CONFLICTS WITH THE RADICAL SIGN. A CASE STUDY WITH PATRICIA

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This work takes its cue from the Patricia's case. Patricia, a Spanish secondary school mathematics teacher, on attempting to understand the definition of equivalent radicals expresses a conflict. She states that the equality $\sqrt[6]{3^2} = \sqrt[3]{3}$ cannot be true, since in the expression on the left the index of the root is even, so that it has two opposing roots, two solutions, whereas in the expression on the right the index is odd so it only has one root, which means that the two expressions have a different number of roots. In order to explain Patricia's conflict we took into account a framework with two fundamental references. One of these is the change in meaning of the signs when passing from arithmetic to algebra, reflected in a traditional way of teaching. Traces of this tradition are found in texts such relevant as Euler's algebra. Euler associated, for example, the expression $\sqrt{4}$ to the set of two results, $+2$ and -2, and the expression $\sqrt{a}$ to one of the two roots of a. (Euler, 1770, p. 44, p. 150). Another reference is the dual (operational/structural) nature of mathematical conceptions (Sfard, 1991). In arithmetic the expressions as $\sqrt{4}$, for example, are perceived as an indicated operation (operational conception), while in algebra the expressions as $\sqrt{a}$ are perceived as objects (structural conception). According to Sfard, the operational conception is the first to develop, whereas the structural conception needs external interventions (of a teacher or a textbook). In our study we have identified characteristic features in the $\sqrt{\ }$ sign teaching in current and representative Spanish textbooks and also, omissions that reinforce the double value (first conception) of the $\sqrt{\ }$ sign (Roach, Gibson & Weber, 2004). The data that we have obtained from the answers to a specific questionnaire and the interview with Patricia show that she has an operational knowledge of the $\sqrt{\ }$ sign and that this conception is consistent with what is shown in the studied textbooks.

References

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