"THE MAIEUTICAL DOGGY": A WORKSHOP FOR TEACHERS

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This paper deals with the contents, application and results of a workshop designed for elementary education teachers, which was structured around meta-cognitive principles taken from Socratic maieutics. We argue that the open-type mathematical task, around which the workshop revolved, was crucial in allowing its participants to perform a maieutical activity, but it also acted as an obstacle to that.

BACKGROUND

Since Flavell's work, experts in mathematical education have expressed a positive view towards the efficacy of meta-cognition (mcn) for the purpose of learning Mathematics, and have developed teaching proposals based upon meta-cognitive (mc) practices. These proposals rest on the assumption “that mcn demands to be taught explicitly" (Desoete, 2007, p. 709). Two decades earlier, Schoenfeld (1992) had already pondered the centrality of teaching and teachers in this kind of instructive processes. This position was later taken, among others, by Hartamn and Sternberg (1993, quoted in Desoete, 2007), who believe that in mcn-based teaching teachers have a central role to play, going as far as setting themselves as an example of the way mc tools are to be implemented. Despite this, "[teachers] still pay little attention to explicit mcn teaching" (Ibid, p. 709). This is probably due to the scarcity of offerings in support of Math teachers in order to acquire the skill and mastery needed for the deployment of mc tools (Kozulin, 2005).

In this regard, this paper presents the contents, application and results of a workshop designed for active elementary education teachers, which was structured around mc principles taken from Socratic maieutics. The workshop, given in the context of a program for professional development, was constituted by two three-hour sessions, supplemented with individual meetings for every individual who took the workshop; this paper deals with the first session.

While there are reports regarding proposals for the professional development of teachers, which are based on several pedagogical methodologies and epistemological points of view, few of them (among them, Sowder's, in 2007, who uses 'the Socratic model') expound upon and analyze ways of offering professional training that are based on the use of mc practices, such as the one approached here.

THEORETICAL AND INTERPRETIVE FRAMEWORK

Mc activities in the Math classroom. A classification

For analyzing the mathematical activities proposed herein, the following mc categories have been used (described and typified in Rigo, Páez & Gómez, 2010):

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	In reference to the Task (T)	*In reference to the Person (P)*
Specific Type (S)	How did I solve it? What did I base it on? What is its degree of difficulty?	How confident am I in the solution I propose? What do I base it on?
Generic Type (G)	Processes for transfer to other tasks	Awareness of what I do not know about the subject

Table 1. Examples of mc activities. Variables (T and P) and types (S and G).

Maieutics: "giving birth to truth"

It is a pedagogical method conceived by Socrates and expounded in Plato's *Meno* dialogue. This study has identified three moments of maieutics (Rigo, 2011):

Construction moment. A task is put forward with the foreknowledge that the student will solve it incorrectly or limitedly, but also that he/she will feel a high degree of confidence about the resolution proposed. *De-construction moment.* The teacher confronts the student with cognitive conflicts which the student then uses for reconsidering his/her resolution (S, T mc) and understand his/her mistake. *Re-construction moment.* The teachers guide the student in the building of a new solution, one which allows him/her to understand what he/she does not know about the subject (G, P mc). Within this process, two types of conflict can be distinguished: a *cognitive* one, when the student must confront the contradictions that emerge from his/her wrong answer, and an *mc* one, which emerges when he/she is constrained to acknowledge his/her ignorance about a subject he/she thought he/she knew about.

THE "MAIEUTICAL DOGGY": A WORKSHOP FOR TEACHERS

About the design of the workshop and its application

At the workshop it was expected that, starting out from an open task, from responses to a written questionnaire (Q), from collective discussions, and from talks delivered by researchers (R), participants would construct a mathematical definition of an intuitive notion (which is not defined) and that this cognitive process may serve as a reference to do some mc (T, P) reflecting, specifically of a maieutical nature. For maieutical purposes, it was essential to promote among students the emergence of cognitive and mc conflicts.

About the task

For the workshop's purposes, the task was essential. In the case dealt with here, a task called "the doggy" was chosen, in which a line figure in the shape of a dog is drawn upon a grid (Fig. 1). Data are given in graphic form and the solution must be presented in the same way:

"We found a pill that causes things to grow to twice their size. The dog drawn here is going to eat the pill. What will he be like after eating it? Draw him". This is an open task of an exploratory nature (Ponte, 2005) in which students, adopting an autonomous attitude (Ibid.), must deploy their intuitive and mathematical knowledge in order to signify the idea of size (not defined in the statement) and make its meaning concrete in the graphic register.

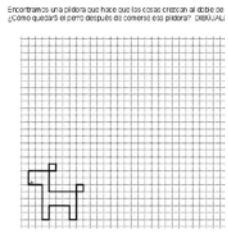


Fig. 1. The doggy task

The doggy is an application of the problem Socrates poses to the slave in *Meno*, concerning the duplication of areas. It involves several notions having to do with the idea of size: similarity, shape, area, reason, proportion, Pythagoras' theorem, √2, which are part of the arithmetic and geometry concepts that are studied in elementary and secondary schools. Students from different educational levels, including higher education students and active teachers, have been asked to solve the doggy, so many of the problem's possible resolutions have already been identified and systematized in previous studies (Gómez, 2007). This permitted anticipating possible answers from participants and planning which cognitive and mc conflicts to promote among them.

About the workshop's structure and contents

As per the maieutical method, the Workshop was divided in three moments:

Construction moment. The first solution

The student is asked to solve the task in the Q, to justify his solution and to meditate about the degree of certainty he/she has in his/her solution, about his/her knowledge of proportionality and about the task's degree of difficulty (S, P mc).

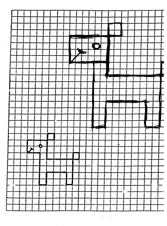
De-construction moment. The cognitive conflicts

Based on their previous analytical work, researchers organize the presentation of various resolutions to the task, showing first those centered on shape, followed by those centered on area and finally those resolutions in which harmonizing shape and area duplication is sought. With this process is expected to promote the generation of cognitive and mc conflicts that allow teachers to gain awareness of the limits of their resolution(s) and of their ideas regarding the duplication of size.

- Resolutions centered on shape. Two types stand out:

-- Duplication of the length of the sides of the original figure (Fig. 2). R: In previous studies, the higher the educational level, the greater the frequency of this figure, known as the Big Dog (BD). But isn't BD too big? And what about BD's ear and tail?

Aren't they too large? Some students thought about other options. One of them is shown in Figure 3.



--A doubling of the perimeter, keeping an eye on the unit square.

R: The ear and the tail of the dog in Figure 3 have an area of two. Is this a reasonable answer to the task?

Fig. 2. Big Dog

Addressing those who continue to defend the BD solution, despite the fact that it has four times the area: Why pay attention only to the shape and not to the area? Consider the case of circles. What would the criterion be for choosing a circle with twice the area, if all circles have the same shape?

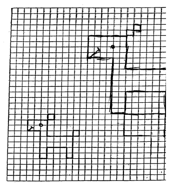


Fig. 3

- Resolutions based on area. The following is presented, among others:

-- One dimension increases, in order to arrive to a figure with twice the area (Fig. 4). R: The design has taken into consideration that, in order to obtain a rectangle with twice the area, it suffices to increase one of its sides to twice the length. But, must it maintain its similarity; i.e., the proportions between its sides? Or is retention of a dog shape enough? Could doubling the size be synonymous with doubling the area? What happens with the segments?

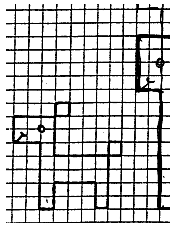


Fig. 4

- Resolutions that harmonize doubling of the area with preservation of the shape. Among other solutions, one that harmonizes area and shape is presented, even though it entails working outside the metrics induced by the grid (v. Fig. 6 and explanation on p. 6).

Re-construction moment. Construction of a mathematical solution and identification of mathematical contents that were considered to be known

At this point in the Workshop, the visual 'demonstration' that Socrates presents in the *Meno* Dialogue, in which he makes use of the diagonal to build a square that is twice the area of a given square, is introduced. Considering this construction, the teacher is asked to attempt a new solution to the task.

-- Resolution that formally harmonizes the doubling of the area with the preservation of the shape: "The Socratic dog". The teacher is expected to draw a dog such as the one presented in Fig. 5 and that he/she justifies his/her solution mathematically. Finally, the teacher is asked to draw a circle that is twice the area of a unit circle and to write down (in Q) his maieutical reflections (G, P mc).

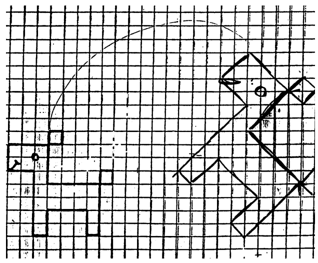


Fig. 5. The Socratic dog

Empirical results encountered in the Workshop

Some of the empirical findings are presented in this part of the paper. The analysis (carried out using the two video recordings of the session, their written transcripts and the results of the Q) is made taking into account the mc and maieutical categories mentioned in the theoretical framework, and is based upon qualitative case studies, as per the recommendations made by experts (Sowder, 2007).

Of the thirteen participants in the Workshop, eleven proposed BD as a first solution. Another person proposed the Socratic dog and one a 'Cubist dog' with twice the area.

Those who opted for BD based their solution on several beliefs and considerations. Some of these are:

- An erroneous belief, which more than half the teachers tacitly maintained, was that the doubling of the segments would result in a doubling of the area (or 'size'); i.e., they thought that the growth of area in a square is linear or proportional to the growth of its sides. We call this here the "spontaneous idea of proportionality", because it coincides with the one that guided the immediate response that the slave gave Socrates in *Meno*. Pedro, as many of his companions, after doubling the sides of the figure, asserts that "each square in the original is equal to two in the enlarged drawing"; he does not realize that in doubling the figure's perimeter, the area ('the square') is multiplied by four (and not by two as he suggests); i.e., he does not conceive that bi-dimensional magnitudes (area) behave differently than uni-dimensional ones (segments) and that 'size' is related to the former.

- Other teachers responded thinking that the task was a routine scaling exercise: "I thought the activity called for doing what is proposed in the Secondary school curriculum: the application of scales", commented José, as did other teachers.

- Another consideration, defended by three quarters of the group was that the task as a problem was not correctly enunciated, since the idea of size is not defined: "the statement is ambiguous and the parameters within which the student is expected to provide a solution have not been well established. How awful!", said Rita.

The truth of these beliefs was brought into question (and some of them even resulted in mathematical contradictions) as the session progressed and different solutions and meanings of size were produced and pondered. The mathematical activity displayed by the teachers for the purpose of responding to such questionings and contradictions, together with the mc ponderings they carried out in connection with that activity, determined different patterns of participation in the Workshop. In the section that follows, three of these patterns are described and illustrated by some cases.

Pattern of participation with maieutical activity (MA)

Teachers who displayed MA participation developed:

Cognitive activities: teachers were involved in the analysis and appraisal of the different resolutions and ideas concerning size that were produced in the course of the workshop session; they were sensitive of the cognitive conflicts that derived from analysis and drew mathematical challenges to solve them. This allowed their first-offered resolution to evolve as the session progressed.

An autonomous attitude: teachers defined and responsibly assumed a characterization based upon what was meant by doubling size, a characterization which they then attempted to represent graphically.

Meta-cognitive activities: The cognitive activities described above allowed the teachers to become aware of some of their conceptual problems with regards to the notions involved and the difficulty of the task.

Lino, one of the teachers who had an MA participation, recounts: "when I finished [BD], I realized that the ear and the tail had grown by four… The problem asks to double the size of the figure and I made it four times bigger". The ear and the tail, which are both one square in area in the original figure, were the trigger that revealed the contradiction which he unconsciously incurred; in order to solve the conflict and self-regulate his solution, he set for himself the challenge to "draw a figure that was proportional [to the original one] and then make it twice as big as the original… [that is, if] the original area is one, now it must be two across the figure".

With an autonomous and precise idea of what it meant to 'grow to twice the size', that challenge brought him to another: in order to arrive to twice the area while preserving the shape, he had to transcend the $\mathbb{N}$ domain metrics induced by the grid; thinking perhaps only in the $\mathbb{Q}$ domain and possibly ignoring $\sqrt{2}$, he arrived to the idea that "the grid got in the way".

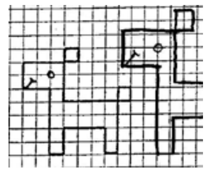


Fig. 6

He then made a figure that was a qualitative approximation of the expected response (Fig. 6). In his final reflection, he clarifies: "I said I was 50% convinced by my own solution, because I was not sure; now I see I was right, because my solution lacked arguments. The problem is not as straightforward as I initially thought it to be".

It is significant that all the teachers who had an MA participation doubled the unit circle by resorting to the use of the diagonal line, successfully transferring the diagonal method in rectilinear figures to the circular figure.

Pattern of participation with incipient maieutical activity (IMA)

Most teachers did not understand and even rejected the open character of the task. This is possibly due to the presence of a school sub-culture that is firmly entrenched among teachers, which is shaped around beliefs of what math tasks ought to be. Some of these beliefs, which Rita expressed with great emphasis (v. p. 5), prevented them from carrying out fully maieutical activities: by not assuming an autonomous position regarding the concept of size, they did not set the challenge for themselves to solve it graphically, neither did they fully involve themselves in discussing the different solutions, nor were they sensitive to the questionings that emerged. In this context of scarce autonomy, their cognitive activity was merely incipient; as a result, their mc activity was also incipient, thus allowing them to have just IMA participation. José, for example, after proposing BD in his first attempt, posited in his next intervention that, for the purpose of solving the problem, "he would have to consult a dictionary in order to determine what size means". With that approach, he systematically considered that every solution to the task was valid, since it depended on what 'size' was understood to mean. Even though he carried out some mathematical tasks, such as the Socratic dog, he developed them solely as a school exercise and they were not useful to him as a reference for appraising his previous mathematical activities and for his mc ponderings.

Pattern of participation without maieutical activity (WMA)

Two teachers assumed that the doggy task was a routine school exercise, one that was also badly formulated. These beliefs, which they held without the possibility of negotiation, really weighed them down because, under such a position, they became refractory to all questionings that emerged in the course of the session, which resulted in a severe reduction in their mathematical activity and, therefore, in their mc activity as well, both being features that define WMA participation. Juan, for example, at the beginning of the Q states that he feels great assuredness concerning his knowledge of proportionality and area; he further considers that the problem is very accessible and is 100% sure of his solution. In his interventions, he maintains that “BD is correct, despite the fact that it is too big… [since] all the area grows exponentially". He does not allow himself to look at other solutions in order to reflect about his, nor in order to identify what he ignores about the topic. It is possible that Juan, feeling insecure about his mathematical ability, was afraid to find himself exposed; in order to avoid this, he held tightly to his beliefs, something which probably made him feel secure, a meta-affective context that stabilized such beliefs (see Goldin, 2002). Sowder (2007) correctly remarks that, in these processes, teachers are often anxious about and reluctant to change, something that professional developers need to be aware of.

FINAL CONSIDERATIONS

A little over half of the workshop's attendants had a maieutical participation. The task was a key point: on the one hand, because the empirical and analytical work that one of the authors had previously carried out on it made it possible to plan for the cognitive and mc conflicts that characterize maieutical education; on the other hand, because of its exploratory, open nature. But this, in turn, was the source of conflict that prevented other participants from getting involved in it, not just because of the relatively difficult mathematical challenges demanded by its solution, but because it went against the grain of teaching beliefs firmly entrenched in them and because it forced them to make autonomous mathematical decisions which they do not seem accustomed to making. Several authors, among them Nelson and Hammerman (quoted in Sowder, 2007), point out the need to increase knowledge in preparing and training math teachers. The contents of this paper may contribute to this end.

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