SU(2) treatment and CFT's

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Contents

- LQG, Isolated Horizons, Chern-Simons theory and black hole entropy
- New perspective: Black hole entropy from CFT's
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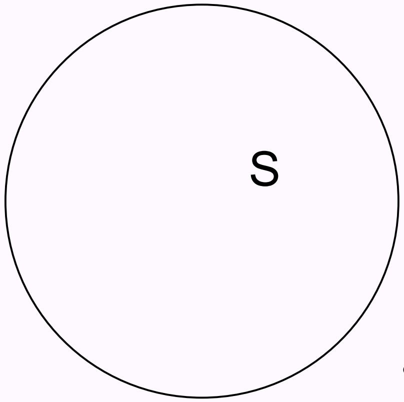
LQG, Isolated Horizons, Chern-Simons theory and black hole entropy

Ashtekar, Baez, Corichi, Krasnov (2000): Quantum geometry of IH and BH Entropy

- The black hole is introduced in an effective way: the horizon is introduced as an inner boundary of the spacetime manifold.



Boundary conditions (at the classical level):

- SU(2)-connection A_a on the bulk $\longrightarrow$ U(1)-connection W_a on S
- U(1)-Chern-Simons Theory on S

LQG, Isolated Horizons, Chern-Simons theory and black hole entropy

Quantizing…

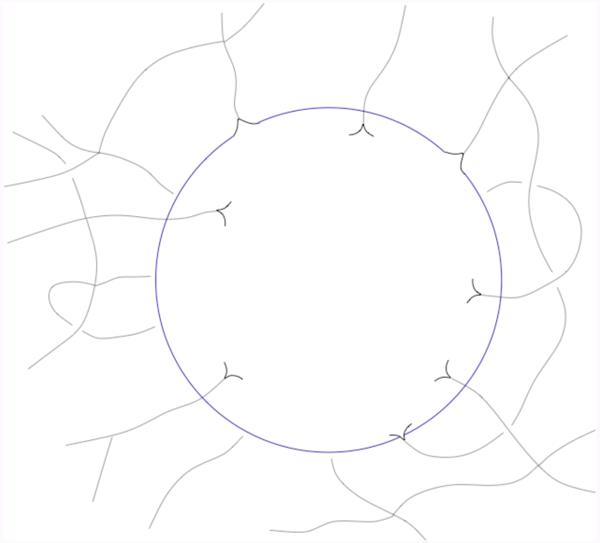
Bulk:

- SU(2) spin network $\longrightarrow$ j_i labels
- finite number of edges piercing the horizon (punctures) $\longrightarrow$ m_i labels

Horizon:

- U(1)-Chern-Simons theory on a punctured horizon $\longrightarrow$ a_i labels

$$\sum_{i=1}^{N} a_i = 0 \mod \kappa$$



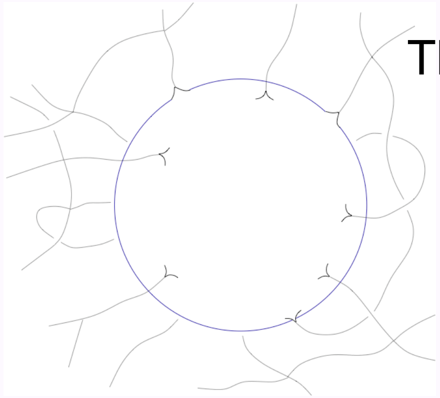
$$\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_S$$

Boundary conditions:

$$-2m_I = a_I \mod \kappa$$

LQG, Isolated Horizons, Chern-Simons theory and black hole entropy

Black hole Entropy



The area of the Horizon is fixed to be in the interval $[A-\delta, A+\delta]$

$$\mathcal{H}^A = \mathcal{H}_V^A \otimes \mathcal{H}_S^A$$

$dim\ \infty$ $\quad$ $dim\,\mathfrak{n}(A)$

$$S = k_B \ln \mathfrak{n}(A)$$

$\mathfrak{n}(A)$ can be computed as the number of sequences $\vec{a} = (a_1, a_2, ..., a_N)$

of non zero integers such that $\sum_{i=1}^{N} a_i = 0 \mod \kappa$ and :

Boundary Conditions from the bulk

1) $\exists\ \vec{m} = (m_1, m_2, ..., m_N)$ such that $-2m_I = a_I \mod \kappa$

2) $\exists\ \vec{j} = (j_1, j_2, ..., j_N)$ such that $m_I \in \{-j_I, -j_I+1, ..., j_I\}$

and $A(\vec{j}) := 8\pi\gamma\ell_P^2 \sum_{I=1}^{N} \sqrt{j_I(j_I+1)} \in [A-\delta, A+\delta]$

LQG, Isolated Horizons, Chern-Simons theory and black hole entropy

We define n_j as the number of times that the label j appears in a sequence

$$\underbrace{(j_1, j_1, \ldots, j_1}_{n_{j_1}\text{- times}}, \underbrace{j_2, \ldots, j_2}_{n_{j_2}\text{- times}}, \ldots, \underbrace{j_n, \ldots, j_n}_{n_{j_{max}}\text{- times}}) \equiv \{n_j\}_{j=1}^{j_{max}}$$

Bulk LQG: $$A(\vec{j}) := 8\pi\gamma\ell_P^2 \sum_{I=1}^{N} \sqrt{j_I(j_I+1)}$$

$$\{n_j\}_{j=1}^{j_{max}} \rightarrow \{n_{|m|}\} \xrightarrow{k=2|m|} \{n_k\} \xrightarrow{-2m_I = a_I} \text{U(1)-Chern-Simons} \rightarrow \mathfrak{n}(A)$$

Reordering factor

$$\mathfrak{n}(A) = \sum_{\{n_k\}} \overbrace{\frac{(\sum_k n_k)!}{\prod_k n_k!}}^{} P(\{n_k\})$$

Fernando's talk : $$P(\{n_k\}) = \frac{1}{2\pi}\int_0^{2\pi} d\theta \prod_k (2\cos k\theta)^{n_k}$$

New perspective: Black hole entropy and CFT's

Motivation:

- **E. Witten**: connection between the Hilbert space of a generally covariant theory and the space of conformal blocks of a conformal invariant theory.

First application of this idea: R.K. Kaul and P. Majumdar Phys. Lett B. 439 (1998)

SU(2) Chern-Simons ⟺ SU(2)-(CFT) Wess-Zumino-Witten

- SU(2) CS theory on the horizon
- SU(2)-WZW theory on a 2-sphere

Computing the number of conformal blocks of the SU(2)-WZW:

$$N^{\mathcal{P}} = \sum_{r_i} N_{j_1 j_2}^{r_1} N_{r_1 j_3}^{r_2} \ldots N_{r_{n-2} j_{n-1}}^{j_n} \quad \longrightarrow \quad \text{Entropy}$$

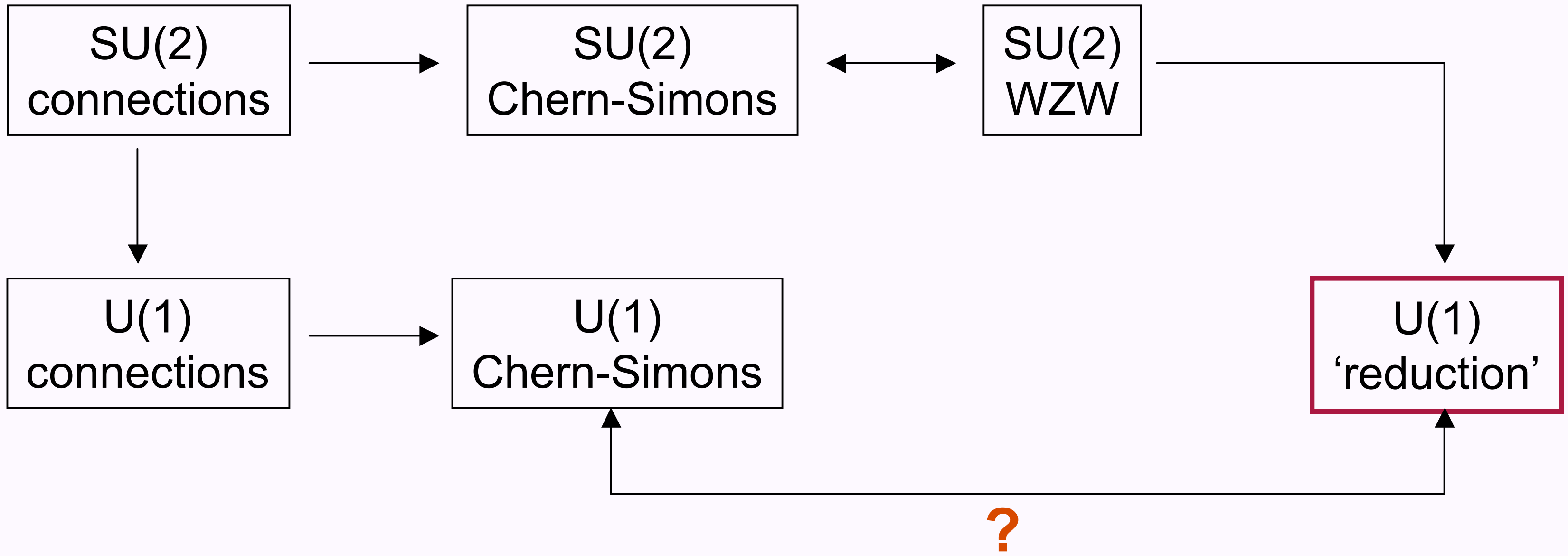
$$[j_i] \otimes [j_l] = \bigoplus_r N_{il}^r [j_r]$$

PROBLEM: In LQG the horizon d.o.f. are described by a U(1)-CS theory

New perspective: Black hole entropy and CFT's

How to account for the U(1) degrees of freedom?

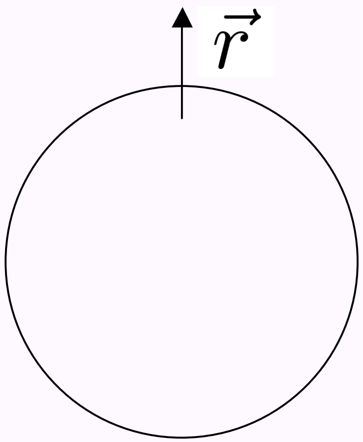
GEOMETRIC SYMMETRY REDUCTION



New perspective: Black hole entropy and CFT's

SYMMETRY REDUCTION

In ABCK it is done by fixing a unit vector field $\vec{r}$ on the horizon



$$W_a := -\frac{1}{\sqrt{2}} \underline{\Gamma}_a^i r_i$$

This $\vec{r}$ picks out a U(1) sub-bundle Q of the SU(2) bundle.

New perspective: Black hole entropy and CFT's

In general terms

- Given a fiber bundle P[SU(2),S] with connection w, a homomorphism $\lambda : U(1) \subset SU(2) \longrightarrow SU(2)$ induces a bundle reduction from P[SU(2), S] to Q[U(1), S] with a U(1) "reduced" conexión w'.

- All the congujacy classes of homomophisms $\lambda : U(1) \longrightarrow SU(2)$ are represented in

$$Hom(U(1), T(SU(2))$$

where $T(SU(2)) = \{diag(z, z^{-1}) \mid z = e^{i\theta} \in U(1)\}$ is the invariant torus of SU(2)

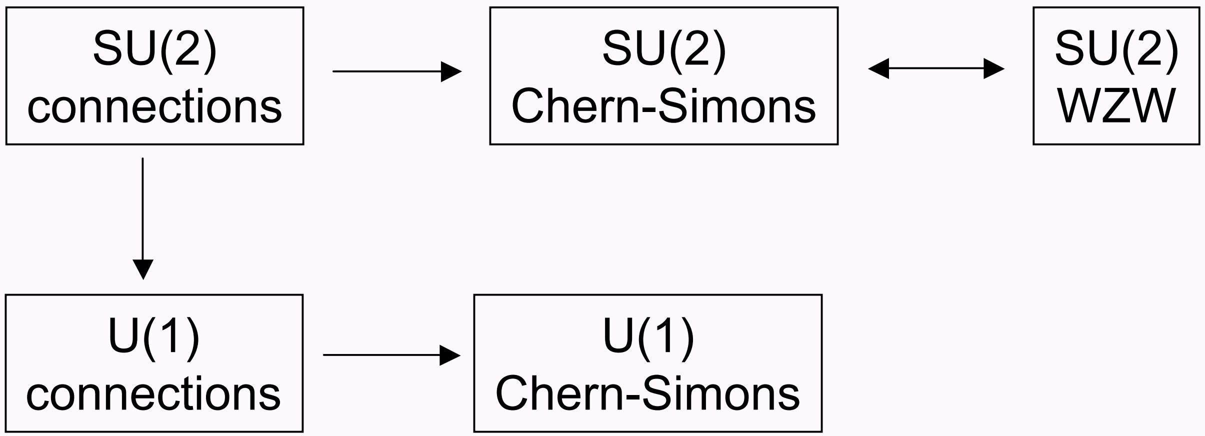
- The homomorphism in $Hom(U(1), T(SU(2))$ can be characterized by

$$\lambda_p : \; z \mapsto diag(z^p, z^{-p}) \qquad p \in \mathbb{Z}$$

- Considering the action of the Weyl group $(diag(z, z^{-1}) \mapsto diag(z^{-1}, z))$ $p \in \mathbb{N}_0$

- λ_p with $p \in \mathbb{N}_0$ characterize all the possible ways of carry out the symmetry breaking from SU(2) to U(1) .

New perspective: Black hole entropy and CFT's



New perspective: Black hole entropy and CFT's

SU(2) Chern-Simons $\longleftrightarrow$ SU(2)-WZW R.K. Kaul and P. Majumdar Phys. Lett B. 439 (1998)

$$\mathcal{P} = \{j_1, j_2, ..., j_N\}$$

Verlinde formula

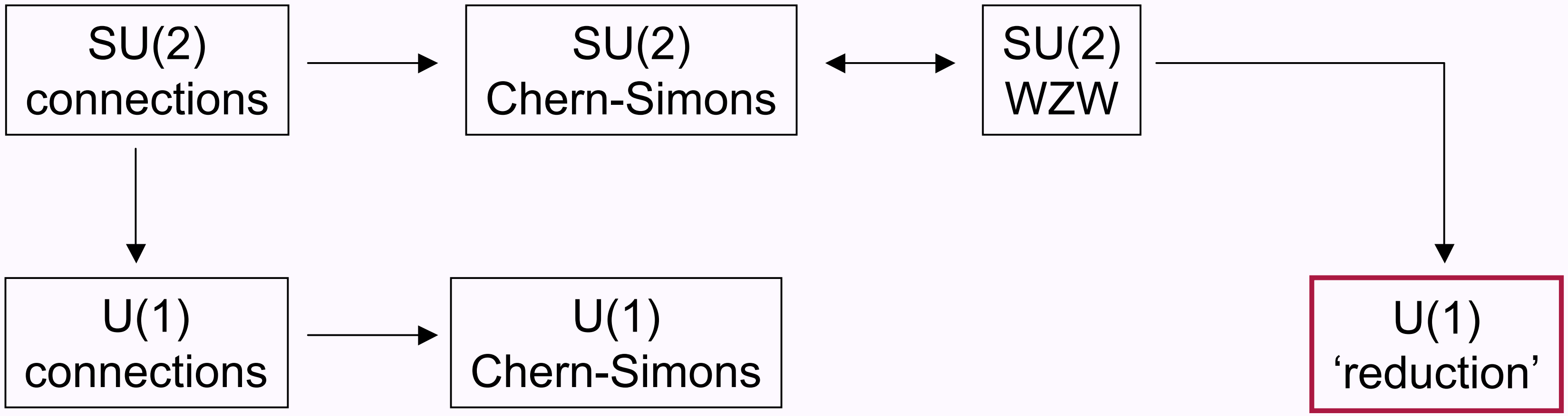
$$N^{\mathcal{P}} = \sum_{r_i} N^{r_1}_{j_1 j_2} N^{r_2}_{r_1 j_3} ... N^{j_n}_{r_{n-2} j_{n-1}} \qquad [j_i] \otimes [j_l] = \bigoplus_r N^r_{il} [j_r]$$

We can re-write this expressions using the theory of characters: $\chi_i \chi_j = \sum_r N^r_{ij} \chi_r$

Using that $\langle \chi_i | \chi_j \rangle = \delta_{ij}$ being $\langle \chi_i | \chi_j \rangle_{SU(2)} = \frac{1}{\pi} \int_0^{2\pi} d\theta \sin^2 \theta \chi_i \chi_j$

$$\Rightarrow \quad N^{\mathcal{P}} = \langle \chi_{j_1} ... \chi_{j_N} | \chi_0 \rangle = \int_0^{2\pi} \frac{d\theta}{\pi} \sin^2 \theta \prod_{I=1}^{N} \frac{\sin[(j_I + 1)\theta]}{\sin \theta}$$

New perspective: Black hole entropy and CFT’s



New perspective: Black hole entropy and CFT's

Applying the symmetry reduction…

- We want to restrict the representations $\mathcal{P} = \{j_1, j_2, ..., j_N\}$ to a set of U(1) representations $\Longleftrightarrow$ symmetry reduction locally at each puncture

- Each SU(2) irrep j can be seen as the direct sum of 2j+1 U(1) irreps:

$$e^{ij\theta} \oplus e^{i(j-1)\theta} \oplus ... \oplus e^{i(1-j)\theta} \oplus e^{-ij\theta}$$

- An explicit symmetry reduction can be made using the homomorphisms λ_p

- This corresponds to pick out a U(1) representation $e^{ip\theta} \oplus e^{-ip\theta}$ with some $p \leq j$

- We consider these U(1) (reducible) representations instead of the irreducible ones as a consequence of the reduction from SU(2)

New perspective: Black hole entropy and CFT's

Applying the symmetry reduction…

We have a set of U(1) representations $\mathcal{P}_{U(1)} = \{p_1, p_2, ..., p_N\}$ (one at each puncture)

$$\tilde{\eta}_{p_I} = e^{ip_I\theta} + e^{-ip_I\theta} = 2\cos p_I\theta$$

We have to couple they to the U(1) gauge invariance representation.

The characters of the U(1) irrep η_i are orthonormal:

$$\langle \eta_i | \eta_j \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta \eta_i^* \eta_j = \delta_{ij}$$

Then:

$$N_{U(1)}^{\mathcal{P}} = \langle \tilde{\eta}_{p_1} ... \tilde{\eta}_{p_N} | \eta_0 \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta \prod_I^N 2\cos p_I\theta$$

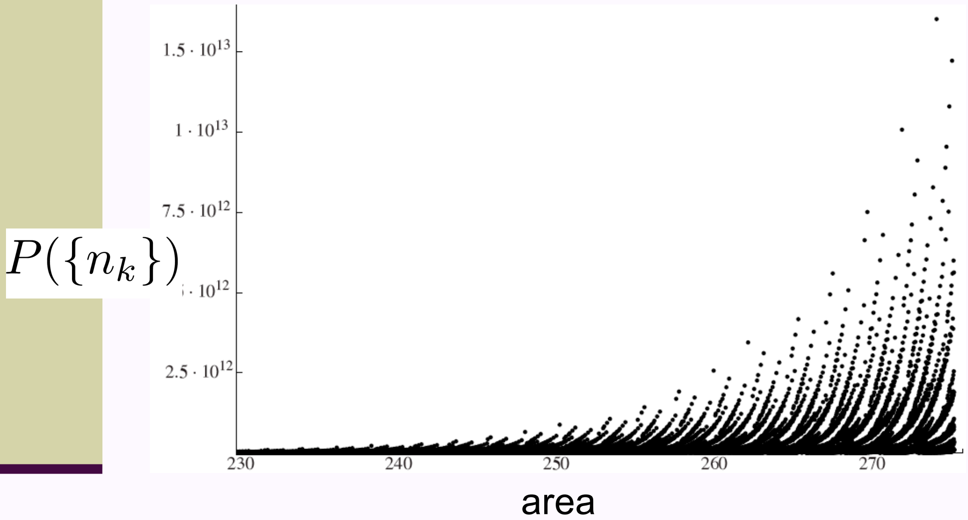
Remembering…..

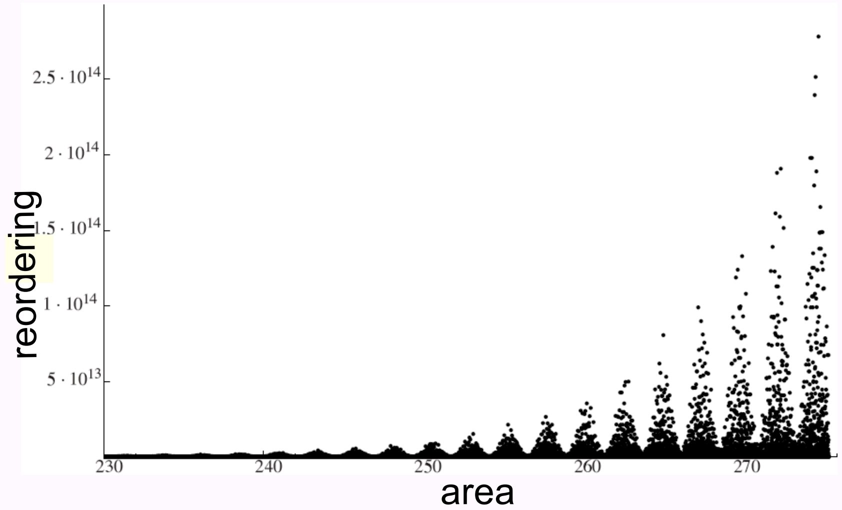
$$P(\{n_k\}) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \prod_k (2\cos k\theta)^{n_k}$$

Same expression if we identify p_I with k_I

New perspective: Black hole entropy and CFT’s

$$\mathfrak{n}(A) = \sum_{\{n_k\}} \overbrace{\frac{(\sum_k n_k)!}{\prod_k n_k!}}^{\text{Reordering factor}} P(\{n_k\})$$





- The contribution form CS or (CFT)WZW gives a linear behavior of the entropy + logarithmic correction
- The ‘discretization effect’ comes from LQG specific features

Conclusions

- We propose a specific way of implementing Witten's analogy in the framework of black hole entropy in LQG.

- The results seem to indicate that the analogy works in this particular case.

- This is a motivation for the search of a deeper relation between the description of black holes in LQG and a conformal field theory.

- The picture obtained is consistent with the physical problem.

- Is consistent with the general idea of a conformal symmetry driving the linear behaviour of entropy plus some extra quantum effects at the Planck scale specific from LQG.

New perspective: Black hole entropy and CFT's