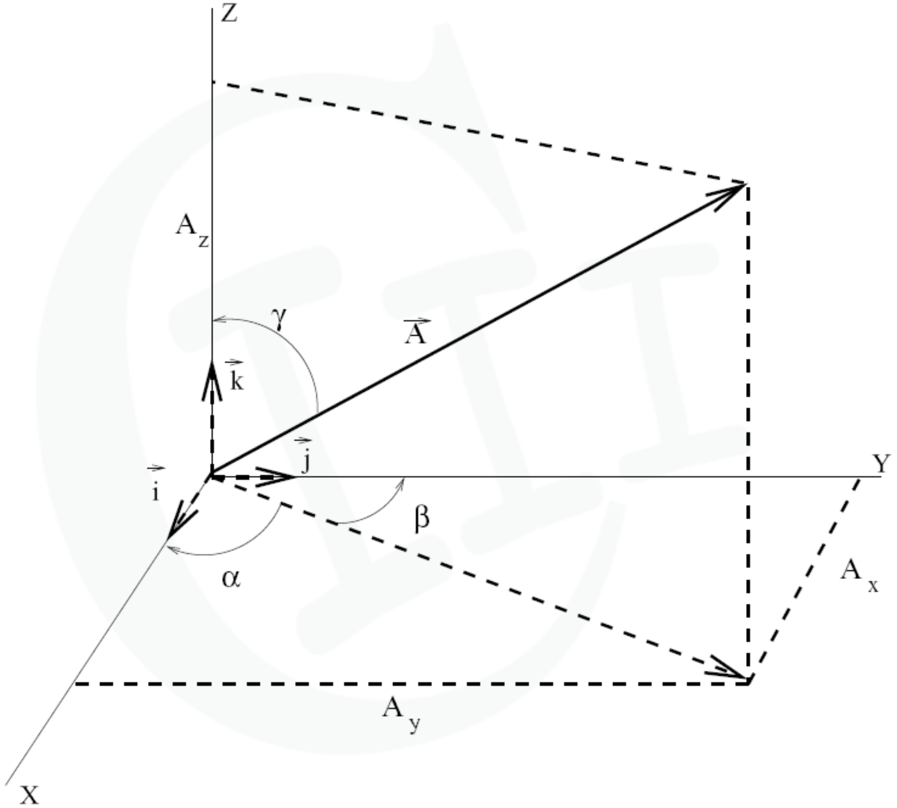
VECTORES

$$\vec{i} \equiv (1,0,0), \quad \vec{j} = (0,1,0) \quad y \quad \vec{k} = (0,0,1).$$



COMPONENTES DE UN VECTOR

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

PRODUCTOR DE UN ESCALAR POR UN VECTOR

$$\lambda\vec{v} = (\lambda v_x, \lambda v_y, \lambda v_z)$$

MODULO DE UN VECTOR (Pitágoras)

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

COSENOS DIRECTORES

$$cos\alpha = \frac{A_x}{|A|} \quad cos\beta = \frac{A_y}{|A|} \quad cos\gamma = \frac{A_z}{|A|}$$

SUMA DE DOS VECTORES

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$
$$\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$$

$$\vec{C} = C_x\vec{i} + C_y\vec{j} + C_z\vec{k} = (A_x + B_x)\vec{i} + (A_y + B_y)\vec{j} + (A_z + B_z)\vec{k}$$

PRODUCTO ESCALAR

$$\vec{A}\cdot\vec{B} = A_xB_x(\vec{i}\cdot\vec{i}) + A_yB_y(\vec{j}\cdot\vec{j}) + A_zB_z(\vec{k}\cdot\vec{k}) = A_xB_x + A_yB_y + A_zB_z$$

- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- $\vec{A}\cdot\vec{B} = 0$ si $\vec{A}\perp\vec{B}$
- $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

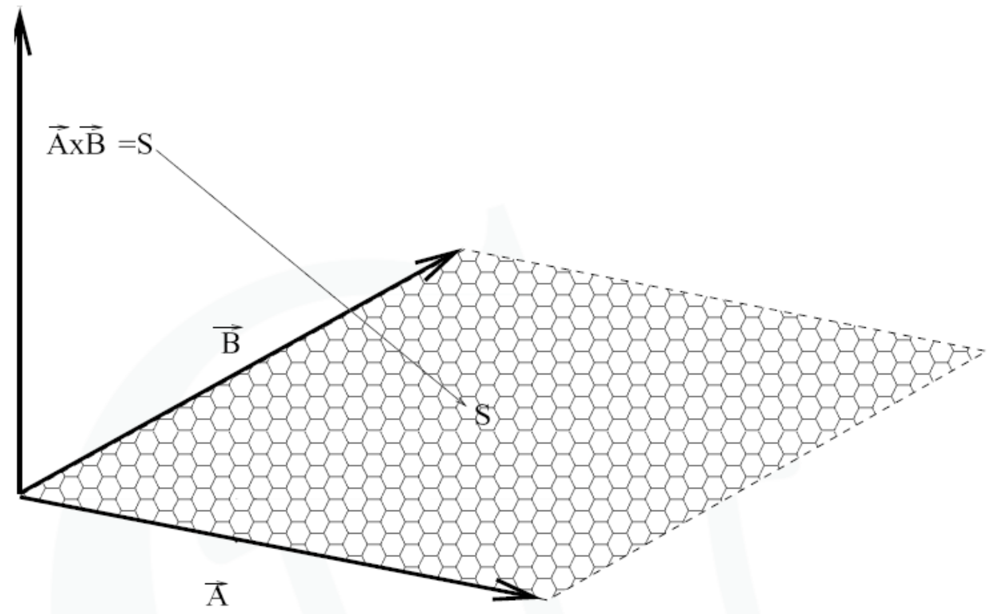
MODULO DE UN VECTOR (Producto escalar consigo mismo)

$$|\vec{v}| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\vec{A}\cdot\vec{i} = |\vec{A}||\vec{i}|cos\theta = Acos\theta$$

PRODUCTO VECTORIAL

- $(\vec{A}+\vec{B})\times\vec{C}=\vec{A}\times\vec{C}+\vec{B}\times\vec{C}$
- $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$
- $(\vec{A}\times\vec{B})\times\vec{C}\neq\vec{A}\times(\vec{B}\times\vec{C})$
- $\vec{A}\times\vec{B}=0$ si $\vec{A}\parallel\vec{B}$

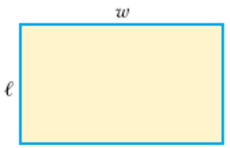
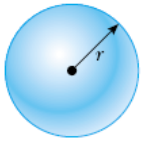
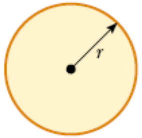
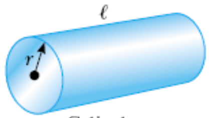
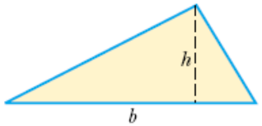
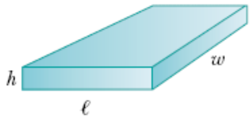


$$\vec{k}\times\vec{i}=\vec{j} \qquad \vec{i}\times\vec{j}=\vec{k} \qquad \vec{j}\times\vec{k}=\vec{i}$$

$$\vec{A}\times\vec{B}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A}\times\vec{B}=(A_x\vec{i}+A_y\vec{j}+A_z\vec{k})\times(B_x\vec{i}+B_y\vec{j}+B_z\vec{k})=$$
$$=(A_yB_z-A_zB_y)\vec{i}+(A_zB_x-A_xB_z)\vec{j}+(A_xB_y-A_yB_x)\vec{k}$$

TABLE B.2 Useful Information for Geometry

Shape	Area or Volume	Shape	Area or Volume
 Rectangle	Area = ℓw	 Sphere	Surface area = $4\pi r^2$ Volume = $\frac{4\pi r^3}{3}$
 Circle	Area = πr^2 (Circumference = $2\pi r$)	 Cylinder	Lateral surface area = $2\pi r\ell$ Volume = $\pi r^2\ell$
 Triangle	Area = $\frac{1}{2}bh$	 Rectangular box	Area = $2(\ell h + \ell w + hw)$ Volume = ℓwh

$$\frac{d}{dx}(ax^n) = nax^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\sin ax) = a\cos ax$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

TABLE B.3 Some Trigonometric Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \csc^2\theta = 1 + \cot^2\theta$$

$$\sec^2\theta = 1 + \tan^2\theta \qquad \sin^2\frac{\theta}{2} = \tfrac{1}{2}(1 - \cos\theta)$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos^2\frac{\theta}{2} = \tfrac{1}{2}(1 + \cos\theta)$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \qquad 1 - \cos\theta = 2\sin^2\frac{\theta}{2}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} \qquad \tan\frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

Table B.5 lists some useful indefinite integrals. Table B.6 gives Gauss's probability integral and other definite integrals. A more complete list can be found in various handbooks, such as *The Handbook of Chemistry and Physics,* CRC Press.

TABLE B.5 Some Indefinite Integrals (An arbitrary constant should be added to each of these integrals.)

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \qquad (\text{provided } n \neq -1)$$

$$\int \frac{dx}{x} = \int x^{-1}\,dx = \ln x$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)$$

$$\int \frac{x\,dx}{a+bx} = \frac{x}{b} - \frac{a}{b^2}\ln(a+bx)$$

$$\int \frac{dx}{x(x+a)} = -\frac{1}{a}\ln\frac{x+a}{x}$$

$$\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\frac{a+x}{a-x} \qquad (a^2 - x^2 > 0)$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\frac{x-a}{x+a} \qquad (x^2 - a^2 > 0)$$

$$\int \frac{x\,dx}{a^2 \pm x^2} = \pm\tfrac{1}{2}\ln(a^2 \pm x^2)$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} = -\cos^{-1}\frac{x}{a} \qquad (a^2 - x^2 > 0)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2})$$

$$\int \frac{x\,dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \sqrt{a^2-x^2}\,dx = \tfrac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}\right)$$

$$\int x\sqrt{a^2-x^2}\,dx = -\tfrac{1}{3}(a^2-x^2)^{3/2}$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \tfrac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln(x + \sqrt{x^2 \pm a^2})\right]$$

$$\int x(\sqrt{x^2 \pm a^2})\,dx = \tfrac{1}{3}(x^2 \pm a^2)^{3/2}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \ln ax\,dx = (x\ln ax) - x$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int \frac{dx}{a + be^{cx}} = \frac{x}{a} - \frac{1}{ac}\ln(a + be^{cx})$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \tan ax\,dx = \frac{1}{a}\ln(\cos ax) = \frac{1}{a}\ln(\sec ax)$$

$$\int \cot ax\,dx = \frac{1}{a}\ln(\sin ax)$$

$$\int \sec ax\,dx = \frac{1}{a}\ln(\sec ax + \tan ax) = \frac{1}{a}\ln\left[\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)\right]$$

$$\int \csc ax\,dx = \frac{1}{a}\ln(\csc ax - \cot ax) = \frac{1}{a}\ln\left(\tan\frac{ax}{2}\right)$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \frac{dx}{\sin^2 ax} = -\frac{1}{a}\cot ax$$

$$\int \frac{dx}{\cos^2 ax} = \frac{1}{a}\tan ax$$

$$\int \tan^2 ax\,dx = \frac{1}{a}(\tan ax) - x$$

$$\int \cot^2 ax\,dx = -\frac{1}{a}(\cot ax) - x$$

$$\int \sin^{-1} ax\,dx = x(\sin^{-1} ax) + \frac{\sqrt{1-a^2x^2}}{a}$$

$$\int \cos^{-1} ax\,dx = x(\cos^{-1} ax) - \frac{\sqrt{1-a^2x^2}}{a}$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

$$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$$