BLACK HOLES IN

LOOP QUANTUM GRAVITY

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BLACK HOLES AND QUANTUM GRAVITY ?

Black Holes are, as Chandrasekhar used to say:
"... the most perfect objects there are in The Universe: the only elements in their construction are our concepts of space and time. Since GR predicts a single family of solutions, they are the simplest as well." **They are the crown of classical physics in terms of their simplicity and beauty.**

But, Bekenstein and Hawking told us that :

i) Black Holes satisfy some 'thermodynamic-like laws'.

$$\delta M = \frac{\kappa}{8\pi G}\,\delta A \;\;\Rightarrow\;\; M \leftrightarrow E, \quad \kappa \leftrightarrow T\,, \quad A \leftrightarrow S$$

ii) When one invokes quantum mechanics ($\hbar$) then something 'weird' happens:

$$E = M$$

$$T = \frac{\kappa\, \hbar}{2\pi} \ ,$$

and

$$S = \frac{A}{4\, G \hbar}$$

The black holes appear to have thermodynamic properties!

But, what are the underlying degrees of freedom responsible for entropy?

The standard wisdom is that only with a full marriage of the Quantum and Gravity will we be able to understand this.

Different approaches:

- **String Theory**
- **Causal Sets**
- **Entanglement Entropy**
- **Loop Quantum Gravity: This Workshop!!**

MOTIVATION

- **How do we characterize black holes in equilibrium?**
- **What are quantum horizon states?**
- **Which states should we count?**
- **How does the entropy behave?**
- **Large BH: Bekenstein-Hawking entropy**
- **What happens when we look at small BH's?**

PLAN OF THE TALK

1. Some History

2. Classical Preliminaries

3. Quantum Preliminaries

4. Quantum Horizon Geometry

5. Counting and Entropy

Work of many people, including A. Ashtekar, J. Baez, AC, M. Domagala, J. Lewandowski, K. Meissner, J. Engle, E. Fernandez-Borja, J. Diaz-Polo, K. Krasnov, R. Kaul, A. Ghosh, P. Majumdar, P. Mitra, A. Perez, C. Rovelli, H. Sahlmann and more ...

1. SOME HISTORY

- **96' Krasnov and Rovelli consider punctures as horizon degrees of freedom.**
- **97' Isolated Horizon boundary conditions understood.**
- **99' Quantum Horizon Geometry fully understood (ABK).**
- **00' Logarithmic corrections computed**
- **02' Possible relation to QNM proposed (SO(3) *vs* SU(2))**
- **04' Error in original ABK computation found. A new counting proposed (DLM)**
- **05'- Several new countings proposed (GM, Dreyer et al, ...)**
- **06' Direct counting of small BH states. New structures found.**
- **07'- Complete counting in terms of number theory. Relation with CFT.**

The Beginning

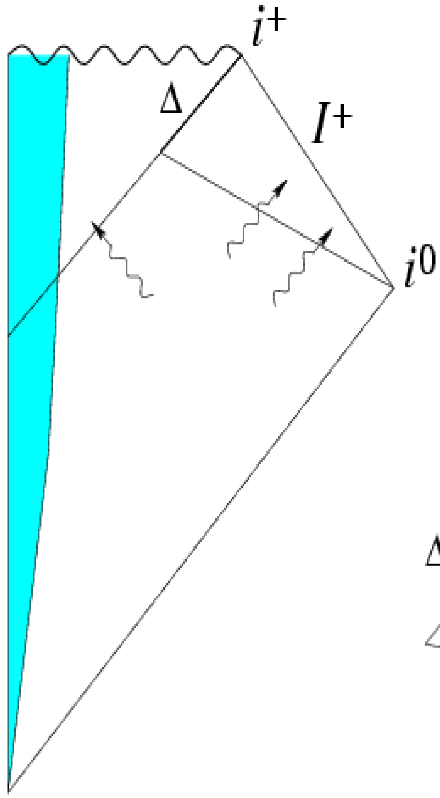
Physically, one is interested in describing black holes in equilibrium. That is, equilibrium of *the horizon*, not the exterior. Can one capture that notion via boundary conditions?

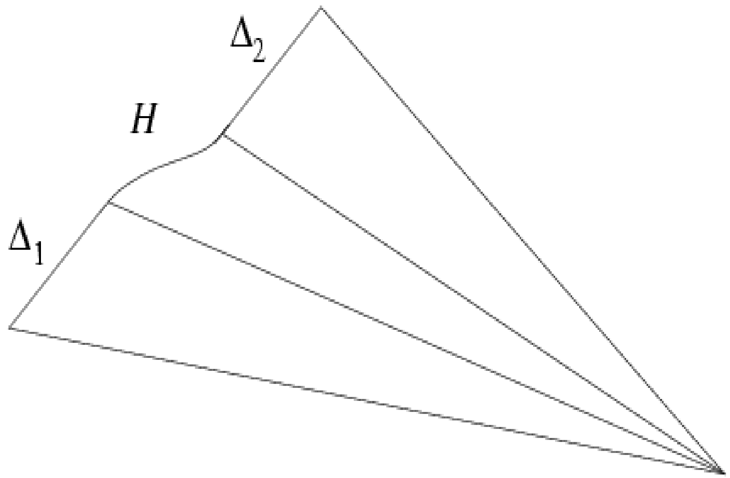
Yes! Answer: Isolated Horizons

Isolated horizon boundary conditions are imposed on an inner boundary of the region under consideration.

The interior of the horizon is cut out. In this a physical boundary?
No! but one can ask whether one can make sense of it:

What is then the physical interpretation of the boundary?





• The boundary Δ, the 3-D isolated horizon, provides an effective description of the degrees of freedom of the inside region, that is cut out in the formalism.

• The boundary conditions are such that they capture the intuitive description of a horizon in equilibrium and allow for a consistent variational principle.

• The quantum geometry of the horizon has independent degrees of freedom that fluctuate 'in tandem' with the bulk quantum geometry.

• The quantum boundary degrees of freedom corresponding to a macrostate (completely characterized by all multipole moments) are then responsible for the entropy.

• The entropy thus found can be interpreted as the entropy assigned by an 'outside observer' to the (2-dim) horizon $S = \Sigma \cap \Delta$.

• Some issues: is this the entropy 'contained by the horizon'? Is there a 'holographic principle' in action? Can we associate an entanglement entropy between the interior and the exterior?

ISOLATED HORIZONS

An isolated horizon is a null, non-expanding horizon Δ with some notion of translational symmetry along its generators. Technically we consider Weakly Isolated Horizons (WIH). We will also restrict ourselves to Type I, spherically symmetric, horizons (see Engle's talk for type II). There are two main consequences of the boundary conditions:

• The gravitational degrees of freedom induced on the horizon are captured in a $U(1)$ connection,

$$W_a = -\frac{1}{2}\,\Gamma^i_a\, r_i$$

• The total symplectic structure of the theory (and this is true even when most matter is present) gets split as,

$$\Omega_{\rm tot} = \Omega_{\rm bulk} + \Omega_{\rm hor}$$

with

$$\Omega_{\rm hor} = \frac{a_0}{8\pi^2\, G\gamma} \oint_S \mathrm{d}W \wedge \mathrm{d}W'$$

• The 'connection part' and the 'triad part' at the horizon must satisfy the condition,

$$F_{ab} = -\,\frac{2\pi\,\gamma}{a_0}\, E^i_{ab}\, r_i,$$

which is called the 'horizon constraint'.

CONSTRAINTS

The Hamiltonian formalism tells us is a natural way what is gauge and what not. In particular, with regard to the constraints we know that:

• The relation between curvature and triad, the horizon constraint, is equivalent to Gauss' law.

• Diffeomorphims that leave S invariant are gauge (their vector field are tangent to S).

• The scalar constraint must have $N|_{\rm hor} = 0$. Thus, the scalar constraint leaves the horizon untouched; any gauge and diff-invariant observable *is* a Dirac observable!

In the quantum theory of the horizon we have to implement these facts.

QUANTUM THEORY: THE BULK

A canonical description:

$$A^i_a \quad SU(2)\,\text{connection} \quad ; \quad E^a_i \quad \text{triad}$$

with $A^i_a = \Gamma^i_a - \gamma\, K^i_a$. **Loop Quantum gravity on a manifold without boundary is based on two fundamental observables of the fundamental variables :**

Holonomies, $h_e(A) := \mathcal{P}\exp(\int_e A)$

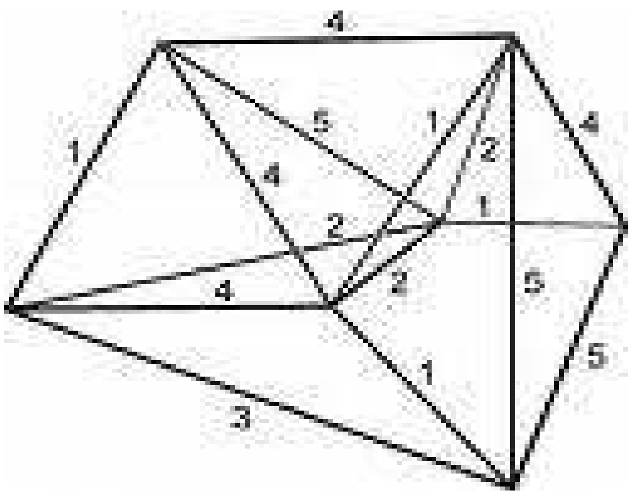
and

Electric Fluxes, $E(f, S) := \int_S \mathrm{d}S^{ab} E^i_{ab}\, f^i$

The main assumption of Loop Quantum Gravity is that these quantities become well defined operators. LOST Theorem: There is a unique represntation on a Hilbert space of these observabes that is *diffeomorphism invariant*.

Hilbert space:

$$\mathcal{H}_{\text{AL}} = \oplus_{\text{graphs}} \mathcal{H}_{\Upsilon} = \text{Span of all Spin Networks } |\Upsilon, \vec{j}, \vec{m}\rangle \tag{1}$$

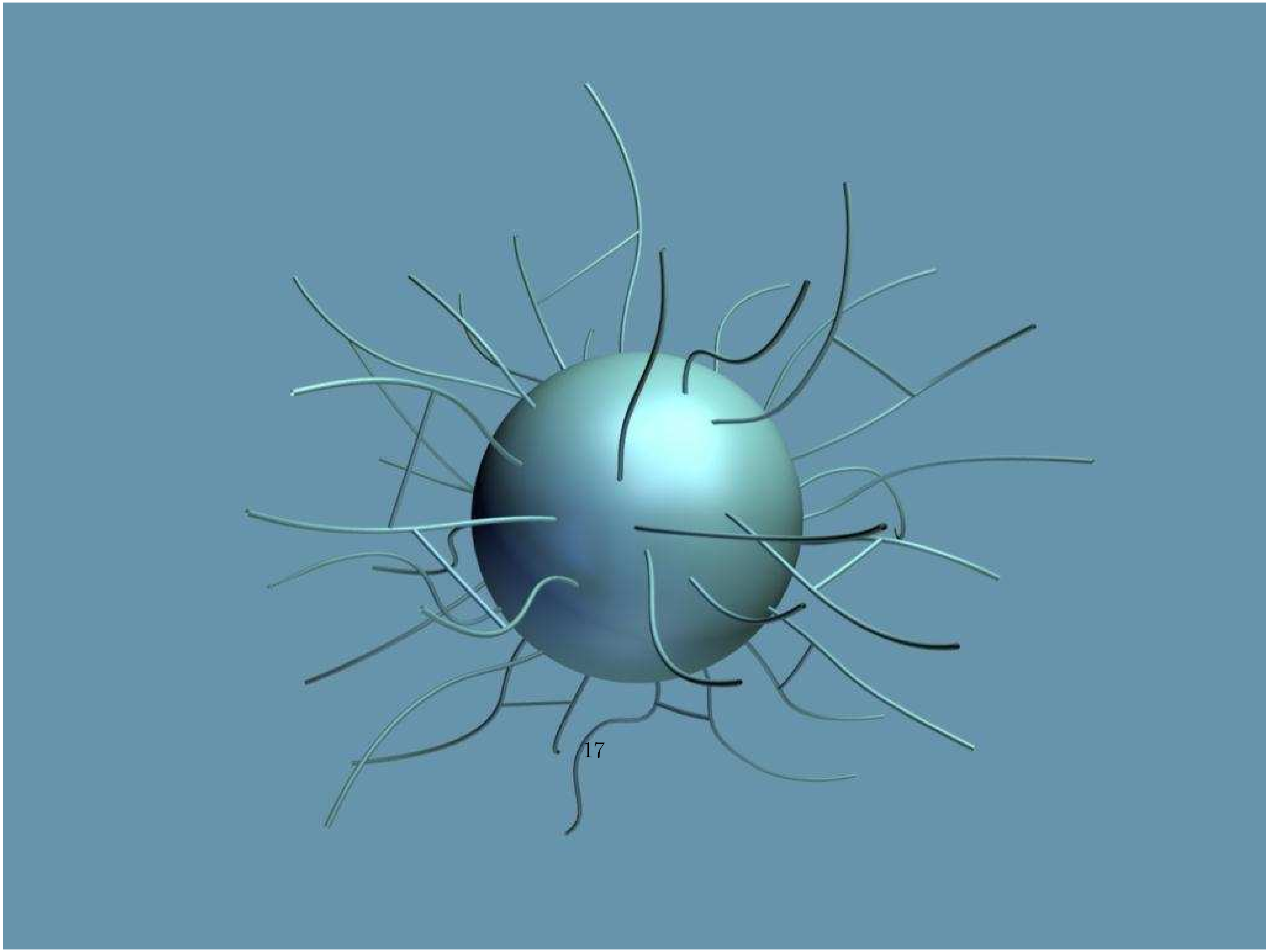


A Spin Network $|\Upsilon, \vec{j}, \vec{m}\rangle$ is a state labelled by a graph Υ, and some colourings $(\vec{j}, \vec{m})$ associated to edges and vertices.

The spin networks have a very nice interpretacion. They are the eigenstates of the quantizad geometry, such as the area operator,

$$\hat{A}[S] \cdot |\Upsilon, \vec{j}, \vec{m}\rangle = 8\pi\ell_{\mathrm{Pl}}^2 \gamma \sum_{\text{edges}} \sqrt{j_i(j_i + 2)}\, |\Upsilon, \vec{j}, \vec{m}\rangle \tag{2}$$

One sees that the edges of the graph, excite the quantum geometry of the surface S at the intersection points between S and Υ.



HORIZON QUANTUM THEORY

Total Hilbert Space is of the form:

$$\mathcal{H} = \mathcal{H}_{\rm V} \otimes \mathcal{H}_{\rm S}$$

where $\mathcal{H}_{\rm S}$, the surface Hilbert Space, can be built from $U(1)$ Chern Simons Hilbert spaces for a sphere with punctures. This comes about since the symplectic structure of the horizon is that of CS for W.

The conditions on $\mathcal{H}$ that we need to impose are: Invariance under diffeomorphisms of S and the quantum condition on Ψ:

$$\left(\mathrm{Id} \otimes \hat{F}_{ab} + \frac{2\pi\,\gamma}{a_0}\,\hat{E}^i_{ab}\,r_i \otimes \mathrm{Id}\right) \cdot \Psi = 0$$

For technical reason one considers the exponential of this, which is object that is well defined in the CS theory.

Then, the theory we are considering is a quantum theory, with an isolated horizon of fixed area a_0 (and in general, other multiple-moments).

Physical state will be such that, in the bulk, they satisfy the ordinary constraints (big assumption!!) and, at the horizon, the quantum horizon condition is satisfied.

This condition relates the horizon quantum geometry to the bulk quantum geometry.

ENTROPY

We are given a black hole of area a_0. What entropy can we assign to it? Let us take the microcanonical viewpoint. We shall count the number $\mathcal{N}$ of microstates compatible with the macrostate, such that they satisfy:

- **The area eigenvalue $\langle \hat{A} \rangle \in [a_0 - \delta, a_0 + \delta]$**
- **The quantum horizon condition.**

The entropy $\mathcal{S}$ will be then given by

$$\mathcal{S} = \ln \mathcal{N}.$$

The challenge now is to identify those states that satisfy the two conditions, and count them.

CHARACTERIZATION OF THE STATES

There is a convenient way of characterizing the states by means of the spin network basis. If an edge of a spin network with label j_i ends at the horizon S, it creates a puncture, with label j_i. The area of the horizon will be the area that the operator on the bulk assigns to it: $A = 8\pi\gamma\ell_{\rm Pl}^2\sum_i\sqrt{j_i(j_i+1)}$.

Is there any other quantum number associated to the punctures p_i? Yes! the eigenstates of $\hat{E}_{ab}$ that are also half integers m_i, such that $-|j_i| \leq m_i \leq |j_i|$. The quantum horizon condition relates these eigenstates to those of the horizon Chern-Simons theory. The requirement that the horizon is a sphere (topological) then imposes a 'total projection condition' on $m's$:

$$\sum_i m_i = 0$$

A 'configuration' the quantum horizon is then characterized by a set of punctures p_i and to each one a pair of half integer (j_i, m_i).

The counting has three steps:

i) Given the classical area a_0, find the possible sets $\{n_k\}$ of configurations of j's compatible with it.

ii) Given such a configuration, $\{n_k\}$, find the degeneracy $R(\{n_k\})$ associated the possible orderings.

If we are given N punctures and two assignments of labels (j_i, m_i) and (j'_i, m'_i). Are they physically distinguishable? or a there some 'permutations' of the labels that give indistinguishable states?

That is, what is the statistics of the punctures?

As usual, we should let the theory tell us. One does not postu-

late any statistics. If one treats in a careful way the action of the diffeomorphisms on the punctures one learns that when one has a pair of punctures with the same labels j and m, then the punctures are indistinguishable and one should not count them twice. In all other cases the states are distinguishable.

iii) Given the degeneracy induced by the 'statistics', one has to find the degeneracy associated to the number of horizon states compatible with the configurations $\{n_k\}$. This step involves a choice. Are we going to keep track of both labels j_i, m_i? or are we just going to count horizon state, labelled by m's, that could come from some j's. This is the distinction between the DLM and GM countings (see talks by Lewandowski and Ghosh). Since this is the step that knows about the horizon thery, it is at this point that a relation with CFT can be found.

Need to understand the assumptions of different countings.

THE COUNTING: A SIMPLIFIED CASE

We start with an isolated horizon, with an area a_0 and ask how many states are there compatible with the two conditions, and taking into account the distinguishability of the states.
First Approach: Count just the different configurations and forget about $\sum_i m_i = 0$. Thus, given $\{n_k\} = \{n_{1/2}, n_1, n_{3/2}, \ldots, n_{k/2}\}$,we count the number of states:

$$N = \frac{(\sum_k n_k)!}{\Pi_j\,(n_j!)}\,\Pi_j\,(2j+1)^{n_j} \tag{3}$$

The first factor is the degenerary $R(\{n_k\})$, and the second comes from $P(\{n_k\})$. Taking the *large area approximation* $A >> \ell_{\rm Pl}$**, and using the Stirling approximation. One gets:**

$$S = \frac{A}{4\ell_{\rm Pl}^2}\,\frac{\gamma_0}{\gamma} \tag{4}$$

with γ_0 the solution to $\sum_j (2j+1) e^{2\pi\,\gamma_0 \sqrt{j_i(j_i+1)}} = 1$.

(and $\sum_j 2\, e^{2\pi\, \gamma_M \sqrt{j_i(j_i+1)}} = 1$ for DLM).
The introduction of the projection constraint introduces a first correction to the entropy area relation as

$$S = \frac{A}{4\ell_{\mathrm{Pl}}^2}\, \frac{\gamma_0}{\gamma} - \frac{1}{2}\ln(A) + \ldots$$

- **If we want to make contact with the Bekenstein-Hawking we have to chose $\gamma = \gamma_0$.**
- **The logarithmic correction is universal.**
- **The formalism can be generalized to more general situations, and the result is** *the same***:**
  - **Maxwell, Dilatonic and Yang Mills Couplings.**
  - **Cosmological, Distortion and Rotation (See Engle's talk).**
  - **Non-minimal Couplings ($S \neq A/4$).**
  - **Topological 'theta term' (see Perez' talk).**

COUNTING BY NUMBERS: ENTROPY QUANTIZATION

A brute force approach is to tell a computer how to count for a range of area a_0 at the Planck scale.

Both the oscillations found with a large value of δ as well as these structures in the 'spectrum' posses the same periodicity

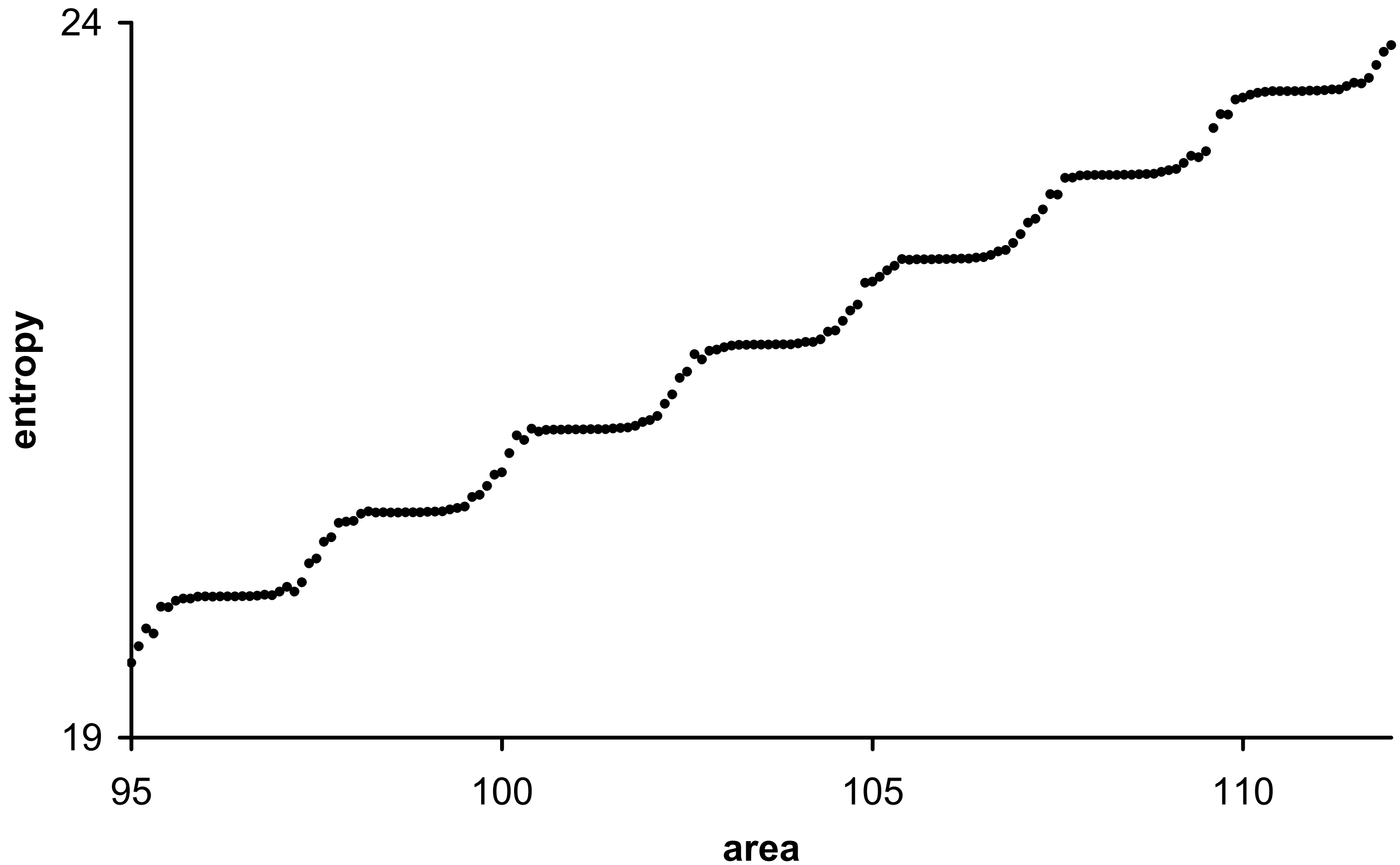
$$\delta A_0 \approx 2.41\ \ell_p^2$$

Is there any physical significance to this periodicity?

we chose the interval:

$$2\,\delta = \Delta A_0$$

With this choice, the plot of the entropy *vs* area becomes:


24
19
entropy
95
100
105
110
area

WHAT DOES THIS MEAN?

Instead of oscillations, Entropy seems to increase in discrete steps.

Furthermore, the height of the steps seems to approach a constant value as the area of the horizon grows, thus implementing in a rather subtle way the conjecture by Bekenstein that entropy should be equidistant for large black holes.

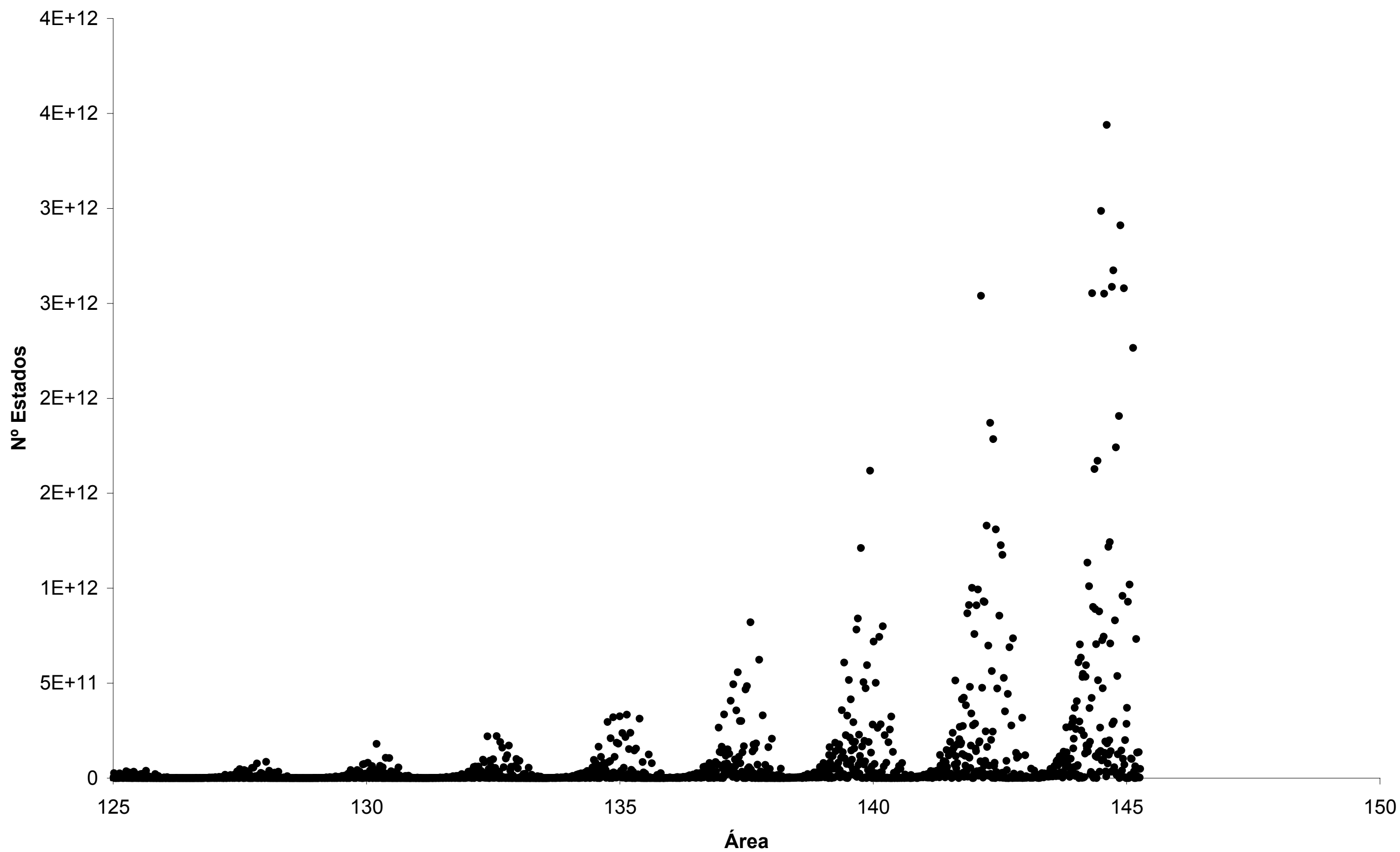
This result is robust: Independent of the counting!

Is there any way of understanding this? Maybe

While the constant number in which the entropy of large black holes 'jumps' is:

$$\Delta S \mapsto 2\,\gamma_0\,\ln(3)$$

(???!!)


4E+12
4E+12
3E+12
3E+12
2E+12
2E+12
1E+12
5E+11
0
Nº Estados
125
130
135
140
145
150
Área

Some recent analytic understanding (See Sahlmann's talk, also Agullo, Fernandez-Borja, Diaz-Polo) on the origin of these 'bands'.

The problem of characterizing $\{n_k\}$ has been completely solved recently. Use of number theory and related tools (generating functions, asymptotics), has been fundamental (see Barbero's talk).

The expectation that BH entropy can be related to CFT given its importance in relating CS and WZW in SU(2), has been recently cristalized. Exciting progress and many things to understand (see Diaz-Polo's talk).

CONCLUSIONS

- **Isolated Horizons provide a consistent framework to incorporate black holes.**
- **One can consistently quantize the theory**
- **Entropy is** *finite* **and the dominant term is linear in Area.**
- **Any black hole of interest is included**
- **Unexpected features appear by considering Planck size horizons.**
- **Contact with Bekenstein's heuristic model, and Mukhanov-Bekenstein in a subtle manner**
- **Recent exciting analytical understanting allows to examine these issues in detail.**

- **Is there more?**

OUTLOOK

- **We have not dealt with the singularity**
- **Ashtekar-Bojowald 'paradigm' for an extended quantum space-time**
- **Based on expectations about singularity resolution coming from LQC (?)**
- **Hawking radiation?**
- **Information Loss Puzzle (see Varadarajan's talk)**
- **Full theory: How to specify quantum black holes from the full theory?**