MR3563304 37B15 68Q80
Liu, Weibin [**Liu, Wei Bin**] (PRC-WUHAN-MS);
Ma, Jihua [**Ma, Ji Hua**] (PRC-WUHAN-MS)

Limit averages of continuous functions under the action of cellular automata. (English summary)

Monatsh. Math. **181** (2016), *no. 4,* 869–874.

In this paper the authors consider averages of continuous functions under the action of cellular automata, which were proposed by M. Boyle, D. Lind and D. J. Rudolph in 1988 [Trans. Amer. Math. Soc. **306** (1988), no. 1, 71–114 (Question 5.3); MR0927684]. They prove that, at non-shift-periodic points, the averages point-wise converge to the integration with respect to the uniform Bernoulli measure.

Let $\mathcal{A}=\{0,1,\dots,q-1\}$ $(q\geq 2)$, let $\mathbb{Z}$ be the set of integers, and let $\mathcal{A}^{\mathbb{Z}}=\{(\dots x_{-1}x_0x_1x_2\dots):x_i\in\mathcal{A},\forall i\in\mathbb{Z}\}$ denote the space of bi-infinite sequences over $\mathcal{A}$. Let $\mathcal{CA}(r)$ denote the set of all cellular automata $F\colon\mathcal{A}^{\mathbb{Z}}\to\mathcal{A}^{\mathbb{Z}}$ of radius r on $\mathcal{A}^{\mathbb{Z}}$ and $\#\mathcal{CA}(r)$ its cardinality. Finally, let $\varphi\colon\mathcal{A}^{\mathbb{Z}}\to\mathbb{R}$ be a continuous function. The authors prove that if $x\in\mathcal{A}^{\mathbb{Z}}$ is not periodic under the action of the shift operator σ then

$$\lim_{r\to\infty}\frac{1}{\#\mathcal{CA}(r)}\sum_{F\in\mathcal{CA}(r)}\varphi\circ F(x)=\int_{\mathcal{A}^{\mathbb{Z}}}\varphi(t)d\lambda(t),$$

where λ is the uniform Bernoulli measure on $\mathcal{A}^{\mathbb{Z}}$. If $x\in\mathcal{A}^{\mathbb{Z}}$ is σ-periodic with minimal period $p\geq 1$, then

$$\lim_{r\to\infty}\frac{1}{\#\mathcal{CA}(r)}\sum_{F\in\mathcal{CA}(r)}\varphi\circ F(x)=\frac{1}{q^p}\sum_{v\in\mathcal{A}^p}\varphi(v^\infty).$$

Vladimir García-Morales

References

1. Boyle, M., Lind, D., Rudolph, D.: The automorphism group of a shift of finite type. Trans. Am. Math. Soc. **306**(1), 71–114 (1988) MR0927684
2. Hedlund, G.A.: Endomorphisms and automorphisms of the shift dynamical system. Math. Syst. Theory **3**, 320–375 (1969) MR0259881
3. Kitchens, B.P.: Symbolic Dynamics. Springer, Berlin (1998) MR1484730
4. Kurka, P.: Topological and symbolic dynamics. Cours Spécialisés–Collection SMF (2003) MR2041676
5. Lind, D., Marcus, B.: An Introduction to Symbolic Dynamics and Coding. Cambridge University Press, Cambridge (1995) MR1369092
6. Wolfram, S.: Theory and Applications of Cellular Automata. World Scientific, Singapore (1986) MR0857608

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.