Non-stable classes of analytic functions

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A function ϕ , analytic in $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, is said to be an inner function if $\phi \in H^{\infty}$ and I has a radial limit $I(e^{i\theta})$ of modulus one for almost every $e^{i\theta} \in \partial \mathbb{D}$.

K. Stephenson in [4], introduced the notion of weak subordination: if f and g are meromorphic functions in Δ , then f is weakly subordinate to g, written $f \prec^w g$, provided that there exist analytic functions I and $\omega : \Delta \to \Delta$, with I an inner function, so that $f \circ I = g \circ \omega$. A class X of meromorphic functions is said to be stable if $f \prec^w g$ and $g \in X$ imply that $f \in X$. All the known examples of spaces of analytic functions which are not stable belong to:

(A) The spaces which contain polynomials and do not contain all the inner functions.

(B) The spaces $X \neq Hol(\Delta)$, which contain \mathcal{B} , the space of Bloch functions.

Then it is natural to ask whether or not does exist a space X which does not belong to the classes (A) and (B) and such that X is not stable. In this paper we shall prove that the answer to this question is positive. In fact, among other things, we prove that certain spaces of Dirichlet type and the Bergman spaces A^p , (0 are not stable.

This is a joint work with Daniel Girela.

References

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