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The author gives a rigorous proof of three conjectures stated in a 2003 paper by Y. Wu [Int. J. Mod. Phys. A 18 (2003), no. 29, 5363, doi:10.1142/S0217751X03015222], where a new regularization technique for quantum field theory was formulated. Let m, n be positive integers. Then, asymptotically, as $m \to \infty$

$$\sum_{k\geq 1}(-1)^k \binom{m}{k} k \log k \quad \sim \quad \frac{1}{\log m}$$

$$\sum_{k\geq 1}(-1)^k \binom{m}{k} \log k \quad \sim \quad \log\log m + \gamma$$

$$\sum_{k\geq 1}(-1)^{k-1} \binom{m}{k} \frac{1}{k^n} \quad \sim \quad \frac{\log^n m}{n!}$$

where γ is the Euler-Mascheroni constant. The validity of these conjectures was numerically checked by Wu up to precision 10^{-3} but a proof of them was lacking. The conjectures are a key step in the derivation of the simple forms of the irreducible loop integrals arising in the theory.

In this paper under review, the above expressions are shown to be valid. Besides, the asymptotic estimates are significantly improved in all three cases. The key to the proof is the study of the asymptotic properties of the following integral, introduced by the author

$$I(\alpha, m) = m \int_0^\infty y^\alpha e^{-y}(1-e^{-y})^{m-1} dy$$

where α is, generally, a complex number. Skillful use of some elementary methods and inequalities, detailed in the Appendix, leads one to overcome the most difficult part of the proof, which involves bounding the extra contributions of different intervals on which integration is carried.

Reviewed by Vladimir García-Morales