CONGESTION INDUCED BY THE STRUCTURE OF MULTIPLEX NETWORKS

Albert Solé-Ribalta, Sergio Gómez and Alex Arenas

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Introduction

In this paper, we focus our study on the transportation congestion problem in multiplex networks. We prove analytically that the structure of multiplex networks can induce congestion for flows that otherwise will be decongested if the individual layers were not interconnected. We provide explicit equations for the onset of congestion and approximations that allow to compute this onset from individual descriptors of the individual layers. The observed cooperative phenomenon reminds the Braess' paradox in which adding extra capacity to a network when the moving entities selfishly choose their route can in some cases reduce overall performance, see our recent publication on this topic [1].

In a multiplex network with N nodes and L layers, the onset of congestion is attained when a node i in layer α is required to process elements at its maximum processing rate, τ. Analitically, the critical injection rate of the system, ρ_c, becomes

$$\rho_c = \tau L^{-1} \frac{N-1}{\mathcal{B}^*}, \tag{1}$$

where $\mathcal{B}^* \equiv \max_{i\alpha} \mathcal{B}_{i\alpha}$ and $\mathcal{B}_{i\alpha}$ is the betweenness of node i in layer α. In a multiplex, $\mathcal{B}_{i\alpha}$ depends on intra-layer paths, inter-layer paths, and on the migration of shortest paths between layers (more efficient layers contain a larger proportion of the starting and ending routes). This last contribution unbalances, in a highly non-linear way, the distribution of shortest paths among the layers. However, some approximations are possible to grasp the effect of the different contributions to the onset of congestion in multiplex structures. In particular, the fraction of shortest paths fully contained within layers, λ, is basically 1, and so, the main factor influencing the traffic dynamics is the migration of shortest paths from the less efficient layer (the one with larger shortest paths) to the most efficient one. Under this situation, we can approximate the interconnected betweenness of node i in layer α, $\mathcal{B}^{i\alpha}$, in terms of the betweenness of node i of layer α, $\mathcal{B}^i_{(\alpha)}$, when layer α is considered as a single layer network:

$$\mathcal{B}^{i\alpha} \approx \lambda \mu_\alpha \mathcal{B}^i_{(\alpha)}\,, \tag{2}$$

where $\mu_\alpha < 1$ is the fraction of shortest paths using only layer α, satisfying $\sum_\alpha \mu_\alpha = 1$. The effect of the product of $\lambda\mu_\alpha$ is to precisely account for the fraction of all shortest paths that traverse only layer α in the multiplex.

Taking advantage of Eq. (2), we can compute the congestion induced by a multiplex as the situation in which a multiplex network reaches congestion with less load than the worst of its layers when operating individually. In a multiplex with two layers 1 and 2 (being 2 the most efficient), this limiting situation is obtained when $\rho_c < \rho_c^{(1)}$, and consequently:

$$\frac{1}{L\lambda\mu_2} \lesssim \frac{\mathcal{B}^*_{(2)}}{\mathcal{B}^*_{(1)}}. \tag{3}$$

Evaluation on several multiplex topologies show that the boundaries approximated by Eq. (3) determine accurately the regions where the multiplex induces congestion, see Figure 1.

References

[1] A. Solé-Ribalta, S. Gómez, and A. Arenas. Congestion induced by the structure of multiplex networks. *Physical Review Letters*, 116(10):108701, 2016.

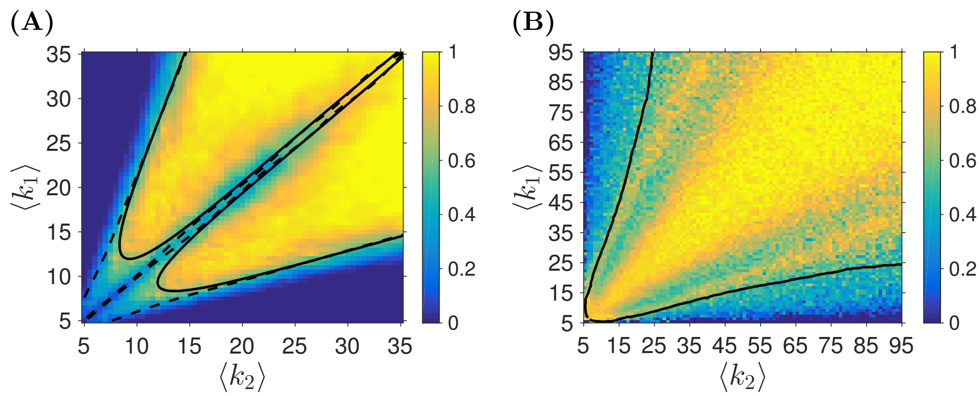

Figure 1: Probability of obtaining a multiplex configuration that induces congestion when: **(A)** the multiplex is composed of two Erdős-Rényi layers; **(B)** the topology is a Random Geometric Multiplex. In both networks topologies each layer has 500 nodes. The number of simulations points is 50^2, and for each point we generate 10^2 configurations fixing $\langle k_1 \rangle$ and $\langle k_2 \rangle$. The colors indicate the probability of observing that the onset of congestion of the multiplex satisfies $\rho_c < \min(\rho_c^{(1)}, \rho_c^{(2)})$. Lines show the accuracy of Eq. 3 in detecting the region where multiplex structure induces congestion. Solid lines represent the expression when the real value of λ is used and dashed lines when we approximate λ by 1.