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Review text:

In the field of exactly solvable models of statistical mechanics, the eight-vertex model is a 2D model related to the completely anisotropic XYZ Heisenberg spin chain. Baxter solved the (finite-size) model on a lattice with an even number of sites N . However, in spite of more than 40 years of works on the eight-vertex/ XYZ model, a complete description of the transfer matrix spectrum and eigenstates for N odd was lacking.

The work under review aims to fill this gap. Building on a previous work [G. Niccoli, *J. Phys. A* 46 (2013), no. 7, 075003; MR3020364], the authors consider the mapping of the model with N odd to the transfer matrix of a dynamical six-vertex model with antiperiodic boundary conditions that can then be solved by means of E. K. Sklyanin's quantum version of the separation of variables approach [in *Nonlinear equations in classical and quantum field theory* (Meudon/Paris, 1983/1984), 196-233, Lecture Notes in Phys., 226, Springer, Berlin, 1985, MR0802110; in *Integrable and superintegrable systems*, 8-33, World Sci. Publ., Teaneck, NJ, 1990, MR1091258; in *Quantum group and quantum integrable systems*, 63-97, Nankai Lectures Math. Phys., World Sci. Publ., River Edge, NJ, 1992, MR1239668]. This strategy allows the authors to handle the eight-vertex model not only under periodic boundary conditions for N odd, but also under integrable boundary twists by some Pauli matrices.

The article culminates in Section 5 with the complete determination of the periodic eight-vertex transfer matrix spectrum and eigenstates for N odd. The authors show that the spectrum is not simple but doubly degenerated and coincides with half of the spectrum of the corresponding antiperiodic dynamical six-vertex transfer matrix. To each of the eigenvalues correspond two linearly independent eigenstates. As in the twisted case (whose spectrum is, however, simple), these eigenstates can be constructed by means of the dynamical six-vertex basis obtained through Sklyanin's approach, in terms of solutions of a system of discrete equations involving the inhomogeneity parameters of the model.

Reviewed by Vladimir García-Morales