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Stability of cellular automata trajectories revisited: branching walks and Lyapunov profiles. (English summary)
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It is well known that dynamical instabilities often do not affect all components of a spatially extended system to the same extent. In this paper the authors investigate this phenomenon, focusing on cellular automaton (CA) dynamics. They specifically undertake here the study of how fast defects spread spatiotemporally, addressing the non-equilibrium defect accumulation dynamics on a CA trajectory. The latter is a branching walk process in which a defect creates a successor on any neighborhood site whose update it affects. On an infinite lattice, defects accumulate at different exponential rates in different directions, giving rise to a Lyapunov profile. This profile quantifies instability of a CA evolution and is connected in this work to the theory of large deviations.

The authors study Lyapunov profiles generated from random initial states. They introduce explicit and computationally feasible variational methods to compute the Lyapunov profiles for periodic configurations, leading to some kind of Floquet theory for CAs. The approach is a mixture of a few rigorous results and an empirical analysis, and leads to a classification of elementary CA rules according to whether the defects spread, collapse, or neither spread nor collapse. An inviting list of 10 open problems, related to the approach presented in this article, is given at the end.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.