20Y OAT	FR (France)	1.75	Annually	AA	2039
FR 25Y OAT	FR (France)	3.25	Annually	AA	2045
AT 2Y BUND	AT (Austria)	3.5	Annually	AA	2021
AT 3Y BUND	AT (Austria)	0	Annually	AA	2022
AT 4Y BUND	AT (Austria)	0	Annually	AA	2023
AT 5Y BUND	AT (Austria)	1.65	Annually	AA	2024
AT 6Y BUND	AT (Austria)	1.2	Annually	AA	2025
AT 7Y BUND	AT (Austria)	0.75	Annually	AA	2026
AT 8Y BUND	AT (Austria)	0.5	Annually	AA	2027
AT 9Y BUND	AT (Austria)	0.75	Annually	AA	2028
AT 10Y BUND	AT (Austria)	0.5	Annually	AA	2029
AT 15Y BUND	AT (Austria)	2.4	Annually	AA	2034
AT 20Y BUND	AT (Austria)	4.15	Annually	AA	2037
AT 25Y BUND	AT (Austria)	3.15	Annually	AA	2044
AT 30Y BUND	AT (Austria)	1.5	Annually	AA	2047
IE 3Y T-BOND	IE (Ireland)	0	Annually	A	2022
IE 4Y T-BOND	IE (Ireland)	3.9	Annually	A	2023
IE 5Y T-BOND	IE (Ireland)	3.4	Annually	A	2024
IE 6Y T-BOND	IE (Ireland)	5.4	Annually	A	2025
IE 7Y T-BOND	IE (Ireland)	1	Annually	A	2026
IE 9Y T-BOND	IE (Ireland)	0.9	Annually	A	2028

Table 3.5: Sovereign bonds considered (cont.)

CODE	ISSUER	CPN	FREQ	RATING	MAT. YEAR
IE 12Y T-BOND	IE (Ireland)	1.35	Annually	A	2031
IE 15Y T-BOND	IE (Ireland)	1.3	Annually	A	2033
IE 20Y T-BOND	IE (Ireland)	1.7	Annually	A	2037
IE 30Y T-BOND	IE (Ireland)	2	Annually	A	2045
IT 2Y BTP	IT (Italy)	0.05	Annually	BBB	2021
IT 3Y BTP	IT (Italy)	1.2	Annually	BBB	2022
IT 4Y BTP	IT (Italy)	0.95	Annually	BBB	2023
IT 5Y BTP	IT (Italy)	1.85	Annually	BBB	2024
IT 6Y BTP	IT (Italy)	1.45	Annually	BBB	2025
IT 7Y BTP	IT (Italy)	1.6	Annually	BBB	2026
IT 8Y BTP	IT (Italy)	2.2	Annually	BBB	2027
IT 9Y BTP	IT (Italy)	2	Annually	BBB	2028
IT 10Y BTP	IT (Italy)	3	Annually	BBB	2029
IT 15Y BTP	IT (Italy)	2.45	Annually	BBB	2033
IT 20Y BTP	IT (Italy)	2.95	Annually	BBB	2038
IT 25Y BTP	IT (Italy)	4.75	Annually	BBB	2044
IT 30Y BTP	IT (Italy)	3.45	Annually	BBB	2048

Table 3.6: Sovereign bonds considered (cont.)

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	86.99	9.11	0.53	0.05	0.08	0.03	0.05	0.01	3.15
AA	0.43	90.37	4.37	0.61	0.1	0.04	0.05	0.02	4.01
A	0.04	0.71	90.56	3.67	0.3	0.12	0.01	0.06	4.53
BBB	0.01	0.08	1.52	89.26	2.38	0.4	0.06	0.17	6.12
BB	0.00	0.05	0.11	2.85	82.72	3.61	0.56	0.56	9.52
B	0	0.02	0.08	0.12	1.58	78.03	4.03	3.6	12.54
CCC	0	0	0.12	0.21	0.59	13.17	43.46	26.82	15.63

Table 3.7: Average one-year transition matrix for global corporates (1981-2017) adjusted from Vazza and Kraemer (2017)[55].

	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	95.79	4.21	0	0	0	0	0	0	0
AA	0	96.33	2.49	1.18	0	0	0	0	0
A	0	0.73	95.26	3.89	0.12	0	0	0	0
BBB	0	0	1.5	95.69	2.53	0.19	0.09	0	0
BB	0	0	0	0.72	91.39	5.53	0.54	1.63	0.18
B	0	0	0	0	1.79	91.01	2.21	1.1	3.89
CCC	0	0	0	0	0	45.74	39.36	14.89	0

Table 3.8: Average one-year transition matrix for global sovereign (1993-2017) adjusted from Witte (2017)[57].

	AAA	AA	A	BBB	BB	B	CCC
0.25	-0.7356	-0.7212	-0.7076	-0.6775	-0.5671	-0.3158	9.2934
0.5	-0.6226	-0.6082	-0.5946	-0.5645	-0.4541	-0.2028	9.4064
1	-0.4474	-0.4302	-0.4111	-0.3797	-0.2645	0.0791	6.0096
2	-0.3325	-0.3027	-0.2725	-0.2163	-0.0004	0.5559	5.1232
3	-0.2592	-0.2107	-0.1684	-0.0825	0.2525	0.9249	5.1663
4	-0.1364	-0.0775	-0.0122	0.1201	0.5741	1.4657	5.5168
5	0.0066	0.0848	0.1595	0.3257	0.8911	1.9498	5.9662
6	0.1636	0.2707	0.3583	0.5566	1.1921	2.3082	5.9328
7	0.3307	0.4667	0.5671	0.7974	1.5031	2.6767	5.9095
8	0.4771	0.6395	0.7399	0.9818	1.6928	2.8740	5.8112
9	0.6115	0.8003	0.9006	1.1543	1.8706	3.0593	5.7009
10	0.7279	0.9431	1.0434	1.3087	2.0304	3.2266	5.5726
12	0.9116	1.1205	1.2341	1.5033	2.2362	3.4808	5.6549
15	1.1517	1.3510	1.4846	1.7596	2.5096	3.8267	5.7430
20	1.4578	1.6680	1.8081	2.0807	2.8510	4.1835	5.7271
25	1.6035	1.8068	1.9380	2.2145	2.9794	4.2931	5.6752
30	1.6122	1.8086	1.9308	2.2114	2.9708	4.2658	5.4864

Table 3.9: Zero corporate rates of credit curves obtained on 26 April 2019 (valuation day) from Reuters Eikon Database.

	AAA	AA	A	BBB	BB	B	CCC
0.25	-0.7390	-0.7197	-0.7020	-0.6753	-0.5686	-0.3398	8.5495
0.5	-0.6260	-0.6067	-0.5890	-0.5623	-0.4556	-0.2268	8.6625
1	-0.5113	-0.4869	-0.4611	-0.4347	-0.3281	-0.0049	5.2496
2	-0.4820	-0.4420	-0.4026	-0.3553	-0.1518	0.3604	4.6153
3	-0.4074	-0.3476	-0.2934	-0.2187	0.1001	0.7282	4.7214
4	-0.2703	-0.1972	-0.1167	0.0038	0.4375	1.2653	5.0784
5	-0.1120	-0.0174	0.0752	0.2310	0.7748	1.7808	5.5816
6	0.0490	0.1719	0.2769	0.4675	1.0789	2.1436	5.5752
7	0.2160	0.3672	0.4847	0.7099	1.3890	2.5123	5.5748
8	0.3679	0.5458	0.6631	0.9004	1.5827	2.7107	5.4906
9	0.5207	0.7254	0.8426	1.0919	1.7774	2.9100	5.4073
10	0.5366	0.7679	0.8851	1.1463	1.8352	2.9724	5.1871
12	0.5578	0.7807	0.9116	1.1761	1.8743	3.0507	5.1114
15	0.5897	0.7997	0.9514	1.2209	1.9330	3.1681	4.9979
20	0.6161	0.8348	0.9948	1.2626	1.9935	3.2302	4.6701
25	0.6458	0.8619	1.0162	1.2858	2.0120	3.2147	4.5325
30	0.6755	0.8890	1.0375	1.3091	2.0305	3.1993	4.3950

Table 3.10: Zero sovereign rates of credit curves obtained on 26 April 2019 (valuation day) from Reuters Eikon Database.

	Mean	Std	10%	90%
Senior Secured	0.635	0.34	0.193	1
Senior Subordinated	0.294	0.335	0	0.823
Senior Unsecured	0.486	0.375	0.014	1
Junior or Subordinated	0.274	0.343	0	0.972
Food	0.692	0.4	0.008	1
Mining	0.623	0.346	0.196	1
Oil	0.545	0.369	0.058	1
Clothes, Textiles and Footware	0.625	0.345	0.156	1
Consumer Durables	0.605	0.396	0.031	1
Chemicals	0.698	0.373	0.1	1
Drugs, soap, perfume, tobacco	0.594	0.422	0.096	1
Construction and Materials	0.584	0.399	0.01	1
Steel	0.551	0.41	0	1
Fabricated Products	0.709	0.376	0.015	1
Machinery	0.624	0.375	0.094	1
Automotive	0.657	0.385	0.007	1
Transport	0.517	0.362	0.037	1
Utilities	0.864	0.259	0.364	1
Retail	0.54	0.403	0.014	1
Financial	0.564	0.417	0.007	1
Other	0.561	0.397	0.009	1

Table 3.11: Recovery rates data obtained from Altman and Kalotay (2014)[1].

	MODEL 1		MODEL 2		MODEL3		MODEL 4	
	Stoxx	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**
Portfolio 1 value 10,689,760.18 €								
IRC (VaR 99.9%)	479,100 € 4.48%	522,170 € 4.88%	660,100 € 6.18%	762,900€ 7.14%	1,072,600 € 10.03%	911,710 € 8.53%	949,460 € 8.88%	1,133,700€ 10.61%
IRC (ES 99.9%)	613,760 € 5.74%	707,010 € 6.61%	913,230 € 8.54%	1,042,800 € 9.76%	1,440,000 € 13.47%	1,292,600 € 12.09%	1,414,100 € 13.23%	1,783,500 € 16.68%
Portfolio 2 value 9,181,462.21 €								
IRC (VaR 99.9%)	348,240 € 3.79%	339,000 € 3.69%	330610 € 3.60%	332,560 € 3.62%	376,460 € 4.10%	373,670 € 4.07%	358,420 € 3.90%	370,600 € 4.04%
IRC (ES 99.9%)	396,130 € 4.31%	395,530 € 4.31%	389,880 € 4.25%	391,420 € 4.26%	414,980 € 4.52%	413,150 € 4.50%	396,920 € 4.32%	396,860 € 4.32%
Portfolio 3 value 67,030,331.05 €								
IRC (VaR 99.9%)	2,788,500 € 4.16%	2,849,200 € 4.25%	3,515,300 € 5.24%	3,313,100 € 4.94%	5,162,500 € 7.70%	3,573,900 € 5.33%	3,557,200 € 5.31%	3,834,500 € 5.72%
IRC (ES 99.9%)	4,993,900 € 7.45%	5,089,000 € 7.59%	5,078,800 € 7.58%	5,431,400 € 8.10%	8,886,800 € 13.26%	5,864,400 € 8.75%	5,526,600 € 8.24%	6,475,200 € 9.66%
Portfolio 4 value 69,972,262.37 €								
IRC (VaR 99.9%)	5,762,300 € 8.24%	5,712,100 € 8.16%	6,788,700 € 9.70%	6,377,700 € 9.11%	10,091,000 € 14.42%	6,386,900 € 9.13%	7,195,600 € 10.28%	7,183,500 € 10.27%
IRC (ES 99.9%)	8,904,300 € 12.73%	8,978,600 € 12.83%	9,624,400 € 13.75%	9,569,500 € 13.68%	12,208,000 € 17.45%	9,745,600 € 13.93%	11,374,000 € 16.26%	11,675,000 € 16.69%
Portfolio 5 value 73,915,989.95 €								
IRC (VaR 99.9%)	11,403,000 € 15.43%	11,894,000 € 16.09%	12,860,000 € 17.40%	13,490,000 € 18.25%	14,729,000 € 19.93%	13,823,000 € 18.70%	13,770,000 € 18.63%	15,761,000 € 21.32%
IRC (ES 99.9%)	13,573,000 € 18.36%	14,332,000 € 19.39%	15,573,000 € 21.07%	16,496,000 € 22.32%	18,419,000 € 24.92%	17,425,000 € 23.57%	17,024,000 € 23.03%	19,878,000 € 26.89%
Portfolio 6 value 71,275,242.17 €								
IRC (VaR 99.9%)	45,617,000 € 64.00%	46,803,000 € 65.67%	47,720,000 € 66.95%	48,198,000 € 67.62%	47,785,000 € 67.04%	48,571,000 € 68.15%	46,183,000 € 64.80%	47,302,000 € 66.37%
IRC (ES 99.9%)	49,529,000 € 69.49%	50,058,000 € 70.23%	50,528,000 € 70.89%	50,678,000 € 71.10%	50,592,000 € 70.98%	50,805,000 € 71.28%	49,722,000 € 69.76%	50,394,000 € 70.70%

Table 3.12: Complete range of IRC results (Unconditional thresholds and stochastic RRs).

	MODEL 1		MODEL 2		MODEL 3		MODEL 4	
	Stoxx	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**
Portfolio 1	0.30%	0.58%	1.42%	1.65%	2.53%	1.96%	3.07%	4.22%
Portfolio 2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Portfolio 3	1.21%	1.09%	0.00%	0.00%	0.21%	0.67%	1.91%	0.32%
Portfolio 4	3.07%	3.12%	1.90%	2.44%	1.10%	2.62%	4.82%	4.20%
Portfolio 5	-2.20%	-2.45%	-2.17%	-2.08%	-1.51%	-0.87%	-1.25%	-0.12%
Portfolio 6	-19.42%	-21.07%	-22.34%	-22.99%	-22.41%	-23.51%	-20.18%	-21.71%

Table 3.13: IRC variation when comparing the model with deterministic and stochastic RRs.

	MODEL 1		**MODEL 2**		**MODEL 3**		**MODEL 4**	
	Stoxx	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**
Portfolio 1	-0.02%	0.25%	0.26%	0.19%	0.33%	0.02%	0.85%	1.54%
Portfolio 2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Portfolio 3	0.25%	0.49%	0.00%	0.00%	0.07%	0.45%	2.88%	0.12%
Portfolio 4	0.07%	0.23%	0.08%	0.07%	0.16%	0.12%	1.26%	1.40%
Portfolio 5	0.16%	0.01%	0.19%	0.08%	0.81%	0.63%	0.88%	1.43%
Portfolio 6	0.57%	0.03%	-1.33%	1.07%	0.14%	0.64%	0.24%	0.41%

Table 3.14: IRC variation when increasing one basis point (0.01%) the probability of default.

	MODEL 1		**MODEL 2**		**MODEL 3**		**MODEL 4**	
	Stoxx	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**
Portfolio 1	-0.14%	-0.09%	-0.09%	-0.26%	-0.11%	-0.39%	-0.18%	-0.31%
Portfolio 2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Portfolio 3	-0.34%	-0.35%	-1.19%	-0.80%	-2.01%	-1.10%	-0.75%	-0.75%
Portfolio 4	-0.37%	-0.30%	0.00%	0.00%	-1.22%	-1.19%	-0.13%	-0.92%
Portfolio 5	0.00%	-0.26%	0.19%	-0.12%	0.69%	0.18%	0.68%	1.21%
Portfolio 6	0.57%	0.03%	-1.01%	-0.33%	-1.26%	-0.78%	0.24%	-0.42%

Table 3.15: IRC variation when decreasing one basis point the probability of default.

Portfolio 4 value 69,972,262.37 €	**MODEL 1**		**MODEL 2**		**MODEL 3**		**MODEL 4**	
	Stoxx	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**	**Stoxx**	**PCA**
VaR condit. to F(perc.90%)	380,680 € 0.54%	501,470 € 0.72%	477,440 € 0.68%	545,600 € 0.78%	633,770 € 0.91%	575,130 € 0.82%	3,322,500 € 4.75%	611,280 € 0.87%
VaR condit. to F(perc.10%)	13,624,000 € 19,47%	13,541,000 € 19.35%	14,476,000 € 20.69%	14,325,000 € 20.47%	21,699,000 € 31.01%	15,807,000 € 22.59%	28,325,000 € 40.48%	25,706,000 € 36.74%

Table 3.16: IRC values obtained under the conditional thresholds approach

Number of overshootings	Fewer than 5	5	6	7	8	9	10 or more
Addend	0.00	0.40	0.50	0.65	0.75	0.85	1.00

Table 3.17: Applicated addend according to the Article 366 [21].

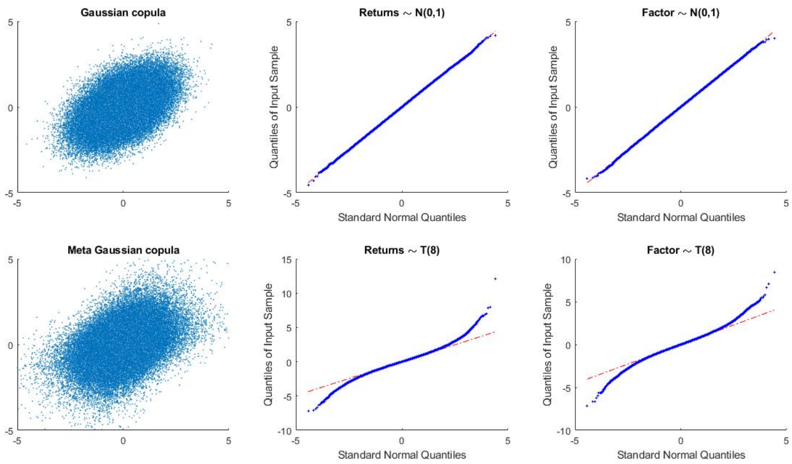

Figure 3.15: Conditional Gaussian copula representation with its correspondent common factor and the returns in a QQ-plot by row.

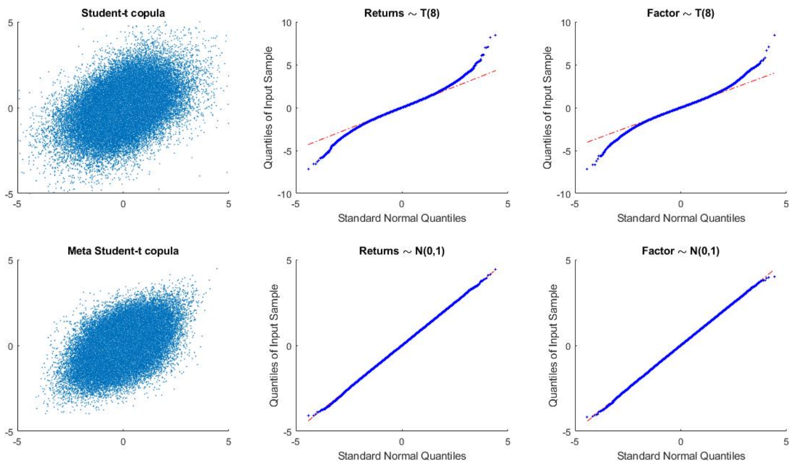

Figure 3.16: Conditional Student-t copula representation with its correspondent common factor and the returns in a QQ-plot by row.

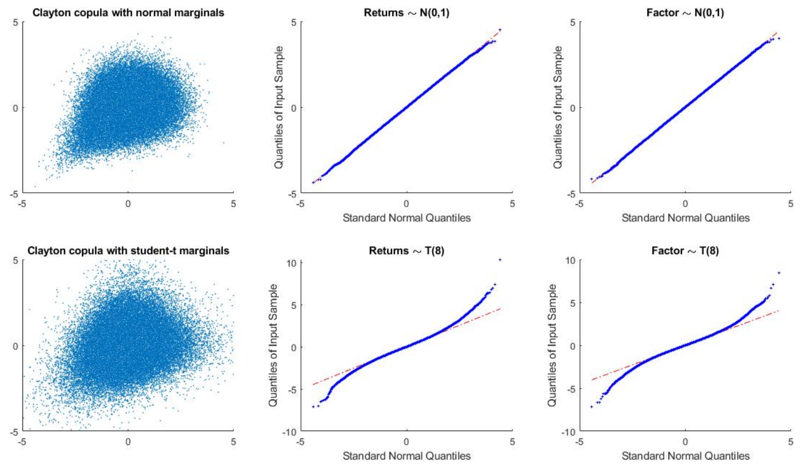

Figure 3.17: Conditional Clayton copula representation with its correspondent common factor and the returns in a QQ-plot by row.

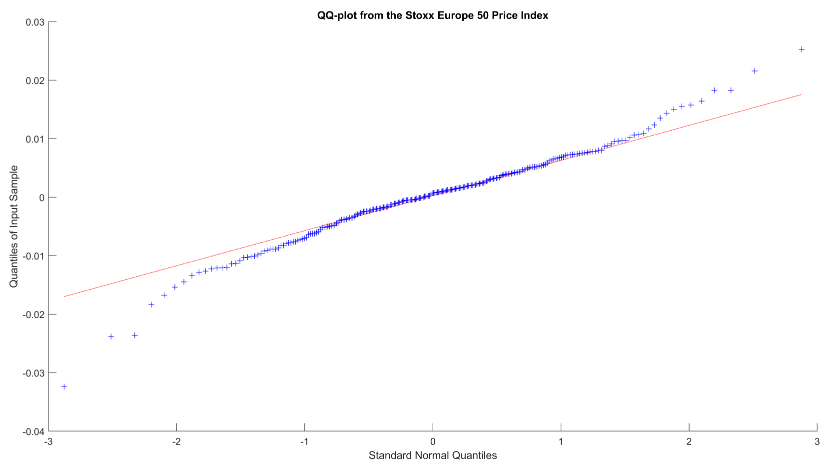

Figure 3.18: QQ-plot comparing the empirical distribution of Stoxx Europe 50 Index versus a theoretical standard normal

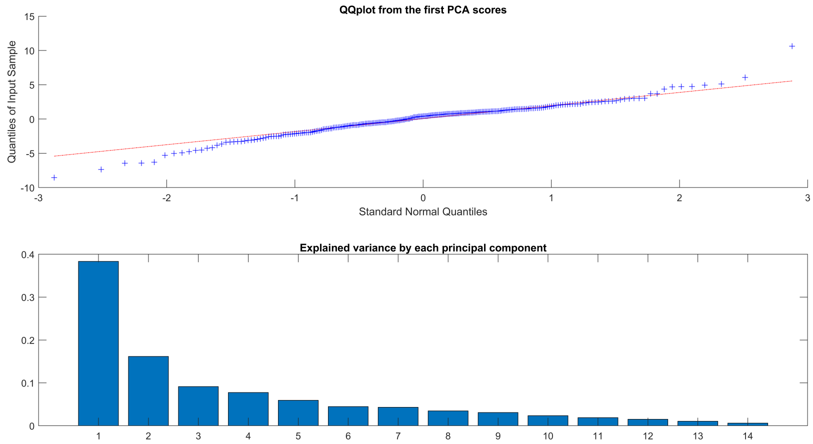

Figure 3.19: QQ-plot of the first PCA scores versus a theoretical standard normal (upper subplot) and the individual explained variance by each PC (lower subplot).

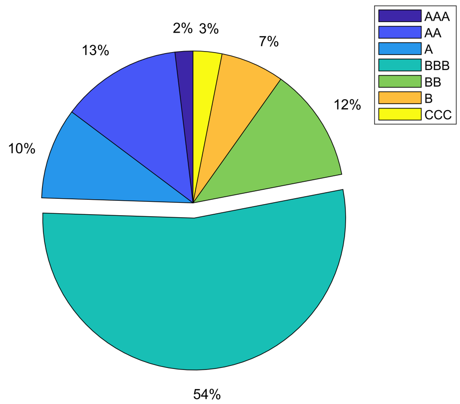

Figure 3.20: Portfolio 1 - percentage of credit rating

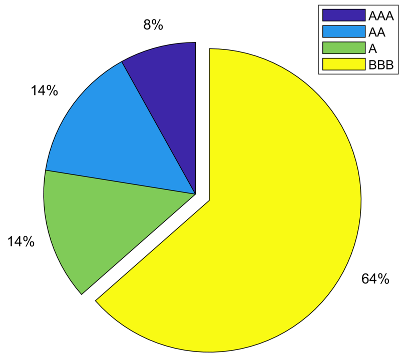

Figure 3.21: Portfolio 2 - percentage of credit rating

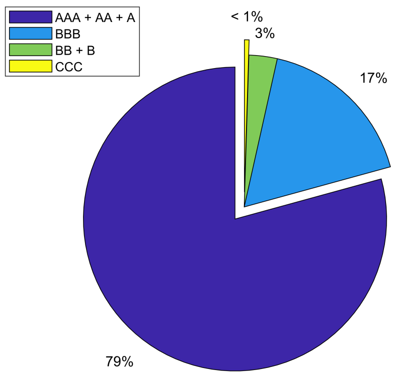

Figure 3.22: Portfolio 3 - percentage of credit rating

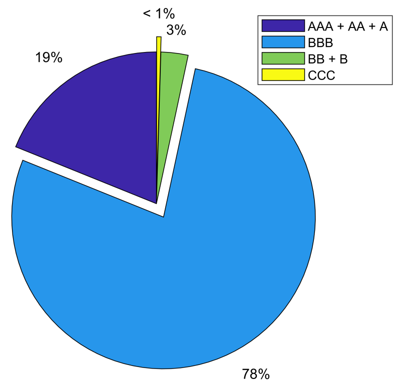

Figure 3.23: Portfolio 4 - percentage of credit rating

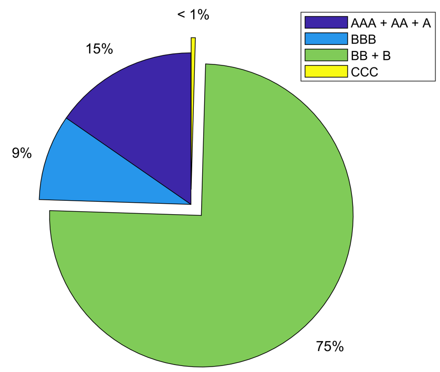

Figure 3.24: Portfolio 5 - percentage of credit rating

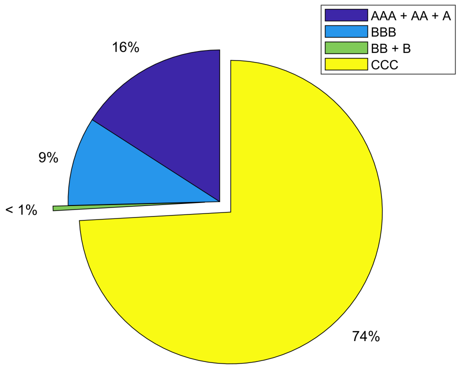

Figure 3.25: Portfolio 6 - percentage of credit rating

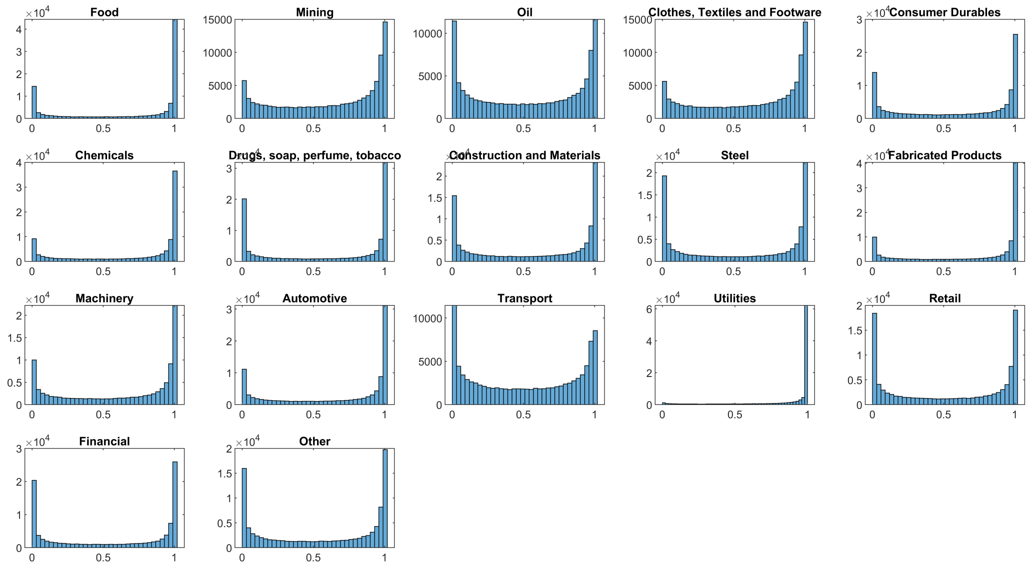

Figure 3.26: Histograms of RRs simulated by industry

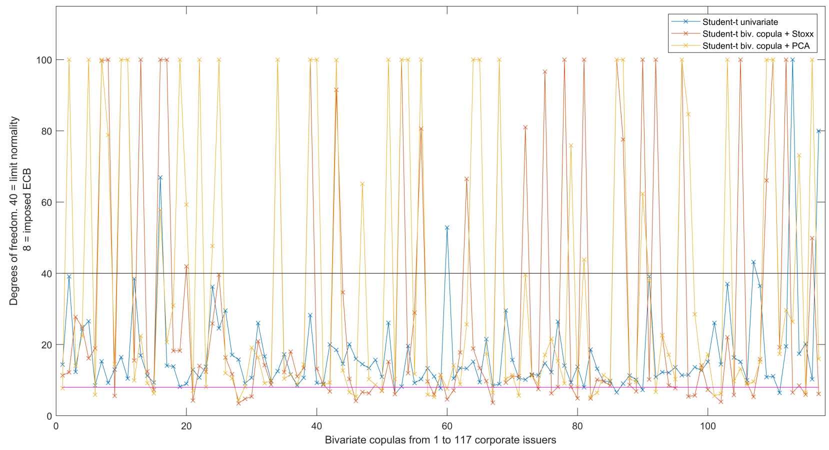

Figure 3.27: Corporate issuer's degrees of freedom estimation

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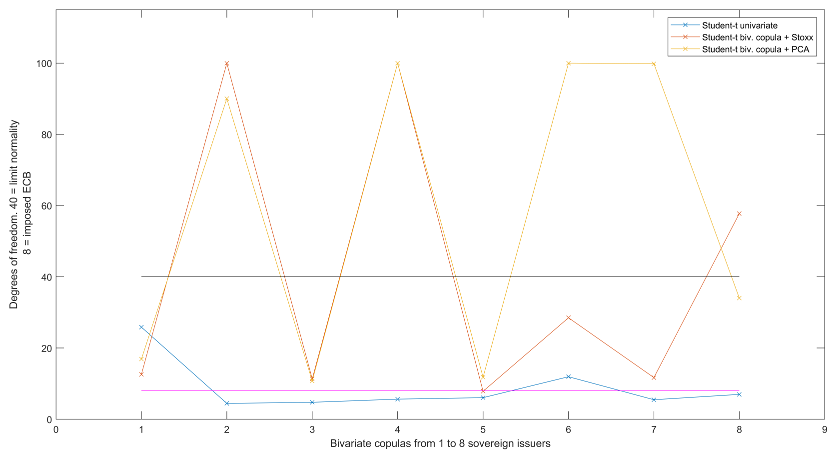

Figure 3.28: Sovereign issuer's degrees of freedom estimation

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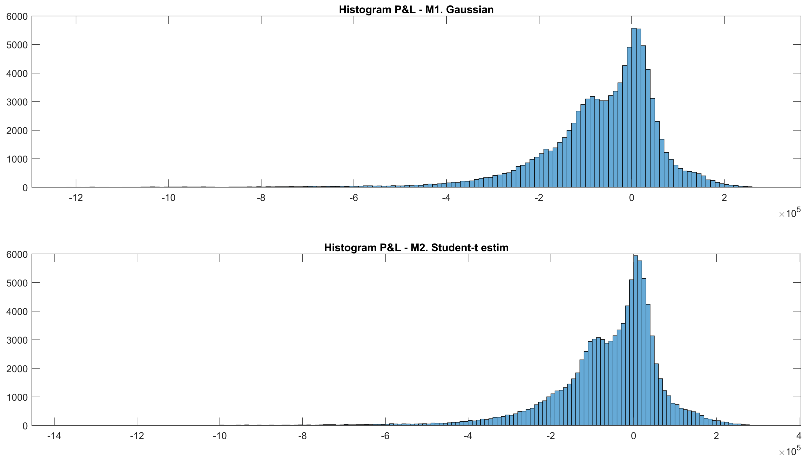

Figure 3.29: Histograms of the application of Model 1 and 2 - Portfolio 4
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