Emergent Horizons in the Laboratory

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Event Horizon

Collapsing matter

Singularity

Light cones, light rays

Event horizon

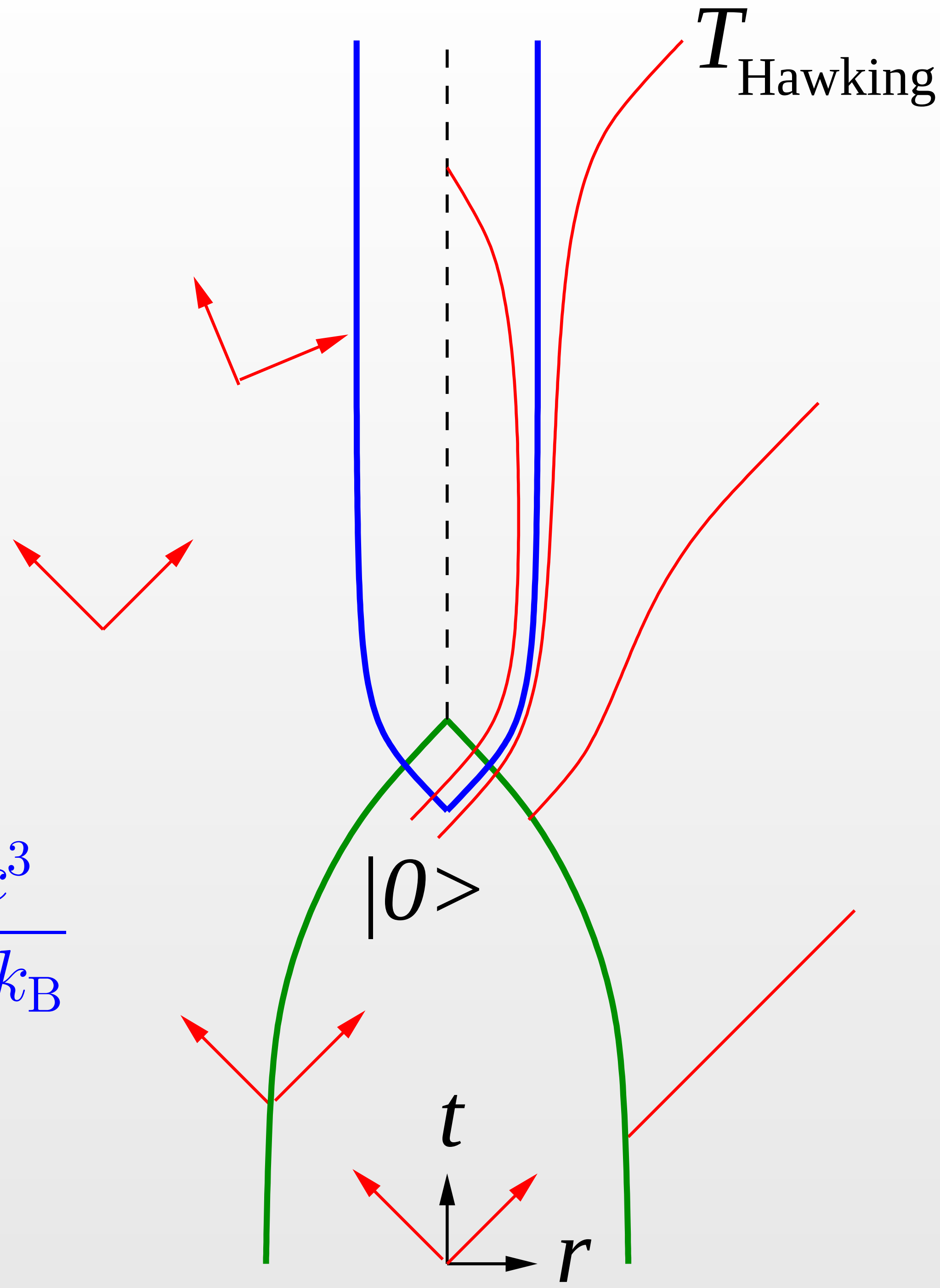
Distortion of quantum fluctuations

$\longrightarrow$ Hawking effect

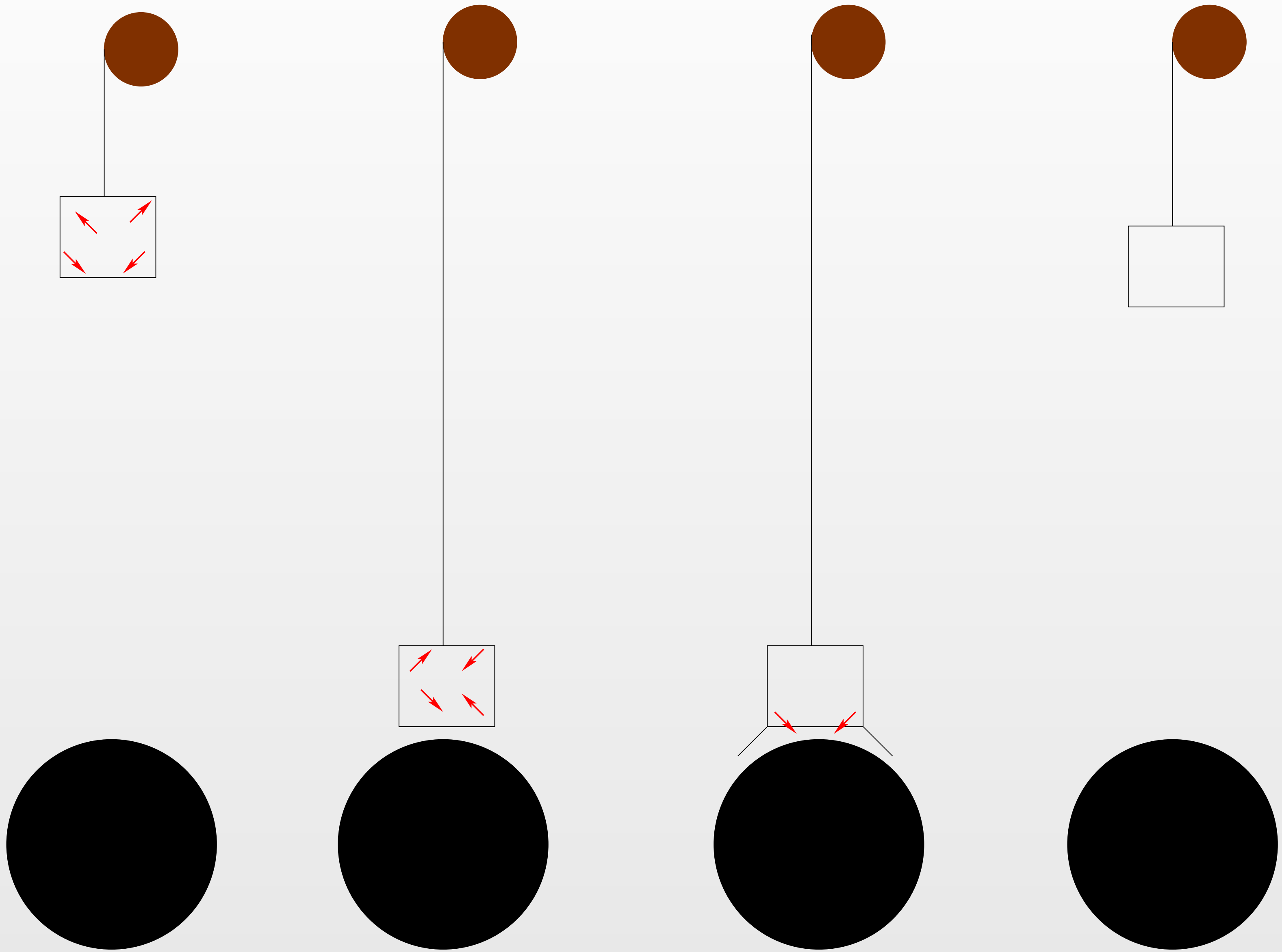
$$T_{\text{Hawking}} = \frac{1}{8\pi M} \frac{\hbar\, c^3}{G_{\text{N}} k_{\text{B}}}$$

gedanken experiments

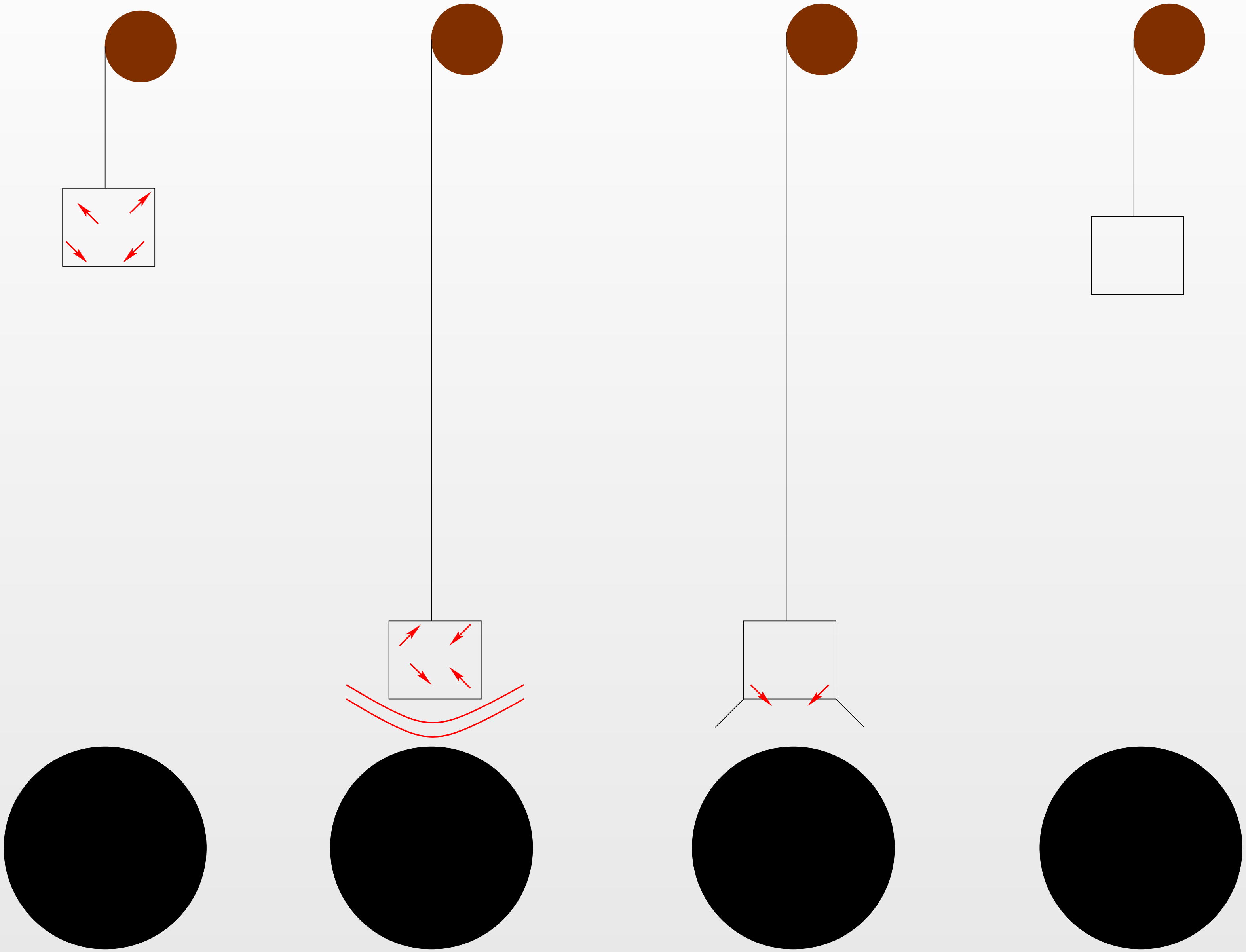
$\longrightarrow$ Black hole entropy…



Perpetuum mobile (2^{nd} kind)?



Resolution: Quantum Effects



W. G. Unruh, R. M. Wald, Phys. Rev. D **25**, 942 (1982)

Transplanckian Problem

Collapsing matter

Singularity

Light cones, light rays

Event horizon

Distortion of quantum fluctuations

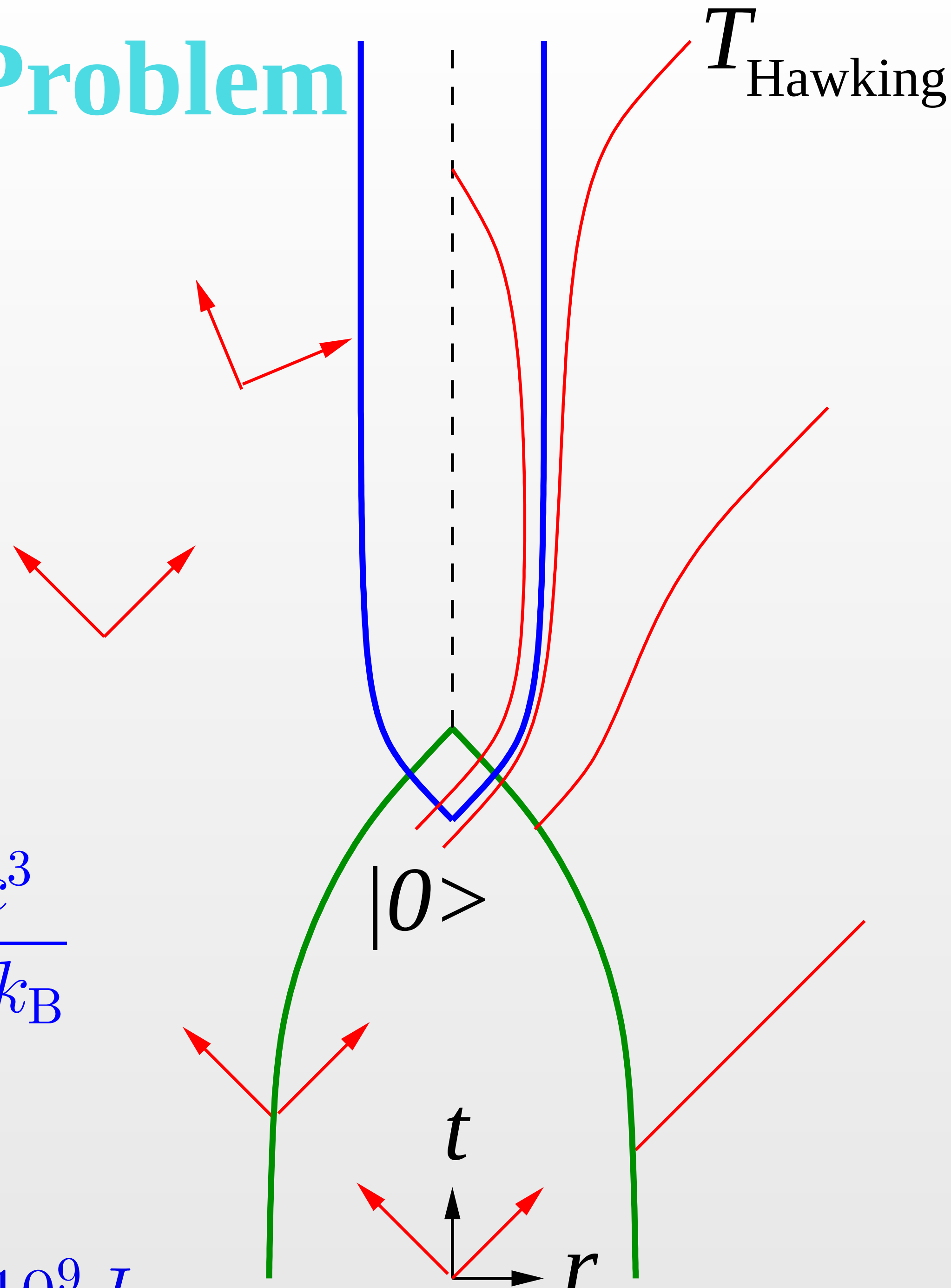
$\longrightarrow$ Hawking effect

$$T_{\rm Hawking} = \frac{1}{8\pi M}\frac{\hbar\, c^3}{G_{\rm N} k_{\rm B}}$$

Problem: red-shift

$\longrightarrow$ Planck scale

$$E_{\rm Pl} = \sqrt{\hbar c^5/G_{\rm N}} \approx 2\times 10^9 J$$



Bill Unruh's Idea

Sound waves in irrotational flow $\delta\boldsymbol{v} = \boldsymbol{\nabla}\phi$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{v}_0\right)\frac{\varrho_0}{c_{\mathrm{s}}^2}\left(\frac{\partial}{\partial t} + \boldsymbol{v}_0\cdot\boldsymbol{\nabla}\right)\phi = \boldsymbol{\nabla}\cdot(\varrho_0\boldsymbol{\nabla}\phi)$$

Scalar field ϕ in curved space-time

$$\Box_{\mathrm{eff}}\phi = \frac{1}{\sqrt{-g_{\mathrm{eff}}}}\partial_\mu\left(\sqrt{-g_{\mathrm{eff}}}\,g_{\mathrm{eff}}^{\mu\nu}\partial_\nu\phi\right) = 0$$

Painlevé-Gullstrand-Lemaître metric

$$g_{\mathrm{eff}}^{\mu\nu} = \frac{1}{\varrho_0 c_{\mathrm{s}}}\begin{pmatrix} 1 & \boldsymbol{v}_0 \\ \boldsymbol{v}_0 & \boldsymbol{v}_0\otimes\boldsymbol{v}_0 - c_{\mathrm{s}}^2\mathbf{1}\end{pmatrix}$$

Phonons (quantized)	$\leftrightarrow$	Quantum fields
Fluid flow (classical)	$\leftrightarrow$	Gravitational field
Euler equation	$\neq$	Einstein equations

W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981)

Generalizations

General linearized low-energy effective action for scalar Goldstone-mode quasi-particles (e.g., phonons)

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi)G^{\mu\nu}(\underline{x}) + \mathcal{O}(\phi^3) + \mathcal{O}(\partial^3)$$

Analogy to quantum fields in curved space-times

$G^{\mu\nu} \to g_{\text{eff}}^{\mu\nu}\sqrt{-g_{\text{eff}}} \to$ **Universal properties**

e.g., phonons, ripplons, magnons. . .

Similarly for (non-scalar) photons in certain media

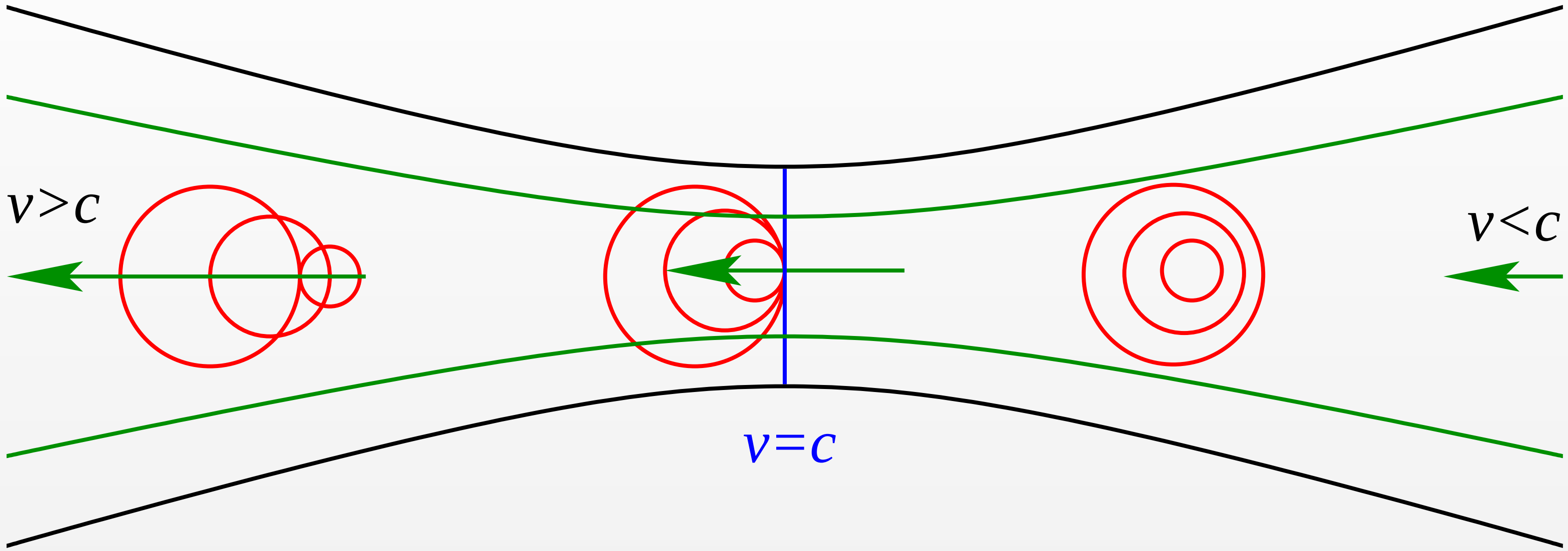
$$g_{\text{eff}}^{\mu\nu} = g_{\text{M}}^{\mu\nu} + (\varepsilon - 1)\, u^\mu\, u^\nu$$

$\to$ Gordon metric

[Barceló, Corley, Fischer, Jacobson, Liberati, Unruh, Visser, and many others]

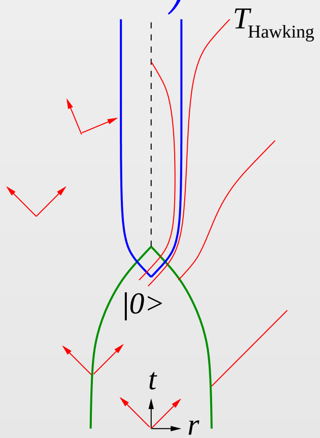
R. S., Class. Quantum Grav. **25**, 114011 (2008)

De Laval Nozzle



Fluid flow, Wall, Sound waves, Event horizon

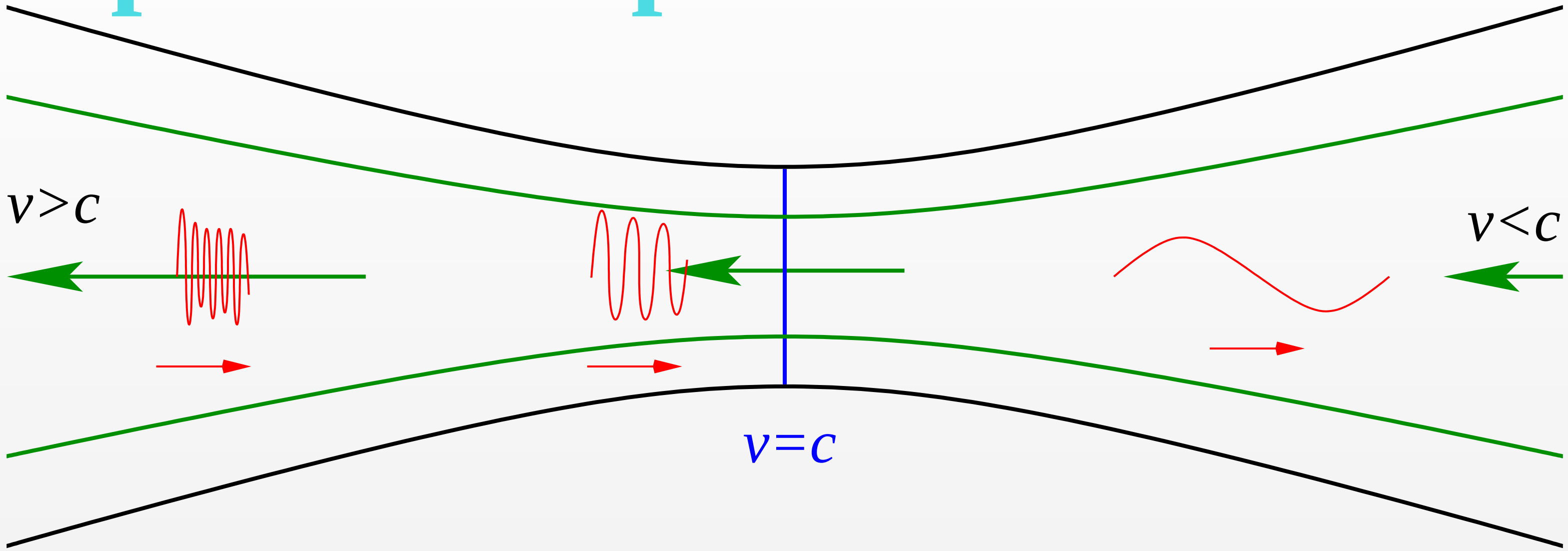
$$T_{\mathrm{Hawking}} = \frac{\hbar}{2\pi\, k_{\mathrm{B}}} \left| \frac{\partial}{\partial r} \left(v_0 - c_{\mathrm{s}} \right) \right| = \mathcal{O}(\mathrm{nK} \ldots \mathrm{K})$$



Toy model for underlying theory
(including quantum gravity)

Experimentally measurable?!?

Impact of Dispersion Relation



Small k $\rightarrow$ Euler equation

$\rightarrow$ linear dispersion $\omega = c_s k$

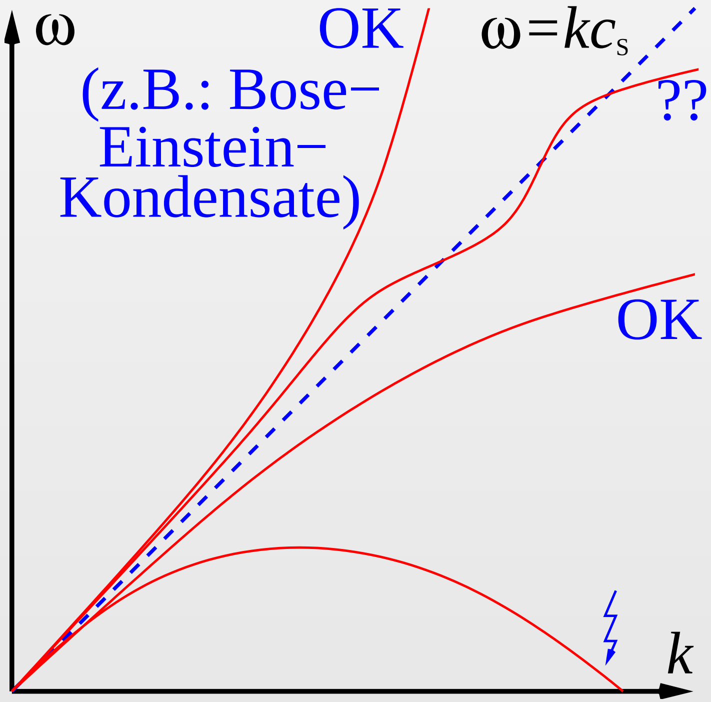
Large k $\rightarrow$ deviations:

- sub-sonic
- super-sonic

W. G. Unruh and R. S.,

Phys. Rev. D **71**, 024028 (2005)

[Corley, Jacobson, Parentani etc.]



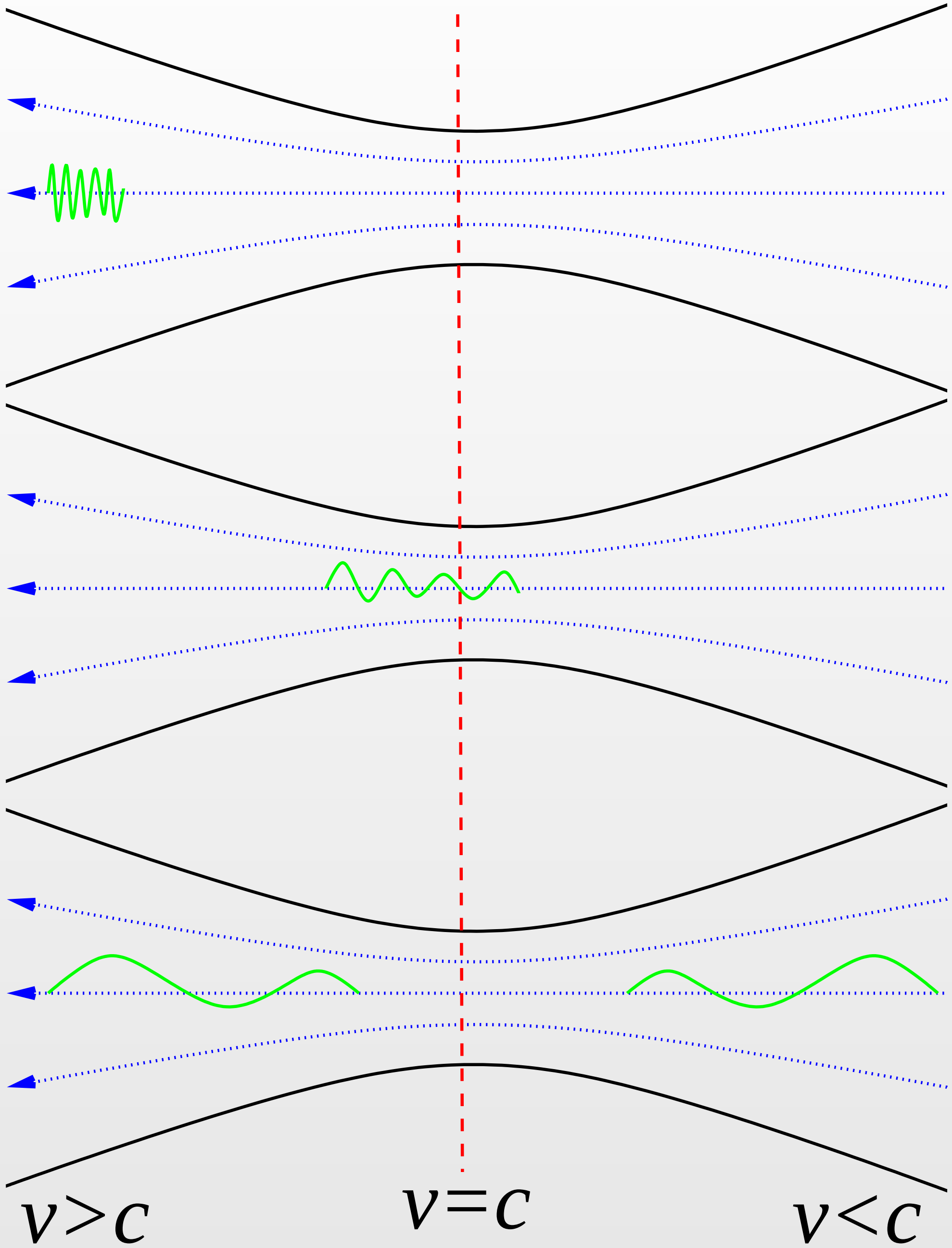
Supersonic Dispersion

Initial wave-packet in its ground state is ripped apart into:

Hawking radiation plus infalling partner

Entanglement (squeezed state) $\rightarrow$ thermality

Caution: Bekenstein entropy does not apply!



Eddington-Finkelstein Metric

Metric $ds^2 = (1 - 2M/r)dV^2 - 2dV\,dr$

$$\left(2\partial_V\partial_r + \partial_r\left[1 - \frac{2M}{r} + f(\partial_r^2)\right]\partial_r\right)\Phi = 0\,,$$

Dispersion relation $f(\partial_r^2) \to$ Hawking temperature

$$T_{\text{Hawking}}(\omega) = \frac{v_{\text{group}}(\omega)v_{\text{phase}}(\omega)}{8\pi M}$$

Ultra-violet catastrophe?

$$\omega^2 = c^2k^2(1 + \ell_{\text{P}}^2k^2) \quad \text{vs} \quad \omega = \frac{ck}{\sqrt{1 - \ell_{\text{P}}^2k^2}}$$

Cf. special relativity $E = mc^2/\sqrt{1 - v^2/c^2}$

R. S. and W. G. Unruh, Phys. Rev. D **78**, (R)041504 (2008)

Miles Instability

Lab system: Ω (observer at rest at $r \uparrow \infty$)

Local fluid system: ω (freely falling observer)

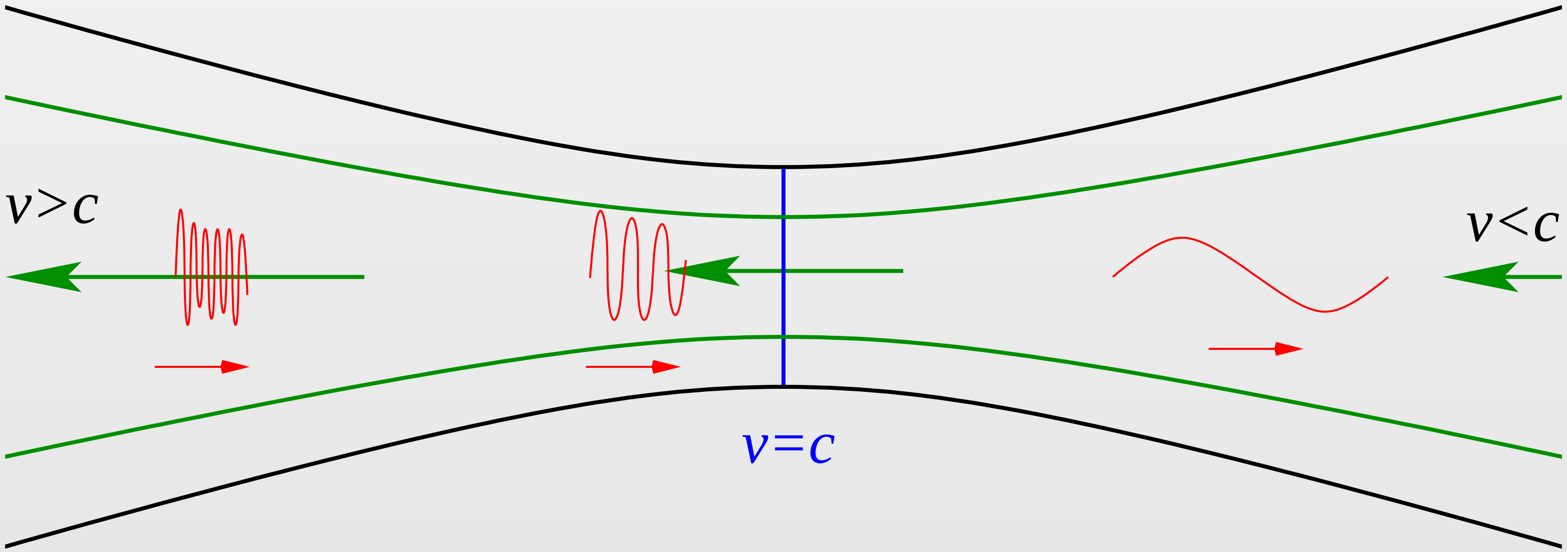
$$(\Omega + vk)^2 + i\gamma\Omega = \omega^2(k)$$

Miles instability

W. G. Unruh and R. S.,

Phys. Rev. D **71**, 024028 (2005)

[Corley, Jacobson, Parentani etc.]

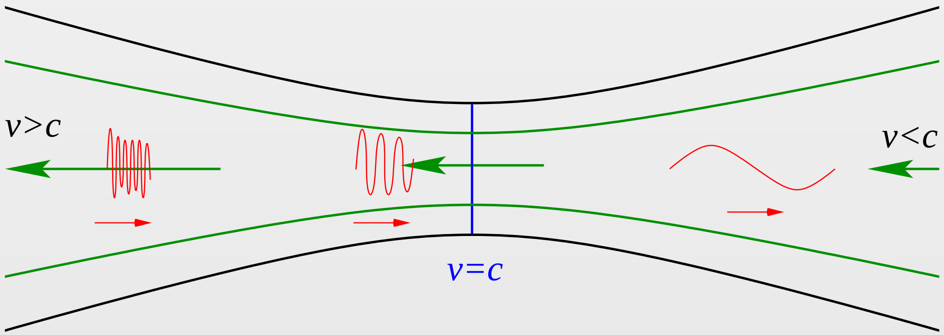


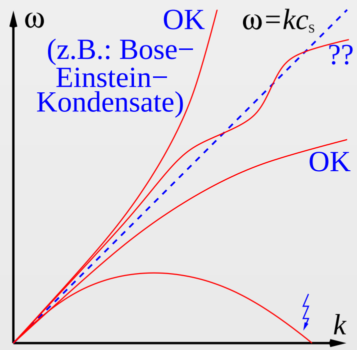
Lessons for Quantum Gravity?

Hawking radiation is quite robust (i.e., independent of the microscopic structure) for a large class of systems and does not require the Einstein equations.

[Unruh, Jacobson, Corley, RS, Parentani etc.]

But there are also (physical) examples, which show deviations from the Hawking effect –
e.g., depending on whether the "space-time foam" is freely falling or prefers the static frame…





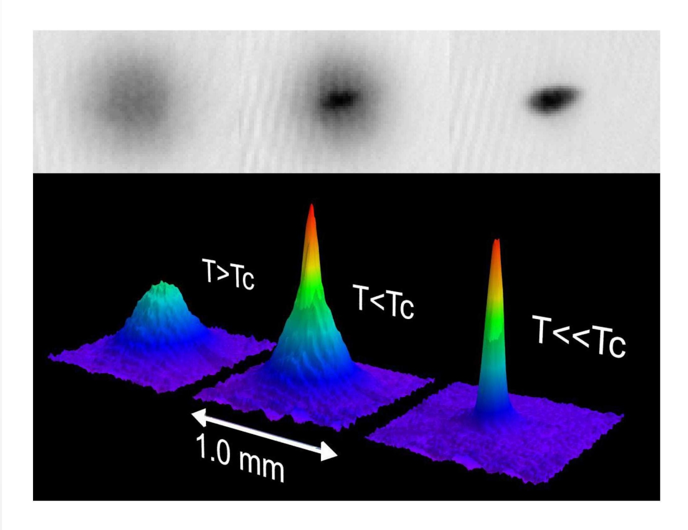
Impact of (non-linear) interactions not understood…

Kinematics $\rightarrow$ Dynamics

Kinematics of phonons:
quantitative analogy $g_{\text{eff}}^{\mu\nu}$
Dynamics of background:
only qualitative analogy

E.g., Bose-Einstein condensates

http://cua.mit.edu/ketterle-group



Euler equation (classical)	$\leftrightarrow$	Einstein equations
Phonons (quantum)	$\leftrightarrow$	Gravitons(?)
Healing length	$\leftrightarrow$	Planck length?
Many-body Hamiltonian	$\leftrightarrow$	Quantum gravity?
Zero-point pressure	$\leftrightarrow$	Cosmological constant?

R. S., M. Uhlmann, Y. Xu and U. R. Fischer, Phys. Rev. D **72**, 105005 (2005)

R. Balbinot et al, Phys. Rev. Lett. **94**, 161302 (2005)

Lessons for Quantum Gravity?

- we can quantize (linearized) phonons for small k
- beyond linear order: UV divergences (non-ren.)
- sum of zero-point fluctuations of phonon modes (extrapolating Euler equation to large k) up to cut-off does **not** yield correct result

  [R. S., Proceedings of Science (QG-Ph) 036 (2007)]
- quantization of vorticity from Euler equation??
  - Bose-Einstein condensates $\xi \leftrightarrow m \leftrightarrow \Gamma$
  - superfluid Helium $\xi_{\text{roton}} \not\leftrightarrow m \leftrightarrow \Gamma$
- several cut-off scales ($\longrightarrow$ Planck scale?)
  - breakdown of Euler $\omega^2 = c^2k^2(1 \pm k^2\xi^2)$
  - circulation quantum Γ of one vortex
  - UV cut-off $\ll \xi$

Summary

Analogy: gravity $\leftrightarrow$ condensed matter

- Robustness of Hawking radiation
  exceptions: friction, UV-catastrophe…
- Black-hole information “paradox”?
- Experiments: Hawking radiation in the lab?
- Kinematics $\rightarrow$ dynamics: quantum fluids
  quantum back reaction, quantization of vorticity
- Outlook: cosmic horizons
  expanding fluids, phase transitions
  inflation as a quantum phase transition?

Apparent Horizon

Expanding condensate

Sound waves

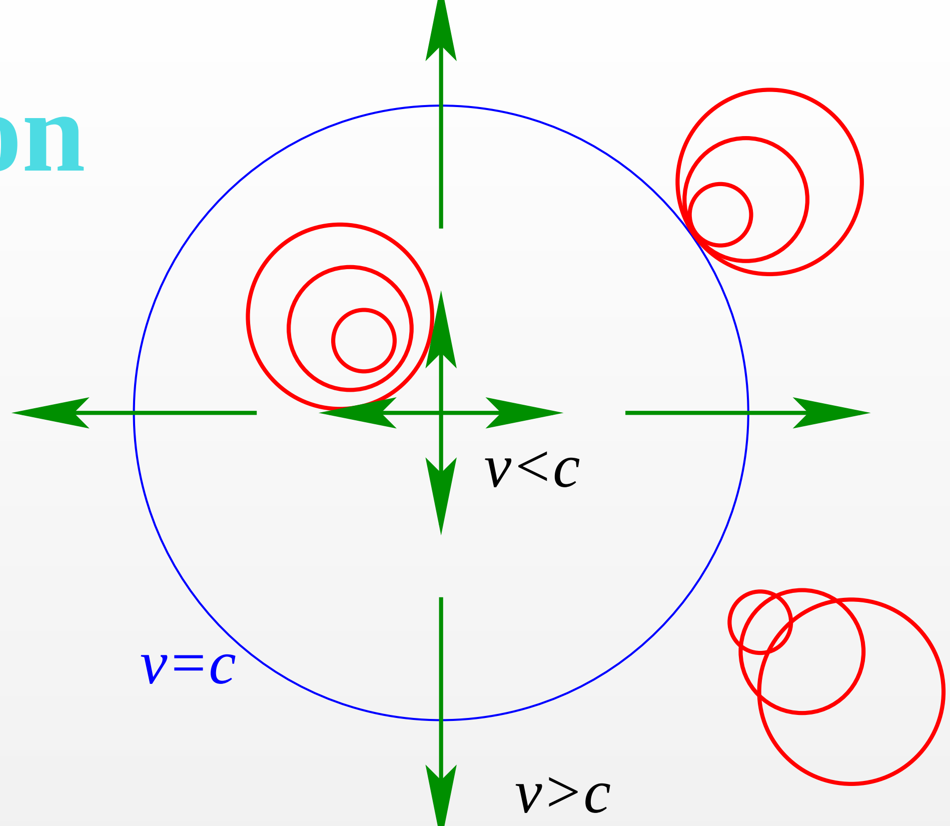
Apparent horizon at

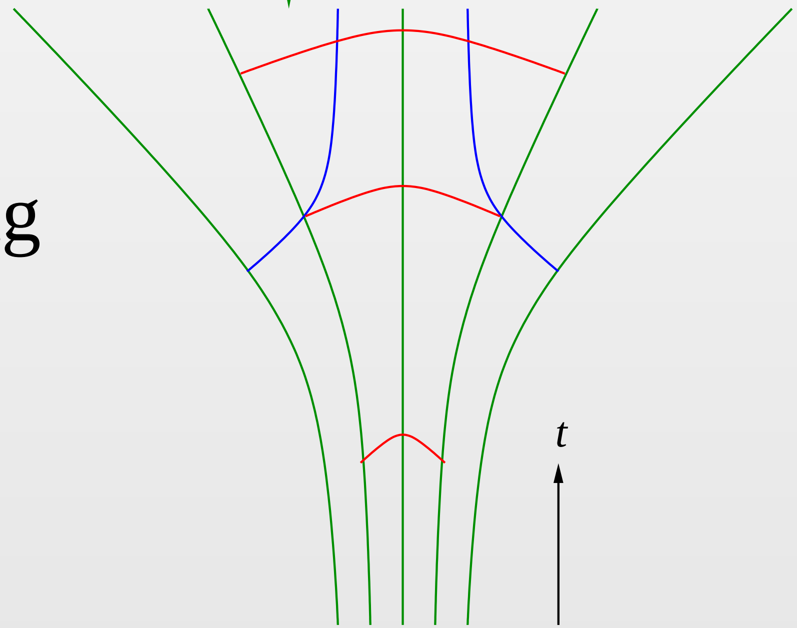
$v_{\text{Fluid}} = c_{\text{Sound}}$

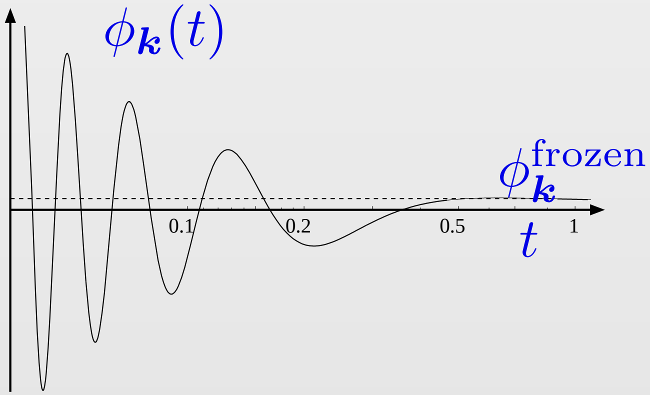
Analogous to
expanding universe

Phonon modes:

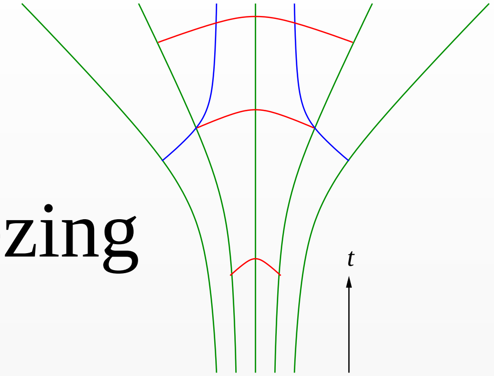
+ oscillation (initially)
+ horizon crossing $\longrightarrow$ freezing



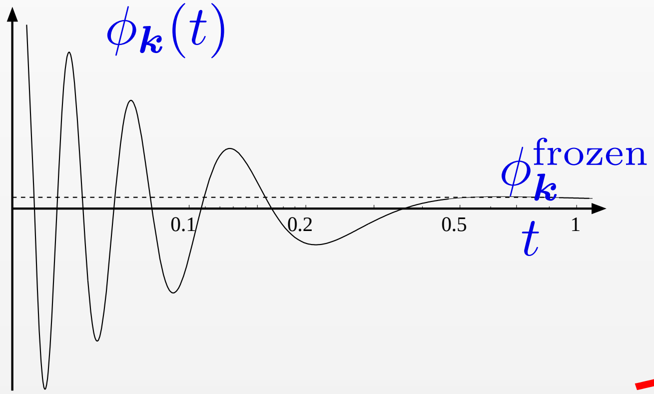


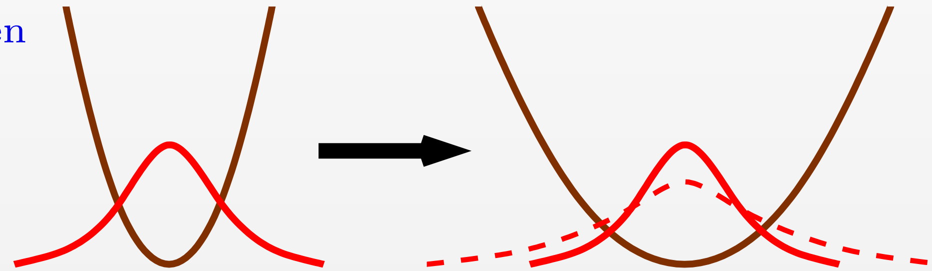


Quantum Fluctuations



Oscillation → horizon crossing → freezing





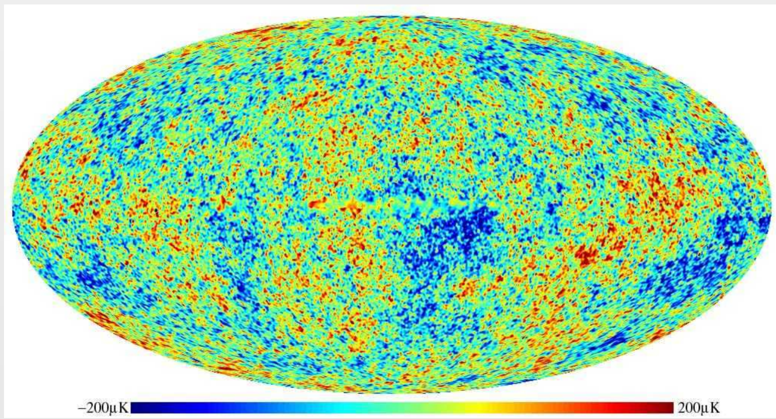
Amplification of quantum fluctuations (squeezing)
(in complete analogy to early universe, e.g., WMAP)

Quantum limit (accuracy)
of time-of-flight imaging

$\langle \delta\hat{\varrho}(\boldsymbol{r})\delta\hat{\varrho}(\boldsymbol{r}')\rangle = \mathcal{O}(1\%)$

M. Uhlmann, Y. Xu, and R. S.,

New J. Phys. **7**, 248 (2005)



Dynamical Phase Transition

Temperature $T = 0$

External parameter

$g = g(t) \approx g_c$

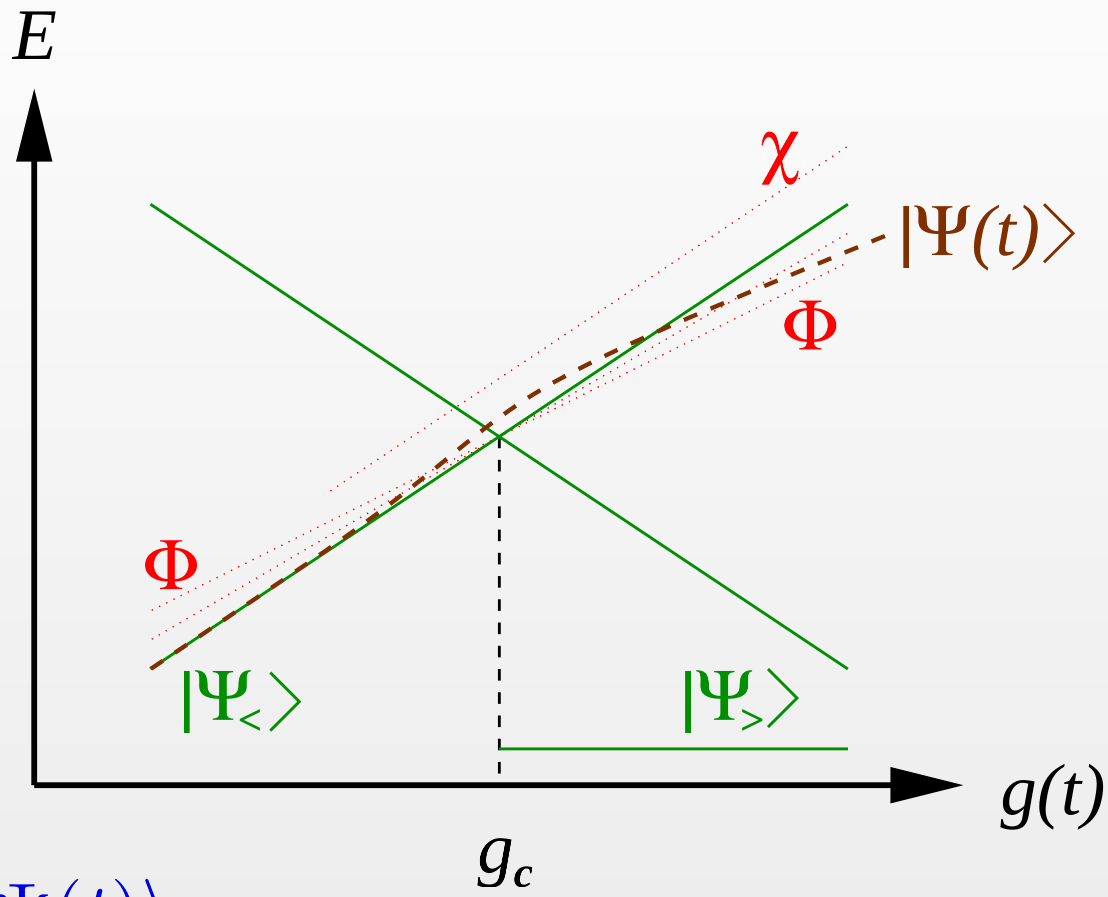
Two competing ground states

$|\Psi_<\rangle$ and $|\Psi_>\rangle$

Quasi-particle excitations

χ and Φ (unstable)

Actual quantum state $|\Psi(t)\rangle$

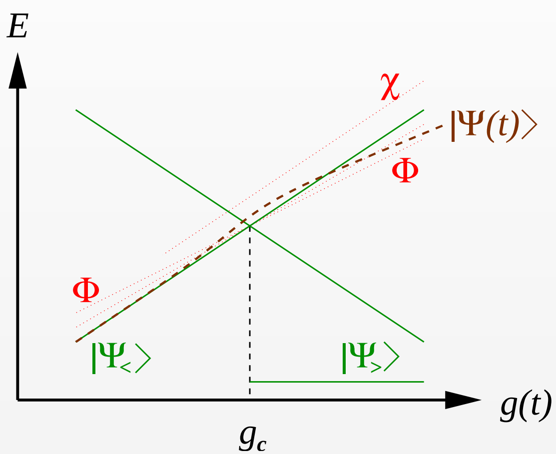


Sweep through (1^{st}-order) transition $(E_n - E_0) \downarrow 0$

$\rightarrow$ response time diverges at critical point g_c

$\rightarrow$ non-equilibrium dynamics $g(t)$

Particle Horizon



Effective Hamiltonian

$$\mathcal{H} = \frac{1}{2}[\alpha\Pi^2 + \beta(\boldsymbol{\nabla}\Phi)^2]$$

Speed of sound

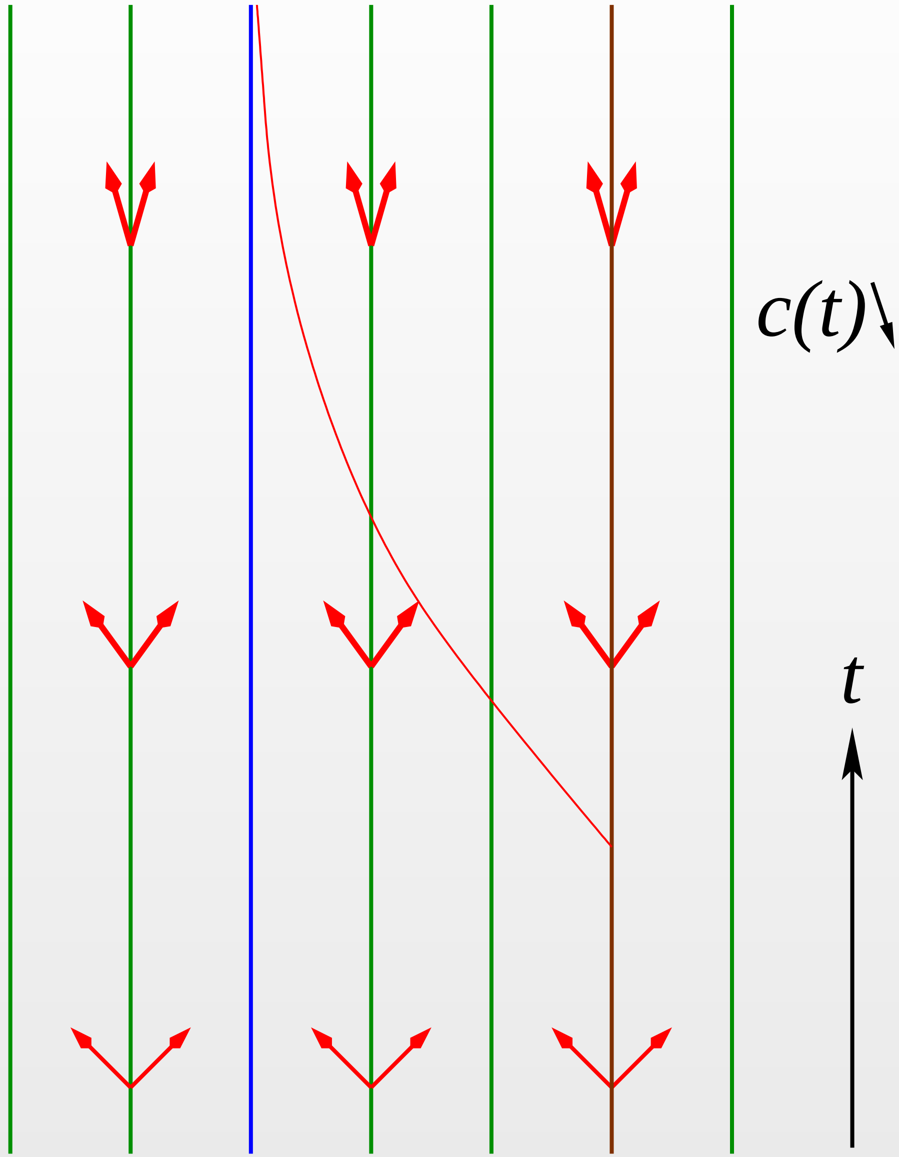
$c_s^2 = \alpha\beta \downarrow 0$ for $g \to g_c$

Homogeneous medium

Particle trajectory

Sound cones/waves

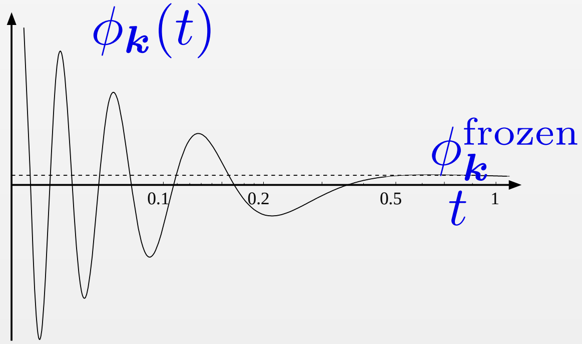
Particle horizon $\longrightarrow$ analogy to expanding universe

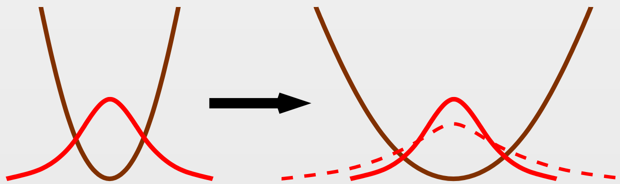


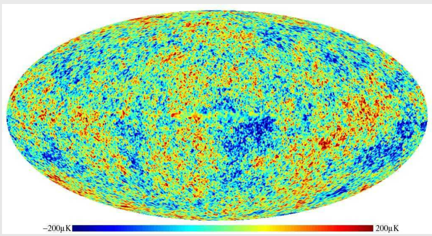
Fluctuations

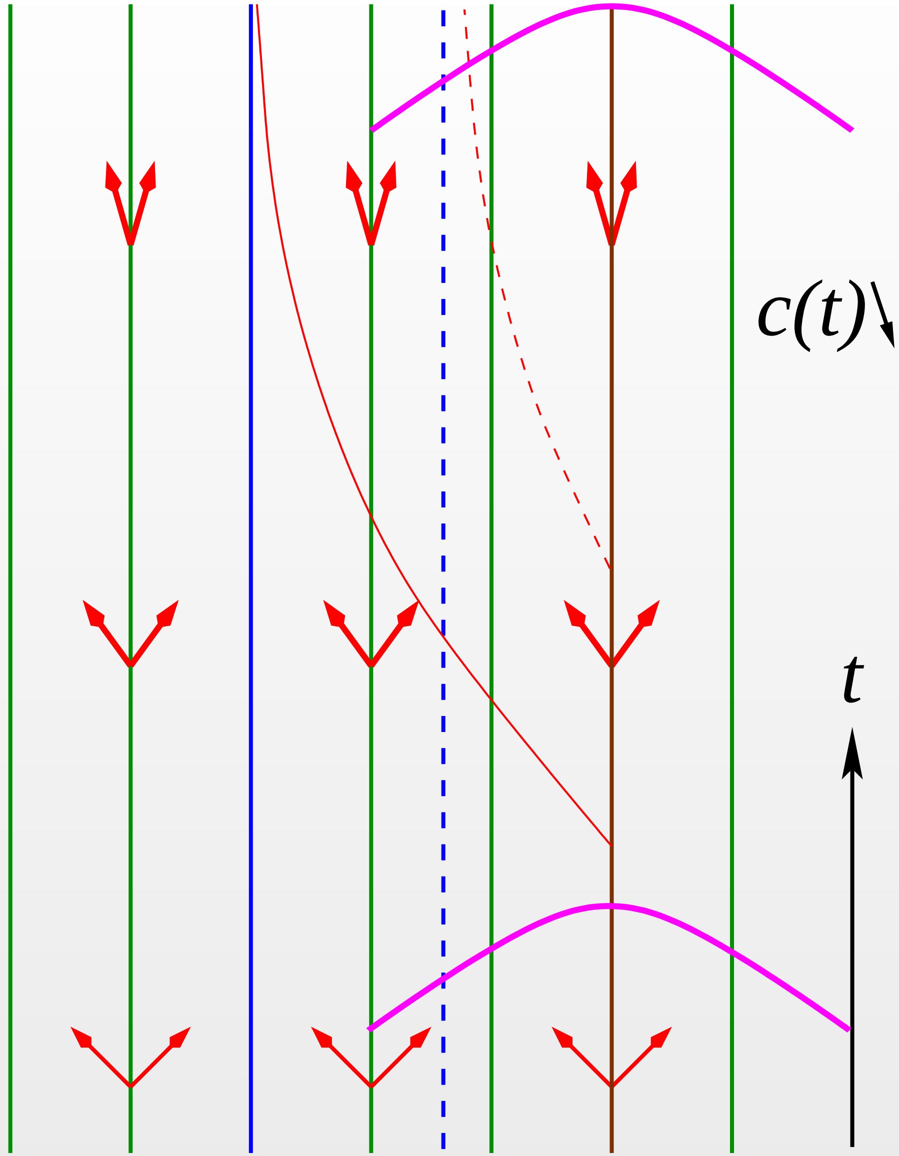
Phonon mode

+ oscillation (initially)
+ horizon crossing
+ freezing





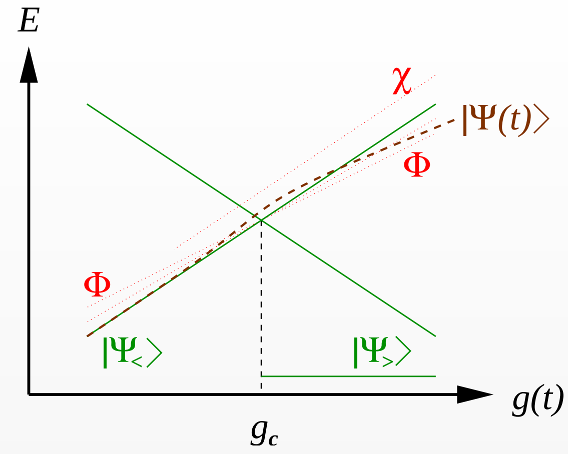




→ universal behavior

R. S., Phys. Rev. Lett. **95**, 135703 (2005)

Bose-Einstein condensate



$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\left(\frac{1}{g(t)}\,\dot{\Phi}^2 - \frac{\varrho_0}{m}\,(\boldsymbol{\nabla}\Phi)^2\right)$$

Phase fluctuations Φ, density fluctuations $\delta\varrho$

Mass of atoms/molecules m, background density ϱ_0

Coupling $g(t)$ (e.g., Feshbach resonance)

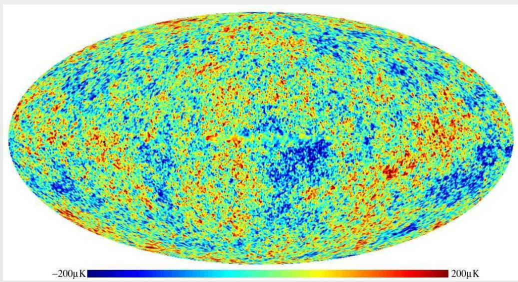
Critical point at $g_c = 0$ (repulsive $\to$ attractive)

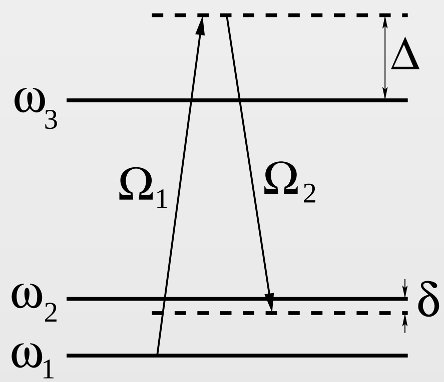
Linear sweep $g(t) \propto t$

$\to$ frozen spectra:

$\sigma(\Phi) = k^{-4/3}$,

$\sigma(\delta\varrho) = k^{4/3}$

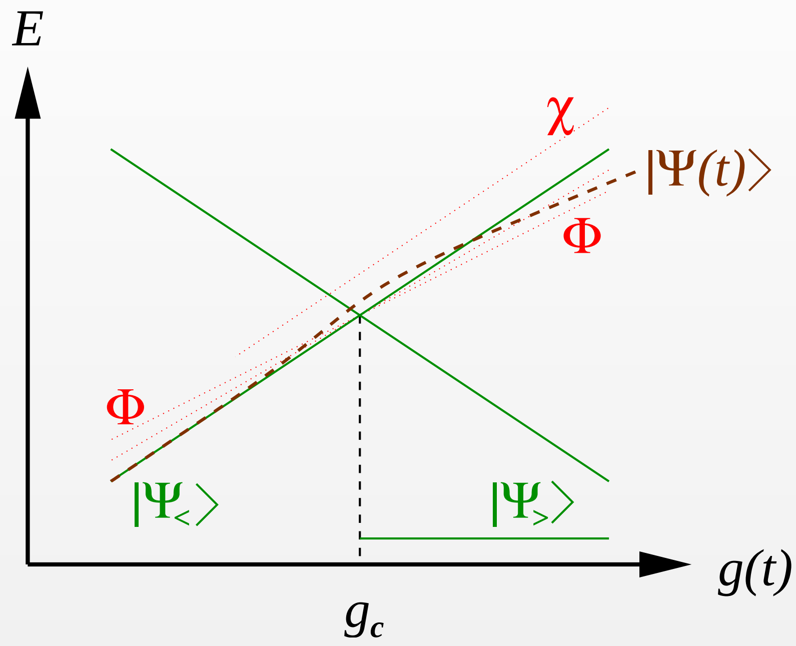




R. S., Phys. Rev. Lett. **95**, 135703 (2005)

Similarities to Cosmic Inflation

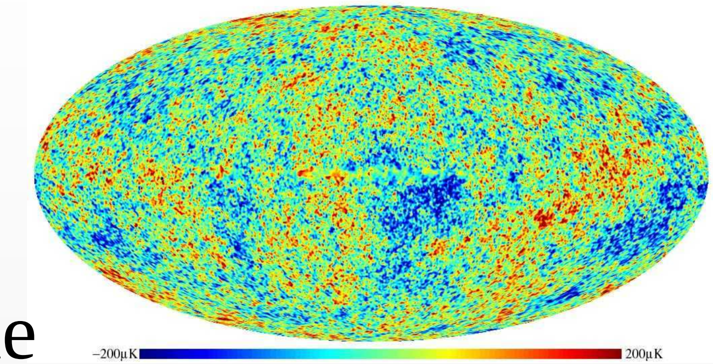
- Release of energy
  $\longrightarrow$ (p)re-heating
- Robust against initial
  (small-scale) perturbations
- Universality
  (no fine-tuning)
- Amplification of quantum fluctuations



But: different spectrum in general, e.g., $\sigma(\Phi) = k^{-4/3}$

- Preferred frame (rest frame of medium)
- No unique/constant propagation speed
- Neglect of (quantum) back-reaction

Speculations...



Postulate:

- No (locally) preferred frame
- Unique/constant propagation speed

$$\mathcal{A}_{\text{eff}} = \frac{1}{2} \int dt\, d^3 r\, \frac{\dot{\Phi}^2 - (\boldsymbol{\nabla}\Phi)^2}{t^2}$$

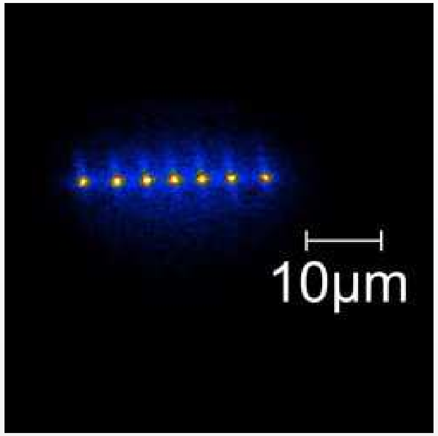
- $\leftrightarrow$ scale-invariance $\mathcal{A}[\lambda t, \lambda \boldsymbol{r}] = \mathcal{A}[t, \boldsymbol{r}]$
- Dominated by (quantum) back-reaction?

$\rightarrow$ correct $1/k^3$-spectrum (conformal de Sitter metric)

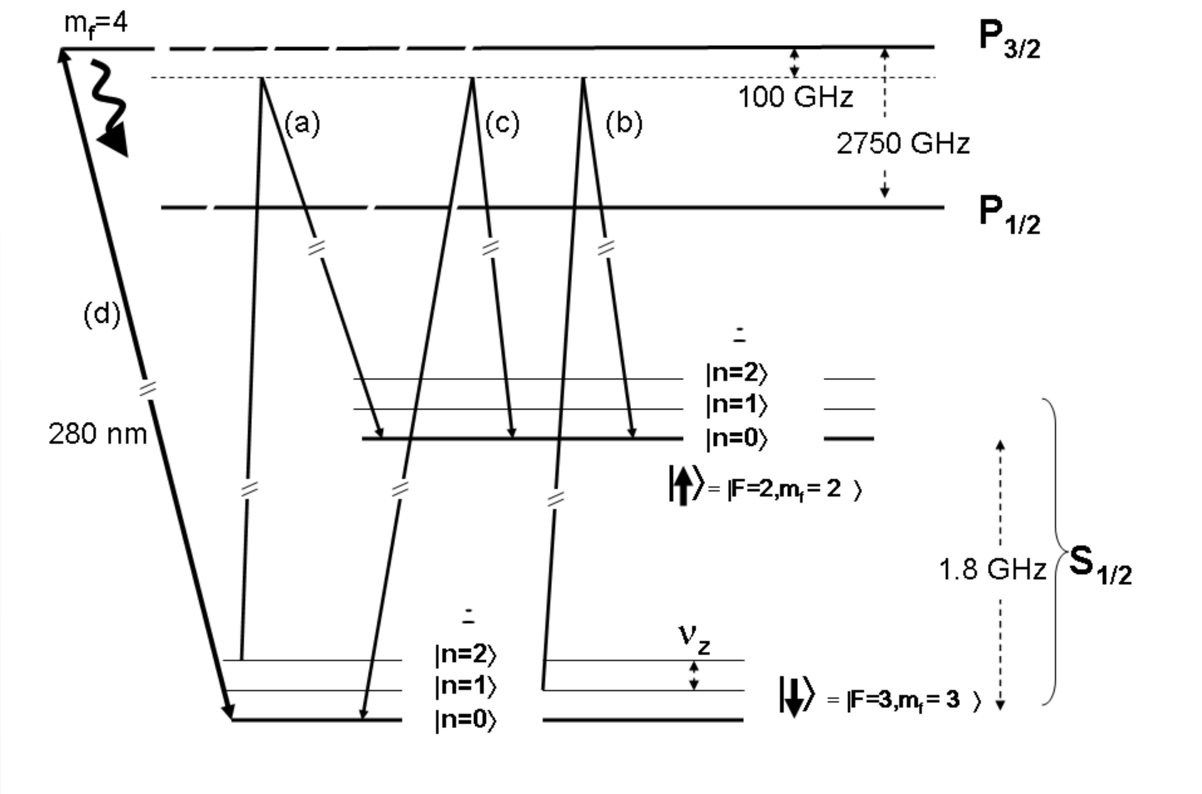
Was cosmic inflation just a phase transition?

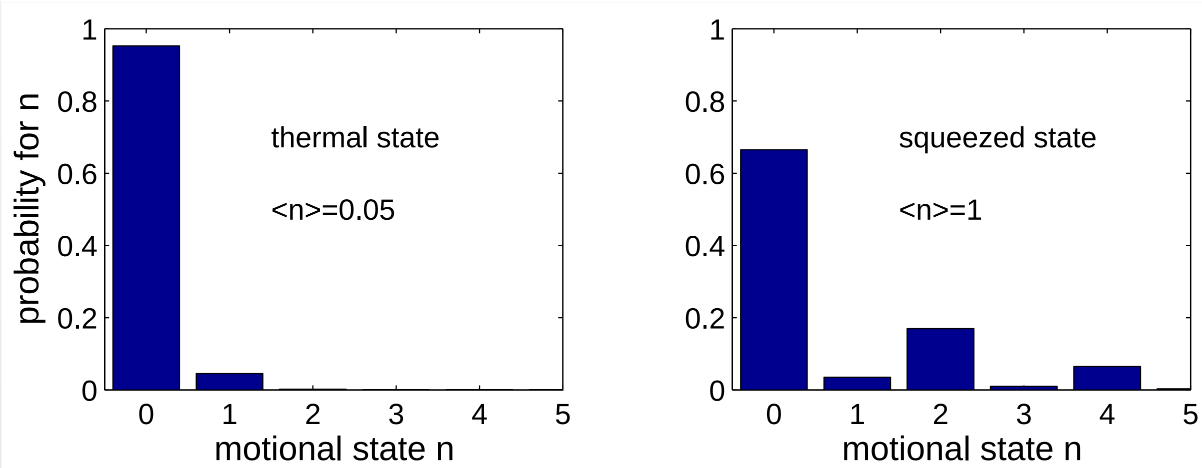
R. S., Phys. Rev. Lett. **95**, 135703 (2005)

Ion Trap



Expansion or Contraction of Ion chain → Expanding Universe → Phonon pair creation
Hawking???





R. S. *et al.*, Phys. Rev. Lett. **99**, 201301 (2007).

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