

Probabilidad **diferida** de fallecimiento .

Persona de 30 años sobreviva hasta los 40 y fallezca entre los 40 y los 45 con función de distribución de X =tiempo hasta fallecimiento (edad de fallecimiento)

conocemos

$$F(x) = \frac{x^2}{10000} \quad 0 \leq x \leq 100$$

forma 1

$$F(x) = \frac{x^2}{10000} \quad 0 \leq x \leq 100 \quad \Rightarrow F'(x) = f(x)$$

$$\begin{aligned} m/n Q_x &= 10/5 Q_{30} = \frac{P(x+m < X < x+m+n)}{P(X > m)} = \frac{P(40 < X < 45)}{P(X > 30)} = \\ &= \frac{F(45) - F(40)}{1 - F(30)} = \frac{0,2025 - 0,16}{1 - 0,09} = \frac{0,0425}{0,91} = 0,0467 \end{aligned}$$

$$F(x) = \frac{x^2}{10000} \quad 0 \leq x \leq 100 \quad \Rightarrow F'(x) = f(x) = \frac{x}{5000}$$

$$F(45) = \frac{45^2}{10000} = \frac{2025}{10000} = 0,2025 \rightarrow P(X < 45)$$

$$F(40) = \frac{40^2}{10000} = \frac{1600}{10000} = 0,16 \rightarrow P(X < 40)$$

$$F(30) = \frac{30^2}{10000} = \frac{900}{10000} = 0,09 \rightarrow S(30) = 1 - F(30) = 1 - 0,09 = 0,91$$

$$F(30) = P(X < 30) \rightarrow S(30) = P(X > 30)$$

forma 2 con función de densidad de X

$$m/n q_x = 10/5 q_{30} = \frac{P(x+m < X < x+m+n)}{P(X > m)} = \frac{P(40 < X < 45)}{P(X > 30)} =$$

$$\frac{\int_{40}^{45} f(x) dx}{\int_{30}^{100} f(x) dx} = \frac{\left[\frac{x^2}{10000} \right]_{40}^{45}}{\left[\frac{x^2}{10000} \right]_{30}^{100}} = \frac{\frac{2025}{10000} - \frac{1600}{10000}}{\frac{10000}{10000} - \frac{900}{10000}} = \frac{425}{9100} = 0,0467$$

forma 3 mediante supervivencia

$$\begin{aligned} m/n q_x &= 10/5 q_{30} = \frac{P(x+m < X < x+m+n)}{P(X > m)} = \frac{P(40 < X < 45)}{P(X > 30)} = \\ &= \frac{S(x+m) - S(x+m+n)}{S(x)} = \frac{S(40) - S(45)}{S(30)} = \frac{0,84 - 0,7975}{0,91} = \frac{0,425}{0,91} = 0,0467 \end{aligned}$$

$$S(40) = 1 - F(40) = 1 - 0,16 = 0,84$$

$$S(45) = 1 - F(45) = 1 - 0,2025 = 0,7975$$

$$S(30) = 1 - F(30) = 1 - 0,09 = 0,91$$

forma 4

$$m/n q_x = 10/5 q_{30} = \frac{P(X > x+m)}{P(X > x)} \cdot \frac{P(x+m < X < x+m+n)}{P(X > x+m)}$$

$$\frac{P(X > x+m)}{P(X > x)} = 10 p_{30} = \frac{P(X > 40)}{P(X > 30)} = \frac{S(40)}{S(30)} = \frac{0,84}{0,91} = 0,9230769231$$

$$\frac{P(x+m < X < x+m+n)}{P(X > x+m)} = 5 q_{40} = \frac{F(45) - F(40)}{S(40)} = \frac{0,2025 - 0,16}{0,84} = \frac{0,0425}{0,84} = 0,0505952381$$

$$S(40) = 1 - F(40) = 1 - 0,16 = 0,84$$

$$S(45) = 1 - F(45) = 1 - 0,2025 = 0,7975$$

$$S(30) = 1 - F(30) = 1 - 0,09 = 0,91$$

$$m/n q_x = 10 p_{30} \cdot 5 q_{40} = 0,9230769231 \cdot 0,0505952381 = 0,0467$$

forma 5 mediante diferencia de probabilidades de fallecimiento

$$m/n q_x = {}_{m+n}q_x - {}_m q_x = 10/5 q_{30} = 15 q_{30} - 10 q_{30}$$

$$15 q_{30} = \frac{F(45) - F(30)}{S(30)} = \frac{0,2025 - 0,09}{0,91} = \frac{0,1125}{0,91} = 0,1236263737$$

$$10 q_{30} = \frac{F(40) - F(30)}{S(30)} = \frac{0,16 - 0,09}{0,91} = \frac{0,07}{0,91} = 0,07692307692$$

$$S(40) = 1 - F(40) = 1 - 0,16 = 0,84$$

$$S(45) = 1 - F(45) = 1 - 0,2025 = 0,7975$$

$$S(30) = 1 - F(30) = 1 - 0,09 = 0,91$$

$$m/n q_x = 10/5 q_{30} = 15 q_{30} - 10 q_{30} = 0,1236263737 - 0,07692307692 = 0,0467$$

forma 6 mediante diferencia de probabilidades de supervivencia

$$m/n q_x = {}_{m+n}q_x - {}_m q_x = 10/5 q_{30} = {}_{15}q_{30} - {}_{10}q_{30} =$$

$$= (1 - {}_{15}P_{30}) - (1 - {}_{10}P_{30}) = {}_{10}P_{30} - {}_{15}P_{30}$$

$${}_{15}P_{30} = \frac{S(45)}{S(30)} = \frac{0,7975}{0,91} = 0,8763736264$$

$${}_{10}P_{30} = \frac{S(40)}{S(30)} = \frac{0,84}{0,91} = 0,9230769231$$

$$S(40) = 1 - F(40) = 1 - 0,16 = 0,84$$

$$S(45) = 1 - F(45) = 1 - 0,2025 = 0,7975$$

$$S(30) = 1 - F(30) = 1 - 0,09 = 0,91$$

$$m/n q_x = 10/5 q_{30} = {}_{10}P_{30} - {}_{15}P_{30} = 0,9230769231 - 0,8763736264 = 0,0467$$