Finitary Lie algebras, their derivation algebras and their inner ideals

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Finitary Lie algebras are one of the main sources of examples of simple Lie algebras with minimal inner ideals. They are algebras of finite rank operators on vector spaces, and have been broadly studied by several authors.

For our purposes, they appear as the infinite dimensional simple components of the Jordan socle of a nondegenerate Lie algebra. Moreover, since a strongly prime Lie algebra with nonzero Jordan socle can be sandwiched, via the adjoint mapping, between its socle and the algebra of derivations of its socle, it is an interesting problem to determine how these derivation algebras can be, at least in the infinite dimensional case over an algebraically closed field of zero characteristic.

On the other hand, minimal inner ideals are the basic pieces of the socle of a nondegenerate Lie algebra. As in the associative and Jordan theories, the socle of a nondegenerate Lie algebra can be defined as the sum of all the minimal inner ideals of the Lie algebra and has nice properties, such as being an ideal of the Lie algebra, being a direct sum of ideals, and satisfying the descending chain condition on principal inner ideals. It is therefore interesting to know if a Lie algebra has minimal inner ideals and how they look like. Since most of the examples of Lie algebras with socle are in the class of finitary Lie algebras, we study each model of finitary simple Lie algebra and describe its (minimal) inner ideals in detail.

References

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