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RG flows and bifurcations. (English summary)
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Quantum field theories at high energies have more degrees of freedom than theories at low energies and information is lost as we flow from the former to the latter. This statement is formalized by the celebrated C-theorem, which establishes the existence of a positive real function, the C-function, that depends on the coupling constants of the theory and the scale μ, and such that (a) it monotonically decreases under the renormalization group (RG) flow and (b) at fixed points of the RG flow the C-function is a constant independent of μ. This was proved by A. B. Zamolodchikov in 1986 [Pis′ma Zh. Èksper. Teoret. Fiz. **43** (1986), no. 12, 730–732; MR0865077]. The author of the paper under review recently proposed [J. High Energy Phys. **2016**, no. 1, 020, front matter+38 pp.; MR3471560] that the spectra of UV and IR theories be viewed as measuring degrees of freedom in a similar way to that of the C-function.

In this intriguing article, the author connects the analysis of fixed points of most general non-supersymmetric RG flows to bifurcation analysis of nonlinear dynamical systems. One of the main goals of the paper is to shed light on the nature of transitions that accompany marginality crossing. Bifurcations, happening at those transitions, are linked to the non-analytic behavior of the C-function or its derivatives. The author first develops in detail the well-known stability analysis of fixed points and the dictionary of both local and global bifurcations found in 2D dynamical systems (which constitute the simplest, but highly nontrivial, examples). In the RG flows considered, in view of the fact that most fixed points in question are hyperbolic, the saddle-node and the transcritical bifurcations take a most prominent role. Even in those cases, however, since there can be many fixed points, complicated heteroclinic connections can appear and interesting dynamical and topological tools, such as the Conley index and, most remarkably, the so-called 'connection matrices' (here discussed by the author) prove useful for classifying and obtaining detailed insight into these situations.

Examples of RG flows considered are the $O(N)$ model in three dimensions, three-dimensional quantum electrodynamics (one control parameter and, hence, codimension-1 bifurcations) and four-dimensional quantum chromodynamics (two control parameters, with the possibility of codimension-2 bifurcations). Surprisingly, even in these familiar theories, new phases and fixed points are predicted by the approach presented. The paper is clearly written and has plenty of novel ideas and nice predictions that open new realms for exploration. *Vladimir García-Morales*