1. A country had 30 millions of inhabitants, and ten years later, it has grown to 33 millions of inhabitants. Assuming a continuous logistic growth, calculate the value of *r* for a carrying capacity *K* of 100 millions of inhabitants. Which population is expected in 10 years' time? Plot a graph of the expected population for the considered values of *K* and r as a function of the starting population.

```
Remove["Global`*"]
```

```
DSolve[{P'[t] == r P[t] (1 - P[t] / 100), P[0] == 30}, P, t]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{P \to \text{Function}\left[\{t\}, \frac{300\, e^{r t}}{7 + 3\, e^{r t}}\right]\right\}\right\}$$

$$\left\{\left\{P \to \text{Function}\left[\{t\}, \frac{300\, e^{r t}}{7 + 3\, e^{r t}}\right]\right\}\right\}[[1, 1, 2]]$$

$$\text{Function}\left[\{t\}, \frac{300\, e^{r t}}{7 + 3\, e^{r t}}\right]$$

```
f = %
```

$$\text{Function}\left[\{t\}, \frac{300\, e^{r t}}{7 + 3\, e^{r t}}\right]$$

```
Solve[f[10] == 33., r, Reals]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

```
{{r → 0.0139113}}
```

```
r = 0.013911280246271779`;
f[20]
```

```
36.1451
```

```
Remove[P];
DSolve[{P'[t] == r P[t] (1 - P[t] / 100), P[0] == P0}, P, t]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

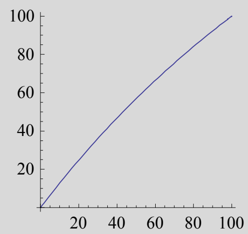
$$\left\{\left\{P \to \text{Function}\left[\{t\}, \frac{100. \times 2.71828^{0.0139113\, t}}{2.71828^{0.0139113\, t} + 1. \left(\frac{100. - 1.\, P0}{P0}\right)^{1.}}\right]\right\}\right\}$$

$$\left\{\left\{P \to \text{Function}\left[\{t\}, \frac{100. \times 2.71828^{0.0139113\, t}}{2.71828^{0.0139113\, t} + 1. \left(\frac{100. - 1.\, P0}{P0}\right)^{1.}}\right]\right\}\right\}[[1, 1, 2]]$$

$$\text{Function}\left[\{t\}, \frac{100. \times 2.71828^{0.0139113\,t}}{2.71828^{0.0139113\,t} + 1.\left(\frac{100.-1.\,P0}{P0}\right)^{1.}}\right]$$

```
f = %;
```

```
Plot[f[20], {P0, 0, 100}, AspectRatio → Automatic]
```



Another way (with x=P/K and without clicking "get solution" on the suggestion bar):

```
K = 100;
Remove[x, r]
DSolve[{x'[t] == r x[t] (1 - x[t]), x[0] == 30 / K}, x, t];
s = x /. %[[1]]
```

$$\text{Function}\left[\{t\}, \frac{3\,e^{r\,t}}{7 + 3\,e^{r\,t}}\right]$$

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

```
Clear[r]
N[Solve[s[10] == 33 / K, r, Reals]];
r = r /. %[[1]]
```

0.0139113

Which population is expected in 10 years' time?

```
x[20] * K
```

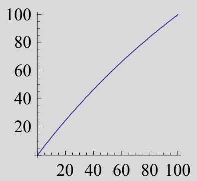
36.1451

Plot a graph of the expected population for the considered values of *K* and r as a function of the starting population.

```
Remove[x, Po]
DSolve[{x'[t] == r x[t] (1 - x[t]), x[0] == Po / K}, x, t];
x = x /. %[[1]]
Plot[K x[20], {Po, 0, 100}, AspectRatio → Automatic]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

$$\text{Function}\left[\{t\},\ \frac{2.71828^{0.0139113\,t}}{2.71828^{0.0139113\,t} + 1.\left(\frac{100. - 1.\,Po}{Po}\right)^{1.}}\right]$$



Exercise 4 (unit 1):

4. Consider an initial population $P_0 = 10$ millions with a discrete logistic growth with carrying capacity $K = 30$ and $r = 3.7$. Plot the population values until $t = 20$. Which is the expected population for $t = 20$? Do the same computations when $P_0 = 10.01$ and compare the results.

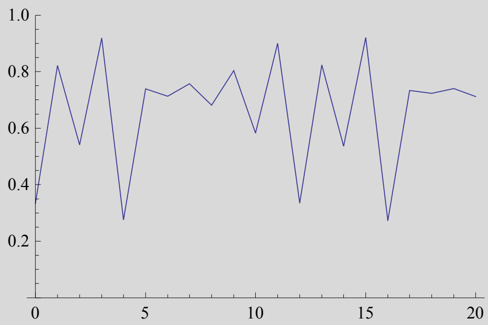
```
Remove["Global`*"]
```

```
K = 30; r = 3.7;
f = Function[x, r x (1 - x)]
```

```
Function[x, r x (1 - x)]
```

```
NestList[f, 10 / K, 20]
A = ListPlot[%, Joined → True, DataRange → {0, 20}, PlotRange → {0, 1}]
%%[[21]] K
```

$\{\frac{1}{3}$, 0.822222, 0.54084, 0.918829, 0.275955, 0.739274, 0.713168, 0.75687, 0.680866, 0.803964, 0.583143, 0.899423, 0.334706, 0.823909, 0.536807, 0.919987, 0.27236, 0.733265, 0.723673, 0.739891, 0.712074$\}$

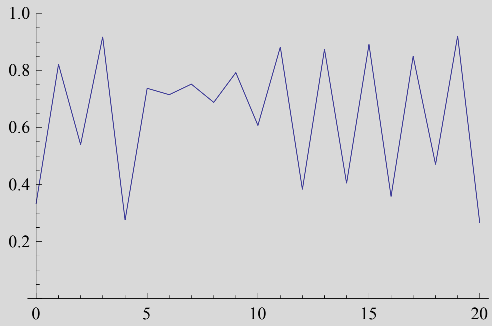



```
21.3622
```

Now P_0 = 10.01.

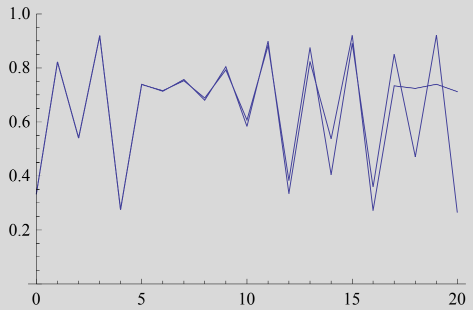
```
NestList[f, 10.01 / K, 20]
B = ListPlot[%, Joined → True, DataRange → {0, 20}, PlotRange → {0, 1}]
%%[[21]] K
```

```
{0.333667, 0.822633, 0.53986, 0.919121, 0.275048, 0.737767, 0.715828,
 0.752648, 0.688826, 0.793076, 0.607194, 0.882485, 0.38371, 0.874963,
 0.404789, 0.891459, 0.358011, 0.850405, 0.470701, 0.921824, 0.266639}
```




```
7.99918
```

```
Show[A, B]
```



- We obtain very different results.

Exercise 10 (unit 1):

10. Given the following demand and supply functions:

$$D(p) = 100 - 3\,p, \qquad S(p) = -20 + 2\,p,$$

consider the following mechanism of adjustment of the price to excess demand or supply:

$$\frac{d\,p}{d\,t} = \alpha(D(p(t)) - S(p(t))),$$

where $\alpha > 0$ is the reactivity of price to excess demand or supply. Describe the behavior of price over time for an initial price $p_0 = 45$ when $\alpha = 0.1$, $\alpha = 0.35$ and $\alpha = 0.4$.

In[199]:=

```
Remove["Global`*"]
```

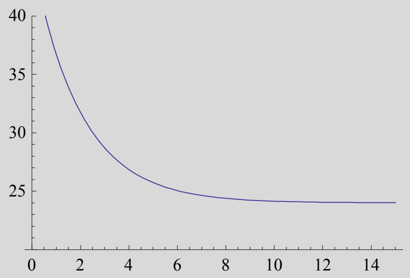
In[200]:=

```
De = Function[{p}, 100 - 3 p];
Su = Function[{p}, -20 + 2 p];
P0 = 45;
α = 0.1;
DSolve[{P'[t] == α (De[P[t]] - Su[P[t]]), P[0] == P0}, P, t];
pr = P /. %[[1]]
Plot[pr[t], {t, 0, 15}, PlotRange → {20, 40}]
```

Out[205]=

$\text{Function}\left[\{t\},\ 24.\ e^{-0.5\,t}\left(0.875 + 1.\ e^{0.5\,t}\right)\right]$

Out[206]=



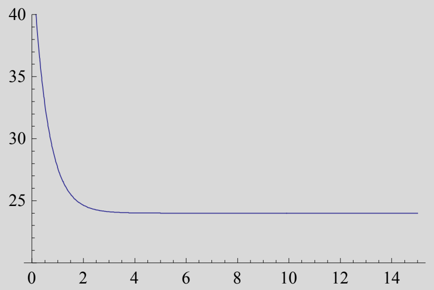
In[207]:=

```
Remove[P];
α = 0.35;
DSolve[{P'[t] == α (De[P[t]] - Su[P[t]]), P[0] == P0}, P, t];
pr = P /. %[[1]]
Plot[pr[t], {t, 0, 15}, PlotRange → {20, 40}]
```

Out[210]=

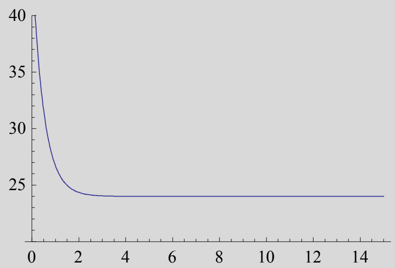
$\text{Function}\left[\{t\},\ 24.\ e^{-1.75\,t}\left(0.875 + 1.\ e^{1.75\,t}\right)\right]$

Out[211]=



In[212]:=
```
Remove[P];
α = 0.41;
DSolve[{P'[t] == α (De[P[t]] - Su[P[t]]), P[0] == P0}, P, t];
pr = P /. %[[1]]
Plot[pr[t], {t, 0, 15}, PlotRange → {20, 40}]
```

Out[215]= Function[{t}, 24. $e^{-2.05\,t}$ (0.875 + 1. $e^{2.05\,t}$)]

Out[216]=


```
Solve[De[p] == Su[p], p]
```

{{p → 24}}

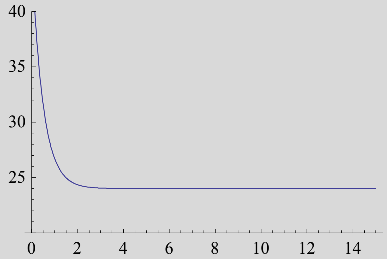
- The price tends to the price of equilibrium.

```
(*We don't need to solve the differential equation every
 time we change the value of α. We can do it this way: *)
```

```
Remove["Global`*"]
```

In[219]:=
```
De = Function[{p}, 100 - 3 p];
Su = Function[{p}, -20 + 2 p];
Remove[P];
P0 = 45;
DSolve[{P'[t] == α (De[P[t]] - Su[P[t]]), P[0] == P0}, P, t];
pre = P /. %[[1]]
α = 0.1;
Plot[pre[t], {t, 0, 15}, PlotRange → {20, 40}]
```

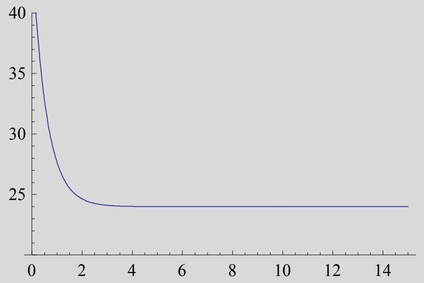
Out[224]= Function[{t}, 24. $e^{-2.05\,t}$ (0.875 + 1. $e^{2.05\,t}$)]

Out[226]=


```
Limit[24 e^(-0.5 t) (0.875 + e^(0.5 t)), t → Infinity]
```

24.

```
α = 0.35;
Plot[pre[t], {t, 0, 15}, PlotRange → {20, 40}]
```



In[227]:=

```
α = 0.41;
Plot[pre[t], {t, 0, 15}, PlotRange → {20, 40}]
```

Out[228]=