The Stabilizing Role of Government Size
Technical Appendix

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1. The model

1.1. Households

1. Two types of consumers: a fraction λ of rule-of-thumb consumers and $(1-\lambda)$ of optimizing households. In our basic model in Section 3 of the paper $\lambda = 0$.
2. Separable utility function (as Baxter and King, 1993):

$$U(c_t, 1-l_t, g_t^c) = \frac{\left(c_t(1-l_t)^{\gamma}\right)^{1-\sigma} - 1}{1-\sigma} + \Gamma(g_t^c) \tag{1}$$

3. Budget restriction for optimizing households:

$$\begin{aligned} &\frac{B_t}{(1+i_t)} + P_t(1+\tau_t^c)c_t \\ = \quad & P_t(1-\tau_t^w)w_t l_t + B_{t-1} + P_t(1-\lambda)g_t^s + \int_0^1 \Omega_{it} di \end{aligned} \tag{2}$$

where Ω_{it} are firms profits and

$$l_t = \int_0^1 l_{it} di, \qquad w_t l_t = \int_0^1 w_{it} l_{it} di \tag{3}$$

1.2. Firms technology

1.

$$y_{it} = A k_{it}^{\alpha} l_{it}^{1-\alpha} - \kappa_i \tag{4}$$

1.3. The Government

1. Government budget constraint:

$$P_t \tau_t^w w_t l_t + P_t \tau_t^k \int_0^1 r_{it} k_{it} di + P_t \tau_t^c c_t = P_t (g_t^c + g_t^s) \tag{5}$$

where r_{it} is the equivalent to the rental cost of capital that would rationalize the quantity of capital used by the firm and it is defined below.

2. Model solution

2.1. Optimizing Households:

$$\max_{c_{ot}, l_{ot}, B_{t+1}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{(c_{ot}(1 - l_{ot})^{\gamma})^{1-\sigma} - 1}{1 - \sigma} + \beta^t \Gamma(g_t^c) \right\} \tag{6}$$

subject to

$$\begin{aligned} & \frac{B_t}{(1 + i_t)} + P_t (1 + \tau_t^c) c_{ot} \\ = \; & P_t (1 - \tau_t^w) w_t l_{ot} + B_{t-1} + P_t (1 - \lambda) g_t^s + \int_0^1 \Omega_{it} di \end{aligned} \tag{7}$$

The Lagrangian

$$\begin{aligned} L_t \; = \; & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(c_{ot}(1 - l_{ot})^{\gamma})^{1-\sigma} - 1}{1 - \sigma} + \Gamma(g_t^c) \right. \\ & + \mu_t \left((1 - \tau_t^w) w_t l_{ot} + \frac{B_{t-1}}{P_{t-1} \pi_t} + (1 - \lambda) g_t^s + \int_0^1 \frac{\Omega_{it}}{P_t} di \right. \\ & \left. \left. - \frac{B_t}{P_t} \frac{1}{1 + i_t} - (1 + \tau_t^c) c_t \right) \right\} \end{aligned} \tag{8}$$

First order conditions

$$\frac{\partial L_t}{\partial c_{ot}} = (1 - l_{ot})^{\gamma} \left(c_{ot} (1 - l_{ot})^{\gamma} \right)^{-\sigma} - \mu_t (1 + \tau_t^c) = 0 \tag{9}$$

$$\frac{\partial L_t}{\partial l_{ot}} = -\gamma c_{ot} (1 - l_{ot})^{(\gamma - 1)} \left(c_{ot} (1 - l_{ot})^{\gamma} \right)^{-\sigma} + \mu_t (1 - \tau_t^w) w_t = 0 \tag{10}$$

$$\frac{\partial L_t}{\partial B_t} = -\mu_t E_t \left(\frac{1}{1+i_t} \right) + \beta E_t \mu_{t+1} \frac{1}{\pi_{t+1}} = 0 \tag{11}$$

Now (9) can be expressed as,

$$\frac{(c_{ot}(1-l_{ot})^{\gamma})^{1-\sigma}}{c_{ot}} = \mu_t (1+\tau_t^c) \tag{12}$$

or

$$\mu_t = \frac{(c_{ot}(1-l_{ot})^{\gamma})^{1-\sigma}}{(1+\tau_t^c)c_{ot}} = \frac{c_{ot}^{-\sigma}(1-l_{ot})^{\gamma(1-\sigma)}}{(1+\tau_t^c)} \tag{13}$$

From equation (10)

$$\mu_t (1-\tau_t^w) w_t = \frac{\gamma\,(c_{ot}(1-l_{ot})^{\gamma})^{1-\sigma}}{(1-l_{ot})} \tag{14}$$

but, using (13),

$$\frac{(1-\tau_t^w)}{(1+\tau_t^c)} w_t = \frac{\gamma c_{ot}}{(1-l_{ot})} \tag{15}$$

which also applies to the labour supply to each firm

$$\frac{(1-\tau_t^w)}{(1+\tau_t^c)} w_{it} = \frac{\gamma c_{ot}}{(1-l_{oit})} \tag{16}$$

From (11)

$$1 = \beta E_t \left(\frac{\mu_{t+1}}{\mu_t} \frac{1+i_t}{\pi_{t+1}} \right) \tag{17}$$

or, using (13),

$$1 = \beta E_t \left(\frac{(1+\tau_t^c) c_{ot+1}^{-\sigma} (1-l_{ot+1})^{\gamma(1-\sigma)}}{(1+\tau_{t+1}^c) c_{ot}^{-\sigma} (1-l_{ot})^{\gamma(1-\sigma)}} \frac{1+i_t}{\pi_{t+1}} \right) \tag{18}$$

which is the Euler equation.

2.2. *Rule-of-thumb consumers*

$$\max_{c_{rt}, l_{rt}} \frac{(c_{rt}(1-l_{rt})^{\gamma})^{1-\sigma} - 1}{1-\sigma} + \beta^t \Gamma(g_t^c) \tag{19}$$

subject to

$$P_t(1+\tau_t^c)c_{rt} = P_t(1-\tau_t^w)w_t l_{rt} + P_t\lambda g_t^s$$

The Lagrangian

$$L_t = \frac{(c_{rt}(1-l_{rt})^\gamma)^{1-\sigma}-1}{1-\sigma} + \Gamma(g_t^c) + \lambda_{rt}\left[(1-\tau_t^w)w_t l_{rt} + \lambda g_t^s - (1+\tau_t^c)c_{rt}\right]$$

First order conditions

$$\frac{\partial L_t}{\partial c_{rt}} = (1-l_{rt})^\gamma \left(c_{rt}(1-l_{rt})^\gamma\right)^{-\sigma} - \lambda_{rt}(1+\tau_t^c) = 0 \tag{20}$$

$$\frac{\partial L_t}{\partial l_{rt}} = -\gamma c_{rt}(1-l_{rt})^{(\gamma-1)} \left(c_{rt}(1-l_{rt})^\gamma\right)^{-\sigma} + \lambda_{rt}(1-\tau_t^w)w_t = 0 \tag{21}$$

Combinind these two FOCs

$$\frac{(1-\tau_t^w)}{(1+\tau_t^c)}w_t = \frac{\gamma c_{rt}}{(1-l_{rt})} \tag{22}$$

which can be used in the budget restrictions to solve for l_{rt}

$$\frac{1}{\gamma}(1-\tau_t^w)w_t(1-l_{rt}) = (1-\tau_t^w)w_t l_{rt} + \lambda g_t^s \tag{23}$$

or

$$(1-l_{rt}) = \gamma l_{rt} + \frac{\gamma\lambda g_t^s}{(1-\tau_t^w)w_t} \tag{24}$$

Then

$$1 - \frac{\gamma\lambda g_t^s}{(1-\tau_t^w)w_t} = (1+\gamma)\, l_{rt} \tag{25}$$

and

$$l_{rt} = \frac{1}{1+\gamma}\left[1 - \frac{\gamma\lambda g_t^s}{(1-\tau_t^w)w_t}\right] \tag{26}$$

Using again the budget constraint we can solve for c_{rt}

$$c_{rt} = \frac{(1-\tau_t^w)w_t}{1+\gamma}\left[1 - \frac{\gamma\lambda g_t^s}{(1-\tau_t^w)w_t}\right] + \lambda g_t^s \tag{27}$$

$$= \frac{(1-\tau_t^w)w_t}{(1+\tau_t^c)(1+\gamma)} + \frac{\lambda g_t^s}{(1+\tau_t^c)(1+\gamma)} \tag{28}$$

2.3. Alternative specifications of the utility function (Guo and Harrison, 2006)

Guo and Harrison (2006) consider the two follwoing specifications of the utility funrtion:

$$U_2 = \ln c_t + \gamma \ln(1 - l_t) + \Gamma(g_t^c) \tag{29}$$

$$U_3 = \ln c_t - \Upsilon_3 \frac{l_t^{1+\gamma_g}}{1+\gamma_g} + \Gamma(g_t^c) \tag{30}$$

Notice that the first one (U_2) correspond to our utility function when $\sigma = 1$.

Optimizing consumers

In these cases the FOCs imply that the intratemporal conditions are given by

$$\frac{(1-\tau_t^w)}{(1+\tau_t^c)} w_t = \frac{\gamma c_{ot}}{(1-l_{ot})} \tag{31}$$

and

$$\frac{(1-\tau_t^w)}{(1+\tau_t^c)} w_t = \Upsilon_3 c_{ot} l_{ot}^{\gamma_g} \tag{32}$$

wheras the intertemporal condition in both cases is

$$1 = \beta E_t \left(\frac{c_{ot}(1+\tau_t^c)}{c_{ot+1}(1+\tau_{t+1}^c)} \frac{1+i_t}{\pi_{t+1}} \right) \tag{33}$$

Non-optimizing consumers

$$\max_{c_{rt}, l_{rt}} \ln c_{rt} - \Upsilon_3 \frac{l_{rt}^{1+\gamma_g}}{1+\gamma_g} + \Gamma(g_t^c) \tag{34}$$

subject to

$$P_t(1+\tau_t^c)c_{rt} = P_t(1-\tau_t^w) w_t l_{rt} + P_t \lambda g_t^s$$

The Lagrangian

$$L_t = \ln c_{rt} - \Upsilon_3 \frac{l_{rt}^{1+\gamma_g}}{1+\gamma_g} + \Gamma(g_t^c) + \lambda_{rt} \left[(1-\tau_t^w) w_t l_{rt} + \lambda g_t^s - (1+\tau_t^c) c_{rt} \right]$$

The first order conditions are

$$\frac{\partial L_t}{\partial c_{rt}} = \frac{1}{c_{rt}} - \lambda_{rt}(1+\tau_t^c) = 0 \tag{35}$$

$$\frac{\partial L_t}{\partial l_{rt}} = -\Upsilon_3 l_{rt}^{\gamma_g} + \lambda_{rt}(1-\tau_t^w)w_t = 0 \tag{36}$$

Combinind these two FOCs

$$\frac{(1-\tau_t^w)}{(1+\tau_t^c)} w_t = \Upsilon_3 c_{rt} l_{rt}^{\gamma_g} \tag{37}$$

which can be used in the budget restrictions to solve for l_{rt}

$$1 = \Upsilon_3 l_{rt}^{\gamma_g+1}\left(1 + \frac{\lambda g_t^s}{(1-\tau_t^w)w_t l_{rt}}\right) \tag{38}$$

Given l_{rt}, the solution of c_{rt} is

$$c_{rt} = \frac{(1-\tau_t^w)w_t}{\Upsilon_3 l_{rt}^{\gamma_g}} \tag{39}$$

and then

$$l_t = \lambda l_{rt} + (1-\lambda) l_{ot} \tag{40}$$

and

$$c_t = \lambda c_{rt} + (1-\lambda) c_{ot} \tag{41}$$

2.4. *Firms*

We follow Woodford (2004 and 2006) and Christiano (2004), assuming that capital and labour are firm-specific.

The aggregator.

Final good is produced using intermediate goods by a competitive/representative firm (the aggregator) that solves the following problem

$$\max_{y_{it}} P_t \left[\int_0^1 (y_{it})^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it} y_{it} di \tag{42}$$

where

$$\left[\int_0^1 (y_{it})^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} = y_t \tag{43}$$

First order condition

$$\frac{\partial}{\partial y_{it}} = P_t \frac{\varepsilon}{\varepsilon - 1} \frac{\varepsilon - 1}{\varepsilon} (y_{it})^{\frac{-1}{\varepsilon}} \left[\int_0^1 (y_{it})^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{1}{\varepsilon-1}} - P_{it} = 0 \tag{44}$$

Therefore

$$\frac{P_{it}}{P_t} = \left(\frac{y_t}{y_{it}} \right)^{\frac{1}{\varepsilon}} \tag{45}$$

or

$$y_{it} = y_t \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \tag{46}$$

Zero profits implies

$$P_t y_t = \int_0^1 P_{it} y_{it} di = y_t P_t^{\varepsilon} \int_0^1 (P_{it})^{1-\varepsilon} di \tag{47}$$

or

$$P_t = \left[\int_0^1 (P_{it})^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{48}$$

Intermediate good firms

The presented discounted value of profits of the intermediate good firm is given by

$$\max_{e_{it+j}, k_{it+j+1}, l_{it+1}} E_t \sum_{j=0}^{\infty} \rho_{t,t+j} \beta^j \left((1 - \tau_{t+j}^k) \left[\frac{P_{it+j}}{P_{t+j}} y_{it+j} - w_{it+j} l_{it+j} \right] - e_{it+j} \right) \tag{49}$$

subject to

$$k_{it+j+1} = \Phi \left(\frac{e_{it+j}}{k_{it+j}} \right) k_{it+j} + (1 - \delta) k_{it+j} \tag{50}$$

$$y_{it+j} = y_{t+j} \left(\frac{P_{it+j}}{P_{t+j}} \right)^{-\varepsilon} \tag{51}$$

and

$$y_{it+j} = A_t k_{it+j}^{\alpha} l_{it+j}^{1-\alpha} - \kappa \tag{52}$$

The Lagrangian of this problem is

$$L_{it} = E_t \sum_{j=0}^{\infty} \beta^j \left\{ \rho_{t,t+j} \left((1-\tau^k_{t+j}) \left[\left(A_{t+j} k^{\alpha}_{it+j} l^{1-\alpha}_{it+j} - \kappa \right)^{\frac{\varepsilon-1}{\varepsilon}} y^{1/\varepsilon}_{t+j} - w_{it+j} l_{it+j} \right] - e_{it+j} \right) \right. \tag{53}$$

$$\left. -\mu^k_{it+j}\, k_{it+j+1} - \Phi \left(\frac{e_{it+j}}{k_{it+j}} \right) k_{it+j} - (1-\delta) k_{it+j} \right\} \tag{54}$$

and the FOCs are

$$\frac{\partial L_{it}}{\partial e_{it}} = -\rho_t + \mu^k_{it} \Phi'_{it} = 0 \tag{55}$$

$$\frac{\partial L_{it}}{\partial l_{it}} = \rho_t \frac{\varepsilon - 1}{\varepsilon} \left(\frac{A_t k^{\alpha}_{it} l^{1-\alpha}_{it} - \kappa}{y_t} \right)^{-\frac{1}{\varepsilon}} (1-\alpha) k^{\alpha}_{it} l^{-\alpha}_{it} - w_{it} = 0 \tag{56}$$

$$\begin{aligned} \frac{\partial L_{it}}{\partial k_{it+1}} = -\mu^k_{it} + \beta E_t \rho_{t+1} (1-\tau^k_{t+1}) \frac{\varepsilon-1}{\varepsilon} \left(\frac{A_{t+1} k^{\alpha}_{it+1} l^{1-\alpha}_{it+1} - \kappa}{y_{t+1}} \right)^{-\frac{1}{\varepsilon}} \alpha k^{\alpha-1}_{it+1} l^{1-\alpha}_{it+1} \\ + \beta E_t \mu^k_{it+1} \left(\Phi_{it+1} - \Phi'_{it+1} \frac{e_{it+1}}{k_{it+1}} + (1-\delta) \right) = 0 \end{aligned} \tag{57}$$

From equation (55)

$$\mu^k_{it} = \rho_{t,t} \left[\Phi'_{it} \right]^{-1} \tag{58}$$

or

$$q_t \equiv \frac{\mu^k_{it}}{\rho_{t,t}} = \left[\Phi'_{it} \right]^{-1} \tag{59}$$

From equation (56)

$$w_{it} = \frac{\varepsilon - 1}{\varepsilon} \left(\frac{y_{it}}{y_t} \right)^{-\frac{1}{\varepsilon}} (1-\alpha) A_t k^{\alpha}_{it} l^{-\alpha}_{it} \tag{60}$$

Taking into account that

$$P_{it} = \frac{\varepsilon - 1}{\varepsilon} P_t mc_{it} \tag{61}$$

and

$$\left(\frac{y_{it}}{y_t} \right)^{-\frac{1}{\varepsilon}} = \frac{P_{it}}{P_t} \tag{62}$$

then

$$w_{it} = (1-\alpha) mc_{it} A_t k_{it}^{\alpha} l_{it}^{-\alpha} \tag{63}$$

and

$$\begin{aligned} \mu_{it}^k &= \beta E_t \mu_{it+1}^k \left(\Phi_{it+1} - \Phi'_{it+1} \frac{e_{it+1}}{k_{it+1}} + (1-\delta) \right) \\ &+ \beta E_t \rho_{t+1} (1-\tau_{t+1}^k) r_{it+1} \end{aligned} \tag{64}$$

where

$$r_{it+1} \equiv mc_{it+1} \alpha A_{t+1} k_{it+1}^{\alpha-1} l_{it+1}^{1-\alpha} \tag{65}$$

Price setting: nominal inertia in the standard case

If some firms change prices only every some periods, the analysis above is not adequate, since there is not a symmetric equilibrium in which $P_{it} = P_t$. We follow Calvo's model of nominal inertia (see Calvo, 1983): a percentage ϕ of firms set

$$P_{it} = \overline{\pi} P_{it-1} \tag{66}$$

whereas the rest of the firms $(1-\phi)$ select $\widetilde{P}_{it}$ to maximize the value of their shares, that is, the present discount value of future profits:

$$\begin{aligned} PV(\widetilde{P}_{it}) &= E_t \left\{ \sum_{j=0}^{\infty} \rho_{t,t+j} (\beta\phi)^j \Omega_{t+j}(\widetilde{P}_{it}) \right. \\ &\left. + \sum_{j=0}^{\infty} \rho_{t,t+j} \beta^j \phi^{(j-1)} (1-\phi) PV(\widetilde{P}_{it+j}) \right\} \end{aligned} \tag{67}$$

Since the terms in $PV(\widetilde{P}_{it+j})$ do not depend on $\widetilde{P}_{it}$ (there is no cost of changing prices nor any state dependence on $\widetilde{P}_{it+j}$), the relevant term of the maximization of $PV(\widetilde{P}_{it})$ is the first. Therefore,

$$\max_{\widetilde{P}_{it}} E_t \sum_{j=0}^{\infty} \rho_{t,t+j} (\beta\phi)^j \left[\widetilde{P}_{it} \overline{\pi}^j y_{it+j} - P_{t+j} mc_{t+j} (y_{it+j} + \kappa) \right] \tag{68}$$

subject to

$$y_{it+j} = \left(\widetilde{P}_{it} \overline{\pi} \right)^{-\varepsilon} P_{t+j}^{\varepsilon} y_{t+j} \tag{69}$$

Substituting (69) in (68)

$$\max_{\widetilde{P}_{it}} E_t \sum_{j=0}^{\infty} \rho_{t,t+j} (\beta\phi)^j P_{t+j}^{\varepsilon} y_{t+j} \left[\left(\widetilde{P}_{it} \overline{\pi} \right)^{1-\varepsilon} - P_{t+j} mc_{t+j} \left(\left(\widetilde{P}_{it} \overline{\pi} \right)^{-\varepsilon} + \frac{\kappa}{P_{t+j}^{\varepsilon} y_{t+j}} \right) \right] \tag{70}$$

The first order condition is

$$\frac{\partial}{\partial \widetilde{P}_{it}} = E_t \sum_{j=0}^{\infty} \rho_{t,t+j} (\beta\phi)^j P_{t+j}^{\varepsilon} y_{t+j} \left[(1-\varepsilon) \left(\widetilde{P}_{it} \overline{\pi} \right)^{\varepsilon} + P_{t+j} mc_{t+j} \varepsilon \overline{\pi}^j \left(\overline{\pi}^j \widetilde{P}_{it} \right)^{-\varepsilon-1} \right] = 0 \tag{71}$$

or

$$\begin{aligned} & -(1-\varepsilon) \left(\widetilde{P}_{it} \right)^{-\varepsilon} \sum_{j=0}^{\infty} \overline{\pi}^{j(1-\varepsilon)} \rho_{t,t+j} (\beta\phi)^j P_{t+j}^{\varepsilon} y_{t+j} \\ = \; & \varepsilon \left(\widetilde{P}_{it} \right)^{-\varepsilon-1} \sum_{j=0}^{\infty} \overline{\pi}^{-j\varepsilon} P_{t+j}^{\varepsilon+1} mc_{t+j} y_{t+j} \rho_{t,t+j} (\beta\phi)^j \end{aligned} \tag{72}$$

or

$$\widetilde{P}_{it} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{\infty} (\beta\phi)^j E_t \left[\rho_{t,t+j} P_{t+j}^{\varepsilon+1} mc_{t+j} y_{t+j} \overline{\pi}^{-j\varepsilon} \right]}{\sum_{j=0}^{\infty} (\beta\phi)^j E_t \left[\rho_{t,t+j} P_{t+j}^{\varepsilon} y_{t+j} \overline{\pi}^{j(1-\varepsilon)} \right]} \tag{73}$$

and the aggregate price index at t, using (48) is

$$P_t = \left[\phi \left(\overline{\pi} P_{t-1} \right)^{1-\varepsilon} + (1-\phi) \widetilde{P}_t^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{74}$$

Notice that for all price-adjusting firms $\widetilde{P}_{it} = \widetilde{P}_t$.

Price setting: nominal inertia when labour and capital are firm-specific

We shall further assume that labour and capital cannot be instantaneously reallocated across firms, so that, the marginal cost of firms adjusting prices ($mc_{it,t}$) differs from the average marginal cost at time t (mc_t). Following Woodford (2004 and 2006) and Christiano (2004), in this case the coefficient which measures the response of inflation ($\widehat{\pi}_t$, in deviations from steady state) to changes in marginal costs ($\widehat{mc}_t$, also in deviations) departs from the one in the standard Calvo model since now it includes a multiplicative term:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1-\beta\phi)(1-\phi)}{\phi} \frac{1 - \phi\beta\kappa_1}{(1 + \varepsilon \frac{\alpha}{1-\alpha})(1 - \phi\beta\kappa_1) + \frac{\alpha}{1-\alpha} \phi\beta\kappa_2} \widehat{mc}_t \tag{75}$$

where κ_1 and κ_2 are the solutions, together with ψ, of the following system of equations:

$$\psi = \frac{\frac{\alpha}{1-\alpha}(1-\beta\phi)}{(1 + \varepsilon \frac{\alpha}{1-\alpha})(1 - \phi\beta\kappa_1) + \frac{\alpha}{1-\alpha} \phi\beta\kappa_2} \tag{76}$$

$$\kappa_2 = \frac{\Xi \phi \kappa_1}{\beta \kappa_1 \phi - 1} \tag{77}$$

and

$$(1 + \beta + [1 - \beta(1 - \delta)]) \frac{1}{1 - \alpha} \frac{1}{\Phi''} \kappa_2 + \beta \kappa_2 (\kappa_1 + \alpha) - (1 - \phi) \psi \kappa_2 \left(\beta \kappa_2 - \Xi \right) = 0 \tag{78}$$

where

$$\Xi = -\left(1 - \beta(1 - \delta)\right) \frac{1}{1 - \alpha} \frac{1}{\Phi''} \varepsilon \tag{79}$$

2.5. Equilibrium

$$k_{it+1} = \Phi\left(\frac{e_{it}}{k_{it}}\right)k_{it} + (1-\delta)k_{it} \tag{E.1}$$

$$1 = \beta E_t\left(\frac{(1+\tau_t^c)c_{ot+1}^{-\sigma}(1-l_{ot+1})^{\gamma(1-\sigma)}}{(1+\tau_{t+1}^c)c_{ot}^{-\sigma}(1-l_{ot})^{\gamma(1-\sigma)}}\frac{1+i_t}{\pi_{t+1}}\right) \tag{E.2}$$

$$\frac{(1-\tau_t^w)}{(1+\tau_t^c)}w_t = \frac{\gamma c_t}{(1-l_t)} \tag{E.3}$$

$$c_t = \frac{\lambda}{(1+\tau_t^c)}\left[\frac{(1-\tau_t^w)w_t}{1+\gamma} + \frac{\lambda g_t^s}{1+\gamma}\right] + (1-\lambda)c_{ot} \tag{E.4}$$

$$l_t = \frac{\lambda}{1+\gamma}\left[1 - \frac{\gamma\lambda g_t^s}{(1-\tau_t^w)w_t}\right] + (1-\lambda)l_{ot} \tag{E.5}$$

$$q_{it} = \left[\Phi'\left(\frac{e_{it}}{k_{it}}\right)\right]^{-1} \tag{E.6}$$

$$q_{it} = E_t\left[\beta\frac{\rho_{t,t+1}}{\rho_{t,t}}\left((1-\tau_{t+1}^k)r_{it+1} + q_{it+1}\left[\Phi\left(\frac{e_{it+1}}{k_{it+1}}\right) + (1-\delta) - \Phi'\left(\frac{e_{it+1}}{k_{it+1}}\right)\frac{e_{it+1}}{k_{it+1}}\right]\right)\right] \tag{E.7}$$

$$w_{it} = mc_{it}(1-\alpha)A_t k_{it}^{\alpha} l_{it}^{-\alpha} \tag{E.8}$$

$$r_{it} = mc_{it}\alpha A k_{it}^{\alpha-1} l_{it}^{1-\alpha} \tag{E.9}$$

$$E_t\sum_{j=0}^{\infty}\rho_{t,t+j}(\beta\phi)^j P_{t+j}^{\varepsilon}y_{t+j}\left[(1-\varepsilon)\left(\widetilde{P}_{it}\overline{\pi}\right)^{\varepsilon} + P_{t+j}mc_{it+j}\varepsilon\overline{\pi}^j\left(\overline{\pi}^j\widetilde{P}_{it}\right)^{-\varepsilon-1}\right] = 0 \tag{E.10}$$

$$P_t = \left[\phi\left(\overline{\pi}P_{t-1}\right)^{1-\varepsilon} + (1-\phi)\widetilde{P}_t^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{E.11}$$

$$\pi_t \equiv \frac{P_t}{P_{t-1}} \tag{E.12}$$

$$\tau_t^w w_t l_t + \tau_t^k \int_0^1 r_{it} k_{it} di + \tau_t^c c_t = g_t^c + g_t^s \tag{E.13}$$

$$y_t = c_t + e_t + g_t^c \tag{E.14}$$

$$y_t = A k_t^\alpha l_t^{1-\alpha} - \kappa \tag{E.15}$$

$$\frac{E_t \rho_{t,t+j+1}}{E_t \rho_{t,t+j}} = \frac{E_t u_{ct+j+1}(\cdot)}{E_t u_{ct+j}(\cdot)} \tag{E.16}$$

- Notice that $\widetilde{P}_{it} = \widetilde{P}_t$
- All τ_t^w, $\tau_t^k$and τ_t^c will be assumed constant for all t unless said otherwise.
- Notice that for the alternative specifications of the utility function (U_2 and U_3), equation E.2 should be replaced by

$$1 = \beta E_t \left(\frac{c_{ot}(1+\tau_t^c)}{c_{ot+1}(1+\tau_{t+1}^c)} \frac{1+i_t}{\pi_{t+1}} \right) \tag{E.2b}$$

and equation E.3 for $U_3$by

$$\frac{(1-\tau_t^w)}{(1+\tau_t^c)} w_t = \Upsilon_3 c_t l_t^{\gamma_g} \tag{E.3c}$$

- Additionally, for U_3 we have that

$$1 = \Upsilon_3 l_{rt}^{\gamma_g+1} \left(1 + \frac{\lambda g_t^s}{(1-\tau_t^w) w_t l_{rt}} \right) \tag{80}$$

Given l_{rt}, the solution of c_{rt} is

$$c_{rt} = \frac{(1-\tau_t^w) w_t}{\Upsilon_3 l_{rt}^{\gamma_g}} \tag{81}$$

and then E.5 is replaced by

$$l_t = \lambda l_{rt} + (1-\lambda) l_{ot} \tag{E.5c}$$

and E.4 by

$$c_t = \lambda c_{rt} + (1 - \lambda) c_{ot} \tag{E.4c}$$

3. Linearization of the model around the steady state

3.1. Linearization of E.1

First we linearize $\Phi\left(\frac{e_t}{k_t}\right)k_t$. Using Taylor, and taking into account the symmetry of the steady state equilibrium,

$$\Phi\left(\frac{e_{it}}{k_{it}}\right)k_{it} \simeq \Phi\overline{k} + \left[\Phi - \Phi'\frac{\overline{e}}{\overline{\overline{k}}}\right](k_{it} - \overline{k}) + \Phi'(e_{it} - \overline{e})$$

Therefore

$$k_{it+1} = \Phi\overline{k} + \left[\Phi - \Phi'\frac{\overline{e}}{\overline{\overline{k}}} + 1 - \delta\right](k_{it} - \overline{k}) + \Phi'(e_{it} - \overline{e}) + (1-\delta)\overline{k}$$

In steady state:

$$\overline{k} = \Phi\overline{k} + (1-\delta)\overline{k} \Rightarrow \Phi = \delta$$

Then

$$\begin{aligned}(1 + \widehat{k}_{it+1})\overline{k} \quad &= \quad \Phi\overline{k} + (1-\delta)\overline{k} + \Phi'\overline{e}(1 + \widehat{e}_{it}) - \Phi'\overline{e} + \\ &\left[\Phi - \Phi'\frac{\overline{e}}{\overline{\overline{k}}} + 1 - \delta\right](1 + \widehat{k}_{it})\overline{k} - \\ &\left[\Phi - \Phi'\frac{\overline{e}}{\overline{\overline{k}}} + 1 - \delta\right]\overline{k}\end{aligned}$$

or

$$\overline{k}\widehat{k}_{it+1} = \Phi'\overline{e}\widehat{e}_{it} + \left[\Phi - \Phi'\frac{\overline{e}}{\overline{\overline{k}}} + 1 - \delta\right]\widehat{k}_{it}\overline{k}$$

$$\widehat{k}_{it+1} = \left[1 - \frac{\overline{e}}{\overline{\overline{k}}}\right]\widehat{k}_{it} + \frac{\overline{e}}{\overline{\overline{k}}}\widehat{e}_{it}$$

As $\Phi' = 1$.Since $\widehat{k}_t = \int_0^1 \widehat{k}_{it}di$ and $\widehat{e}_t = \int_0^1 \widehat{e}_{it}di$, integrating over all $i \in (0,1)$ we have that

$$\widehat{k}_{t+1} = \left[1 - \frac{\overline{e}}{\overline{\overline{k}}}\right]\widehat{k}_t + \frac{\overline{e}}{\overline{\overline{k}}}\widehat{e}_t \tag{L.1}$$

3.2. Linearization of E.2

$$1 = \beta E_t \left(\frac{(1+\tau^c_t) c^{-\sigma}_{ot+1} (1 - l_{ot+1})^{\gamma(1-\sigma)}}{(1+\tau^c_{t+1}) c^{-\sigma}_{ot} (1 - l_{ot})^{\gamma(1-\sigma)}} \frac{1+i_t}{\pi_{t+1}} \right) \tag{E.2}$$

Under the assumption τ^c is constant

$$-\sigma \widehat{c}_{ot} - \gamma(1-\sigma) \frac{\bar{l}_o}{(1-\bar{l}_o)} \widehat{l}_{ot} \tag{82}$$

$$= \quad -\sigma \widehat{c}_{ot+1} - \gamma(1-\sigma) \frac{\bar{l}_o}{(1-\bar{l}_o)} \widehat{l}_{ot+1} + \frac{\bar{i}}{1+\bar{i}} \widehat{i}_t - \widehat{\pi}_{t+1} \tag{83}$$

Then

$$\widehat{c}_{ot} = \widehat{c}_{ot+1} - \frac{1}{\sigma} \left(\frac{\bar{i}}{1+\bar{i}} \widehat{i}_t - \widehat{\pi}_{t+1} \right) - \gamma \frac{(\sigma - 1)}{\sigma} \frac{\bar{l}_o}{(1-\bar{l}_o)} \Delta \widehat{l}_{ot+1} \tag{L.2}$$

3.3. Linearization of E.3

$$\frac{(1-\tau^w_t)}{(1+\tau^c_t)} w_t = \frac{\gamma c_t}{(1-l_t)} \tag{E.3}$$

Under the assumption τ^c and τ^w are constant

$$\widehat{w}_t = \widehat{c}_t + \frac{\bar{l}}{(1-\bar{l})} \widehat{l}_t \tag{L.3}$$

3.4. Linearization of E.4

$$c_t = \frac{\lambda}{(1+\tau^c_t)} \left[\frac{(1-\tau^w_t) w_t}{1+\gamma} + \frac{\lambda g^s_t}{1+\gamma} \right] + (1-\lambda) c_{ot} \tag{E.4}$$

In steady state

$$\bar{c} = \lambda \bar{c}_r + (1-\lambda) \bar{c}_o \tag{84}$$

Therefore

$$\widehat{c}_t = \frac{\lambda}{\bar{c}} \left(\frac{(1-\tau^w)\overline{w}}{(1+\gamma)} \widehat{w}_t + \frac{\lambda \overline{g^s}}{(1+\gamma)} \widehat{g}^s_t \right) + \frac{(1-\lambda)\bar{c}_o}{\bar{c}} \widehat{c}_{ot} \tag{L.4}$$

3.5. Linearization of E.5

$$l_t = \frac{\lambda}{1+\gamma}\left[1 - \frac{\gamma\lambda g_t^s}{(1-\tau_t^w)w_t}\right] + (1-\lambda)l_{ot} \tag{E.5}$$

In steady state

$$\bar{l} = \lambda\bar{l}_r + (1-\lambda)\bar{l}_o \tag{85}$$

Therefore

$$\widehat{l}_t = \frac{\lambda}{\bar{l}}\left(-\frac{\gamma\lambda\overline{g^s}}{(1+\gamma)(1-\tau^w)\overline{w}}(\widehat{g}_t^s - \widehat{w}_t)\right) + \frac{(1-\lambda)\bar{l}_o}{\bar{l}}\widehat{l}_{ot} \tag{L.5}$$

3.6. Linearization of E.6

Using Taylor

$$\Phi'\left(\frac{e_{it}}{k_{it}}\right) \simeq \Phi'\left(\frac{\overline{e}}{\overline{k}}\right) + \Phi''\frac{\overline{e}}{\overline{k}}(\widehat{e}_{it} - \widehat{k}_{it})$$

Therefore E.5 can be written as

$$\frac{1}{\overline{q}}(1-\widehat{q}_{it}) = \Phi'\left(\frac{\overline{e}}{\overline{k}}\right) + \Phi''\frac{\overline{e}}{\overline{k}}(\widehat{e}_{it} - \widehat{k}_{it})$$

Assuming that $\Phi'\left(\frac{\overline{e}}{\overline{k}}\right) = 1$, then

$$-\widehat{q}_{it} = \Phi''\frac{\overline{e}}{\overline{k}}(\widehat{e}_{it} - \widehat{k}_{it})$$

which allows us to obtain the following aggregate expression

$$-\widehat{q}_t = \Phi''\frac{\overline{e}}{\overline{k}}(\widehat{e}_t - \widehat{k}_t) \tag{L.6}$$

3.7. Linearization of E.7

The steady state is of E.6 is ($\overline{q} = 1$)

$$1 = \beta(1-\overline{\tau}^k)\overline{r} + \beta\left[\Phi\left(\frac{\overline{e}}{\overline{k}}\right) + (1-\delta) - \Phi'\left(\frac{\overline{e}}{\overline{k}}\right)\frac{\overline{e}}{\overline{k}}\right]$$

Define

$$z_{it+1} = \frac{e_{it+1}}{k_{it+1}}$$

and using Taylor

$$\Phi(z_{it+1}) \simeq \Phi(\overline{z}) + \Phi'(\overline{z})(z_{it+1} - \overline{z})$$

$$\begin{aligned}\Phi'(z_{it+1})z_{it+1} \quad &\simeq \quad \Phi'(\overline{z})\overline{z} + \left(\Phi''(\overline{z})\overline{z} + \Phi'(\overline{z})\right)(z_{it+1} - \overline{z}) = \\ &\quad\ \Phi''(\overline{z})\overline{z}z_{it+1} + \Phi'(\overline{z})z_{it+1} - \Phi''(\overline{z})\overline{z}^2\end{aligned}$$

Then, we can write E.6 as

$$q_{it} = \beta E_t \left\{ \frac{\mu_{t+1}}{\mu_t} \left[(1-\tau^k) r_{it+1} + q_{it+1} \left((1-\delta) + \Phi - \Phi'\overline{z} - \Phi''(\overline{z})\overline{z}z_{it+1} + \Phi''(\overline{z})\overline{z}^2 \right) \right] \right\}$$

or

$$\begin{aligned} q_{it} \quad = \quad & \beta E_t \left(\frac{\mu_{t+1}}{\mu_t} (1-\tau_t^k) r_{it+1} \right) - \beta \Phi''(\overline{z})\overline{z} E_t \left(\frac{\mu_{t+1}}{\mu_t} q_{it+1} z_{it+1} \right) + \\ & \beta \left((1-\delta) + \Phi - \Phi'\overline{z} - \Phi''\overline{z}^2 \right) E_t \left(\frac{\mu_{t+1}}{\mu_t} q_{it+1} \right) \end{aligned}$$

Now, taking a linear approximation around the steady state and since $\beta E_t \left(\frac{\mu_{t+1}}{\mu_t} \frac{1+i_t}{\pi_{t+1}} \right) = 1$,

$$\begin{aligned} (1+\widehat{q}_{it}) \quad = \quad & \beta(1-\overline{\tau}^k)\overline{r} E_t \left[1 + E_t \widehat{\pi}_{t+1} - \frac{\overline{i}}{1+\overline{i}} \widehat{i}_t - \frac{\overline{\tau^k}}{1-\overline{\tau^k}} \widehat{\tau}_{t+1}^k + \widehat{r}_{it+1} \right] \\ & -\beta \Phi'' \overline{z}^2 \left[1 + E_t \widehat{\pi}_{t+1} - \frac{\overline{i}}{1+\overline{i}} \widehat{i}_t + \widehat{q}_{it+1} + \widehat{z}_{it+1} \right] + \\ & +\beta \left((1-\delta) + \Phi - \Phi'\overline{z} - \Phi''\overline{z}^2 \right) \left[1 + E_t \widehat{\pi}_{t+1} - \frac{\overline{i}}{1+\overline{i}} \widehat{i}_t + \widehat{q}_{it+1} \right] \end{aligned}$$

That using

$$\widehat{z}_{it+1} = \widehat{e}_{it+1} - \widehat{k}_{it+1}$$

and the steady-state restriction, can be written as follows:

$$\begin{aligned} \widehat{q}_{it} \quad = \quad & (E_t \widehat{\pi}_{t+1} - \frac{\overline{i}}{1+\overline{i}} \widehat{i}_t) + \beta(1-\overline{\tau}^k)\overline{r} E_t \left(\widehat{r}_{it+1} - \frac{\overline{\tau^k}}{1-\overline{\tau^k}} \widehat{\tau}_{t+1}^k \right) + \\ & \left(1 - (1-\overline{\tau}^k)\overline{r} \right) E_t \widehat{q}_{it+1} - \beta \Phi'' \left(\frac{\overline{e}}{\overline{k}} \right)^2 E_t (\widehat{e}_{it+1} - \widehat{k}_{it+1}) \end{aligned}$$

Integrating over all $i \in (0,1)$ we have that

$$\widehat{q}_t = (E_t\widehat{\pi}_{t+1} - \frac{\bar{i}}{1+\bar{i}}\widehat{i}_t) + \beta(1-\overline{\tau}^k)\overline{r}E_t\left(\widehat{r}_{t+1} - \frac{\overline{\tau^k}}{1-\overline{\tau^k}}\widehat{\tau}^k_{t+1}\right) + \left(1-(1-\overline{\tau}^k)\overline{r}\right)E_t\widehat{q}_{t+1} - \beta\Phi''\left(\frac{\overline{e}}{\overline{\overline{k}}}\right)^2 E_t(\widehat{e}_{t+1} - \widehat{k}_{t+1}) \quad \text{(L.7)}$$

3.8. Linearization of E.8

Integrating over all $i \in (0,1)$ we have that

$$\widehat{w}_t = \widehat{mc}_t + \widehat{a}_t + \alpha\widehat{k}_t - \alpha\widehat{l}_t \quad \text{(L.8)}$$

3.9. Linearization of E.9

Integrating over all $i \in (0,1)$ we have that

$$\widehat{r}_t = \widehat{mc}_t + \widehat{a}_t + (\alpha-1)\widehat{k}_t + (1-\alpha)\widehat{l}_t \quad \text{(E.9)}$$

3.10. New Phillips curve when capital and labour are firm-specific

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{(1-\beta\phi)(1-\phi)}{\phi}\frac{1-\phi\beta\kappa_1}{(1+\varepsilon\frac{\alpha}{1-\alpha})(1-\phi\beta\kappa_1) + \frac{\alpha}{1-\alpha}\phi\beta\kappa_2}\widehat{mc}_t \quad (86)$$

3.11. Fiscal Policy: linearization of fiscal rule for transfers

In the steady state we assume that the variation of public debt is zero ($\overline{b} = 0$) :

$$\overline{\tau}^w\overline{w}\overline{l} + \overline{\tau}^k\overline{r}\overline{k} + \overline{\tau}^c\overline{c} - (\overline{g}^c + \overline{g}^p + \overline{g}^s) = 0 \quad (87)$$

then linearizing the budget constraint

$$P_t\tau^w_t w_t l_t + P_t\tau^k_t\int_0^1 r_{it}k_{it}di + P_t\tau^c_t c_t - P_t(g^c_t + g^s_t) = -\frac{B_t}{(1+i_t)} + B_{t-1} \quad \text{(E.13)}$$

we have that

$$\overline{\tau}^w\overline{w}\overline{l}(\widehat{w}_t + \widehat{\tau}^w_t + \widehat{l}_t) + \overline{\tau}^k\overline{r}\overline{k}(\widehat{r}_t + \widehat{\tau}^k_t + \widehat{k}_t) + \overline{\tau}^c\overline{c}\left(\widehat{\tau}^c_t + \widehat{c}_t\right) - \overline{g}^c\widehat{g}^c_t - \overline{g}^s\widehat{g}^s_t = -\frac{1}{(1+\overline{i})}\left(b_t - \overline{b}\right) + \frac{1}{\overline{\pi}}\left(b_{t-1} - \overline{b}\right) \quad (88)$$

In our simulations $\widehat{\tau}_t^w = \widehat{\tau}_t^k = \widehat{\tau}_t^c = 0$. We assume that the fiscal rule in transfers is given by

$$\widehat{g}_t^s = \alpha_b^s \left(b_t - \overline{b}\right) + \alpha_y^s \widehat{y}_t + \varepsilon_t^s \tag{89}$$

In order to have the minimun distortion we can use a low value of α_b^s (equal to 0.1), which guarantee the equilibrium in all simulation and $\alpha_y^s = 0$. Finally, we may allow government consumption to rect to output deviations, that is

$$\widehat{g}_t^c = \alpha_y^c \widehat{y}_t + \varepsilon_t^c \tag{90}$$

Unless explicitelly mentioned, we assume that $\alpha_y^c = \varepsilon_t^c = 0$.

3.12. *Linearization of E.14*

$$\overline{y}\widehat{y}_t = \overline{c}\widehat{c}_t + \overline{e}\widehat{e}_t + \overline{g}^c \widehat{g}_t^c$$

3.13. *Linearization of E.15*

$$\widehat{y}_{it} = \frac{\overline{y} + \kappa}{\overline{y}} \left(\widehat{a}_t + \alpha \widehat{k}_{it} + (1 - \alpha)\widehat{l}_{it}\right) \tag{91}$$

To obtain an alternative expresion notice that:

$$\overline{y}^g = A\overline{k}^\alpha \overline{l}^{1-\alpha} \tag{92}$$

Then

$$\overline{wl} + \overline{r}\overline{k} + \kappa = \overline{y}^g \tag{93}$$

Using the expressions for w and r

$$\frac{\varepsilon - 1}{\varepsilon}(1 - \alpha)\overline{y}^g + \frac{\varepsilon - 1}{\varepsilon}\alpha \overline{y}^g + \kappa = y^g \tag{94}$$

which can be simplified to

$$\kappa = \frac{\overline{y}^g}{\varepsilon} \tag{95}$$

Since $y^g = y + \kappa$

$$\kappa = \frac{\overline{y} + \kappa}{\varepsilon} \tag{96}$$

or

$$\kappa = \frac{\overline{y}}{\varepsilon - 1} \tag{97}$$

This expression allows us to obatin an expression for y^g in terms of y

$$\overline{y}^g = \overline{y} + \kappa = \overline{y}^g = \overline{y} + \frac{\overline{y}}{\varepsilon - 1} = \frac{\varepsilon}{\varepsilon - 1}\overline{y} \tag{98}$$

Therefore

$$\widehat{y}_{it} = \frac{\overline{y} + \kappa}{\overline{y}} \left(\widehat{a}_t + \alpha \widehat{k}_{it} + (1 - \alpha)\widehat{l}_{it} \right) = \frac{\varepsilon}{\varepsilon - 1} \left(\widehat{a}_t + \alpha \widehat{k}_{it} + (1 - \alpha)\widehat{l}_{it} \right) \tag{99}$$

Integrating over all $i \in (0, 1)$ we have that

$$\widehat{y}_t = \frac{\varepsilon}{\varepsilon - 1} \left(\widehat{a}_t + \alpha \widehat{k}_t + (1 - \alpha)\widehat{l}_t \right) \tag{100}$$

3.14. Monetary policy: linearization of the interest rate rule

$$i_t = \rho_r i_{t-1} + (1 - \rho_r)\bar{i} + (1 - \rho_r)\rho_\pi(\pi_t - \overline{\pi}) + (1 - \rho_r)\rho_y \widehat{y}_t$$

$$\bar{i}(1 + \widehat{i}_t) = \rho_r \bar{i}(1 + \widehat{i}_{t-1}) + (1 - \rho_r)\bar{i} + (1 - \rho_r)\rho_\pi(\overline{\pi}(1 + \widehat{\pi}_t) - \overline{\pi}) + (1 - \rho_r)\rho_y \widehat{y}_t$$

$$\bar{i}\widehat{i}_t = \rho_r \bar{i}\widehat{i}_{t-1} + (1 - \rho_r)\rho_\pi(\overline{\pi}\widehat{\pi}_t) + (1 - \rho_r)\rho_y \widehat{y}_t$$

3.15. Linearization of E.17

$$\frac{1}{\Upsilon_3 l_{rt}^{\gamma_g + 1}} = \left(1 + \frac{\lambda g_t^s}{(1 - \tau_t^w) w_t l_{rt}} \right) \tag{E.17}$$

$$-(\gamma_g + 1)\widehat{l}_{rt} = \frac{1}{\Upsilon_3 \bar{l}_r^{\gamma_g + 1}} \frac{\lambda \overline{g^s}}{(1 - \tau^w)\overline{w}\bar{l}_r} (\widehat{g^s}_t - \widehat{w}_t - \widehat{l}_{rt}) \tag{101}$$

3.16. Linearization of E.18

$$c_{rt} = \frac{(1 - \tau_t^w) w_t}{\Upsilon_3 l_{rt}^{\gamma_g}} \tag{E.18}$$

$$\widehat{c}_{rt} = \widehat{w}_t - \gamma_g \widehat{l}_{rt} \tag{102}$$

3.17. Linearization of E.4c

$$\widehat{c}_t = \frac{\lambda \bar{c}_r}{\bar{c}} \widehat{c}_{rt} + \frac{(1-\lambda)\bar{c}_o}{\bar{c}} \widehat{c}_{ot} \tag{103}$$

3.18. Linearization of E.5c

$$\widehat{l}_t = \frac{\lambda \bar{l}_r}{\bar{l}} \widehat{l}_{rt} + \frac{(1-\lambda)\bar{l}_o}{\bar{l}} \widehat{l}_{ot} \tag{104}$$

4. The simulated model (AD): the steady state.

$$\frac{\bar{e}}{\bar{k}} = \delta \tag{SS.1}$$

$$\frac{(1-\tau_t^w)}{(1+\tau_t^c)}\overline{w} = \frac{\gamma\bar{c}}{(1-\bar{l})} \tag{SS.2}$$

$$\beta^{-1} = \frac{1+\bar{\imath}}{\bar{\pi}} \tag{SS.3}$$

$$\bar{q} = \left[\Phi'\left(\frac{\bar{e}}{\bar{k}}\right)\right]^{-1} \tag{SS.4}$$

$$1 = \beta(1-\bar{\tau}^k)\bar{r} + \beta\left[1-\delta\right] \tag{SS.5}$$

(Notice that we have made use here of SS.4 and SS.12)

$$\bar{w} = \overline{mc}(1-\alpha)A\bar{k}^{\alpha}\bar{l}^{-\alpha} \tag{SS.6}$$

$$\bar{r} = \overline{mc}\alpha A\bar{k}^{\alpha-1}\bar{l}^{1-\alpha} \tag{SS.7}$$

$$\overline{mc} = \frac{\varepsilon-1}{\varepsilon} \tag{SS.8}$$

$$\bar{\tau}^w\overline{w}\bar{l} + \bar{\tau}^k\bar{r}\bar{k} + \bar{\tau}^c\bar{c} = (\overline{g}^c + \overline{g^s}) \tag{SS.9}$$

$$\overline{y} = \bar{c} + \bar{e} + \overline{g}^c \tag{SS.10}$$

$$\overline{y} = \frac{\varepsilon-1}{\varepsilon}A\bar{k}^{\alpha}\bar{l}^{1-\alpha} \tag{SS.11}$$

$$\bar{c} = \frac{\lambda}{(1+\tau^c)}\left[\frac{(1-\tau^w)\overline{w}}{1+\gamma} + \frac{\lambda\overline{g}^s}{1+\gamma}\right] + (1-\lambda)\bar{c}_o \tag{SS.12}$$

$$\bar{l} = \frac{\lambda}{1+\gamma}\left[1 - \frac{\gamma\lambda\overline{g^s}}{(1-\tau^w)\overline{w}}\right] + (1-\lambda)\bar{l}_o \tag{SS.13}$$

- Again Assuming that $\theta = 0$.
- *Exogenous variables (5)*: $\overline{\pi}$ and most fiscal variables: $\bar{\tau}^k$, $\bar{\tau}^c$, $\frac{\overline{g^c}}{\overline{y}}$, $\frac{\overline{g^s}}{\overline{y}}$.
- *Endogenous variables* (13): $\bar{c}$, $\bar{l}$, $\overline{w}$, $\bar{i}$, $\bar{q}$, $\bar{e}$, $\bar{k}$, $\bar{r}$, $\overline{mc}$, $\bar{y}$, τ^w, $\bar{c}_o$, $\bar{l}_o$.
- In the resolution $\bar{l}$ ($\bar{l} = 0.33$) is given; then γ is made endogenous. We also impose that $\bar{\tau}^w = \bar{\tau}^k$ and $\bar{\tau}^c = 0$.
- Notice that for the alternative specification of the utility function (U_3), equation SS.2 should be replaced by

$$\frac{(1-\tau^w)}{(1+\tau^c)}\overline{w} = \Upsilon_3 \bar{c}\bar{l}^{\gamma_g} \tag{SS.2c}$$

A new equation is added,

$$1 = \Upsilon_3 \overline{l_r}^{\gamma_g+1}\left(1 + \frac{\lambda\overline{g}^s}{(1-\tau^w)\overline{w}\overline{l_r}}\right) \tag{SS.14}$$

SS.13 is replaced by

$$\bar{c} = \lambda\left[\frac{(1-\tau^w)\overline{w}}{(1+\tau^c)} + \frac{l}{\Upsilon_3\overline{l_r}^{\gamma_g}}\right] + (1-\lambda)\bar{c}_o \tag{SS.13c}$$

and SS.14 by

$$\bar{l} = \lambda\overline{l_r} + (1-\lambda)\bar{l}_o \tag{SS.14}$$

5. The simulated model (AD): deviations from the steady state.

$$\widehat{k}_{t+1} = \left[1 - \frac{\overline{e}}{\overline{\overline{k}}}\right]\widehat{k}_t + \frac{\overline{e}}{\overline{\overline{k}}}\widehat{e}_t \qquad \text{(AD.1)}$$

$$\widehat{c}_{ot} = \widehat{c}_{ot+1} - \frac{1}{\sigma}\left(\frac{\overline{i}}{1+\overline{i}}\widehat{i}_t - \widehat{\pi}_{t+1}\right) - \gamma\frac{(\sigma-1)}{\sigma}\frac{\overline{l}_o}{(1-\overline{l}_o)}\Delta\widehat{l}_{ot+1} \qquad \text{(AD.2)}$$

$$\widehat{w}_t = \widehat{c}_t + \frac{\overline{l}}{(1-\overline{l})}\widehat{l}_t \qquad \text{(AD.3)}$$

$$\widehat{c}_t = \frac{\lambda\overline{c}_r}{\overline{c}}\left(\frac{(1-\tau^w)\overline{w}}{(1+\gamma)\overline{c}_r}\widehat{w}_t + \frac{\lambda\overline{g^s}}{(1+\gamma)\overline{c}_r}\widehat{g}^s_t\right) + \frac{(1-\lambda)\overline{c}_o}{\overline{c}}\widehat{c}_{ot} \qquad \text{(AD.4)}$$

$$\widehat{l}_t = \frac{\lambda\overline{l}_r}{\overline{l}}\left(-\frac{\gamma\lambda\overline{g^s}}{\overline{l}_r(1+\gamma)(1-\tau^w)\overline{w}}(\widehat{g}^s_t - \widehat{w}_t)\right) + \frac{(1-\lambda)\overline{l}_o}{\overline{l}}\widehat{l}_{ot} \qquad \text{(AD.5)}$$

$$\widehat{q}_t = -\Phi''\frac{\overline{e}}{\overline{\overline{k}}}(\widehat{e}_t - \widehat{k}_t) \qquad \text{(AD.6)}$$

$$\begin{aligned}\widehat{q}_t \quad = \quad & \frac{\widehat{\pi}_{t+1}}{\overline{\pi}} - \frac{\overline{i}}{1+\overline{i}}\widehat{i}_t + (1-\overline{\tau}^k)\overline{r}\beta E_t\left(\widehat{r}_{t+1} - \frac{\overline{\tau^k}}{1-\overline{\tau^k}}\widehat{\tau}^k_{t+1}\right) + \\ & \left(1-(1-\overline{\tau}^k)\overline{r}\right)E_t\widehat{q}_{t+1} - \beta\delta\Phi''\left(\frac{\overline{e}}{\overline{\overline{k}}}\right)E_t(\widehat{e}_{t+1} - \widehat{k}_{t+1})\end{aligned} \qquad \text{(AD.7)}$$

(Notice that we have substituted out AD.1, using: $\frac{\overline{e}}{\overline{\overline{k}}} = \delta$)

$$\widehat{w}_t = \widehat{mc}_t + \widehat{a}_t + \alpha\widehat{k}_t - \alpha\widehat{l}_t \qquad \text{(AD.8)}$$

$$\widehat{r}_t = \widehat{mc}_t + \widehat{a}_t + (\alpha-1)\widehat{k}_t + (1-\alpha)\widehat{l}_t \qquad \text{(AD.9)}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{(1-\beta\phi)(1-\phi)}{\phi}\frac{1-\phi\beta\kappa_1}{(1+\varepsilon\frac{\alpha}{1-\alpha})(1-\phi\beta\kappa_1) + \frac{\alpha}{1-\alpha}\phi\beta\kappa_2}\widehat{mc}_t \qquad \text{(AD.10)}$$

$$\overline{\tau}^w\overline{w}\overline{l}(\widehat{w}_t+\widehat{\tau}_t^w+\widehat{l}_t)+\overline{\tau}^k\overline{r}\overline{k}(\widehat{r}_t+\widehat{\tau}_t^k+\widehat{k}_t)+\overline{\tau}^c\overline{c}\left(\widehat{\tau}_t^c+\widehat{c}_t\right)-\overline{g}^c\widehat{g}_t^c-\overline{g}^s\widehat{g}_t^s \tag{AD.11}$$
$$= -\frac{1}{(1+\bar{i})}\left(b_t-\bar{b}\right)+\frac{1}{\bar{\pi}}\left(b_{t-1}-\bar{b}\right)$$

$$\widehat{y}_t=\frac{\overline{c}}{\overline{y}}\widehat{c}_t+\frac{\overline{e}}{\overline{y}}\widehat{e}_t+\frac{\overline{g}^c}{\overline{y}}\widehat{g}_t^c \tag{AD.12}$$

$$\widehat{y}_t=\frac{\varepsilon-1}{\varepsilon}\left(\widehat{a}_t+\alpha\widehat{k}_t+(1-\alpha)\widehat{l}_t\right) \tag{AD.13}$$

Fiscal and monetary reaction functions:

$$\widehat{i}_t=\rho_r\widehat{i}_{t-1}+(1-\rho_r)\rho_\pi\frac{\pi_t}{\overline{\pi}}+(1-\rho_r)\rho_y\widehat{y}_t \tag{AD.14}$$

$$\widehat{g}_t^c=\alpha_y^c\widehat{y}_t+\varepsilon_t^c \tag{AD.15}$$

$$\widehat{g}_t^s=\alpha_y^s\left(b_t-\overline{b}\right)+\varepsilon_t^s \tag{AD.16}$$

$$\widehat{c}_{ot}=E_{t-1}\widehat{c}_{ot}+\varepsilon_t^e \tag{AD.17}$$

$$\widehat{e}_t=E_{t-1}\widehat{e}_t+\varepsilon_t^e \tag{AD.18}$$

$$\widehat{l}_{ot}=E_{t-1}\widehat{l}_{ot}+\varepsilon_t^\lambda \tag{AD.19}$$

$$\widehat{\pi}_t=E_{t-1}\widehat{\pi}_t+\varepsilon_t^\pi \tag{AD.20}$$

$$\widehat{q}_t=E_{t-1}\widehat{q}_t+\varepsilon_t^q \tag{AD.21}$$

$$\widehat{r}_t=E_{t-1}\widehat{r}_t+\varepsilon_t^r \tag{AD.22}$$

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \varepsilon_t^a \tag{AD.23}$$

- Variable order in the system, when $\widehat{\tau}^k = \widehat{\tau}^w = \widehat{\tau}^c = 0$:

$\widehat{k}_{t+1}$	$\widehat{e}_t$	$E_t\widehat{c}_{ot+1}$	$\widehat{c}_{ot}$	$\widehat{l}_t$	$\widehat{i}_t$	$\widehat{w}_t$	$\widehat{\pi}_t$	$\widehat{q}_t$	$\widehat{r}_t$	$\widehat{l}_{ot}$	$\widehat{mc}_t$	$\widehat{c}_t$	$\widehat{y}_t$	$\widehat{g}_t^c$	b_t	$\widehat{g}_t^s$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$E_t\widehat{e}_{t+1}$	$E_t\widehat{l}_{ot+1}$	$E_t\widehat{\pi}_{t+1}$	$E_t\widehat{q}_{t+1}$	$E_t\widehat{r}_{t+1}$	$\widehat{a}_t$											
18	19	20	21	22	23											

- *Number of equations*: 23
- *Endogenous variables (23)*: $\widehat{k}_{t+1}$, $\widehat{e}_t$, $E_t\widehat{c}_{ot+1}$, $\widehat{c}_t$, $\widehat{l}_t$, $\widehat{i}_t$, $\widehat{w}_t$, $\widehat{\pi}_t$, $\widehat{q}_t$, $\widehat{r}_t$, $\widehat{c}_{ot}$, $\widehat{mc}_t$, $\widehat{l}_{ot}$, $\widehat{y}_t$, $\widehat{g}_t^c$, b_t, $\widehat{g}_t^s$, $\widehat{e}_{t+1}$, $E_t\widehat{l}_{ot+1}$, $E_t\widehat{\pi}_{t+1}$, $E_t\widehat{q}_{t+1}$, $E_t\widehat{r}_{t+1}$ and $\widehat{z}_t$.
  (Notice that $\widehat{k}_{t+1}$is a variable endogenously set at t. Thus they are not expectational and do not require auxiliary equations. They enter in g0, whereas $\widehat{k}_t$ does so in g1).
- In our simulations $\varepsilon_t^c = \varepsilon_t^s = 0$ and $\alpha_y^c = 0$, unless said otherwise.
- From Christiano (2004) κ_1 and κ_2 are the solutions, together with ψ, of the following system of equations:

$$\psi = \frac{\frac{\alpha}{1-\alpha}(1-\beta\phi)}{(1+\varepsilon\frac{\alpha}{1-\alpha})(1-\phi\beta\kappa_1) + \frac{\alpha}{1-\alpha}\phi\beta\kappa_2}$$

$$\kappa_2 = \frac{\Xi\phi\kappa_1}{\beta\kappa_1\phi - 1}$$

  and

$$(1+\beta+[1-\beta(1-\delta)])\frac{1}{1-\alpha}\frac{1}{\Phi''}\kappa_2 + \beta\kappa_2(\kappa_1+\alpha) - (1-\phi)\psi\kappa_2\left(\beta\kappa_2 - \Xi\right) = 0$$

  where

$$\Xi = -\left(1-\beta(1-\delta)\right)\frac{1}{1-\alpha}\frac{1}{\Phi''}\varepsilon$$

- Notice that under the alternative specifications of the utility functions (U_2 and U_3), equation AD.2 changes to

$$\widehat{c}_{ot} = \widehat{c}_{ot+1} - \frac{\bar{i}}{1+\bar{i}}\widehat{i}_t + \widehat{\pi}_{t+1} \tag{AD.2b}$$

- For U_3, equation AD.3 is given by

$$\widehat{w}_t = \widehat{c}_t + \gamma_g \widehat{l}_t \tag{AD.3c}$$

AD.4 by

$$\widehat{c}_t = \frac{\lambda \bar{c}_r}{\bar{c}} \widehat{c}_{rt} + \frac{(1-\lambda)\bar{c}_o}{\bar{c}} \widehat{c}_{ot} \tag{105}$$

and AD.5 by

$$\widehat{l}_t = \frac{\lambda \bar{l}_r}{\bar{l}} \widehat{l}_{rt} + \frac{(1-\lambda)\bar{l}_o}{\bar{l}} \widehat{l}_{ot} \tag{106}$$

- Additionally two new variables ($\widehat{c}_{rt}$ and $\widehat{l}_{rt}$) and two new equations are added

$$-(\gamma_g + 1)\widehat{l}_{rt} = \frac{1}{\Upsilon_3 \bar{l}_r^{\gamma_g+1}} \frac{\lambda \overline{g^s}}{(1-\tau^w)\overline{w}\bar{l}_r} (\widehat{g^s}_t - \widehat{w}_t - \widehat{l}_{rt}) \tag{AD.25}$$

and

$$\widehat{c}_{rt} = \widehat{w}_t - \gamma_g \widehat{l}_{rt} \tag{AD.26}$$