NONLOCAL DENSITY CORRELATIONS AS A SIGNATURE OF THE HAWKING EFFECT FROM ACOUSTIC BHS IN BECS (PART 1)

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TOWARDS THE OBSERVATION OF HAWKING RADIATION IN CONDENSED MATTER SYSTEMS

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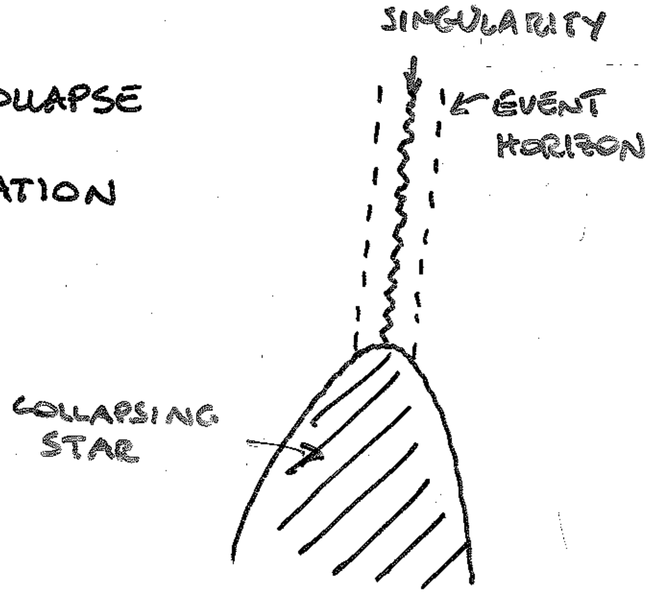
THE HAWKING EFFECT

GRAVITATIONAL COLLAPSE AND BH FORMATION



A QUANTUM FIELD IS EXPANDED IN TERMS OF A COMPLETE SET OF POSITIVE FREQUENCY MODES

$$\hat{f} = \sum_i \left[\hat{a}_i u_i + \hat{a}_i^{+} u_i^{*} \right], \quad \hat{a}_i |0\rangle = 0 \quad \forall i$$

SUCH DECOMPOSITION IS NOT UNIQUE IN A CURVED SPACETIME..... IN GRAVITATIONAL COLLAPSE ONE HAS <u>TWO</u> DIFFERENT STATIONARY REGIONS:

- IN THE PAST, BEFORE THE STAR STARTS TO COLLAPSE

$$\hat{f} = \sum_i \left[\hat{a}_i^{IN} u_i^{IN} + \hat{a}_i^{+\,IN} u_i^{*\,IN} \right], \quad \hat{a}_i^{IN} |0\rangle_{IN} = 0 \quad \forall i$$

- IN THE FUTURE, AFTER THE BH HAS FORMED

$$\hat{f} = \sum_i \left[\hat{a}_i^{OUT} u_i^{OUT} + \hat{a}_i^{OUT\,+} u_i^{*\,OUT} \right], \quad \hat{a}_i^{OUT} |0\rangle_{OUT} = 0 \quad \forall i$$

ARE THESE TWO DECOMPOSITIONS EQUIVALENT ?

THE MATHEMATICS OF THE HAWKING EFFECT CONCERNS ESTABLISHING A RELATION BETWEEN $|0\rangle_{IN}$ AND $|0\rangle_{OUT}$

ONE COMMENT ABOUT THE MODES:

- u_i^{IN} ARE DEFINED IN THE (TOPOLOGICALLY TRIVIAL) SPACETIME OF A STATIC STAR
- $$u_i^{OUT} \equiv \underset{\substack{\uparrow \\ \text{EXTERNAL} \\ \text{REGION}}}{u_i^{EXT}} \otimes \underset{\substack{\uparrow \\ \text{BH REGION}}}{u_i^{BH}}$$

BEING BOTH SETS OF MODES COMPLETE WE CAN EXPRESS ONE IN TERMS OF THE OTHER

$$u^{IN} = \sum \left(\alpha^* u^{EXT} - \beta u^{EXT} + \gamma^* u^{BH} - \gamma u^{BH*} \right)$$

BOGOLIUBOV COEFFS.

$$\Rightarrow \quad \hat{a}^{IN} = \sum \left(\alpha\, \hat{a}^{EXT} + \beta^* \hat{a}^{EXT+} + \gamma\, \hat{a}^{BH} + \gamma^* \hat{a}^{BH+} \right)$$

FROM $\hat{a}^{IN} |0\rangle_{IN} = 0$ IT FOLLOWS THAT

$$\hat{a}^{IN} |0\rangle_{OUT} \equiv \hat{a}^{IN} |0\rangle_{EXT} \otimes |0\rangle_{BH} \neq 0 \quad \text{IF} \quad \beta(\gamma) \neq 0 .$$

MEANING THAT $|0\rangle_{IN} \neq |0\rangle_{OUT}$ ...

THE PHYSICS OF THE HAWKING EFFECT

THE NATURAL BOUNDARY CONDITION FOR OUR PROBLEM IS TO HAVE A STATE WITH NO PARTICLES IN THE PAST, I.E. $|0\rangle_{IN}$

THE RESULTS OF THE CALCULATIONS SHOW THAT

$$|0\rangle_{IN} \sim e^{\sum e^{-\frac{\pi\omega}{\kappa}} \hat{a}^{EXT+} \hat{a}^{BH+}} |0\rangle_{OUT} \sim \sum_N e^{-\frac{\pi\omega}{\kappa}} |N_\omega^{EXT}\rangle \otimes |N_\omega^{BH}\rangle$$

$|0\rangle_{IN}$ CONTAINS $\boxed{\text{CORRELATED}}$ PAIRS OF QUANTA, SITUATED RESPECTIVELY OUTSIDE AND INSIDE THE HORIZON

(HAWKING QUANTA) (PARTNER)

RESTRICTING MEASUREMENTS TO THE EXTERIOR REGION (AS IS <u>ALWAYS</u> THE CASE IN THE GRAVITATIONAL CASE) WE <u>LOOSE</u> SUCH CORRELATIONS

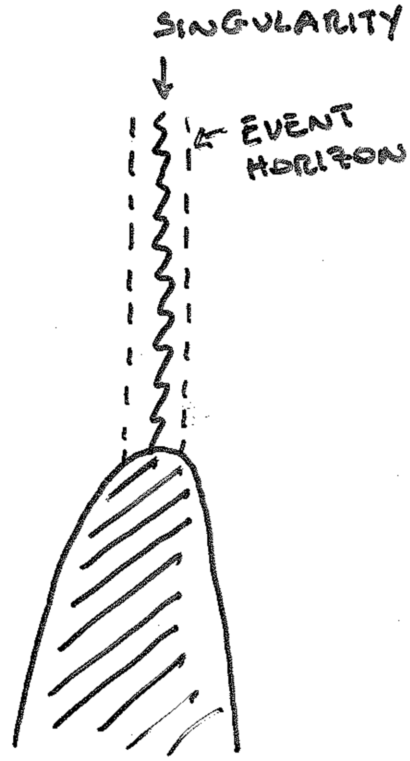
$\Rightarrow$ MATHEMATICALLY WE TRACE OVER THE BH DEGREES OF FREEDOM, THUS CONVERTING A PURE STATE $|0\rangle_{IN}$ INTO A DENSITY MATRIX

$$\rho = Tr_{BH} |0\rangle_{IN} \langle 0_{IN}| \sim \sum e^{-\frac{\hbar\omega}{T_H}} |N_\omega^{EXT}\rangle\langle N_\omega^{EXT}|$$

WHICH IS THERMAL AT THE (HAWKING) TEMPERATURE $\boxed{T_H = \frac{\hbar\kappa}{2\pi}}$

HAWKING '74 (A BRIEF SUMMARY)

GRAVITATIONAL COLLAPSE AND BH FORMATION



$$\hat{f} = \sum_i \left[\hat{a}_i u_i + \hat{a}_i^+ u_i^* \right], \quad \hat{a}_i |0\rangle = 0 \ \forall i$$

IF WE PREPARE A QUANTUM FIELD TO BE IN THE VACUUM IN THE PAST $|0\rangle_{IN}$

SUCH STATE WON'T BE PERCEIVED AS EMPTY BY OBSERVERS SITUATED OUTSIDE THE EVENT HORIZON

THE REASON IS, ESSENTIALLY, TOPOLOGICAL:

WE CANNOT ACCESS THE BH REGION $\Rightarrow$ WE TRACE OVER THE DEGREES OF FREEDOM IN THE INTERIOR

THUS $\underset{\text{PURE}}{|0\rangle_{IN}} \longrightarrow \underset{\text{MIXED}}{\rho}$ WHICH IS <u>THERMAL</u> AT THE HAWKING TEMPERATURE $\boxed{T_H = \frac{\hbar \kappa}{2\pi k_B}}$ SURFACE GRAVITY

AND WE LOOSE $\boxed{\text{BH-EXT CORRELATIONS}}$ NECESSARY TO RECOVER THE PURITY OF $|0\rangle_{IN}$

IN THE ANALOG CASE BOTH BH AND EXT ARE ACCESSIBLE: THE <u>CHARACTERISTIC SIGNATURE OF THE HAWKING EFFECT IN BH-EXT CORRELATIONS</u> CAN BE USED TO CHECK EXPERIMENTALLY ITS EXISTENCE

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WHAT IS THE HAWKING SIGNATURE IN BH-EXT CORRELATIONS?

LESSON FROM GRAVITY (METHODOLOGY)

A SCALAR FIELD ($\Box f=0$) IN SCHWARZSCHILD (SPHERICALLY SYMMETRIC) DECOMPOSES AS $f=\sum \frac{f_\ell(t,r)}{r} Y_{\ell m}$

RESTRICTING TO $\ell=0$ (HAWKING RADIATION IS 90% IN S-WAVE) WE EFFECTIVELY REDUCE 4D → 2D: A 2D FIELD $f^{(2)} \equiv f_0(t,r)$ WITH WAVE EQUATION

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^{*2}} - \underbrace{\left(1-\frac{2M}{r}\right)\frac{2M}{r^3}}_{\substack{\text{S-WAVE POTENTIAL}\\ \text{INDUCED BY THE}\\ \text{SPACETIME CURVATURE}}} \right) f^{(2)} = 0$$

NEGLECTING THE POTENTIAL ($\simeq 0$ ON THE HORIZON WHERE THE INTERESTING PHYSICS TAKES PLACE) WE ARE LEFT WITH A 2D FIELD FREE AND CONFORMAL

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^{*2}} \right) f^{(2)} \underset{x^\pm = t\pm r^*}{=} -\partial_{x^+}\partial_{x^-} f^{(2)} = 0$$

WHERE EVERYTHING IS ANALYTICALLY CALCULABLE!

POSITIVE FREQUENCY MODES

$$e^{-i\omega x^\pm} \quad \begin{matrix} + \leftarrow \text{INGOING} \\ - \leftarrow \text{OUTGOING} \end{matrix}$$

TWO-POINT CORRELATOR

$$\langle x^\pm | \hat{f}^{(2)}(x) \hat{f}^{(2)}(x') | x^\pm \rangle = -\frac{\hbar}{4\pi} \ln (x^+ - x^{+\prime})(x^- - x^{-\prime})$$

NOW CONSIDER GRAVITATIONAL COLLAPSE + BH FORMATION

IN THE 'IN' REGION (BEFORE THE COLLAPSE) $|0\rangle_{IN}$ IS DEFINED BY EXPANDING $\hat{f}$ WITH RESPECT TO $e^{-i\omega x^{\pm}_{IN}}$

SUCH MODES PROPAGATE IN THE DYNAMICAL REGION AND FINALLY EMERGE IN THE 'OUT' ~~BH~~ ⊕ EXT AS:

- THE INGOING SECTOR IS TRIVIAL $e^{-i\omega x^+_{IN}} \to e^{-i\omega x^+_{OUT}}$
- IN THE OUTGOING ONE

$$e^{-i\omega x^-_{IN}} \to e^{-i\omega\left(\pm e^{-\kappa x^-_{OUT}}\right)}$$

BH ($+$), EXT ($-$)

$=$ KRUSKAL COORD. REGULAR AT THE HORIZON ($x^-_{OUT} \to +\infty$)

: THESE ARE ~~[illegible]~~ NOT POSITIVE FREQUENCY x^-_{OUT} MODES!

INFINITE REDSHIFT AT THE HORIZON

TO DETECT BH-EXT CORRELATIONS WE LOOK AT THE $x^-, x^{-\prime}$ COMPONENT OF THE POINT-SPLIT STRESS TENSOR ($x^{\pm}_{OUT} \equiv x^{\pm}$)

$$_{IN}\langle 0| \partial_{x^-} \hat{f}^{(2)}(x^+, x^-)\, \partial_{x^{-\prime}} \hat{f}^{(2)}(x^{+\prime}, x^{-\prime}) |0\rangle_{IN}$$

$$= \partial_{x^-}\partial_{x^{-\prime}}\left[-\frac{\hbar}{4\pi} \ln \Delta x^-_{IN}\, \Delta x^+_{IN}\right] \underset{\text{BH-EXT}}{=} \frac{\hbar}{4\pi} \frac{\kappa^2}{4\cosh^2\left(\frac{\kappa(x^{-\prime} - x^-)}{2}\right)}$$

WHICH HAS A $\boxed{\text{PEAK}}$ WHEN $x^-_{IN} = -x^{-\prime}_{IN}$ (CORRELATED PAIRS OF PARTICLES CREATED ON EITHER SIDE ~~SYMMETRICALLY~~ WITH RESPECT TO THE HORIZON)

CARLITZ AND WILLEY
~~[illegible]~~ MASSAR AND PARENTANI

THIS IS THE SIGNATURE OF THE HAWKING EFFECT

NO PEAK FOR BH-BH, EXT-EXT CORR. WHERE $\cosh \to \sinh$

APPLICATION TO BECS

LET'S SEE BRIEFLY HOW THE ANALOGY WORKS

GARAY, ANGLIN, CIRAC, ZOLLER
BARCELÒ, LIBERATI, VISSER

- IN THE DILUTE GAS APPROX. A BOSE GAS [illegible] IS DESCRIBED BY A QUANTUM FIELD $\hat{\psi}$ SATISFYING

$$i\hbar\frac{\partial}{\partial t}\hat{\psi} = \Big(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + \underbrace{V_{EXT}}_{\text{TRAPPING POTENTIAL}} + \underbrace{g}_{\equiv \frac{4\pi\hbar^2 a}{m}}\hat{\psi}^\dagger\hat{\psi}\Big)\hat{\psi}$$

$a \to$ SCATTERING LENGTH

WRITE $\hat{\psi} = \underbrace{\psi}_{\text{~~WAVE FUNCTION OF THE~~ CONDENSATE}} + \hat{\phi} \to$ FLUCTUATION:

IN THE HOMOGENEOUS CASE $|\psi|^2 = n = \text{CONST.}$ IT CAN BE EXPANDED IN PLANE-WAVES $e^{i\vec{k}\cdot\vec{x} - i\omega t}$ WITH A NONLINEAR DISPERSION RELATION: $\omega = c_s k\sqrt{1+\xi^2 k^2}$

$c_s = \sqrt{\frac{ng}{m}}$ SPEED OF SOUND

$\xi = \frac{\hbar}{m c_s}$ HEALING LENGTH

$\Downarrow$

THE LINEAR (RELATIVISTIC) DISPERSION IS VALID FOR LOW MOMENTA $k \ll \frac{1}{\xi}$

CONSIDERING $\hat{\Psi} = e^{i\hat{\theta}} \sqrt{\hat{n}}$

AND EXPANDING AS $\begin{cases} \hat{n} = n + \hat{n}_1 \\ \hat{\theta} = \theta + \hat{\theta}_1 \end{cases}$

THE BACKGROUND (n, θ) SATIFIES GROSS-PITAEVSKI EQ.

WHEREAS FOR THE FLUCTUATIONS

$$\frac{\partial}{\partial t} \hat{n}_1 + \frac{1}{m} \vec{\nabla}\left(\hat{n}_1 \hbar \vec{\nabla}\theta + n\hbar \vec{\nabla}\hat{\theta}_1 \right) = 0$$

$$\hbar \partial_t \hat{\theta}_1 + \frac{1}{m}\hbar^2 \vec{\nabla}\theta \vec{\nabla}\hat{\theta}_1 + g n_1 + \boxed{\frac{\hbar^2}{2m}\left[\frac{1}{2n^{3/2}} \vec{\nabla}(n^{1/2}) \hat{n}_1 - \frac{1}{2\sqrt{n}} \vec{\nabla}^2 \left(\frac{\hat{n}_1}{\sqrt{n}} \right) \right]} = 0$$

THESE TERMS CAN BE NEGLECTED

AS LONG AS $|\vec{x}| \gg \xi$

$\Updownarrow$

HYDRODYNAMICAL APPROX.

WE THUS END UP WITH

$$\hat{n}_1 = -\frac{\hbar}{g}\left(\partial_t \hat{\theta}_1 + \frac{\hbar}{m} \vec{\nabla}\theta \vec{\nabla}\theta_1 \right),$$

$$\Box \hat{\theta}_1 = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \hat{\theta}_1 \right) = 0$$

WITH THE ACOUSTIC METRIC DEFINED BY

$$ds^2 = \frac{n}{mc}\left[-c^2 dt^2 + (d\vec{x} - \vec{v}dt)(d\vec{x} - \vec{v}dt) \right]$$

$\frac{m\xi}{\hbar}$

THIS IS HOW THE ANALOGY WORKS FOR BECS

A QUANTITY OF INTEREST IN THE EXPERIMENTS IS THE (ONE-TIME) DENSITY-DENSITY CORRELATION FUNCTION

$$G_2(x,x') = \langle \hat{\rho}_1(x)\, \hat{\rho}_1(x') \rangle \Big|_{t=t'} =$$

$$= \frac{\hbar^2}{\rho(x)\rho(x')} \lim_{t'\to t} \left[\partial_t \partial_{t'} + \vec{v}(x)\vec{\nabla}_{\vec{x}}\partial_{t'} + \vec{v}(\vec{x}')\partial_t \vec{\nabla}_{\vec{x}'} + v(\vec{x})\vec{v}(\vec{x}')\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{x}'} \right] \langle \hat{\theta}_1(x)\hat{\theta}_1(x') \rangle$$

THE CRUCIAL QUANTITY TO CALCULATE IS THUS THE 2-POINT FUNCTION FOR $\hat{\theta}_1$.

TO COMPUTE IT ANALYTICALLY WE CONSIDER STEPS SIMILAR TO THE PREVIOUS (GRAVITY) EXAMPLE:

- CONSIDER A 1D CONDENSATE (TRANSVERSE SIZE $\ell_\perp \ll \xi$)

$\Rightarrow$ DIMENSIONAL REDUCTION ALONG Y, Z GIVES A 2D FIELD $\hat{\theta}_1^{(2)} = \sqrt{\frac{\ell_\perp^2 m \xi}{\hbar}}\, \hat{\theta}_1$ WITH WAVE EQUATION

$$\left(\Box^{(2)} + V\right)\hat{\theta}_1^{(2)} = 0, \quad ds^2 = \frac{m}{mc}\left[-(c^2 - v^2)dt^2 - 2v\,dt\,dx + dx^2\right]$$

NEGLECTING V WE GET AN EXACTLY SOLVABLE THEORY

WE NEED TO INTRODUCE LIGHT-CONE COORDINATES

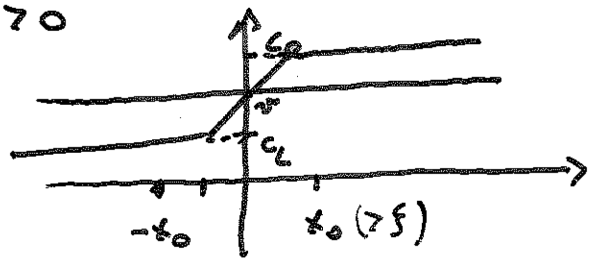
$$x^+ = t \pm \int \frac{dx}{c \mp v}, \quad x^- = t - \int \frac{dx}{c+v}$$ FOR WHICH THE 2-POINT

FUNCTION IS $\langle \hat{\theta}_1^{(2)}(x)\, \hat{\theta}_1^{(2)}(x') \rangle = -\frac{1}{4\pi} \ln (x^+ - x^{+'})(x^- - x^{-'})$

- WE NEED TO CONSIDER A (SIMPLE) FORMATION OF AN ACOUSTIC BLACK HOLE

THE CONDENSATE HAS UNIFORM DENSITY $n_{1D} = n \ell_\perp^2$ AND CONSTANT VELOCITY $\vec{v} = -v \hat{x}$, c IS VARIED:

- $t<0$: $c = c_R = \text{CONST.} > v$ (SUBSONIC FLOW)
- $t>0$



$x = 0$ ACOUSTIC HORIZON

$$\kappa = \frac{1}{2c} \frac{d}{dx}(c^2 - v^2)\Big|_{x=0} = \frac{dc}{dx}\Big|_{x=0}$$

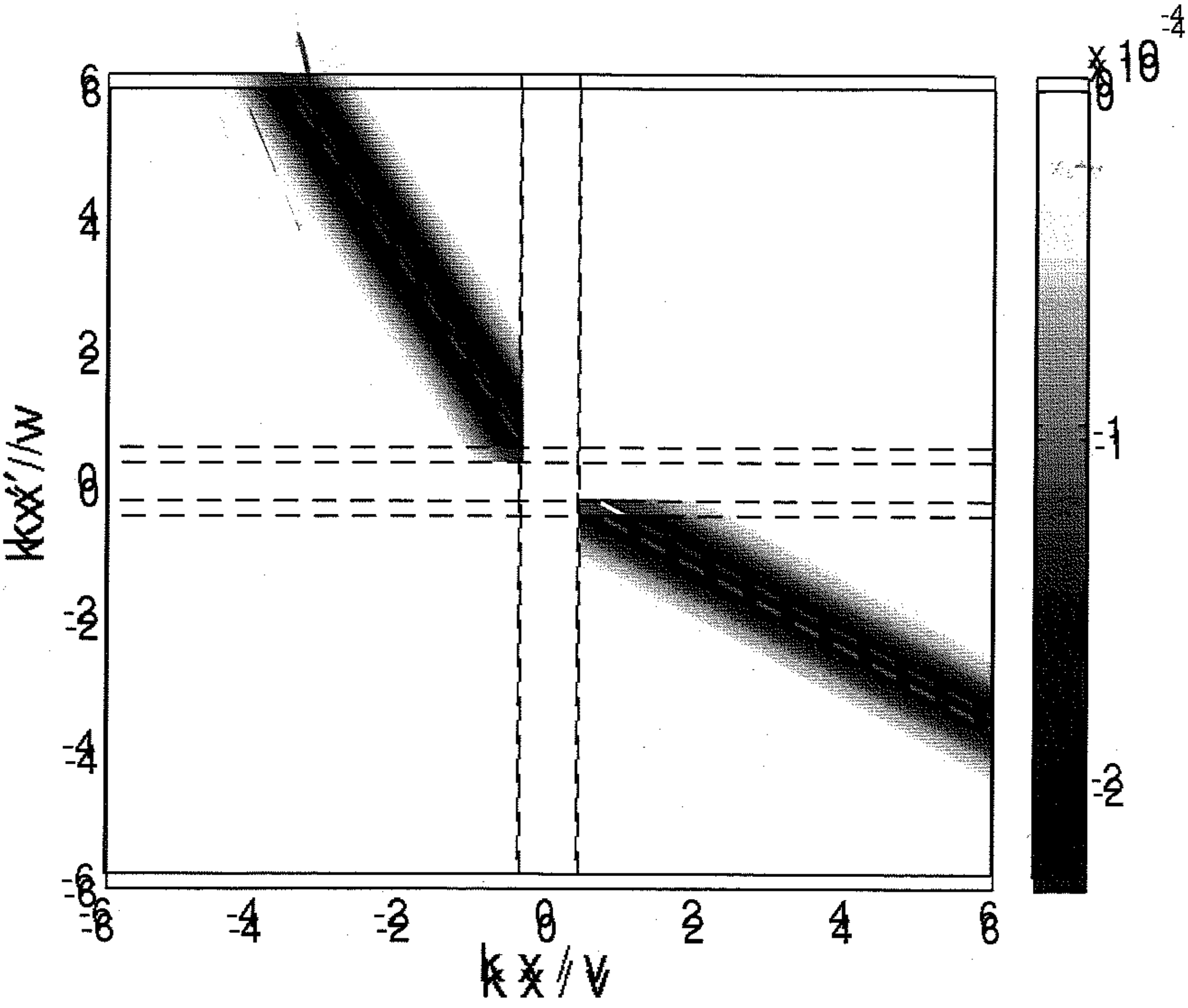
- THE STATE $|0\rangle_{IN}$ REQUIRES THE COORDS. $x^\pm_{IN}$ WHICH ARE RELATED TO $x^\pm_{OUT} \equiv x^\pm$ VIA $x^-_{IN} \sim \mp e^{-\kappa x^-}$, $x^+_{IN} = x^+$

INSIDE HORIZON / OUTSIDE HORIZON

- COLLECTING ALL RESULTS, FOR $x > x_0$, $x' < -x_0$ WE HAVE

$$\frac{G_2^{1D}(x,x')}{n_{1D}^2} = -\frac{\kappa^2 \xi_L \xi_R}{16\pi c_L c_R} \frac{1}{\sqrt{(n_{1D}\xi_R)(n_{1D}\xi_L)}} \frac{c_R c_L}{(c_R - v)(v - c_L)} \frac{1}{\cosh^2\left[\frac{\kappa}{2}\left(\frac{x}{c_R - v} + \frac{x'}{v - c_L}\right)\right]}$$

WITH A PEAK AT $\boxed{\frac{x}{c_R - v} + \frac{x'}{v - c_L} = 0}$



PREDICTION OF THE ANALOGY FOR AN

EXISTING GUIDED Rb-ATOM LASER EXPERIMENT

PHYSICAL INTERPRETATION

ONCE THE HORIZON IS FORMED, CORRELATED PAIRS OF OUTGOING PHONONS [≡ PROPAGATING UPSTREAM IN THE FREE FALLING FRAME] ARE CREATED AT $x \simeq 0$, ONE PROPAGATING TO THE RIGHT (HAWKING QUANTA), THE OTHER TO THE LEFT (PARTNER)

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$x = (c_R - v)\Delta t \qquad\qquad x' = (c_L - v)\Delta t$$

THIS IS THE BASIC PREDICTION OF THE ANALOGY

[TO REFINE IT WE NEED TO CONSIDER THE POTENTIAL V]
SEE TALK BY P. ANDERSON

THE HAWKING SIGNAL IN THE CORRELATIONS PERSISTS IN THE ATOMIC THEORY, AND IS CLEARLY VISIBLE AND DISTINCT FROM ~~AGAINST~~ V POTENTIAL COMPETING EFFECTS (SUCH AS THERMAL EFFECTS).... TALK BY I. CARUSOTTO