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Generalized adding machines and Julia sets. (English, French summaries)

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In this paper the authors define stochastic adding machines based on Cantor systems of numeration, computing the parts of the spectra of the transition operators associated with these machines in different classical Banach spaces. They show that the spectra are connected to fibered Julia sets. The study of these spectra gives information about the dynamical properties of transition operators acting on separable Banach spaces.

The authors first construct the transition matrix S of the stochastic adding machines and use its auto-similarity to show that the spectrum of the transition operator is equal to a fibered Julia set (Theorem 2.1). Then, it is shown that the residual spectrum is either empty or contains a countable subset of the boundary of the fibered Julia set (Theorem 2.2), depending on the underlying Banach space Ω under consideration. Furthermore, if the sequence of strictly positive probabilities characterizing the stochastic adding machine $(p_i)_{i \geq 0}$ does not converge to 1, the point spectrum of the transition operator is empty. If, on the contrary, $\lim_{j \rightarrow \infty} p_j = 1$, the point spectrum is not empty (Theorem 2.3).

Analogous results to Theorems 2.1 and 2.2 were obtained in [E. H. El Abdalaoui and A. Messaoudi, *Fund. Math.* **218** (2012), no. 1, 47–68; [MR2950198](#)] for the specific case of binary numeral systems. The extension of these two results for general Cantor systems of numeration, achieved in the paper under review, is hard and nontrivial. Theorem 2.3 is not obtained for any other system of numeration.

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