Vida residual conjunta T(x,y)

T(x,y)= min (T(x),T(y)) o bien para m : $T(x_1,x_2,\ldots,x_m)$= min $(T(x_1),T(x_2),\ldots T(x_m))$

Función de distribución

$$F_{T_{xy}}(t) = P(T_{xy} \leq t) = {}_tq_{xy} = 1 - {}_tp_{xy}$$

$$F_{T_{xy}}(t) = P(T_{xy} \leq t) = P\big((\min(T(x),T(y))) \leq t\big) = 1 - P\big((\min(T(x),T(y))) > t\big) = 1 - S_{T_{x,y}}(t)$$

$F_{T_{xy}}(t) = {}_tq_{xy}$ **y sabemos** $\;{}_tq_{xy} = {}_tq_x + {}_tq_y - {}_tq_{\overline{xy}}$

Función de densidad

$$f_{T_{xy}}(t) = \frac{d}{dt}F_{T_{xy}}(t) = \frac{d}{dt}\left(1 - {}_tp_{xy}\right) = \frac{d}{dt}\left(1 - {}_tp_x \cdot {}_tp_y\right) = -\frac{d}{dt}\left({}_tp_x \cdot {}_tp_y\right) =$$

$$= f_{T_{xy}}(t) = -\left(\frac{d\,{}_tp_x}{dt} \cdot {}_tp_y + \frac{d\,{}_tp_y}{dt} \cdot {}_tp_x\right)$$

en base al tanto instantáneo de mortalidad

$$f_{T_{xy}}(t) = \Big(\big({}_tp_x \cdot {}_tp_y \cdot \mu(y+t)\big) + \big({}_tp_y \cdot {}_tp_x \cdot \mu(x+t)\big)\Big) =$$

$$f_{T_{xy}}(t) = {}_tp_{xy} \cdot \big(\mu(x+t) + \mu(y+t)\big)$$

el tanto instantáneo conjunto es igual a la suma de los individuales

$${}_{t}p_{xy} = \frac{l(x+t, y+t)}{l(x,y)}$$

$$\mu(x,y) = -\frac{\frac{d}{dt}\,{}_{t}p_{xy}}{{}_{t}p_{xy}} = -\frac{\frac{\frac{d}{dt}[l(x+t,y+t)]}{l(x,y)}}{\frac{l(x+t,y+t)}{l(x,y)}} = -\frac{\frac{d}{dt}[l(x+t,y+t)]}{l(x+t,y+t)}$$

habitual

$${}_{n}q_{xy} = F_{T_{xy}}(n) = \int_0^n f_{T_{xy}}(t)dt = \int_0^n {}_{t}p_{xy}\,\mu(x+t,y+t)dt$$

Vida residual conjunta hasta la extinción

$$T_{\overline{x,y}} = \max\left(T(x), T(y)\right)$$

$$T_{\overline{x,y}} + T_{x,y} = T(x) + T(y)$$

$$T_{\overline{x,y}}.T_{x,y} = T(x).T(y)$$

Función de distribución.

$$F_{T_{\overline{xy}}}(t) = P(T_{\overline{xy}} \le t) = {}_{t}q_{\overline{xy}} = {}_{t}q_{x} \cdot {}_{t}q_{y}$$

Función de densidad

$$f_{T_{\overline{xy}}}(t) = {}_{t}q_{x} \cdot {}_{t}p_{y}.\mu(y+t) + {}_{t}q_{y} \cdot {}_{t}p_{x}.\mu(x+t)$$

Esperanza de vida conjunta hasta la disolución

$$\overline{e}_{xy} = E[T_{xy}] = \int_0^\infty t.f_{T_{xy}}(t)dt = \int_0^\infty t.\,{}_{t}p_{xy}.\mu(x+t,y+t)\,dt$$

$$\overline{e}_{xy} = \frac{\int_0^{\infty} l(x+t, y+t)\, dt}{l(x,y)}$$

Esperanza de vida conjunta abreviada

$$e_{xy} = \frac{\sum_{i=1}^{\infty} l(x+i+1, y+i+1)}{l(x,y)} = \sum_{t=1}^{\infty} {}_t p_{xy}$$

Esperanza de vida conjunta completa

$$\overset{0}{e}_{xy} = \frac{1}{2} + \sum_{t=1}^{\infty} {}_t p_{xy} = \frac{1}{2} e_{xy}$$

Esperanza de vida conjunta hasta la extinción

$$\overline{e}_{\overline{xy}} = E[T_{\overline{xy}}] = \int_0^{\infty} t.f_{T_{\overline{xy}}}(t)dt = \int_0^{\infty} t.{}_t q_x \cdot {}_t p_y.\mu(y+t)dt + \int_0^{\infty} t.{}_t q_y \cdot {}_t p_x.\mu(x+t)dt$$

solo con p

$$\overline{e}_{\overline{xy}} = \int_0^{\infty} t.{}_t p_y.\mu(y+t)dt - \int_0^{\infty} t.{}_t p_y \cdot {}_t p_x.\mu(y+t)dt +$$

$$+\int_0^{\infty} t.{}_t p_x.\mu(x+t)dt - \int_0^{\infty} t.{}_t p_y \cdot {}_t p_x.\mu(x+t)dt =$$

relación esperanzas de vida extinción/disolución

$$\overline{e}_{\overline{xy}} = \overline{e}_x + \overline{e}_y - \overline{e}_{xy}$$

$$\overline{e}_x = \int_o^{\infty} {}_t p_x dt \qquad \overline{e}_y = \int_o^{\infty} {}_t p_y dt \qquad \overline{e}_{xy} = \int_o^{\infty} {}_t p_{xy} dt$$

$$\overline{e}_{\overline{xy}} = \overline{e}_x + \overline{e}_y - \overline{e}_{xy} = \int_0^\infty ({}_tp_x + {}_tp_y - {}_tp_{xy})dt = \int_0^\infty {}_tp_{\overline{xy}}\, dt$$

abreviada y completa

$$e_{\overline{xy}} = e_x + e_y - e_{xy} = \sum_{i=1}^\infty {}_tp_x + \sum_{i=1}^\infty {}_tp_y - \sum_{i=1}^\infty {}_tp_{xy} = \sum_{i=1}^\infty {}_tp_{\overline{xy}}$$

$$\overset{0}{e}_{\overline{xy}} = \overset{0}{e}_x + \overset{0}{e}_y - \overset{0}{e}_{xy} = \frac{1}{2} + e_x + \frac{1}{2} + e_y - \frac{1}{2} - e_{xy} = \frac{1}{2} + \sum_{i=1}^\infty {}_tp_{\overline{xy}}$$