Axial resonance of periodic patterns by using a Fresnel biprism

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This paper proposes a method for the generation of high-contrast localized sinusoidal fringes with spatially noncoherent illumination and relatively high light throughput. The method, somehow similar to the classical Lau effect, is based on the use of a Fresnel biprism. It has some advantages over previous methods for the noncoherent production of interference fringes. One is the flexibility of the method, which allows the control of the fringe period by means of a simple axial shift of the biprism. Second is the rapid axial fall-off in visibility around the high-contrast fringe planes. And third is the possibility of creating fringes with increasing or with constant period as the light beam propagates. Experimental verifications of the theoretical statements are also provided. © 2013 Optical Society of America


1. INTRODUCTION

The formation of self-images, both in the coherent (Talbot effect) and in the incoherent (Lau effect) version, has attracted the attention of scientists since its discovery [1]. This phenomenon has deserved the interest of the scientific community as a basic property of certain wave fields [2–9], but it also has been fruitfully exploited in many other applications [1,10–14]. The Talbot effect appears when a certain class of diffracting objects, Montgomery objects, are illuminated by a monochromatic plane wave. Montgomery objects have the ability to produce wave fields that are periodic along the direction of propagation of the wave. Then at axial distances that are integer multiples of the so-called Talbot distance, the amplitude distribution of the field is the same as the one emerging from the original diffracting object. It is worth mentioning that although it was originally associated with the amplitude or the irradiance of light fields, the self-imaging phenomenon also occurs to other magnitudes as the mutual intensity. This function exhibits axial periodicities for partially coherent fields when some conditions, which generalize those defining a Montgomery object for coherent illumination, are fulfilled [15,16].

The Lau effect, described first in 1948, is the spatially incoherent counterpart of the Talbot effect. The Lau effect is obtained when one allows the superposition in consonance of Talbot fringes generated by a series of mutually incoherent quasi-monochromatic sources. The most common practical implementation of the Lau effect is achieved when two 1D gratings, oriented parallel to each other, are illuminated by the light proceeding from a quasi-monochromatic spatially incoherent planar source. After imaging the planar source onto the first grating by a large-aperture lens, this plays the role of an incoherent source encoded linearly. (However, in general, imaging an incoherent quasi-monochromatic source results in partially coherent illumination; if the aperture of the imaging system is large enough, then the residual spatial coherence can be neglected (critical illumination scheme) [17].) The second grating acts as a diffracting object. The Lau effect is achieved only in Talbot planes (self-imaging planes) and results in the production of high-contrast noncoherent fringe patterns. Similar approaches have been used to produce localized and nonlocalized interference patterns in a wide set of grating interferometers [18], including Talbot and moiré devices [19,20].

On the other hand, the Fresnel biprism is an optical element formed by two thin equal prisms joined at the base [21,22]. When this element is illuminated with the light from a monochromatic point source, it generates two equal-intensity virtual sources. Since these are the images that the biprism generates from the real source, they are mutually coherent and, therefore, generate behind the biprism an axially extended sinusoidal interference pattern, similar to that obtained in a Young experiment. The period of this pattern increases proportionally to the distance between the point source and the observation plane. The high contrast obtained is directly related to the degree of spatial and temporal coherence of the real source. A change in any of these conditions produces a dramatic decay in the visibility of the fringes. This device was designed by Augustin Fresnel to help silence the objections raised by Young’s experiment, among the detractors of the wave theory of light. Contrary to what happens in the double-slit experiment, with the biprism a pure interference pattern is obtained without relying upon diffraction on apertures. The Fresnel biprism has recently found application in its ability to greatly simplify a commonly used technique for measuring ultrashort laser pulses [23], to develop a type of transmission phase-shifting laser microscope [24], to illustrate the wave-particle behavior of light in the single-photon regime [25,26], or, in the case of its x-ray or electron-beam analogue, to implement a biprism interferometer for coherence
measurement of x-ray sources [27,28], as well as for use in interference electron microscopy [29].

In this paper, it is proposed for the first time (to our knowledge) that the Fresnel biprism be used to produce high-contrast fringes with spatially incoherent light. The method for producing such fringes is somehow based on the Lau experiment. Specifically, we propose the use of an array of linear, spatially incoherent sources for the illumination of the Fresnel biprism. This arrangement allows the production of sets of axially extended, mutually incoherent, laterally shifted sinusoidal patterns. Depending on the parameters of the setup, there are some planes where the patterns superpose resonantly, giving rise to fringe patterns of high visibility. These high-visibility, or resonant, patterns are surrounded by patterns with very low visibility.

The paper is organized as follows. In Section 2, we explain the basic parageometrical theory [30,31] behind this interferential phenomenon, and we derive the equations that describe the fringe patterns that can be obtained when an array of linear sources illuminates the biprism. In Section 3, we propose a new interferential architecture for the production of resonant fringes whose period does not change with the field propagation. Section 4 is devoted to studying the effect of the diffraction at the edge of the biprism and to deriving the formula that takes into account its influence on the structure of the interference patterns under issue. Finally, in Section 5, we show the corresponding experimental verifications, and in Section 6 we summarize the main achievements of our research.

It is worth noting that the analysis of the Lau effect has been successfully approached by some authors in the spatial frequency domain by using the optical transfer function formalism and linked to coherence theory [32]. However, here we use a more direct approach in the spatial domain, which is more convenient for the purpose of this paper.

2. BASIC THEORY

Let us start by recalling the case in which a quasi-monochromatic light beam proceeding from a spatially incoherent planar source illuminates a thin object. We assume that the object and source planes are parallel to each other, so that we can define an optical axis, as is done in Fig. 1. If we name as $\eta$ the distance between the source and the object, then we can write the irradiance distribution on an arbitrary plane placed at a distance $z$ beyond the object as [12]

\[
I(\vec{x}, z; \eta) = \frac{1}{M_S^2} I_S(\frac{\vec{x}}{M_S}) \otimes_2 I_0(\vec{x}, z; \eta),
\]

where $\otimes_2$ represents the 2D convolution product performed over the transverse coordinates, $\vec{x} = (x, y)$. In the above equation, $I_S(\vec{x})$ denotes the normalized irradiance distribution of the planar source, $I(\vec{x}, z; \eta)$ is the irradiance distribution generated at the observation plane by a point source placed at the intersection between the optical axis and the planar source, and $M_S = -z/\eta$ is a magnification factor between the source and the observation planes.

This formula has been successfully used in a number of interesting applications, for example, the measurement of the degree of coherence of a light beam [11] or the implementation of lensless correlators [12]. In our case, we aim to design a method for the production of high-visibility periodic patterns with incoherent light.

Thus, we start by studying the patterns produced when a Fresnel biprism is used in our architecture. At this point it is important to remember that when a spherical wavefront proceeding from a point source illuminates a Fresnel biprism, the exiting wavefront is split into two spherical waves that virtually proceed from two virtual point sources placed at the plane of the source and separated by the distance $a = 2\eta(n-1)\delta/[21]$, where $n$ is the refracting index and $\delta$ is the refraction angle of the biprism. Note that we are assuming a thin biprism, so that the angle $\delta$ is considered to be small. The virtual point sources are mutually coherent, thus producing an interference pattern beyond the Fresnel biprism whose irradiance distribution can be written as

\[
I_0(\vec{x}, z; \eta) = 1 + \cos(\frac{2\pi x}{p}).
\]

In this equation we considered the biprism central edge to be aligned with the Cartesian $OY$ direction, and the period of the interference pattern is given by

\[
p = \frac{M}{2u_0},
\]

where $M = (\eta + z)/\eta$ is the magnification factor that accounts for the increase of the period along the axial coordinate and $u_0 = (n-1)\delta/\lambda$. It is worth mentioning that Eq. (2) gives a parageometrical description of the irradiance pattern under issue, thus neglecting the effect of the diffraction at the edge of the biprism and considering only the region of the geometrical superposition of rays coming from the virtual sources through the biprism. A more accurate study will be provided in Section 4.

Next, we substitute Eq. (2) into Eq. (1) to straightforwardly obtain

\[
I(\vec{x}, z; \eta) = \frac{I_S(\vec{0})}{M_S^2} \left[1 + V(z) \cos(\frac{2\pi x}{p} + \Phi(z))\right],
\]

where

\[
\bar{I}_S(\vec{u}) = |\bar{I}_S(\vec{u})| \exp(j\Phi(\vec{u})),
\]

which stands for the Fourier transform of the irradiance distribution of the source.

The visibility of the sinusoidal pattern in Eq. (4) is given by

\[
V(z) = \left|\frac{I_S[\frac{\eta}{M_S}, 0]}{I_S(\vec{0})}\right| = \left|\frac{I_S(-2u_0 \frac{z}{\eta}, 0)}{I_S(\vec{0})}\right|.
\]
Thus, the spatially incoherent planar source produces a fringe pattern whose visibility is given by the Fourier transform of the irradiance variations of the source along the \( OX \) direction (perpendicular to the biprism edge). Note that the visibility depends on the propagation distance \( z \), i.e., it is different in each observation plane.

The above general study can be particularized to the case of special interest in which the quasi-monochromatic source is composed by an array of mutually incoherent, equidistant point sources arranged perpendicular to the biprism edge. For simplicity, we assume \( N \) sources that have the same irradiance, \( I_p \), and are distributed symmetrically to the optical axis. Then, the irradiance distribution of the source is

\[
I_S(x) = \sum_{i=1}^{N} I_p \delta(x - x_i, y), \tag{7}
\]

where

\[
x_i = \left( \frac{N + 1}{2} - 1 \right) x_0, \quad I = 1, \ldots, N, \tag{8}
\]

\( x_0 \) being the separation between neighbor point sources. By using Eq. (1), the irradiance distribution in this case is obtained to be as follows:

\[
I(x, z; \eta) = I_p \sum_{i=1}^{N} I_0(x - M_S x_i, y, z; \eta). \tag{9}
\]

According to Eq. (6), the visibility of the irradiance pattern at any observation plane can be written in this case as

\[
V(z) = \frac{\sin \left( \frac{\pi N M_S x_0}{p} \right)}{N \sin \left( \frac{\pi M_S x_0}{p} \right)}. \tag{10}
\]

Equation (10) is a periodic function that takes significant values only in a discrete set of planes on which the argument of both sin functions achieve an integer multiple of \( \pi \). This condition determines the planes of maximum visibility, and these planes will be named hereafter as the planes of axial resonance. Therefore, the planes of axial resonance appear at distances

\[
z_m = \frac{m \lambda \eta}{2 \pi \eta (n - 1) \delta - m \lambda} \tag{11}
\]

from the biprism plane, where \( m \) is an integer number. Since the relation between \( z_m \) and \( m \) is not linear, the maximum-visibility planes are not equidistant. Note, however, that the position of such planes does not depend on the number of point sources \( N \). Furthermore, Eq. (10) indicates that the greater the number of slits, the higher the axial localization of the planes of resonance.

It is worth noting that, as stated above, the structure of the incoherent source along the \( y \) direction does not influence the visibility of the fringes but only their maximum irradiance. Thus, the same resonances could be obtained if instead of using an array of point sources one used an array of linear sources. It is interesting to note that the result in Eq. (11) can be also obtained by realizing that Eq. (9) represents a superposition of laterally shifted versions of the periodic pattern \( I_0(\bar{x}, z; \eta) \). In planes for which these lateral shifts equal an integer multiple of the period, a high-visibility irradiance distribution is obtained.

On the other hand, in a realistic experimental situation, the width \( \Delta \) of the sources along the \( x \) direction would not be infinitesimal. In that case, the irradiance distribution of the source can be written as the convolution between \( I_S(x) \) in Eq. (7) and a rectangle of width \( \Delta \). Thus, in such a realistic case the visibility at any observation plane is given by

\[
V(z) = \frac{\sin \left( \frac{\pi N M_S x_0}{p} \right)}{N \sin \left( \frac{\pi M_S x_0}{p} \right)} \sin \left( \frac{\Delta M_S}{p} \right). \tag{12}
\]

The main difference between Eqs. (12) and (10) is the \( \sin \) function, which appears because of the finite size of the incoherent sources. This factor causes the visibility of the planes of axial resonance to decrease along the axial direction, as shown in Fig. 2. Note that the lower the modulation (fill factor or duty cycle), \( \Delta / x_0 \), of the set of slit sources, the smoother the variation of this function. Note also that the positions of the planes of axial resonance (maximum, although not unit visibility) are still given by Eq. (11) as far as this variation is smooth. It is also interesting to mention that Eq. (12) describes, in a different context, the Fraunhoffer pattern of a binary diffraction grating.

To conclude this section, we next discuss briefly whether the results obtained with this technique can be extended to other devices which are an alternative to Young’s experiment in producing two coherent sources. Although all of them are capable of generating interference fringes with monochromatic point source illumination, their behavior when using a quasi-monochromatic extended light source is quite different.

Let us consider first the classical Young’s double-slit experiment. It is straightforward to find that in this case the visibility of the fringes is also given by the function in the middle term of Eq. (6), but with a ratio \( M_S / p \) equal to \(-a/\lambda \eta\), \( a \) being the separation between the two slits and \( \eta \) the distance from the source to the slits. Since this ratio is independent of the observation distance \( z \), the visibility of the fringes is the same for any observation plane, in contrast to the results presented in this section for the Fresnel biprism setup. In other words, no

![Fig. 2. (Color online) Absolute value of the visibility of the interference patterns obtained in planes behind the biprism. For the calculation of these curves we assumed a specific setup in which \( n = 1.5, \delta = 0.51^\circ, \lambda = 0.634 \text{ \mu m}, x_0 = 400 \text{ \mu m}, \) and \( \Delta = 60 \text{ \mu m}. \)](image)
axial modulation of the contrast is obtained in the double-slit configuration.

However, some other alternatives to Young’s interferometer share the axial modulation for the visibility of the interference fringes that we have presented here when using incoherent quasi-monochromatic sources. In fact, any interferometer fulfilling the condition stated in Eq. (1) and showing some variation in the ratio $M_2/p$ with the propagation distance $z$ may present some axial modulation for the contrast of the interference fringes, as stated in Eq. (6). This is, among others, the case of a Wollaston prism. But in this case, the device cannot work with unpolarized light unless the prism is located between two crossed (or parallel) linear polarizers, thus providing a lower light gathering power compared with the Fresnel biprism.

Other devices that do not allow us to obtain the axial resonance of sinusoidal patterns by the proposed technique are Lloyd’s mirror and the Kösters prism. In Lloyd’s mirror, a shift of the point source normal to the mirror causes the same displacement of its image, but in the opposite direction, so that the separation between the two sources generated by the device changes. Therefore, a shift of the source makes a change in the period of the interference pattern instead of just a lateral displacement of the fringes. In other words, for Lloyd’s mirror the condition stated in Eq. (1) is not fulfilled and, consequently, Lloyd’s mirror does not show the capability of generating resonant superposition planes with sinusoidal irradiance variations. The same argument applies to the Kösters prism. This prism is also capable of generating two virtual sources, but due to the fact that one of the sources is generated by a reflection in the semi-reflecting plane surface of the prism, it exhibits the same behavior as Lloyd’s mirror. Some details of this discussion can be found in [33].

### 3. AXIAL RESONANCE OF FAR-FIELD PERIODIC PATTERNS

Our aim here is to obtain periodic fringes whose period does not change with the axial distance, so that the period is the same in all the planes of resonance. Since the period in such planes corresponds to the one provided by the biprism under point source illumination [see Eq. (3)], the only way of achieving this goal is to set the distance $n$ to infinity. In other words, we have to separate infinitely the source from the biprism. Experimentally, this can be performed by simply inserting a converging lens, with focal length $f$, between the sources and the biprism, as shown in Fig. 3, in such a way that the source plane coincides with the front focal plane of the lens.

Following the same reasoning as in the previous section, the irradiance distribution obtained in this case at planes beyond the biprism is given by

$$I(x, z) \propto 1 + V'(z) \cos \left( \frac{2\pi x}{p'} \right),$$

where now the period of the fringe pattern,

$$p' = \frac{1}{2u_0},$$

does not change with $z$. The visibility is given in this configuration by

$$V'(z) = \frac{\sin \left( \frac{\pi N M_2 x_0}{N S} \right)}{N \sin \left( \frac{\pi M_2 x_0}{p} \right)} \sin \left( \frac{\Delta M_2 x_0}{p} \right).$$

From Fig. 4, this equidistance is clear, as is, interestingly, that the decrease of the visibility around a resonant plane is symmetric in the axial direction. Additionally, the axial extent to which the visibility remains above a threshold value (in a sense, the depth of focus of the high-visibility planes) is uniform for every plane of resonance.

### 4. DIFFRACTION EFFECTS AT THE CENTRAL EDGE OF THE BIPRISM

As stated above, the formalism developed in the previous sections gives an approximate description of the irradiance patterns generated by the Fresnel biprism only in the interference region defined by the parageometrical approximation. Furthermore, the effect of the edge diffraction on the biprism has been neglected in previous sections. In this section, we present a more accurate description of the field diffracted by the biprism. As we will show, the full description of the

![Fig. 3. Scheme for the production of far-field periodic resonant patterns.](image)

![Fig. 4. Visibility of fringe patterns behind the biprism when the array of linear sources is placed at infinity by using a converging lens.](image)
irradiance pattern at every transverse plane can be expressed as a modulated version of the parageometrical model predictions. In this way, the results presented in the previous sections are accurate enough that this envelope function is very smooth as compared with the sinusoidal pattern. This approximation gives rise to equations that predict very accurately the axial position of the maxima and minima of visibility in Eq. (4) or (12), and it gives the qualitative behavior of $V(z)$, but it is not accurate enough for a precise calculation of its values.

Accordingly, the fringes obtained with the parageometrical model are modified by diffraction effects, due to the fact that the waves, which diverge from the two virtual sources, are not complete but are abruptly cut off at the arista of the biprism\cite{21,22}, which acts as a straight edge. A reliable evaluation of the fringes’ profile and their visibility requires the exact calculation of the diffraction pattern produced by the biprism. To obtain the amplitude distribution behind the Fresnel biprism when it is illuminated by a spherical wave, one uses the Fresnel–Kirchhoff integral\cite{34}, assuming the usual approximation of a thin object. We have modeled the biprism as a thin object whose transmission function is given by $t(\vec{x}) = t(\vec{\xi}) + t(-\vec{\xi})$, where $t(\vec{\xi})$ can be written as

$$t(\vec{\xi}) = \exp(-j2\pi\alpha x)\text{step}(\vec{\xi}),$$

(17)

step($x$) being the unit step function\cite{35}.

After straightforward mathematical manipulations, the expression of the amplitude distribution beyond the biprism, for the case of a monochromatic point (or linear) source placed at a distance $\eta$ from it, can be obtained as

$$U'_0 = \frac{1}{\sqrt{2M}} \exp\left[-j\frac{\pi}{\lambda z M} (x + \lambda zu_0)^2\right] \times \left\{1 + j + \text{Fres}\left[\frac{2}{\lambda z M} (x + \lambda zu_0)\right]\right\} + \frac{1}{\sqrt{2M}} \exp\left[-j\frac{\pi}{\lambda z M} (x - \lambda zu_0)^2\right] \times \left\{1 + j + \text{Fres}\left[\frac{2}{\lambda z M} (x - \lambda zu_0)\right]\right\},$$

(18)

where

$$\text{Fres}[\alpha] = \int_0^\alpha \exp\left(j\frac{\pi}{2} x^2\right) dx = C(\alpha) + jS[\alpha].$$

(19)

In this definition, $C(x)$ and $S(x)$ are the Fresnel integrals defined as the criterion of Abramowitz and Stegun\cite{36}. Note that this wave model only takes into account the central edge of the biprism, neglecting the external ones. However, the usual practical configurations justify this approach.

From Eq. (18), the irradiance distribution can be written as

$$I_0(\vec{x}, z; \eta) = \frac{\text{env}(\vec{x}, z; \eta)}{2M} \left[1 + V(\vec{x}, z; \eta) \cos\left(\frac{2\pi x}{p}\right)\right].$$

(20)

where the expression of $\text{env}(\vec{x}, z; \eta)$ is

$$\text{env}(\vec{x}, z; \eta) = 1 + C\left[\frac{2}{\lambda z M} (x + \lambda zu_0)^2\right] + S\left[\frac{2}{\lambda z M} (x + \lambda zu_0)^2\right] + C\left[\frac{2}{\lambda z M} (x + \lambda zu_0)^2\right] + S\left[\frac{2}{\lambda z M} (x - \lambda zu_0)^2\right] - C\left[\frac{2}{\lambda z M} (x - \lambda zu_0)^2\right] - S\left[\frac{2}{\lambda z M} (x - \lambda zu_0)^2\right],$$

(21)

and $V(\vec{x}, z; \eta)$ is given by

$$V(\vec{x}, z; \eta) = \frac{1}{\text{env}(\vec{x}, z; \eta)} \times \left\{1 + 2C\left[\frac{2}{\lambda z M} (x + \lambda zu_0)^2\right] \times \left[\frac{2}{\lambda z M} (x - \lambda zu_0)^2\right] - 2S\left[\frac{2}{\lambda z M} (x + \lambda zu_0)^2\right] \times \left[\frac{2}{\lambda z M} (x - \lambda zu_0)^2\right] + C\left[\frac{2}{\lambda z M} (x + \lambda zu_0)^2\right] \times \left[\frac{2}{\lambda z M} (x + \lambda zu_0)^2\right] - C\left[\frac{2}{\lambda z M} (x - \lambda zu_0)^2\right] \times \left[\frac{2}{\lambda z M} (x - \lambda zu_0)^2\right]\right\}.$$ 

(22)

The irradiance diffraction patterns behind the Fresnel biprism are completely described by Eq. (20). The interference terms of Eqs. (2) and (20) have the same period. Thus, following a similar reasoning to that in Section 2, the position of maximum-visibility planes, given by Eq. (11), remains valid. In order to show the deviations of the parageometrical model with respect to the more rigorous prediction presented in Eq. (20), we present in Fig. 5 a representation of the envelope functions, $\text{env}(\vec{x}, z; \eta)$ and $V(\vec{x}, z; \eta)$, for two different transverse planes. As can be seen, in this figure, within the region of superposition of the light coming from the virtual sources in the parageometrical approximation, $V(\vec{x}, z; \eta)$ attains an almost constant unity value, as predicted by the simpler model. However, the effect of the modulating function $\text{env}(\vec{x}, z; \eta)$ may be of certain importance to the accurate description of the interference patterns. Note that this effect is less significant at large distances from the biprism.

For the case of an incoherent source composed by $N$ point (or linear) sources [such as the source that is described in the Eq. (7)], the irradiance distribution, given by the Eq. (1), can be calculated by using Eq. (9). As an example, in Fig. 6, we show the numerically evaluated fringe patterns for the case of $N = 1, 2$, and 5 point (or linear) incoherent sources placed at a distance $\eta = 207.8$ mm from the biprism. As predicted in Section 2, the positions of the planes of maxima visibility do not depend on the number of sources. Nevertheless, the axial modulation of the visibility increases with the number of slits in the sources. The planes of resonance are marked with dashed lines. Next, in Fig. 7 we show the profile of the fringes in a plane of maximum visibility and one of minimum visibility.
When the incoherent source is located at infinity (or in the front focal plane of a converging lens), Eqs. (20)–(22) are still valid. The only change is that for this particular case the parameter $M$ takes the unit value. Also in this case we have calculated the incoherent fringes for the case of a different number of point sources, as shown in Fig. 8.

5. EXPERIMENTAL VERIFICATIONS

To demonstrate the capability of a Fresnel biprism for producing the resonant superposition of interference fringes, we prepared the experimental setup shown in Fig. 9. For the implementation of the quasi-monochromatic mutually incoherent linear sources, we illuminated a 1D binary grating of period $x_0 = 400 \mu m$ and aperture $\Delta = 60 \mu m$, with the light proceeding from an ultrabright red LED whose emission central wavelength was 634 nm with 14 nm spectral FWHM. The Fresnel biprism (BK7 glass) had a refringence angle $\delta$ and a refractive index $n$ such that $\delta = 0.51^\circ$, $\lambda = 0.634 \mu m$, and $\eta = 207.8 \mu m$. To capture the interference patterns we used a 2.5× objective lens (NA = 0.075) and a CCD composed of 765 × 578 square pixels 11 μm on each side. Note that although it was not strictly monochromatic, the light source that we employed had a very narrow spectral bandwidth, and no chromatic aberration effects were expected to occur. In fact, the relative variation in the refractive index $n$ of the biprism used in the spectral

![Figure 5](image1)

![Figure 6](image2)

![Figure 7](image3)
band of the source is lower than 0.05%. Other chromatically induced effects that might affect the matching of the experimental results to the quasi-monochromatic predictions, such as the coherence loss in the periphery of the interference field, are mostly negligible in the situations considered here.

In our experiment we placed the biprism at a distance \( \eta = 207.8 \text{ mm} \) from the grating. Experimental patterns obtained at different axial positions behind the biprism are shown in Fig. 10. In the left column we show patterns of very low visibility, and in the right one we show some planes of resonance. Clearly, the planes of maximum visibility are not equidistant, and the fringe period varies with the axial coordinate, as predicted in Section 2. In Fig. 11 we show the irradiance distribution behind the biprism in the meridian plane (XZ) for various values of \( N (N = 1, 2, \text{ and } 5) \). We can compare these patterns with the ones evaluated numerically from our formula (see Fig. 6). The similarity between the experimental and calculated results is apparent. Next, in Fig. 12 we show the structure of the fringe pattern in a plane of maximum visibility \((z = 105 \text{ mm})\) and also in a plane of minimum visibility \((z = 160 \text{ mm})\). In this case also the similarity to the calculated results (Fig. 7) is apparent.

Finally, we calculated, plane by plane, the visibility of the fringes. For this calculation we obtained the Fourier transform of the irradiance pattern in each plane and evaluated the visibility as two times the quotient between the +1 order and the zero order. In Fig. 13 we show these experimental values as compared with the theoretical ones. The agreement between them is high.

To demonstrate the feasibility of the resonance of far-field fringes, we arranged the experimental setup schematized in Fig. 14. For this experiment we used a source grating composed of \( N = 5 \) slits. The grating was placed at the front focal plane of a converging lens of focal length \( f = 200 \text{ mm} \). With

![Fig. 9. (Color online) Schematic diagram of the experimental setup used for the production of resonant superposition of interference fringe.](image)

![Fig. 10. Interference patterns obtained for \( N = 5 \) at different distances from the Fresnel biprism. The patterns shown in the right column correspond to the first, second, and third planes of resonance, respectively.](image)

![Fig. 11. Experimental irradiance distribution along the axial direction for different numbers of finite-width sources.](image)
this setup we again captured patterns in planes perpendicular to the optical axis. The irradiance distribution in the meridian plane is plotted in Fig. 15. Again we can see the great similarities between the experimental results and the ones calculated with our formula (Fig. 8). Within the represented axial range we could find two planes of resonance (marked with dashed lines in the figure) at distances $z_1 = 35.2$ mm and $z_2 = 70$ mm. Then we measured the period of the fringes in those planes of resonance and obtained $p_1 = (69 \pm 4)$ $\mu$m and $p_2 = (67 \pm 4)$ $\mu$m. As predicted, the period of resonant fringes is kept constant within the error range.

We also measured the visibility of the fringes obtained at different distances from the biprism. These values are represented in Fig. 16, as compared with the theoretical ones. Again the degree of correlation between theoretical and experimental data is very high.

6. CONCLUSIONS

A new system for the generation of localized interference fringes is presented. The proposal works with noncoherent illumination coming from a properly coded spatially incoherent quasi-monochromatic source illuminating a Fresnel biprism. In particular, a set of incoherently illuminated equidistant parallel slits is used to generate resonant overlapping in some singular planes of the periodic interference patterns generated by the biprism from each point in the source plane. This resonance generates in these planes high-contrast highly localized sinusoidal interference patterns. A brief discussion on the possibility of obtaining such a result with some other “Young’s double-slit”-related elements is also presented, and the necessary conditions for achieving this goal are also established.

The technique proposed here presents some advantages over other similar devices. On one hand, the period of the resonant fringes can be tuned by axially displacing the biprism with respect to the source plane, as can be seen easily from Eq. (3). On the other hand, when the number of slits in the grating source is high, the visibility of the fringes falls off very rapidly in the vicinity of the planes of resonance, generating a
highly localized fringe pattern. Finally, we have demonstrated that it is also possible to modify the experimental architecture so that the period of the resonant fringes remains constant, as stated in Section 4. All these features make this setup very attractive for some modern applications using patterned light, for instance, in structured illumination microscopy [37–39].

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REFERENCES AND NOTE

33. See Ref. [17], Chap. 7.
36. M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables (Dover, 1972), Chap. 7.