Strategic behavior in Schelling dynamics: A new result and experimental evidence*

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Abstract

In this paper we experimentally test Schelling's (1971) segregation model and confirm the striking result of segregation. In addition, we extend Schelling's model theoretically by adding strategic behavior and moving costs. We obtain a unique subgame perfect equilibrium in which rational agents facing moving costs may find it optimal not to move (anticipating other participants' movements). This equilibrium is far for full segregation. We run experiments for this extended Schelling model. We find that the percentage of strategic players dramatically increases with the cost of moving and that the degree of segregation depends on the distribution of rational subjects.
Keywords: Subgame perfect equilibrium, segregation, experimental games

1 Introduction

Individuals with similar ideas, habits, preferences, political affiliations or ethnic group tend to join together and create cliques and clusters in communities, which result in segregation at the society scale. Thomas C. Schelling (1971) showed that even individuals with very low mixing aversion may cause a segregated society in dynamic environments.[1] From this piece of research two intriguing results emerge: segregation is the result of individual preferences but also the maximizing social welfare configuration; in sharp contrast, segregation is not supposed to be a desirable outcome. Programs such us "*Moving to the Opportunity*" indicates how strong is the social concern about segregation (for more references see Kling, Liebman and Katz, 2007)

The Schelling segregation benchmark consists of a spatial model in which agents of two well-differentiated types distribute along a line with preferences that depend on the composition of their surrounding neighborhoods.[2] In this model there are no objective neighborhood

[1]Schelling defined a model in which agents, divided into two types, move on a checkerboard according to a given utility function. Within this set up, Schelling shows that segregation occurs even when individuals have very mild preferences for neighbors of their type as long as they are allowed to move in order to satisfy their preferences.

[2]Schelling defined a model in which agents, divided into two types, move on a checkerboard according to a given utility function. Within this set up, Schelling shows that segregation occurs even when individuals

boundaries; everybody defines their neighborhood with respect to their location. An individual moves if she is not content with the type mixture of her neighborhood, moving to where the mixture meets her tolerance level, which is defined as the proportion of individuals of different types in her neighborhood. Schelling's seminal model strikingly predicts a high segregation outcome from the initial situation when agents are myopic and can move without any cost.

Individuals in the Schelling model are myopic[3], that is, are persons who responds to immediate incentives. So, they do not compute too much and they just respond *instictively* (see Rubinstein, 2007). Its a nice question to check whether the dynamic of the model changes when we do not assume such us non–elaborated reasoning.

When we model the Schelling dynamics using strategic instead of myopic players we discover a non expected result: we do find that another less segregated outcomes than the predicted by Schelling are also equilibrium. In presence of multiplicity equilibria the final configuration of the society is uncertain. Consequently the design of social policies becomes subtle.

It would be highly desirable to find an environment in which society members' incentives are aligned around a unique equilibrium.

The assumption of free commuting in the original model is restricted to minimal movement (the nearest place). Therefore, Schelling is assuming that in certain sense, moving is not completely costless. Interestingly we do find that the introduction of any positive cost in the strategic model solves the multiplicity equilibria problem: we do find the unique subgame perfect equilibrium.

This result is not only interesting from a theoretical point of view. In real life, costs of moving, commuting, etc. play a crucial role on society members' decisions.[4] Notice that this realistic touch solves the theoretical problem before aforementioned.

have very mild preferences for neighbors of their type as long as they are allowed to move in order to satisfy their preferences.

[3]We consider myopic agents as those individuals that move according to the Schelling specific rule: individuals move whenever they are not content.

[4]Strategic agents facing moving costs anticipate subsequent movements by other participants, they may find it optimal not to move.

The contribution of this paper is threefold: i) We provide a strategic setting for the classical Schelling model when we show that under positive costs there exist an unique subgame perfect equilibrium. ii) We develop a new experimental setting for studying Schelling segregation models. iii) We propose a methodology to analyze experimental data in which individual decisions are compared to the best response for all observed paths.

In our theoretical setting, we consider a configuration with eight players where nobody is content. This example is the simplest one that compiles the main features of any extended Schelling model where strategic behavior may change the Schelling prediction. In our framework, Schelling would state that three players (the first, fourth and seventh) move, thereby generating a completely segregated configuration. In sharp contrast to the Schelling prediction, when individuals behave according to strategic behavior, we compute the subgame perfect equilibrium of the corresponding finite extensive game and obtain that only two players will move and four will stay in anticipation to the actions of others.[5] As a consequence, we obtain a much less segregated configuration.

This experiment is conducted both using a face-to-face setting and running a laboratory experiment in order to check the robustness of our results. We design a experimental one-shot game where:

- 8 subjects are randomly placed around a real circle describing a nonhappy society in a black, white, black, white configuration.
- Sequentially, and following the initial random sorting, subjects are given the chance to move or stay in order to reach the maximum level of happiness in the form of a fixed monetary payoff.

This simple scenario allows us to explore subjects who move (as predicted by Schelling) or subjects who stay 'even in absence of costs- in anticipation of other subsequent movements, that is, strategic players.

With the spirit of capturing the real decision of moving we add commuting cost to our experiment. Despite the baseline model we run two additional treatments: low and high

[5]Moreover, the remaining two players do not need to move as they are already happy when their turn to decide comes.

commuting costs. Costs are experimentally introduced by placing the money on the floor in front of each subject in the face-to-face experiment and in the computer interface ring in the laboratory experiment. Agents lose their money when moving. All the other features of these treatments are identical to the baseline.

The results we obtain are impressive: i) the percentage of strategic players dramatically increases with the cost of moving, specifically 13 % for 0-euro cost, 34% for a 5-euro cost and 44% for a 20-euro cost; and ii) the final degree of segregation drops with the number of strategic players (with at least one strategic player, the full segregated configuration failed 50 percent of the cases). The most important output of our experiment is that we provide evidence of players who do not select the more segregated equilibrium.

The paper is organized as follows. The related literature is presented in section 2. Section 3 is divided into three subsections. In the first subsection we recall the classic Schelling linear model, in the second subsection we present our extensive Schelling dynamics game and, finally, we introduce two definitions of individual behavior in the third subsection. The experimental design is explained in section 4 and the conclusions and results presented in section 5.

2 Related literature

Using one-dimensional and two-dimensional landscapes, Schelling (1969, 1971a) showed the emergence of high segregation even if individuals in the society had mild preferences for living with neighbors of their own type. Schelling's result is of interest to economists, policy-makers and social scientists in general because it illustrates the emergence of an aggregated phenomenon that cannot be directly foreseen from individual behavior and concerns an important problem: segregation. Considered particularly striking, this result has generated a vast amount of literature from a wide range of scientific trends. Miltaich and Winter (2002) assuming that individual's characteristics are unidimensional find a stable partition that not only is stable but also segregating. Likewise Karni and Schneidler (1990) examine the conditions for segregation and group formation in an example of overlapping generation model. On the other hand, the seminal concept of stochastic stability introduced by Foster and Young (1990), and developed within evolutionary game theory literature, provides insight into Schelling's spatial proximity model. Young (1998, 2001) presents a simple variation of the one-dimensional Schelling model, showing that segregation tends to emerge in the long run, even though a segregated neighborhood is not preferred by any agent. Zhang (2004) extends Young's one-dimensional set-up (1998) into a two-dimensional framework. They argue that complete segregation is the only viable long-run outcome of best-response dynamics if the agents' preferences are biased in favor of their own type. Pancs and Vriend (2007) also find that complete segregation is the only possible long-run outcome in a ring where agents have balanced preferences about the racial composition of the neighborhood. Although the analytical result in Pancs and Vriend (2007) cannot be extended to a two-dimensional setting, through simulations they show that best-response dynamics tend to produce segregation even in a two-dimensional space.

In summary, this branch of the literature shows that even if all individual agents have a strict preference for perfect integration, myopic best-response dynamics may lead to segregation[6]. This finding casts some doubts on the design (ability) of public policies to improve integration by promoting openness and tolerance with respect to diversity.

On the empirical side, a major approach in studies on racial segregation has been to analyze discrimination in housing prices. Specifically, studies from the 1960's such as King and Mieszkowski (1973) tend to find evidence that African-Americans pay more for equivalent housing. However, studies from the 1970's such as Follain and Malpezzie (1981) do not confirm this evidence. Cutler, Glaeser and Vigdor (1999) confirm that the African-American rent premium fell dramatically between 1940 and 1970 and had reversed entirely by 1990.

Another branch of the empirical literature explains segregation through social interaction models. In this literature, the concept of tipping[7] is crucial for understanding the dynamics of segregation. In particular, segregation emerges and persists precisely because such residential patterns resist tipping. Clark and Fossett (2007) provide simulation experiment results crafted to explore the implications of ethnic preferences in multi-group situations.

[6]In the above literature, the main assumption about individual behavior is not full rational behavior.

[7]Tipping is said to occur when some recognizable minority group in a neighborhood reaches a size that motivates the other residents to begin leaving.

They establish that ignoring the role of choice behavior based on own-race preferences is akin to omitting the potentially important influence of racial and ethnic dynamics in residential composition. Using regression discontinuity methods and Census tract data from 1970 through 2000, Card, Mas and Rothstein (2008) find strong evidence that white population flows exhibit tipping-like behavior in most cities of the U.S. This result is consistent with that of Cutler, Glaeser, and Vidgor (1999): tipping points[8] are higher in cities with less tolerant whites.

In sum, the empirical evidence also points to the existence of high segregation even when agent preferences depend on individual choices and every agent prefers to live in a mixed-race neighborhood.

3 An extensive Schelling dynamics game: A new result

3.1 Schelling's linear model

Schelling's (1969, 1971) linear model considers a finite number of individuals distributed along a line[9] where the individuals are of black or white types. All members of the population are assumed to care about the typology of the individuals they live with, *i.e.*, their neighborhood. Everyone is able to move to another location if they are dissatisfied with the type mixture they live in. Specifically, each agent defines her neighborhood as the $d > 0$ individuals on either side of her own location. Therefore each agent's neighborhood is composed of her $2d$ adjacent neighbors. Schelling assumed that every agent prefers to have at least $m \in \{1, \ldots, 2d\}$ neighbors of her same type. We call m the individual's *tolerance* level[10],

[8]Card, Mas and Rothestein (2008) find that the racial share variables have coefficients of 0.53 and 0.65, suggesting that tipping points are higher, but less than proportionately so in cities with higher minority shares. More densely populated cities have lower tipping points. This is consistent with Cutler, Glaeser, and Vigdor (1999) who find a positive relationship between density and segregation. The attitudes index also has a significant negative coefficient, indicating that tipping points are lower in areas where whites have stronger preferences against minority contact.

[9]In the one-dimensional model, Schelling (1971) also refers to the possibility of considering an infinitely continuing line or a ring.

[10]Young (2009) called this concept the agent i's social threshold.

which represents a threshold over the composition of each agent's neighborhood. Therefore agents' preferences over their neighborhoods are defined over the parameters d and m. Through these parameters we can determine whether an individual is happy (if the number of neighbors like her is larger than or equal to m) or unhappy (if the number of neighbors like her is smaller than m).

Let us denote a happy agent by ☺ and an unhappy one by ☹. Non-happy agents move in turns[11] starting from the left to the nearest point that fulfills their neighborhood configuration demand. Schelling defined nearest place as the point reached by passing the smallest number of neighbors on the way. In cases in which an agent has two nearest places at the same distance on both sides, the choice is arbitrary.

Whenever one agent moves, two different situations may arise. First, someone who was happy may become unhappy because like members move out of her neighborhood (or opposite members move in). Second, those who were initially unhappy are now happy as opposite neighbors move away (or like neighbors move nearby). Moreover, the Schelling rule holds that, in each round, any initially unhappy member who is happy when her turn comes will not move. Likewise, anyone who becomes discontent in the previous round will have her turn after all initially discontents have had their innings[12]. This process stops when no agent wants to move anymore or continue ad infinitum.

Note that, in the Schelling setting *nobody anticipates the movements of others.* That is, when their turn to move comes, everyone moves if their neighborhood demands are not met.[13] Although nobody actually prefers segregation to integration, the typical outcome is a highly segregated society. The dynamics in the Schelling's model are an iterative and sequential process of agents choosing myopic responses, where the only restriction to the mobility of agents is to go to the nearest place. The latter implies that moving is costless as any agent can move as many times as she wants.

Let us consider a very simple case to understand the apparently simple dynamics of Schelling's linear model. Suppose that a society is composed of 8 individuals of two types of which four are blacks (B) and four are whites (W). Assume that these individuals are

[11]Should they still be unhappy when their turn comes.

[12]This agent will move if she is still discontent when her turn comes.

[13]This coincides with our definition of a myopic player.

distributed along a ring with the following configuration:

$$\{B, W, B, W, B, W, B, W\} \tag{1}$$

Let us denote the individuals as their location on (??) starting from left to right. Hence, agent 1 is the top black bullet, agent 2 is the next white bullet and so on until agent 8, who will be the last white bullet. This configuration is represented as a connected ring in Figure ??.

Figure 1: A particular initial configuration of four type B individuals and four type W individuals.

Furthermore, suppose that each individual in configuration (??) accepts up to 50% of unlike neighbors[14] over a neighborhood composed of one individual at each side ($d = 1$). Notice that no player is happy in the initial configuration of the society and therefore all of them are willing to move. Let's see now how Schelling's dynamics work.

The first unhappy individual is agent 1 (Figure ??(a)), who is not satisfied with her neighborhood configuration. She may move to two satisfactory positions: either the position between agents 2 and 3 or the position between agents 7 and 8. As stated before, we solve this symmetric case by moving agent 1 to the right. That is, agent 1 moves between agents 2 and 3 (Figure ??(a) below). After agent 1 moves, agents 2, 3 and 8 also become happy. Therefore the next unhappy agent in the ring is individual 4. This agent will move to the location between 5 and 6 (Figure ??b), thereby making agents 5 and 6 happy. The next unhappy agent, and in this case the last one, is agent 7. Agent 7 will move to the position between agents 2 and 1, as it is the nearest position to the right that fulfills her preferences (Figure ??c). Notice that although individuals 2, 3, 5, 6, and 8 were initially unhappy, they do not move because they were happy when their turns to move came. This process ends at

[14] As in the original Schelling model. Schelling (1971, 1978) also considers the possibility that agents accept up to other percentages of unlike neighbors.

this stage because all individuals are happy, and no one wants to move to another location (Figure **??**d). Thus, the society ends up in a situation of *full segregation*.

Figure 2: Figures (a), (b), (c) and (d) illustrate the dynamics of Schelling's myopic response over the particular case of Figure **??**.

3.2 The extensive Schelling dynamics game

Now, we study the impact of strategic behavior in Schelling dynamics with moving costs. We translate the agent's preference into a utility function that depends on the individual's actions and the final configuration of the society. The dynamic structure of this framework is modeled as an extensive game where the players play sequentially.

In particular, each player $i \in \{1, 2, \ldots, N\}$ has two possible actions: either "to stay" at her initial location or "to move" to the nearest space with a neighbor of her same type. Each player prefers to be close to at least one like neighbor. Given these preferences, the payoff for every player is defined as a positive value, M, if the agent ends up with at least one neighbor like her. Recall that in the Schelling model there exists a moving order for each unhappy player. The first unhappy player moves, then the second agent faces the same decision and so on until the last unhappy agent. In our setting, each agent takes her best-response action. That is, one player could find it optimal either "to stay", because she anticipates that the actions of the other players would generate a final configuration where she will become happy, or "to move", otherwise. Both actions are allowed in our model, and an unhappy player does not necessarily have to move.

We extend the game by introducing moving costs of $C > 0$ to modify the value of the payoff function. Thus, if at the very end of the game player i is ☺, she earns the payoff either if she has moved M or if she has not moved $M + C$. If she ends up in an ☹ situation, she gets 0 or C if her action was "to move" or "to stay", respectively. We assume that $M > C > 0$.

Formally, the above description can be viewed as a sequential game denoted by $\Gamma^n = (\mathbf{N} = \{1, 2, \ldots, N\}, A_i = \{\underline{i}, \bar{i}\}_{i \in \mathbf{N}}, \{K\}_{1,\ldots,\sum_{i=0}^{N} 2^i}, \{Z\}_{2^{N-1},\ldots,2^N-1}, u_i : \prod_{i=1}^{N} A_i \to \mathbb{R})$ where

- $\mathbf{N} = \{1, 2, \ldots, N\}$ is the set of players.
- $A_i = \{\underline{i}, \bar{i}\}$ is the set of actions for each player such that $\underline{i}$ means that player i stays at her initial location and $\bar{i}$ that player i moves to the nearest space with someone of her same type.
- K is the set of nodes and Z is the set of terminal nodes. Notice that all nodes in $K - Z$ are information sets for only one player.
- The map $i : \mathbb{H} = K - Z \to \{1, 2, \ldots, N\}$ where $I(k) = i$ such that $\sum_{j=0}^{i-1} 2^j \leq k \leq \sum_{j=0}^{i} 2^j - 1$ determines the agent playing at this node.
- The set of terminal nodes $Z = \{\sum_{j=0}^{N-1} 2^j + 1, \ldots, \sum_{j=0}^{N} 2^j\}$ where payoffs $\{u_i\}_{i \in \mathbf{N}}$ are realized.

Given the sequential structure of Γ^n, we now characterize the subgame perfect equilibrium (SPE henceforth). The equilibrium strategies should specify optimal behavior from any information node up to the end of the game. That is, any agent's strategy should prescribe what is optimal from that node onwards given her opponents' strategies. Then, for each s in $K - Z$ an equilibrium strategy should specify the best action for player $i(s)$ who plays at s. Let us fix $h_s \in A_1 \times \ldots \times A_{i(s)-1}$ as the unique history from the root node reaching s and denote by $\Gamma^n_{h_s}$ the subgame starting at s.

Given the payoff structure of the game at state s, the behavior of player $i(s)$ will depend on both d and m.

Consider first the following case: let {B,W,B,W,B,W,B,W} be an initial configuration. If $d = m = 2$ such configuration will be happy since all their individuals have two neighbours (with distance equals to 2) like her. Nevertheless, if $d = m = 1$ then the same configuration is totally unhappy since no individual has at least one neighbor like her. Therefore, the same configuration can be considered either happy or unhappy depending on the specific tolerance level.

Schelling prescribes a behavior when a player is happy and when she is not. Namely, if an individual is unhappy she will move to the nearest place where she becomes happy. Nevertheless if she is happy she will stay at the same location. Actually, the movement of distance zero is the nearest place where she is (already) happy. Under strategic consideration we may find all the possible responses conditional on the tolerance level. Let us see some examples to understand the scope of individual behavior.

Consider a history of length 6 generating the following configuration $\{W, W, B, W, B, W, B, B\}$ from the initial configuration $\{W, W, B, B, W, W, B, B\}$ with $d = m = 2$. The game is at node s such that $i(s) = 7$. In other words, player 7 with type B is called upon to play. She has one neighbor like her with distance two to the left and another one with distance one to the right. Therefore, she is already happy. Schelling would declare “to stay” as the action played by player 7. But would player 7 play “to stay” if player 8 were rational?

Suppose that player 7 imitates the behavior that Schelling declares, *i.e.*, she does not move. Then the best response for player 8 is “to move” since player 8 is at the last stage of the game and she is still ☹.

The final configuration would be $\{W, W, B, B, W, B, W, B\}$ being the last one player 7. It is easy to compute that player 7 would finish being unhappy. Nevertheless, if player 7 anticipates the best choice that player 8 will choose, then player 7 could see that moving between player 1 and 2 the next configuration would be $\{W, B, W, B, W, B, W, B\}$. In this configuration player 8 is the last B and player 7 is at position 2. Both players have two neighbors like them at distance two. At the last stage player 8 has no incentive to move and this would be the end of the game[15].

Suppose now that the $d = m = 1$ and the initial configutaion is $BWBWBWBW$.

- If $i(s)$ is already ☺ when she is called upon to play, then happiness only comes from the previous movements of players $1, 2, \ldots (i(s) - 1)$. Notice that the next neighbor $i(s) + 1$ has a different type than $i(s)$. In this case, the best response is always “to stay”.

[15] This case illustrates that within our framework, strategic behavior with moving costs, a happy player may have incentives to move. Therefore, a positive cost does not necessarily imply a reduction of movements.

- If $i(s)$ is ☹ when she is called upon to play, then she has to compute her best response in the corresponding subgame Γ_{h_s}. She has to check whether to stay or to move is her best response.

For instance, suppose that $h_5 = (\underline{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5})$ is the history for player 6 at node s. As player 6 is already ☺ when she is called upon to play, since player 4 has moved close to her, then, her best response is "to stay", $\underline{6}$, and her payoff would be $M + C$ given that she did not pay any moving costs.

Suppose now that player 6 is at node s after the new history $h_5^{'} = (\underline{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5})$. In this case, player 6 is ☹. Is "to stay" the best response for player 6? As the action to move results in a lower payoff for any final situation, let's compute if $\underline{6}$ is the best response for player 6.

According to the definition of subgame perfection, it is necessary to compute the best response for players 7 and 8 at Γ_{h_5}. Let's start by studying the best response for player 8:

- Suppose that player 7 moves. Player 8 is then ☺ since player 6 is in her neighborhood. Therefore the best response for player 8 is "to stay".

- Suppose that player 7 does not move. Then the best response for player 8 given the history $(\underline{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7})$ is "to move" since player 8 is at the last stage of the game and she is still ☹.

Now consider the best response of player 7 given the above best response for player 8.

- If player 7 is ☺, then her best response is obviously "to stay".

- If player 7 is ☹, taking into account the best strategy for player 8, the corresponding best response will be "to stay" since it guarantees her the maximum payoff $(M + C)$.

Consequently, as player 7 will not move if agent 6 plays "to stay", she will end up ☹. Hence, her best response is "to move", thus guaranteeing the happiness position.

The above argumentation can be carried out at any (information) node for each player generating the SPE. In the previous examples, any configuration with any tolerance level generates an extensive game with the common property of *no indifference condition* at any information node[16]. Moreover, if there are enough players to guarantee the happiness condition for any type then the final configuration is a happy society. This entails that the full rationality assumption supports a one-round extensive game. Namely, players only need to play once to reach happiness in contrast to the Schelling dynamics that may be infinite.

The following theorem states the existence of a unique SPE for Γ^n for any tolerance level. Moreover, it is studied the case of $d = m = 1$ for the initial configuration $\{B, W, B, W, B, W, B, W\}$. This instance is the simplest one that collects the main features to properly discriminate between strategic versus myopic behavior. Furthermore, the equilibrium path that arises from the unique SPE does not generate the full segregation configuration in contrast to the full segregated outcome reached from Schelling model.

Theorem 1

- *There exists a unique subgame perfect equilibrium in the extensive game* Γ^n.
- *For* $d = m = 1$ *and the initial configuration* $\{B, W, B, W, B, W, B, W\}$*, the final configuration* $\{B, B, W, W, B, B, W, W\}$ *is the consequence of the unique equilibrium path* $(\underline{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$.

The proof is in the Appendix.

The configuration reached by strategic players playing the SPE for a initial configuration $\{B, W, B, W, B, W, B, W\}$ and $d = m = 1$ has two remarkable properties. On the one hand, any player is happy at the end of the sequential game and, on the other hand, half of the players decide to stay, thus avoiding the moving cost. We call this final configuration $\{B, B, W, W, B, B, W, W\}$ a *happy-non-segregated* (HNS henceforth) society. We should point out that this is the only happy configuration when there are moving costs.

3.3 Subject types and outcomes

The above Theorem holds that there are four types of behavior in equilibrium. The first one, Type I, is implemented by players 5 and 6. Given the history $(\underline{1}, \underline{2}, \underline{3}, \bar{4})$, players 5 and 6 are already happy as a result of player 4's action. We can say that their best response, to

[16]Note that the game Γ is a generic game.

stay, is equivalent to what they would have done in the Schelling framework. In other words, both players 5 and 6 would play the same action regardless of whether they were playing a myopic response or the SPE. In both cases they stay in their initial position. This makes it impossible to distinguish between cases where players do not need to move whether they are playing the linear Schelling model or the sequential Schelling dynamics model.

The second type of behavior is performed by players 1, 2, 3, and 7. When these agents are called upon to play, no one is happy, but their best response is to stay. Such behavior implies that they have computed[17] the best response to their associated subgame. Notice that in all of these nodes players play differently than when they act as myopic players. We call such behavior Type II behavior.

The third behavior could appear in opposition to the above strategic behavior. For instance, if player 7 played "to move" she would get a positive payoff M because she ends ups in a happy configuration. Nevertheless, such behavior implies that player 7 did not compute her best response taking into account the best response of player 8. We call such behavior "myopic behavior" or Type III behavior.

Finally, we have Type IV individuals: players 4 and 8. Both players are unhappy and they decide "to move" in contrast with the above set of players $\{1, 2, 3, 7\}$. This is so because their best action is "to move". In other words, if player 4 decided to stay, the best response by the rest of the players would never generate a configuration with player 4 in a happy situation.

The four types described above are summarized in Table 1:

$Type$	$Initial$	$Best - Response$	$Myopic$	$Action$
I	$Happy$	$Stay$	$Stay$	$Stay$
II	$Unhappy$	$Stay$	$Move$	$Stay$
III	$Unhappy$	$Stay$	$Move$	$Move$
IV	$Unhappy$	$Move$	$Move$	$Move$

Table 1: Subject types.

[17] The action of staying in such situations conveys a non-trivial computation.

Although all players play their best response in the equilibrium, we specially distinguish the set of Type II players ($\{1, 2, 3, 7\}$), whom we call *strategic players*. We emphasize that the action "to stay" played by unhappy players demonstrates[18] the ability to compute the best response in the current subgame.

Formally,

Definition 1 (Type II, unhappy-strategic): Given history h, we define as *strategic behavior* those ☹ i who play their best-response "to stay" in Γ_h.

Definition 2 (Type III, unhappy-myopic): Given history h,we define as *myopic behavior* those ☹ i who play "to move" instead of their best-response "to stay" in Γ_h.

We should emphasize that there may exist another type of behavior. Irrational subjects may choose *to move* in a situation in which they are already happy (not-Type I) or subjects who are not happy choose *to stay* when they do not have any chance of being happy unless they move[19] (not-Type IV). Formally,

Definition 3 (Type $\neg$ I and $\neg$ IV): Given history h, we define as *irrational behavior* those ☺ (☹) i who play "to move" ("to stay") when both their myopic and best response is "to stay" ("to move") in Γ_h.

Let us now focus on how the emergence of unhappy-strategic (Type II) versus unhappy-myopic (Type III) behavior may affect the final outcome configuration. We have characterized all possible histories of length $i-1$ such that player i faces the above dilemma: strategic versus myopic response. At each possible decision node of the above family of histories, agent i should play "to stay" as her best response and therefore player i declares herself to be Type II, unhappy-strategic. However, we also consider the other action, "to move", where player i reveals herself to be Type III, unhappy-myopic. We can distinguish between both types since there is a one-to-one correspondence between actions and types, (Types II

[18]The rule of thumb " never move" may explain the same behavior than Type II in our study. Nevertheless we exclude such explanation since our data do not present any individual that follows this rule.

[19]An individual follows the rule of thumb if she is categorized as type II and not-Type IV at the same time. There is not data in our experiment with the above condition.

and III), for the above family of histories. We assume that regardless of the action taken by player i, the remaining players $(i+1,\ldots,8)$ play their subgame best response. A complete characterization is provided in Table C of Appendix C.

The two following conclusions emerge from the above characterization. The first one is that in *the absence of Type II players* (row **4*** in Table C, Appendix C) the Schelling full segregation outcome will always emerge.

The second conclusion makes our results substantially different from the Schelling outcome. In *the presence of one Type II player* the Schelling full segregation outcome would emerge if and only if player 1 is strategic and players 2, 5 and 8 are myopic. See row **5*** in Table C, Appendix C.

Hence, the existence of strategic players in the society may play a crucial role in the prevalence of Schelling predictions. Moreover, the emergence of irrational players Type $\neg$ I and $\neg$ IV may alter the previous idea. Actually, both types of behavior enhance less segregated societies. However, this is not conclusive regarding the final configuration as the irrational action of player i at node k could be balanced out by the actions of subsequent players.

4 Experimental design

In this section we describe the design of the experiments to test first the existence of Type II (unhappy-strategic) players and, second, how their behavior affects the Schelling segregation result.

We start with an experiment of the linear Schelling model without moving costs and then provide an experiment of our extensive game (the Schelling dynamics model with high and low moving costs). The experiment was run at the School of Economics of the University of Granada (Spain) on April 20, 2009. A standard laboratory session at LINEEX, University of Valencia (Spain), was also conducted to check the robustness of our results.

4.1 A face-to-face Schelling ring experiment

Commuting is a key ingredient of the Schelling model. Subjects choices are restricted either "to move" or "to stay". In an attempt to maintain the essence of individual choice, in our game, subjects physically move across a ring. We consider that a face-to-face experiment might capture better the features an individual takes when she decides to move.

We organize individuals into groups of eight around a circle measuring 11.5 feet in diameter (see Figure **??**). To allocate the individuals in each circle, we follow a random sorting scheme where the first individual is assigned the black type, the second individual the white type, and so on[20]. Thus, the initial configuration in each circle is $\{B, W, B, W, B, W, B, W\}$. Each subjects' type is easily identifiable by a black/white scarf.

According to Schelling, players obtain positive utility if they reach a happiness position (i.e., a player has at least one neighbor of her type). We capture the individual's utility function through a fixed prize that might be earned if a subject ends up with a like neighbor (Figure **??** below).

Figure 3: Circle configuration

Notice that in the initial configuration nobody is happy. Therefore, subjects have the opportunity "to stay" or "to move" from their location when their turn comes. Thus,

[20]The experiment was conducted in Spain where the labels "black" and 'white" are meaningless. Therefore there is no reason to be aware of potential framing.

- Subject 1 might decide to stay or to move $\{\underline{1}, \overline{1}\}$;
- After subject 1 has taken her decision, subject 2 (on the left of subject 1) faces an identical decision problem $\{\underline{2}, \overline{2}\}$;
- Then, subject 3, etc.

Before starting the real game, subjects made two trial runs to ensure that they understood the structure of the game. Individuals then play the game five times. In each round a random device replaces subjects in new positions (and therefore the color of their scarves may change). The following payoff schemes were used in the experiment:

a) players would be paid for one of the 5 games (randomly selected);

b) two out of the eight subjects (randomly selected) would be paid; and

c) the prize would be 50 euros (60 US dollars) for the happy subjects selected.

In summary, two out of the eight participants would be randomly selected to be paid 0 euros if they end up being unhappy and 50 euros otherwise.

4.2 Face-to-face and costly Schelling experiment

To implement the extensive Schelling dynamics game, we add commuting costs to the above experiment. In particular, we run two additional treatments in the moving costs experiment:

Treatment 1 (low): 5-euro commuting cost.

Treatment 2 (high): 20-euro commuting cost.

The introduction of costs in the experiment was as easy as possible. We leave the money placed on the floor in front of each subject and they were advised that, if they move, they will loose this (potential) money. The analogy is quite obvious: you leave your home or friends behind when you move. That is, all subjects can see a 5-euro [20-euro] bill at their feet (6 [24] US dollars). Consequently, if they move, they lose the 5 [20] euros.

As before, two out of the eight participants are randomly selected to be paid for the second 5 rounds. Therefore, in the extensive game:

- An individual might earn 0 euros if she ends up unhappy and has moved or 5 (20) euros if she ends up unhappy and has not moved.

- If she ends up with a neighbor like her she might earn 50 euros (if she has moved) or 50 plus 5 (20) euros if she ends up with a neighbor like her and has not moved.

In addition to the 5 rounds (identical to the baseline with the only difference of the costs) we run another additional 5 rounds after a surprise re-start.[21] The only reason to do this was to double the number of observations of these treatments.

4.3 Implementation

The experiment was run at the School of Economics of the University of Granada (Spain) on April 20, 2009. To check the robustness of our main results we run an additional computer interface experiments at the LINEEX experimental laboratory of the University of Valencia (Spain) on February, 2010.

Overall, related to the implementation of the experiment, 6 sessions were run for each treatment with 8 subjects each (5 for the baseline) with a total of 136 participants ($8 \times 6 + 8 \times 6 + 8 \times 5$). The participants were randomly recruited from a subject pool of 191 subjects who had signed up to participate in the experiment.

All the experimental sessions were conducted on the same day and at the same time. Experimental subjects were assigned to each ring randomly and they stay in the same ring during the whole experiment. Seventeen associate professors, teaching assistants and Ph.D. students (working in experimental economics) acted as monitors to conduct the sessions. The monitors received identical training and an instructions booklet[22] (including random assignments for each round, payoffs scheme, etc.). The complete experiment lasted 1 hour and the average earning was 30 euros.

Two important features stand out in our design:

[21] Once agents playing in the experiment finished the 5th round of the game, the monitor informed them that they were going to play five more games again with identical conditions and payoffs.

[22] A copy of the instructions booklet is available upon request.

1. The randomization sorting in each round allows us to obtain different types of actions as first mover, second mover and so on for any given subject.

2. The surprise re-start allows us to study learning. This within-subject analysis enables us to explore how experience may alter subjects' willingness to play more strategically.

5 Results

This section presents the results on the individual behavior of agents playing in the two settings described earlier: a face-to-face Schelling ring experiment (section ??.??) and a face to face costly Schelling experiment (section ??.??). We also provide information on the final level of segregation in the society.

To explore the first inquiry, we measure the proportion of players who played strategically in the three experiments we ran. We should point out that although we have 1160 observations, $(48 \times 10 + 48 \times 10 + 40 \times 5)$, only 374 of them are informative about the type of behavior we are interested in (either myopic or strategic). As we have already mentioned, we are unable to confirm strategic behavior for Type I and Type IV (see Table 1 in section ??.??).

Its also important to emphasize what we can learn from our experimental setting. We can check if a experimental subject fails at least once playing myopically. Its also true that subjects who play myopically in our experiment (given the position, history, etc.) might play strategically in other environments. In sum, we can say that the number of strategic player we are able is unravel is a lower bound.

The rest of the section is divided into two subsections. First, we explore how often unhappy-strategic players (Type II) emerge. Second, we study which outcome we obtain in relation to the number of Type II agents within a circle.

5.1 The emergence of strategic players

We explore how the cost of moving (either 0, 5 or 20 euros) may affect subjects' moving decisions, that is, either "to stay" or "to move". Using the information provided by each circle, we compute for all empirical histories[23] those cases where subjects face the dilemma either "to stay" or "to move" when the situation is not trivial (Type II, unhappy-strategic versus Type III, unhappy-myopic). We find that, on average, subjects do not exhibit myopic behavior in 30% of the cases. That is, they behaved strategically in 30% of the cases (acting as Type II). Furthermore, this percentage rises to 39% when data drawn from the zero cost treatment are removed.[24] Figure **??** compares the proportion of Type III players across treatments.

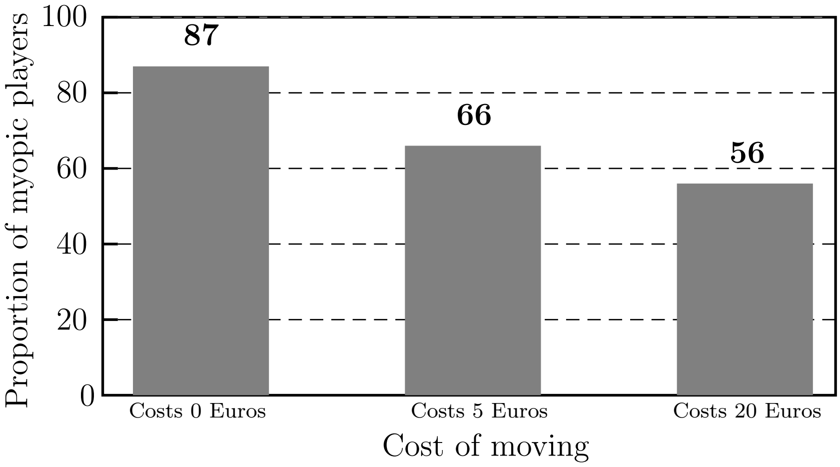

Figure 4: Percentage of myopic players across treatments

The existence of moving costs prevents subjects from being indifferent between their actions (either to stay or to move). In particular, each player, given her history, will calculate her best subgame response before taking any action and find situations in which she is not unhappy, but not moving is her best response. This is because she can anticipate that the best actions of subsequent players will make her happy in the final configuration of the society and hence she does not need to give up her initial endowment (5 or 20 euros). However, we do not find this feature in the 0 cost setting since subjects are indifferent between both actions.

[23] All the empirical histories we obtain in the experiment are reported in Appendix B. In each history we underline who behaves strategically and who deviates from such behavior.

[24] As a result of our randomization, the subjects did not learn at all. When comparing data from treatments 2 and 3 (positive cost treatments), we find that 61% of subjects were myopic players in rounds 1-5, whereas the same 61% applies for rounds 6-10.

In our experiment we find that the percentage of players playing á la Schelling dramatically falls when moving decisions are costly. Figure **??** shows that the number of subjects who play strategically by anticipating what their neighbors would play substantially increases with the costs of moving.[25] In fact, in the high-cost treatment nearly half of the subjects behave strategically (44%).

From the above result we can conclude:

Result 1: The existence of moving costs increases the percentage of subjects playing strategically. In particular, 39% of the subjects in the positive cost settings play the dominant strategy (i.e., not moving).

The repetition of the experiment in the Lab reports very similar results. We conducted low and high-cost treatments with the same number of subjects (8×6 and 8×6) and instructions. Our results are maintained in terms of the number of strategic players. In fact, we found that 62.08% of players behaved as Type III (unhappy-myopic) players in contrast to 61% in the face-to-face experiment.[26]

5.2 Strategic behavior and segregation

Our theoretical model indicates that just one strategic player might be enough to reverse the Schelling output (see second conclusion in section **??**.**??**). Our experimental data show that strategic behavior is not rare but, on the contrary, quite abundant (40%). Hence, Schelling's prediction should be less frequent in our setting. We now explore the final level of segregation achieved in each circle and how the existence of strategic players alters the

[25]We find that the proportion of the zero cost treatment is larger and statistically different from the proportion of the positive cost treatments either using a test of proportions or a Kruskal-Wallis test (in both tests we can reject the hypothesis that the proportions are equal at any level below 0.0002). Further, the proportion of the zero cost treatment is also larger and statistically different than the proportion in the 5 Euros cost treatment (20 Euros treatment), either using a proportion or Krukal-Wallis test, at any level of significance below 0.041 (0.0001). And we reject the null that the proportions of the two positive moving cost treatments are statistically different at the 5% significance level (as the p-value is 0.078).

[26]Using a test of proportions we cannot reject the hypothesis that these proportions are statistically equal at the 5% significance level (p-value is 0.6267).

final configuration. That is, we analyze if the number of strategic players within each clock has an effect on the final level of segregation in the society. As expected, we find that strategic players substantially change the final segregation outcome achieved.

In this experiments we provide evidence from 145 clocks. We consider that each round is an independent observation given that the randomization design process ensures that subjects do not necessarily hold the same position or color in each round they play. Table C (Appendix C) summarizes all the possible histories observed during the experiment. Conditional on the personal history of each subject, we calculate the number of strategic players within each clock. In our sample (145 clocks) we have 61 clocks with no strategic players, 48 clocks with 1 strategic player, 25 clocks with 2 strategic players, and 11 clocks with 3 or more strategic players. We find that the most relevant configurations are:

- the complete segregation outcome defined as (BBBBWWWW), is reached in 68 out of 145 clocks (47% of the cases);
- the HNS (Happy-non-segregated, defined in section **??**.**??**) is achieved in 36 clocks (25% of the cases); and,
- 41 cases in which we obtain neither the full segregation outcome nor the HNS configuration (28% of the cases).

We further explore how the final segregation configuration is related to the presence or absence of strategic players within each clock. In Figure **??**, we plot the relation between segregation and the proportion of strategic players. We present the percentages of the full segregation outcome ($BBBBWWWW$) when we have 0, 1, 2, and 3 or more strategic players within a clock. Likewise, in Figure **??** we report the percentage of non-full segregated outcome and the HNS outcomes ($BBWWBBWW$) within each of these proportions when there are $0, 1, 2, 3$ or more strategic players within a clock.

By comparing the above figures we obtain the following conclusions:

- In the absence of strategic players, we obtain the Schelling full segregated outcome in 67.21% of the cases (see first bar in Figure **??**). We have that 32.79% of the outcomes

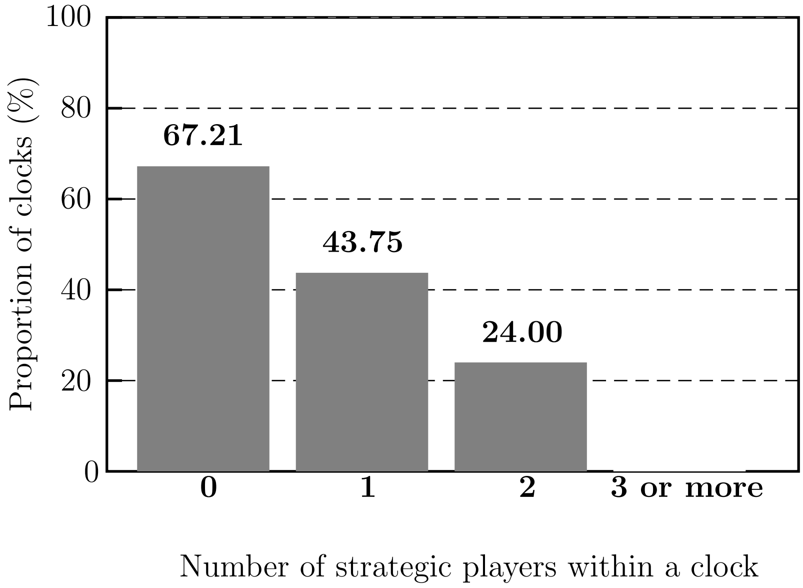

Figure 5: Proportion of clocks with the BBBBWWWW outcome.

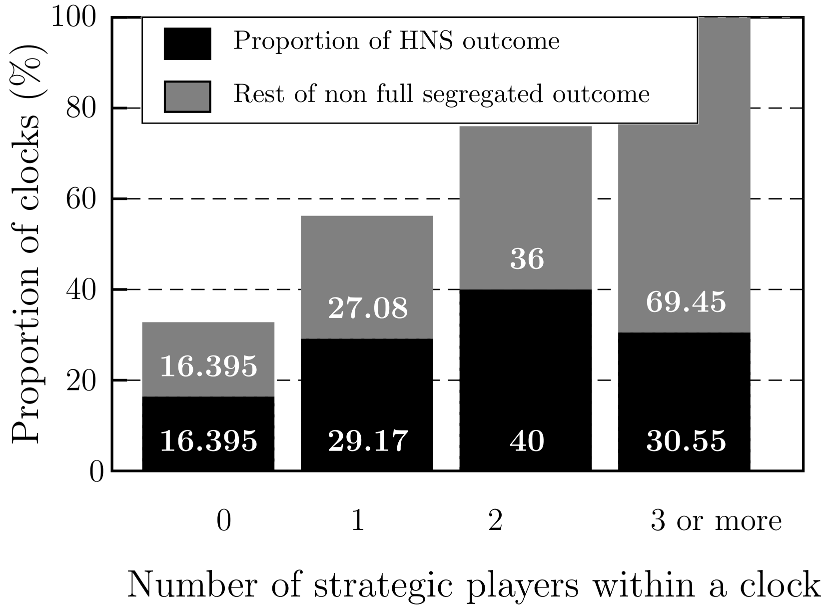

Figure 6: Proportion of clocks with a non full segregated outcome

are non-segregated, of which half are HNS outcomes.[27]

- In sharp contrast with the above conclusion, the existence of 3 or more strategic players completely denies the emergence of a full segregated outcome (see last bar in Figure

[27]We have tested that these two proportions are different from each other using a test of proportions. The test indicates that we can reject the hypothesis that these proportions are statistically different from each other with a p-value of 0.0001. This result is confirmed by a Kruskal-Wallis test.

??).[28]

- Furthermore, the number of HNS outcomes significantly increases with the number of strategic players within a clock (see second, third and fourth bars in Figure **??**).[29]

Hence, the Schelling full segregated outcome seems to be significant and negatively related with the existence of Type II (unhappy-strategic) players within a clock. Likewise, the HNS society configuration seems to be significant and positively related with the existence of strategic players. As the number of Type II players within a clock increases, the higher the probability of ending up with an HNS society (see Figure **??**).

Result 2: As the number of strategic players increases, the appearance of completely segregated outcomes is significantly reduced.

Finally, we discuss the results for all the cases in which we have neither a full segregated, nor the HNS, outcome. These are circles where the final outcome was not complete happiness. Although we do not get a happy society, we also find some interesting trend in these cases. First, note that these configurations are very unlikely to appear in the absence of a strategic player. Second, the combination of Type II and $\neg$ I and $\neg$ IV makes the non-equilibrium outcome much more likely to appear. Hence, in the spirit of theorem **??** we see that the larger the number of Type II the more likely is the HNS equilibrium.

6 Concluding Remarks

The relevance of this paper is twofold. First, we provide an extension of the Schelling linear model where subjects face costly decisions and, as a consequence, strategic playing emerges. We show that this variation affects the basic result of the model by moving from

[28]These two proportions are statistically different from each other with a p-value of 0.

[29]We obtain that the proportion of HNS outcomes (when there are not any strategic player) is statistically different from the proportion of the HNS outcomes (when there is at least one strategic player), using a proportions test, at any level of significance greater than 0.0451 (this is confirmed by a Kruskal-Wallis test). Furthermore, we cannot reject the hypothesis that the proportions of HNS when there are 1, 2, 3 or more strategic players are statistically different at the 5% significant level.

full segregation to clustering. Second, we experimentally test the prediction of the original a Schelling model and the extended model we propose. That is, we develop a geographical experimental setup where we check whether experimental subjects play according to the predicted Schelling myopic behavior or, in sharp contrast, they decide as strategic players.

This paper contains certain features that support the development of an interesting research agenda. The most obvious example is that we provide a new experimental setup to explore the role of locational issues on strategic behavior. Given that subjects learn much more easily to play in this environment, the emergence of strategic behavior is not rare. Moreover, we find that visual experimental setups make subjects more aware of their rivals' strategic playing.

We must recall that different types of behavior was one of the main reasons why we undertook this study. Specifically, we were interested in exploring how differences among players may have an effect on economic outcomes. Our simplified representation of the society is useful in analyzing how subjects play these "sorted games".[30] We plan to extend our research agenda to improve the applicability of our work, including ethnic segregation with income differentiation. In the same vein, and in line with some works in public policy, we are working on Schelling dynamic models with taxation concerning situations where the state designs policies to achieve more desirable configurations.

References

[1] Card, David, Alexandre Mas, and Jesse Rothstein. 2008. "Tipping and the Dynamics of Segregation." *Quarterly Journal of Economics.* 126(1): 177-218.

[2] Clark, William A. V., and Mark Fossett. 2008. "Understanding the social context of the Schelling segregation model." *Proceedings of the National Academy of Science of the United States of America*, 105(11): 14109-4114.

[30] Julie Berry Cullen, Brian A. Jacob and Steven Levit (2006) is an excellent example of how random (public lottery) versus endogenous (parental choice) sorting may have an effect on child academic performance.

[3] Cullen, Julie Berry, Brian A. Jacob, and Steven Levitt. 2006. "The Effect of School Choice on Participants: Evidence from Randomized Lotteries". *Econometrica*, 74(5): 1191-1230.

[4] Cutler, David M., Edward L. Glaeser, and Jacob L. Vigdor. 1999. "The Rise and Decline of the American Ghetto." *Journal of Political Economy*, 107(3): 455-506.

[5] Follain, James R., and Stephen Malpezzi. 1981. "Another look at racial differences in housing prices." *Urban Studies*, 18(2): 195-203.

[6] Foster, Dean P., and H. Peyton Young. 1990. "Stochastic Evolutionary Game Dynamics." *Theoretical Population Biology*, 38(2): 219-232.

[7] Karni, E., and D. Schmeidler. 1990. "Fixed preferences and changing tastes", *American Economic Review* 80(2), 262-267.

[8] King, A. Thomas, and Peter Mieszkowski. 1973. "Racial discrimination, segregation, and the price of housing." *Journal of Political Economy*, 81(3): 590-606.

[9] Kling, Jeffrey R., Jeffrey B. Liebman, and Lawrence F. Katz (2007), "Experimental Analysis of Neighborhood Effects", *Econometrica* 75, 83-119.

[10] Miltaich, I., and E. Winter. 2002. "Stability and Segregation in Group Formation", *Games and Economic Behavior* 38, 318-346.

[11] Osborne, Martin J., and Ariel Rubinstein. 1996. *A Course in Game Theory.* Cambridge, Massachusetts: The MIT Press.

[12] Pancs, Romans, and Vriend Nicolaas J. 2007. "Schelling's spatial proximity model of segregation revisited." *Journal of Public Economics*, 91(1): 1-24.

[13] Rubinstein, Ariel. (2007). Instinctive and Cognitive Reasoning: A Study of Response Times, *Economic Journal*, 117 (2007), 1243-1259.

[14] Schelling, Thomas C. 1969. "Models of Segregation." *American Economic Review, Papers and Proceedings*, 59(2): 488-493.

[15] Schelling, Thomas C. 1971. "Dynamic Models of Segregation." *Journal of Mathematical Sociology*, 1(2): 143-186.

[16] Young, H. Peyton. 1998. *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions.* Princeton, NJ: Princeton University Press.

[17] Young, H .Peyton. 2001. The Dynamics of Conformity. In S.N. Durlauf & H.P. Young (Eds.), *Social Dynamics*, 132-153. Cambridge, Massachusetts: The MIT Press.

[18] Young, H. Peyton. 2009. "Innovation Diffusion in Heterogeneous Populations: Contagion, Social Influence, and Social Learning." *American Economic Review*, 99(5): 1899-1924.

[19] Zhang, Junfu. 2004. "Residential Segregation in an All-integrationist World." *Journal of Economic Behavior & Organization*, 54(4): 533-550.

7 Appendix A

The payoff for player i depends on her action and the final configuration reached after all players have played. If her action was to stay and at the end of game the number of her neighbors like her is at least m then she gets $M + C$. Otherwise she only gets C. If her action was to move and she reached a position with at least m neighbors like her then her payoff is M. In contrary, she will obtain 0. Notice that she will never be indifferent between both actions, to move and to stay. Therefore the property of no indifference holds. In other words, Γ^n is a finite generic extensive game. Consider any possible initial configuration of n individuals of type 0 or 1. This configuration can be viewed as a sequence in $\{0, 1\}^n$. Next lemma assert the existence and unicity of a SPE.

Theorem 1

- *There exists a unique subgame perfect equilibrium in the extensive game* Γ^n.
- *For* $d = m = 1$ *and the initial configuration* $\{B, W, B, W, B, W, B, W\}$*, the final configuration* $\{B, B, W, W, B, B, W, W\}$ *is the consequence of the unique equilibrium path* $(\underline{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$.

Proof. i) The existence of a pure Nash equilibrium of the extensive game Γ^n is a consequence of Zermelo's Theorem since Γ^n is a finite game. Moreover by Osborne and Rubinstein (1996), page 100, this game verifies the non indifference property for any two terminal nodes. Therefore, all the SPE are equivalent in payoff. As the game is generic then the result holds.

Recall that $d \geq m$. If the number of players of each type is at least $m+1$ then the final society is always happy for any initial configuration. This is so since there exist at least a path over the 2^n possible paths of actions of the extensive game with a cluster of these $m+1$ players one after the other. Each player has at m neighbors like her in a neighborhood of ratio d.

The existence of a pure Nash equilibrium of the extensive game associated to the initial configuration $\{B, W, B, W, B, W, B, W\}$ holds since it is a particular case of the above case. At node s of the extensive game, a player, say i_s, will play her best response given history h_s and considering that the rest of the players will play their best response. Therefore, in order to describe the best strategy for each player, we distinguish two sets of histories: those in which player i_s is already happy and those that do not verify the happiness condition. Notice that for the first set of histories, the best response for player i_s is always "to stay" since her payoff is $M + C$ greater than M.

1. The analysis starts with the last player; player 8.

   (a) The histories of length 7 where player 8 is ☺ are:

   - $h(s) = (*, *, *, *, *, \bar{6}, *)$
   - $h(s) = (*, *, *, *, \underline{5}, \underline{6}, \bar{7})$
   - $h(s) = (\bar{1}, \underline{2}, *, *, *, *, *)$

   where the symbol $*$ denotes any possible strategy for the player who has to play at this position. As we already noted, her best-response is "to stay" $\underline{8}$.

   (b) Consider now the histories where player 8 is ☹ :

   - $h(s) = (\bar{1}, \bar{2}, *, *, *, \underline{6}, \underline{7})$
   - $h(s) = (\underline{1}, *, *, *, *, \underline{6}, \underline{7})$.
   - $h(s) = (\bar{1}, \bar{2}, *, *, \bar{5}, \underline{6}, \bar{7})$

In all these cases, as player 8 is the last player, her best response is "to move" $\bar{8}$.

2. To compute the strategy of player 7, we take into account both the history and the best reply of player 8 given the two possible actions of player 7.

   (a) Player 7 is $\smiley$ in the following cases:

   - $h(s) = (*, *, *, *, \bar{5}, *)$
   - $h(s) = (*, *, *, \underline{4}, \underline{5}, \bar{6})$

   Her best response is "to stay" $\underline{7}$.

   (b) Player 7 is $\frownie$ if her history is $(*, *, *, *, \underline{5}, \underline{6})$. As the action "to stay" is dominant over the action "to move", we check if "to stay" is her best response given the corresponding best response of player 8:

   - The two following histories $h(s) = (\bar{1}, \bar{2}, *, *, \underline{5}, \underline{6})$ and $h(s) = (\underline{1}, *, *, *, \underline{5}, \underline{6})$ have in common that for action $\underline{7}$, the best response for player 8 is "to move". Consequently, at the final configuration, player 7 becomes $\smiley$ given the best response of player 8. Therefore, in those cases, the best response for player 7 is "to stay", $\underline{7}$.
   - Nevertheless, the history $h(s) = (\bar{1}, \underline{2}, *, *, *, \underline{6})$ with action $\underline{7}$ presents a path for player 8 within a best response of $\underline{8}$. Therefore player 7 has to move in order to end up with at least one neighbor like her. We conclude that her best response is $\bar{7}$.

   To summarize, depending on the action of player 1 and 2, the path generated by the best responses of players 7 and 8 is either $(\underline{7}, \bar{8})$ or $(\bar{7}, \underline{8})$.

3. Following the same argument as before, the best response for player 6 depends on her history of length 5 and the actions that player 7 and 8 will play afterwards.

   (a) Player 6 is $\smiley$ if:

   - $h(s) = (*, *, *, \bar{4}, *)$
   - $h(s) = (*, *, \underline{3}, \underline{4}, \bar{5})$

thus her best response is “to stay”, $\underline{6}$.

(b) Player 6 is ☹ in the following histories of length 5:

- $h(s) = (\bar{1}, \underline{2}, *, \underline{4}, \underline{5})$. In this situation, player 8 is happy since player 1 moved. Using the conclusion obtained above for players 8 and 7, player 6 anticipates their best response $(\bar{7}, \underline{8})$. Therefore, giving the movement of player 7, the best response for player 6 will be “to stay”, $\underline{6}$.
- $h(s) = (\bar{1}, \underline{2}, \bar{3}, \underline{4}, \bar{5})$. As in the above case, we can conclude that will decide “to stay”, $\underline{6}$.
- $h(s) = (\underline{1}, *, *, \underline{4}, \underline{5})$, $(\bar{1}, \bar{2}, *, \underline{4}, \underline{5})$, $h(s) = (\underline{1}, *, \bar{3}, \underline{4}, \bar{5})$ and $h(s) = (\bar{1}, \bar{2}, \bar{3}, \underline{4}, \bar{5})$. In all these cases, player 8 is unhappy. Now player 6 anticipates that player 7 and 8 will play $(\underline{7}, \bar{8})$. Consequently, player 6 has “to move” to end up with at least one neighbor like her.

As a consequence, depending on the action of player 1, “to stay” or “to move”, the path generated by the best response of players 6, 7 and 8 are $(\bar{6}, \underline{7}, \underline{8})$ and $(\underline{6}, \bar{7}, \underline{8})$, respectively.

4. The history of length 4 and the best strategies for players 6, 7 and 8 dictate the best answer for player 5.

(a) If 5 is ☺, the histories of length 4 are:

- $h(s) = (*, *, \bar{3}, *)$
- $h(s) = (*, \underline{2}, \underline{3}, \bar{4})$

with her best response being “to stay”, $\underline{5}$.

(b) If 5 is ☹, we consider:

- $h(s) = (\bar{1}, \underline{2}, \underline{3}, \underline{4})$ where player 5 will anticipate the action profile $(\underline{6}, \bar{7}, \underline{8})$. Hence her best response is “to move”, $\bar{5}$.
- $h(s) = (\bar{1}, \bar{2}, \underline{3}, \bar{4})$, $h(s) = (\underline{1}, \bar{2}, \underline{3}, \bar{4})$ have in common that player 8 is not happy. Therefore player 5 anticipates $(\bar{6}, \underline{7}, \underline{8})$ and plays “to stay”, $\underline{5}$, as her best response.

Hence the actions of players 1 and 2 condition the action of player 8 (either "to stay" or "to move") and the paths generated by the best response of players 5, 6, 7 and 8 are $(\underline{5}, \bar{6}, \underline{7}, \underline{8})$ or $(\bar{5}, \underline{6}, \bar{7}, \underline{8})$, respectively.

5. Following the same process, player 4 considers:

   (a) If 4 is ☺, the histories of length 3 are:

   - $h(s) = (*, \bar{2}, *)$
   - $h(s) = (\underline{1}, \underline{2}, \bar{3})$

   then her best response is "to stay", $\underline{4}$

   (b) If 4 is ☹, then the histories corresponding to such a situation are:

   - $h(s) = (\bar{1}, \underline{2}, \underline{3})$ where player 4 anticipates $(\bar{5}, \underline{6}, \underline{7}, \underline{8})$ and her best response is "to stay", $\underline{4}$.
   - $h(s) = (\underline{1}, \underline{2}, \underline{3})$ where player 8 is not happy and therefore player 4 anticipates $(\underline{5}, \bar{6}, \underline{7}, \underline{8})$ her best response to be "to move", $\bar{4}$.
   - $h(s) = (\bar{1}, \underline{2}, \bar{3})$ where player 8 and player 5 are both happy. Following the subgame perfection equilibrium path of the corresponding subtree, player 4 anticipates $(\underline{5}, \underline{6}, \bar{7}, \underline{8})$. Her best response is then "to move", $\bar{4}$.

In any situation where player 4 is not happy, she has to move generating the path $(\bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$.

6. The case for player 3 is the following:

   (a) If 3 is ☺, the histories of length 2 are:

   - $h(s) = (\bar{1}, *)$
   - $h(s) = (\underline{1}, \bar{2})$

   then her best response is "to stay", $\underline{3}$.

   (b) If 3 is ☹, then the unique history to be considered is:

   - $h(s) = (\underline{1}, \underline{2})$ where player 3 will anticipate $(\bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$. Thus her best response is "to stay", $\underline{3}$.

Therefore, the best response for player 3 in any situation is "to stay", $\underline{3}$.

7. For the case of player 2, we have only two histories:

   (a) If 2 is ☺:

   - $h(s) = (\bar{1})$

   then her best response is "to stay", $\underline{2}$.

   (b) If 2 is ☹, then the unique history to consider is:

   - $h(s) = (\underline{1})$ where player 2 anticipates $(\underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$, thus her best response is "to stay", $\underline{2}$, because player 8 will move to the space close to her.

   The above situation therefore means that the best reply for player 2 is "to stay", $\underline{2}$.

8. The last case corresponds to player 1 with only one case:

   (a) 1 is ☹ at the initial state so she will anticipate $(\underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$. Her best response is "to stay", $\underline{1}$, because player 8 will move to the space.

   Given the above situation, the best reply for player 1 is "to stay", $\underline{1}$.

The unique equilibrium path of Γ is $(\underline{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$ with a final circle configuration of players 18235467 generating a society of $BWWBBWWB$. Hence the result holds. ■

8 Appendix B

This appendix is devoted to studying some particular paths generated by the subgame best response after a specific history. Specifically, the extensive game has many paths that do not verify the subgame perfect criterion. Any of them establishes at least one player who plays off the equilibrium path, that is, players who are not Type II. Nevertheless, these choices may reach either a happy society where all players end up with at least a neighbor like them or not. The goal of the following discussion is to study such cases that arise in our experimental data by characterizing player types.

Let us start with the second row of Table B in this Appendix, $(\bar{1}, \underline{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})$. This configuration appears in our empirical data with a frequency of 3.33%. The lemma below states the SPE of the subgame after the action of player 1. In the particular case where player 1 is Type III, the generated tree denoted by $\Gamma_{\bar{1}}$ can be solved for the remaining players using the subgame perfect criterion. This allows us to determine each player's type in the path obtained.

Lemma 1 *If player 1 plays $\bar{1}$, then the SPE for $\Gamma_{\bar{1}}$ is $(\underline{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})$. The corresponding output society is $BBWWBBWW$.*

Proof. As player 1 moves, player 8, 2 and 3 become ☺. We can conclude that the best response for all of them is "to stay". Let us check what the best response is for players 4, 5, 6 and 7.

Player 4 will react against the history of length 3: $(\bar{1}, \underline{2}, \underline{3})$. By the proof of the main theorem, we know that player 4 will anticipate the action profile for players 5, 6, 7 and 8: $(\bar{5}, \underline{6}, \underline{7}, \underline{8})$. Therefore, her best response is "to stay", $\underline{4}$. The final output configuration is obtained by the path $(\bar{1}, \underline{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})$, thus generating the society $BBWWBBWW$. ■

Given the above lemma, we can conclude that player 1 is Type III and player 4 is Type II. Nevertheless, given that the remaining players are Type I, we are not able to distinguish between strategic behavior and myopic behavior.

Consider now the following path: $(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$. This path corresponds to the fourth row in Table B. In this case, player 1 is of Type II, but player 2 is of Type III. The next lemma states the subgame path of the corresponding subtree after the history $(\underline{1}, \bar{2})$.

Lemma 2 *If player 1 plays $\underline{1}$ and player 2 plays $\bar{2}$, then the SPE for $\Gamma_{\underline{1},\bar{2}}$ is $(\underline{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$.*

Proof. After the action of player 2, both player 3 and player 4 become ☺. Following the proof of the main result, we can check that if player 5 decides "to stay" it is because she anticipates the best responses of players 6, 7, and 8: $(\bar{6}, \underline{7}, \underline{8})$. With this path, player 5 becomes happy and thus her best response is "to stay". ■

In this case, player 5 plays $\underline{5}$ even when she is not happy after her history $h_5 = (\underline{1}, \bar{2}, \underline{3}, \underline{4})$. This yields Type II behavior for player 5. The final configuration of the final society is $BBWWBBWW$.

The last lemma in this section presents a variation of types and the consequent best response for an information node of a Type III player. This case is shown in the third row of Table B in this Appendix.

Fix the action of player 1, $\bar{1}$. By lemma **??**, the corresponding subgame perfect path of the subtree $\Gamma_{\bar{1}}$ is $(\underline{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})$. In this case, player 4 is of Type II. What is the best response if player 4 were actually of Type III?

Lemma 3 *If players 1, 2, 3 and 4 play* $(\bar{1}, \underline{2}, \underline{3}, \bar{4})$*, the SPE for* $\Gamma_{(\bar{1},\underline{2},\underline{3},\bar{4})}$ *is* $(\underline{5}, \underline{6}, \bar{7}, \underline{8})$*.*

Proof. Notice that player 8 is ☺ given the action of player 1. Moreover, as player 4 is of Type III, she makes players 5 and 6 happy. Player 7 is therefore the last unhappy player. Following the proof of the main result, we can conclude that her best response is "to move". ■

In the above case, there are no Type II individuals. Moreover, a Type IV individual appears: player 7. The final configuration is not a happy configuration: $WBBWWWBB$.

The following table shows the rest of the empirical cases[31] in our experiment. The first column shows the path. The second one gives the players who deviate from the equilibrium path, *i.e.*, Types III, ¬ I and ¬ IV. The third column enumerates the players engaging in strategic behavior (Type II), while the last column presents the final configuration of the society.

[31] A non-rational deviation could be any movement after an immediate movement of your neighbor. Then, any path with two consecutive movements is not an equilibrium path and is not a best response for the last player. As we find some of these cases in our data, we study them one by one.

$Play$	$Types III, \neg I, \neg IV$	$Type II$	$Outcome$
$(\underline{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$		$1, 2, 3, 7$	$BBWWBBWW$
$\boldsymbol{(\bar{1}, \underline{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})}$	**1**	**4**	**BBWWBBWW**
$\boldsymbol{(\bar{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \bar{7}, \underline{8})}$	$1, 4$		$BBBBWWWW$
$\boldsymbol{(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})}$	**2**	$\mathbf{1, 5}$	**BBWWBBWW**
$(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \bar{8})$	$2, 5$	1	$BBBBWWWW$
$(\underline{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$	**3**	$\mathbf{1, 2}$	**BWWBBBWW**
$(\underline{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \bar{7}, \underline{8})$	7	$1, 2, 3$	$BBWBBWWW$
$(\bar{1}, \bar{2}, \underline{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$	$1, 2$	5	$BBWWBBWW$
$(\bar{1}, \bar{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \bar{8})$	$1, 2, 5$		$BBBBWWWW$
$(\bar{1}, \bar{2}, \underline{3}, \underline{4}, \bar{5}, \bar{6}, \underline{7}, \underline{8})$	$1, 2, 5, 6$		$BBWWBBWW$
$(\underline{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5}, \bar{6}, \bar{7}, \underline{8})$	$3, 7$	$1, 2$	$BWWBBWWB$
$(\underline{1}, \underline{2}, \bar{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$	$3, 4$	$1, 2, 7$	$BBWWBBWW$
$(\underline{1}, \underline{2}, \bar{3}, \bar{4}, \underline{5}, \underline{6}, \bar{7}, \underline{8})$	$3, 4, 7$	$1, 2$	$WWBWWBBB$
$(\underline{1}, \underline{2}, \bar{3}, \bar{4}, \underline{5}, \underline{6}, \bar{7}, \bar{8})$	$3, 4, 7, 8$	$1, 2$	$WWBBWWBB$
$(\bar{1}, \underline{2}, \underline{3}, \bar{4}, \bar{5}, \underline{6}, \bar{7}, \underline{8})$	$1, 4, 5$		$BBWWBBWW$
$(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \bar{5}, \bar{6}, \underline{7}, \underline{8})$	$2, 5, 6$	1	$BBWWBBWW$
$(\underline{1}, \bar{2}, \bar{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$	$2, 3$	1	$BBWWBBWW$
$(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \bar{7}, \underline{8})$	$2, 3, 6, 7$	1	$BBWWBBWW$
$(\bar{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6}, \bar{7}, \underline{8})$	$1, 5$	$4, 6$	$BBWBWWWB$
$(\bar{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$	$1, 4, 6$		$BBBWBWWW$
$(\bar{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \underline{8})$	$1, 4, 7$		$BBBWWBWW$
$(\bar{1}, \underline{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \bar{7}, \underline{8})$	$1, 7$	4	$BBWWBWWB$
$(\bar{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$	$1, 5, 6$	4	$BBWBBWWW$
$(\bar{1}, \underline{2}, \underline{3}, \bar{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})$	$1, 4, 5$		$BBWWBBWW$
$(\bar{1}, \underline{2}, \bar{3}, \bar{4}, \underline{5}, \underline{6}, \bar{7}, \underline{8})$	$1, 3, 4$		$BBBBWWWW$
$(\bar{1}, \bar{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})$	$1, 2, 5$		$BBWWWBBW$
$(\bar{1}, \bar{2}, \bar{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$	$1, 2, 3$		$BBWWWBBW$
$(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$	$2, 6$	$1, 5, 7$	$BBWWBWWB$
$(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6}, \bar{7}, \underline{8})$	$2, 6, 7$	1	$BBWWBWWB$
$(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})$	$2, 5, 8$	1	$BBWWWBBW$
$(\underline{1}, \bar{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$	$2, 4$	$1, 7$	$BBWWBWWB$
$(\underline{1}, \bar{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \bar{7}, \underline{8})$	$2, 4, 7$	$1, 7$	$BBWBWWWB$
$(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \bar{7}, \bar{8})$	$2, 5, 7, 8$	1	$BBBBWWWW$
$(\underline{1}, \bar{2}, \underline{3}, \bar{4}, \bar{5}, \underline{6}, \underline{7}, \bar{8})$	$2, 4, 5$	1	$BBBBWWWW$
$(\underline{1}, \bar{2}, \bar{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$	$2, 3$	1	$BWWBBBWW$
$(\underline{1}, \bar{2}, \bar{3}, \underline{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$	$2, 3, 6$	$1, 7$	$BWWWBBWB$
$(\underline{1}, \bar{2}, \bar{3}, \underline{4}, \underline{5}, \underline{6}, \bar{7}, \underline{8})$	$2, 3, 6, 7$	1	$BWWBBWWB$
$(\underline{1}, \bar{2}, \bar{3}, \underline{4}, \underline{5}, \bar{6}, \bar{7}, \underline{8})$	$2, 3, 7$	1	$BBWWBBWW$
$(\underline{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$	$3, 6$	$1, 2, 7$	$BBWWWBBW$
$(\underline{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5}, \underline{6}, \bar{7}, \underline{8})$	$3, 6, 7$	$1, 2$	$BWWBBWWB$
$(\underline{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5}, \underline{6}, \bar{7}, \bar{8})$	$3, 6, 7, 8$	$1, 2$	$BWWWBBWB$
$(\underline{1}, \underline{2}, \bar{3}, \underline{4}, \bar{5}, \bar{6}, \underline{7}, \underline{8})$	$3, 5$	$1, 2$	$BWWBBBWW$
$(\underline{1}, \underline{2}, \bar{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \bar{8})$	$3, 5, 6$	$1, 2$	$BBBWBWWW$
$(\underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8})$	4	$1, 2, 3, 5$	$BWBWBBWW$
$(\underline{1}, \underline{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \bar{8})$	$4, 5$	$1, 2, 3$	$BWWBWWBB$
$(\underline{1}, \underline{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})$	$4, 5, 8$	$1, 2, 3$	$BWBWWBBW$

Table B

9 Appendix C

$Player$	$History$	$InitialConfiguration$	$Action$	$BestResponse$	$FinalConfiguration$
1	0	$BWBWBWBW$	$\underline{1}$	$(\underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$	$BBWWBBWW$
1	0	$BWBWBWBW$	$\bar{1}$	$(\underline{2}, \underline{3}, \underline{4}, \bar{5}, \underline{6}, \underline{7}, \underline{8})$	$BBWWBBWW$
2	$\underline{1}$	$BWBWBWBW$	$\underline{2}$	$(\underline{3}, \bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$	$BBWWBBWW$
2	$\underline{1}$	$BWBWBWBW$	$\bar{2}$	$(\underline{3}, \underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8}))$	$BBWWBBWW$
3	$(\underline{1}, \underline{2})$	$BWBWBWBW$	$\underline{3}$	$(\bar{4}, \underline{5}, \underline{6}, \underline{7}, \bar{8})$	$BBWWBBWW$
3	$(\underline{1}, \underline{2})$	$BWBWBWBW$	$\bar{3}$	$(\underline{4}, \underline{5}, \bar{6}, \underline{7}, \underline{8}))$	$BWWBBBWW$
4	$(\bar{1}, \underline{2}, \underline{3})$	$WBBWBWBW$	$\underline{4}$	$(\bar{5}, \underline{6}, \underline{7}, \underline{8}))$	$WBBWWBBW$
$4*$	$(\bar{1}, \underline{2}, \underline{3})$	$WBBWBWBW$	$\bar{4}$	$(\underline{5}, \underline{6}, \bar{7}, \underline{8})$	$BBBBWWWW$
5	$(\underline{1}, \bar{2}, \underline{3}, \underline{4})$	$BBWWBWBW$	$\underline{5}$	$(\bar{6}, \underline{7}, \underline{8})$	$BBWWBBWW$
$5*$	$(\underline{1}, \bar{2}, \underline{3}, \underline{4})$	$BBWWBWBW$	$\bar{5}$	$(\underline{6}, \underline{7}, \bar{8})$	$BBWWWWBB$
5	$(\bar{1}, \bar{2}, \underline{3}, \underline{4})$	$BBWWBWBW$	$\underline{5}$	$(\bar{6}, \underline{7}, \underline{8})$	$BBWWBBWW$
$5*$	$(\bar{1}, \bar{2}, \underline{3}, \underline{4})$	$BBWWBWBW$	$\bar{5}$	$(\underline{6}, \underline{7}, \bar{8})$	$BBWWWWBB$
5	$(\underline{1}, \underline{2}, \underline{3}, \underline{4})$	$BWBWBWBW$	$\underline{5}$	$(\bar{6}, \underline{7}, \underline{8})$	$BWBWBBWW$
5	$(\underline{1}, \underline{2}, \underline{3}, \underline{4})$	$BWBWBWBW$	$\bar{5}$	$(\underline{6}, \underline{7}, \bar{8})$	$BWWBWWBB$
6	$(\bar{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5})$	$WBBWBWBW$	$\underline{6}$	$(\bar{7}, \underline{8})$	$WBBBWBWW$
6	$(\bar{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5})$	$WBBWBWBW$	$\bar{6}$	$(\underline{7}, \underline{8})$	$WBBWBBWW$
6	$(\bar{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5})$	$WBWBBWBW$	$\underline{6}$	$(\bar{7}, \underline{8})$	$WBBWBBWW$
$6*$	$(\bar{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5})$	$WBWBBWBW$	$\bar{6}$	$(\underline{7}, \underline{8})$	$WBWBBBWW$
7	$(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6})$	$BBWWBWBW$	$\underline{7}$	$\bar{8}$	$BBWWWBWB$
7	$(\underline{1}, \bar{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6})$	$BBWWBWBW$	$\bar{7}$	$\underline{8}$	$BBWWBWWB$
7	$(\underline{1}, \bar{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6})$	$BBWBWWBW$	$\underline{7}$	$\bar{8}$	$BBWWBWWB$
7	$(\underline{1}, \bar{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6})$	$BBWBWWBW$	$\bar{7}$	$\underline{8}$	$BBWBWWWB$
7	$(\underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6})$	$BWBWBWBW$	$\underline{7}$	$\bar{8}$	$BWWBWBWB$
7	$(\underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6})$	$BWBWBWBW$	$\bar{7}$	$\underline{8}$	$BWBWBWWB$
7	$(\underline{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5}, \underline{6})$	$BWWBBWBW$	$\underline{7}$	$\bar{8}$	$BWWBWBWB$
7	$(\underline{1}, \underline{2}, \bar{3}, \underline{4}, \underline{5}, \underline{6})$	$BWWBBWBW$	$\bar{7}$	$\underline{8}$	$BWBWBWWB$
7	$(\underline{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6})$	$BWBBWWBW$	$\underline{7}$	$\bar{8}$	$BWWBBWWB$
7	$(\underline{1}, \underline{2}, \underline{3}, \bar{4}, \underline{5}, \underline{6})$	$BWBBWWBW$	$\bar{7}$	$\underline{8}$	$BWBBWWWB$

Table C