MR3650253 37B15 05A99 68R05 74H05 76T25 82C22

Perrot, Kévin (F-AMU); **Rémila, Éric** (F-SETN-GAT)

Strong emergence of wave patterns on Kadanoff sandpiles. (English summary)

Electron. J. Combin. **24** (2017), *no. 2, Paper* 2.4, 30 *pp.*

The Kadanoff sandpile model is a discrete dynamical system describing the evolution of finitely many sand grains falling from an hourglass (or equivalently from a finite stack of sand grains) to a stable configuration. The repeated application of a simple local rule lets grains move until reaching a fixed point.

In this interesting paper on the Kadanoff sandpile model, the authors prove the emergence of exact wave patterns periodically repeated on fixed points. Remarkably, these regular patterns do not cover the entire fixed point, but eventually emerge from a seemingly disordered segment: grains are added on the left, triggering avalanches that become regular as they fall down the sandpile. The form of stable configurations is asymptotically completely described, although a part of asymptotically null relative size (but infinite absolute size), apparently complicated and non-regular, remains unexplained.

The linear algebraic and combinatorial proof techniques used in proving the main result (Theorem 26 in Section 3) stress the idea that sandpile models are on the edge between discrete and continuous systems. On one hand, there is a seemingly unordered initial segment, interpreted as reflecting the discrete behavior. On the other hand, the asymptotic and ordered part, interpreted as reflecting the continuous behavior, lets a regular and smooth pattern emerge. *Vladimir García-Morales*

References

1. P. Bak and K. Tang, C. Wiesenfeld. Self-organized criticality. *Phys. Rev. A*, 38(1):364–374, Jul 1988. MR0949160
2. A. Björner, L. Lovász, and P. W. Shor. Chip-firing games on graphs. *Eur. J. Comb.*, 12(4):283–291, 1991. MR1120415
3. A. Bjöer and L. Lovász. Chip-firing games on directed graphs. *Journal of Algebraic Combinatorics*, 1:305–328, 1992. MR1203679
4. N. Bourbaki. *General Topology.* Elements of mathematics. Springer, 1998. MR1726872
5. M. Creutz. Cellular automata and self organized criticality. In *in Some New Directions in Science on Computers*, 1996.
6. A. Dartois and C. Magnien. Results and conjectures on the sandpile identity on a lattice. In *Discrete Model for C.S.*, pages 89–102. Discrete Math. and T.C.S., 2003. MR2011023
7. D. Dhar. Self-organized critical state of sandpile automaton models. *Phys. Rev. Lett.*, 64:1613–1616, 1990. MR1044086
8. J. O. Durand-Lose. Parallel transient time of one-dimensional sand pile. *Theor. Comput. Sci.*, 205(1-2):183–193, 1998. MR1638656
9. K. Eriksson. *Strongly convergent games and coexeter groups.* PhD thesis, KTH Stockholm, 1993. MR2714773
10. E. Formenti, B. Masson, and T. Pisokas. Advances in symmetric sandpiles. *Fundam. Inform.*, 76(1-2):91–112, 2007. MR2293052
11. E. Formenti, K. Perrot, and E. Rémila. Computational complexity of the avalanche

problem on one dimensional kadanoff sandpiles. *LNCS*, 8996:21–30, 2014. AUTOMATA'2014. MR3396136
12. E. Formenti, T. Van Pham, T. H. D. Phan, and T. T. H. Tran. Fixed point forms of the parallel symmetric sandpile model. *Theor. Comput. Sci.*, 533:1–14, 2014. MR3192636
13. A. Gajardo and E. Goles. Crossing information in two-dimensional sandpiles. *Theor. Comput. Sci.*, 369(1-3):463–469, 2006. MR2277592
14. A. Gajardo, A. Moreira, and E. Goles. Complexity of Langton's ant. *Discrete Applied Mathematics*, 117(1-3):41–50, 2002. MR1881266
15. D. Gale, J. Propp, S. Sutherland, and S. Troubetzkoy. Further travels with my ant. *Mathematical Entertainments column, Mathematical Intelligencer*, 17:48–56, 1995. MR1347895
16. R. B. Gardner and N. K. Govil. Some generalizations of the Eneström-Kakeya theorem. *Acta Math. Hungar.*, 74(1-2):125–134, 1997. MR1428052
17. E. Goles and M.A. Kiwi. Games on line graphs and sand piles. *Theor. Comput. Sci.*, 115:321–349, 1993. MR1224440
18. E. Goles, M. Latapy, C. Magnien, M. Morvan, and T. H. D. Phan. Sandpile models and lattices: a comprehensive survey. *Theor. Comput. Sci.*, 322(2):383–407, 2004. MR2080235
19. E. Goles and B. Martin. Computational Complexity of Avalanches in the Kadanoff Two-dimensional Sandpile Model. In *Proc. of JAC 2010*, pages 121–132, 12 2010.
20. E. Goles, M. Morvan, and T. H. D. Phan. The structure of a linear chip firing game and related models. *Theor. Comput. Sci.*, 270(1-2):827–841, 2002. MR1871097
21. S. Grauwin, É. Bertin, R. Lemoy, and P. Jensen. Competition between collective and individual dynamics. *Nat. Ac. of Sciences USA*, 106(49):20622–20626, 2009.
22. J. E. Hopcroft, R. Motwani, and J. D. Ullman. *Introduction to automata theory, languages, and computation - international edition (2. ed)*. Addison-Wesley, 2003. MR0645539
23. L. P. Kadanoff, S. R. Nagel, L. Wu, and S. Zhou. Scaling and universality in avalanches. *Phys. Rev. A*, 39(12):6524–6537, Jun 1989. MR1043636
24. L. Levine, W. Pegden, and C. K. Smart. Apollonian structure in the Abelian sandpile. `arXiv:1208.4839`, 2012. MR3494492
25. L. Levine, W. Pegden, and C. K. Smart. The Apollonian structure of integer superharmonic matrices. `arXiv:1309.3267`, 2013. MR3664999
26. L. Levine and Y. Peres. Spherical asymptotics for the rotor-router model in z d. *Indiana Univ. Math. J*, pages 431–450, 2008. MR2400263
27. P. B. Miltersen. The computational complexity of one-dimensional sandpiles. *Theory Comput. Syst.*, 41(1):119–125, 2007. MR2313402
28. C. Moore and M. Nilsson. The computational complexity of sandpiles. *Journal of Statistical Physics*, 96:205–224, 1999. MR1706761
29. K. Perrot, T. H. D. Phan, and T. Van Pham. On the set of Fixed Points of the Parallel Symmetric Sand Pile Model. *DMTCS*, pages 17–28, 2011. AUTOMATA'2011. MR2877001
30. K. Perrot and É. Rémila. Avalanche Structure in the Kadanoff Sand Pile Model. *LNCS*, 6638:427–439, 2011. LATA'2011 `arXiv:1101.5940`. MR2881721
31. K. Perrot and É. Rémila. Transduction on Kadanoff Sand Pile Model Avalanches, Application to Wave Pattern Emergence. *LNCS*, 6907:508–520, 2011. MFCS'2011. MR2881721
32. K. Perrot and É. Rémila. Emergence on decreasing sandpile models. *LNCS*, 9234:419–431, 2015. MFCS'2015. MR3419445
33. K. Perrot and É. Rémila. Kadanoff sand pile model. Avalanche structure and wave

shape. *Theor. Comput. Sci.*, 504:52–72, 2013. MR3107551
34. T. H. D. Phan. Two sided sand piles model and unimodal sequences. *Theor. Inf. Appl.*, 42(3):631–646, 2008. MR2434039
35. W. Weaver. Science and Complexity. *American Scientist*, 36(536), 1948.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.