NOTES IN TRANSFERENCE OF BILINEAR MULTIPLIERS.

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1. Notation and preliminaries.

These notes contain an extended version of the talk given in the III International Course of Mathematical Analysis hold in La Rabida (Huelva, Spain) in September 2007 and they are based on results appeared in [1, 2, 3, 4]. I would like to thank the organizers for the kind hospitality and their nice working atmosphere that all the participants (students and professors) enjoyed during our stay.

Let us start by recalling some classical operators whose bilinear formulation will be considered throughout the paper. Let $f : \mathbb{R} \to \mathbb{C}$ belong to the Schwarzt class, and write

$$H(f)(x) = \lim_{\varepsilon \to 0} \int_{|y|>\varepsilon} \frac{f(x-y)}{y} dy$$

and

$$H^*(f)(x) = \sup_{\varepsilon > 0} |\int_{|y|>\varepsilon} \frac{f(x-y)}{y} dy|,$$

for the *Hilbert* and *maximal Hilbert transform* respectively.

We also write

$$M(f)(x) = \sup_{\varepsilon > 0} \frac{1}{2\varepsilon} \int_{|y|<\varepsilon} |f(x-y)| dy,$$

for *Hardy-Littlewood maximal function* and

$$I_\alpha(f)(x) = \int_{\mathbb{R}} \frac{f(x-y)}{|y|^{1-\alpha}} dy,$$

for the *Fractional Integral* where $0 < \alpha < 1$.

They are very classical operators in Harmonic Analysis and are rather well understood not only in $\mathbb{R}$ but in many other groups and not only for the Lebesgue measure but for weight functions $w(x)dx$. Of course the boundedness in the setting of Lebesgue (and many other function spaces) of these operators (not entering in the extreme cases) is well known. Let recall that there exist constant $A_p, B_p > 0$ such that

$$\|H(f)\|_p \leq A_p \|f\|_p, \quad \|H^*(f)\|_p \leq B_p \|f\|_p \tag{1}$$

for $1 < p < \infty$.

There exists $C_p > 0$ such that

$$\|M(f)\|_p \leq C_p \|f\|_p, \tag{2}$$

for $1 < p \leq \infty$.

Key words and phrases. bilinear Hilbert transform, bilinear maximal functions, transference methods, discretization .

Partially supported by Proyecto MTM 2005-08350.

There exists $D_p > 0$ such that

$$\|I_\alpha(f)\|_q \leq D_p \|f\|_p, \tag{3}$$

for $0 < \alpha < \frac{1}{p}, 1 < p < \infty, \frac{1}{q} = \frac{1}{p} - \alpha$.

There are bilinear versions of these operators that have been studied in the last decade and which will be the aim of our considerations.

Given $f, g : \mathbb{R} \to \mathbb{C}$ belonging to the Schwarzt class we can now define the *bilinear Hilbert transform* by

$$H(f,g)(x) = \lim_{\varepsilon \to 0} \int_{|y|>\varepsilon} \frac{f(x-y)g(x+y)}{y} dy,$$

the *bisublinear maximal Hilbert transform* by

$$H^*(f,g)(x) = \sup_{\varepsilon>0} |\int_{|y|>\varepsilon} \frac{f(x-y)g(x+y)}{y} dy|,$$

the *bisublinear Hardy-Littlewood maximal function* by

$$M(f,g)(x) = \sup_{\varepsilon>0} \frac{1}{2\varepsilon} \int_{|y|<\varepsilon} |f(x-y)g(x+y)| dy,$$

and the *bilinear fractional integral* by

$$I_\alpha(f,g)(x) = \int_{\mathbb{R}} \frac{f(x-y)g(x+y)}{|y|^{1-\alpha}} dy, \quad 0 < \alpha < 1.$$

It has been the effort of several authors and many years to get the range of boundedness for the corresponding bilinear versions. We collect in the following theorem the actual knowledge of the problem.

Theorem 1.1. *Let $1 < p_1, p_2 < \infty$, $0 < \alpha < 1/p_1 + 1/p_2$, $1/q = 1/p_1 + 1/p_2 - \alpha$, $1/p_3 = 1/p_1 + 1/p_2$ and $2/3 < p_3 < \infty$. Then there exist constants A, B, C, D such that*

$$\|H(f,g)\|_{p_3} \leq A \|f\|_{p_1} \|g\|_{p_2} \textit{(Lacey-Thiele, [16, 17, 18])}, \tag{4}$$

$$\|H^*(f,g)\|_{p_3} \leq B \|f\|_{p_1} \|g\|_{p_2} \textit{(Lacey, [15])}, \tag{5}$$

$$\|M(f,g)\|_{p_3} \leq C \|f\|_{p_1} \|g\|_{p_2}. \textit{(Lacey, [15])}, \tag{6}$$

$$\|I_\alpha(f,g)\|_q \leq A \|f\|_{p_1} \|g\|_{p_2}. \textit{ (Kenig-Stein [14], Grafakos-Kalton, [13])}. \tag{7}$$

We would like to consider analogue operators in the periodic or the discrete case and to analyze their boundedness.

In particular, one can define the *bilinear conjugate function* as

$$B(F,G)(e^{it}) = \int_{-\pi}^{\pi} F(t-s)G(t+s) cotag(s/2) \frac{ds}{2\pi}$$

where F and G are polynomials on $\mathbb{T}$.

Using Fourier series expansion of the polynomials, the operator can also be written as

$$B(F,G)(e^{it}) = -i \sum_k (\sum_{n+m=k} sign(n-m) \hat{F}(n) \hat{G}(m)) e^{ikt}$$

where $F(t) = \sum_{-N}^{N} \hat{F}(n) e^{int}$ and $G(t) = \sum_{-M}^{M} \hat{G}(m) e^{imt}$.

The fundamental question is the following: *Is the bilinear conjugate transform bounded from $L^{p_1}(\mathbb{T}) \times L^{p_2}(\mathbb{T}) \to L^{p_3}(\mathbb{T})$ for some values of p_1, p_2, p_3?*.

While the situation in the linear case reduces to adapt the proof of the group $\mathbb{R}$ to the group $\mathbb{T}$ (or to replace the half-space for the disc when using a complex-variable approach), the techniques that were needed for the real line in the bilinear case do not seem to have any easy modification to the periodic setting to obtain the boundedness of the bilinear conjugate function defined in $\mathbb{T}$. However some transference techniques known in the linear case can be adapted to the bilinear one.

Another analogue formulations that we would like to consider are *discrete bilinear Hilbert transform*, the *discrete bisublinear Hardy-Littlewood maximal function* and the *discrete bilinear fractional transform*, defined by

$$H_d(\lambda,\beta)(n) = \lim_{\mathbb{N}\to\infty} \sum_{0<|k|\leq N} \frac{\lambda_{n-k}\beta_{n+k}}{k},$$

$$M_d(\lambda,\beta)(n) = \sup_{N\in\mathbb{N}} \frac{1}{2N} \sum_{0<|k|\leq N} |\lambda_{n-k}||\beta_{n+k}| \text{ and}$$

$$I_d^\alpha(\lambda,\beta)(n) = \sum_{k\neq 0, k\in\mathbb{Z}} \frac{\lambda_{n-k}\beta_{n+k}}{|k|^{1-\alpha}}$$

for finite sequences λ, β respectively.

As above, the fundamental question is the following: *Are they bounded from* $\ell^{p_1}(\mathbb{Z}) \times \ell^{p_2}(\mathbb{Z}) \to \ell^{p_3}(\mathbb{Z})$ *for some values of* p_1, p_2, p_3*?.*

Several methods have been developed to such purposes in the last five years. In fact two different approaches have been applied: The first one is the bilinear formulation of the DeLeeuw method [8] first considered in the paper by Fan and Sato [9] and then developed by O. Blasco and P. Villarroya [1, 5]. The second one is the bilinear formulation of the Coifman- Weiss transference method [7] that has been extensively studied in [2, 3, 4] by O. Blasco, E. Berkson, M.J. Carro and A.T. Gillespie.

We shall only mention one theorem and its application of each of the procedures considered in the just mentioned papers. All the results appearing in Theorem 1.1 can be transferred to both situations periodic and discrete. We will also present a detailed proof for the reader to see the tools used in our approaches. The interested reader can consult the references in the bibliography for a further study of the topic.

2. Methods and applications

Let us start considering the simplest situation, corresponding to bilinear convolution with integrable kernels.

Assume $K \in L^1(\mathbb{R})$ and define

$$B_K(f,g)(x) = \int_{\mathbb{R}} f(x-y)g(x+y)K(y)dy.$$

Writing $f(x-y) = \int_{\mathbb{R}} \hat{f}(\xi)e^{i(x-y)\xi}d\xi$ and $g(x+y) = \int_{\mathbb{R}} \hat{g}(\eta)e^{i(x+y)\eta}d\eta$, we can also use expression:

$$\begin{aligned}
B_K(f,g)(x) &= \int_{\mathbb{R}} f(x-y)g(x+y)K(y)dy \\
&= \int_{\mathbb{R}}\int_{\mathbb{R}}\int_{\mathbb{R}} \hat{f}(\xi)\hat{g}(\eta)K(y)e^{i(x-y)\xi}e^{i(x+y)\eta}d\xi d\eta dy \\
&= \int_{\mathbb{R}}\int_{\mathbb{R}} \hat{f}(\xi)\hat{g}(\eta)(\int_{\mathbb{R}} K(y)e^{-i(\xi-\eta)y}dy)e^{i(\xi+\eta)x}d\xi d\eta \\
&= \int_{\mathbb{R}}\int_{\mathbb{R}} \hat{g}(\eta)\hat{f}(\xi)\hat{K}(\xi-\eta)e^{i(\xi+\eta)x}d\xi d\eta.
\end{aligned}$$

This motivates the following definition.

Definition 2.1. *Let $0 < p_1, p_2, p_3 < \infty$ and $1/p_1 + 1/p_2 = 1/p_3$. Given a bounded measurable function $m(\xi,\eta)$ is said to be a bilinear multiplier on $\mathbb{R}$ of type (p_1,p_2,p_3) if the operator*

$$B_m(f,g)(x) = \int_{\mathbb{R}}\int_{\mathbb{R}} \hat{f}(\xi)\hat{g}(\eta)m(\xi,\eta)e^{i(\xi+\eta)x}d\xi d\eta$$

is bounded from $L^{p_1}(\mathbb{R}) \times L^{p_2}(\mathbb{R})$ to $L^{p_3}(\mathbb{R})$.

The study of bilinear multipliers for smooth symbols (where $m(\xi,\eta)$ is a "nice" regular function) goes back to the work by R.R. Coifman and Y. Meyer in [6].

Let us restrict ourselves to a smaller family of multipliers: The case $m(\xi,\eta) = m'(\xi-\eta)$ where $m'(x)$ is bounded in $\mathbb{R}$.

The simplest case is $m'(x) = \hat{\mu}(x)$ where μ is a Borel regular measure in $\mathbb{R}$. It is elementary to see that m' define a bilinear multiplier on $\mathbb{R}$ of type (p_1,p_2,p_3) whenever $p_3 \geq 1$ and $1/p_1 + 1/p_2 = 1/p_3$.

Indeed, using the expression

$$B_m(f,g)(x) = \int_{\mathbb{R}} f(x-t)g(x+t)d\mu(t)$$

one gets

$$\begin{aligned}
\|B_m(f,g)\|_{p_3} &\leq \int_{\mathbb{R}} \|f(\cdot - t)g(\cdot + t)\|_{p_3} d|\mu|(t) \\
&\leq \int_{\mathbb{R}} \|f(\cdot - t)\|_{p_1}\|g(\cdot + t)\|_{p_2} d|\mu|(t) \\
&= \|f\|_{p_1}\|g\|_{p_2}\int_{\mathbb{R}} d|\mu|(t) = \|\mu\|_1\|f\|_{p_1}\|g\|_{p_2}.
\end{aligned}$$

However, the case where the symbol m' is not smooth has a much shorter story. A very non trivial example is given by $m'(x) = -isign(x)$ which leads to the bilinear Hilbert transform and it was first considered by Lacey and Thiele in [16, 17, 18]) and then extended to other cases in [10, 11]. The solution took many years to be achieved after the formulation of the question by A. P. Calderón in the seventies.

Let us mention a general method to transfer results from $\mathbb{R}$ to $\mathbb{T}$. The approach follows the DeLeeuw method in the linear case and there are two different proofs of the following result.

Theorem 2.2. *(see [1, 9]) Let $m(\xi,\eta)$ be a continous function defining a bilinear multiplier on $\mathbb{R}$ of type (p_1,p_2,p_3) where $1/p_1 + 1/p_2 = 1/p_3$ and $p_3 \geq 1$ i.e. the operator*

$$B_m(f,g)(x) = \int_{\mathbb{R}}\int_{\mathbb{R}} \hat{f}(\xi)\hat{g}(\eta)m(\xi,\eta)e^{i(\xi+\eta)x}d\xi d\eta$$

is bounded from $L^{p_1}(\mathbb{R}) \times L^{p_2}(\mathbb{R})$ to $L^{p_3}(\mathbb{R})$ then the sequence $m_{k,k'} = m(k,k')$ define a bilinear multiplier on $\mathbb{T}$ of type (p_1,p_2,p_3), i.e. the operator

$$\tilde{B}_m(F,G)(t) = \sum_k \Big(\sum_{n+n'=k} \hat{F}(n)\hat{G}(n')m(n,n') \Big) e^{ikt}$$

is bounded from $L^{p_1}(\mathbb{T}) \times L^{p_2}(\mathbb{T})$ to $L^{p_3}(\mathbb{T})$.

Corollary 2.3. *The bilinear conjugate function operator is bounded from $L^{p_1}(\mathbb{T}) \times L^{p_2}(\mathbb{T})$ to $L^{p_3}(\mathbb{T})$ whenever $1 < p_1, p_2 < \infty$, $1/p_1 + 1/p_2 = 1/p_3$ and $p_3 \geq 1$.*

The reader should be aware that the restriction $p_3 \geq 1$ is a limitation of the proof but it can be removed using other approaches (see [4]).

To handle the discrete case, there are also two different techniques (see [1]) or ([4, 2]). We shall select here the second approach using a "discretization" method.

Let us define the mappings $P : \ell^p(\mathbb{Z}) \to L^p(\mathbb{R})$ by

$$\lambda = (\lambda_n) \to f = \sum_{n \in \mathbb{Z}} \lambda_n \chi_{(n-1/4,n+1/4)}$$

and $Q : L^p(\mathbb{R}) \to \ell^p(\mathbb{Z})$ by

$$f \to (\int_{(n-1/4,n+1/4)} f(x)dx)_{n \in \mathbb{Z}}.$$

Clearly $\|P(\lambda)\|_p = C\|\lambda\|_p$ for $0 < p < \infty$ and $\|Q(f)\|_p \leq C\|f\|_p$ for $1 \leq p < \infty$.

Theorem 2.4. *([4]) Let K be integrable in $\mathbb{R}$ and denote*

$$B_K(f,g)(x) = \int_{\mathbb{R}} f(x-y)g(x+y)K(y)dy$$

for f and g simple functions. If

$$K_n = \int_{[-1/4,1/4]} \int_{[n-1/4,n+1/4]} K(x-u)K(x+u)dxdy$$

then

$$QB_K\Big(P(\lambda), P(\beta)\Big)(n) = \sum_{k \in \mathbb{Z}} \lambda_{n-k}\beta_{n+k}K_k$$

for any finite sequences λ and β.

In particular, for $p_3 \geq 1$ one has QB_KP is bounded from $\ell^{p_1}(\mathbb{Z}) \times \ell^{p_2}(\mathbb{Z})$ to $\ell^{p_3}(\mathbb{Z})$ with norm bounded by the norm of B_K as operator from $L^{p_1}(\mathbb{R}) \times L^{p_2}(\mathbb{R})$ to $L^{p_3}(\mathbb{R})$).

Corollary 2.5. *The bilinear discrete Hilbert transform is bounded from $\ell^{p_1}(\mathbb{Z}) \times \ell^{p_2}(\mathbb{Z})$ to $\ell^{p_3}(\mathbb{Z})$ whenever $1 < p_1, p_2 < \infty$, $1/p_1 + 1/p_2 = 1/p_3$ and $p_3 \geq 1$.*

The reader should also be aware that the restriction $p_3 \geq 1$ is again a limitation of the proof but it was removed in [2] to get the $p_3 > \frac{2}{3}$.

Let us finally explain a bit how to get the transference method of Coifman-Weiss in the bilinear setting (see [4, 2, 3]).

Let G be a l.c.a group with Haar measure m, let (Ω, Σ, μ) be a measure space and let R_u be a representation of G in the space of bounded linear operators on $L^p(\mu)$, i.e. $R : G \to L(L^p(\mu), L^p(\mu))$ such that $u \to R_u$ verifies

- $R_u R_v = R_{uv}$ for $u, v \in G$,
- $\lim_{u \to 0} R_u f = f$ for $f \in L^p(\mu)$,
- $\sup_{u \in G} \|R_u\| < \infty$.

Let $K \in L^1(G)$ with compact support. Denote now

$$B_K(\phi,\psi)(v) = \int_G \phi(v-u)\psi(v+u)K(u)dm(u)$$

for ϕ, ψ simple functions defined on G, and assume that, for $0 < p_1, p_2 < \infty$ and $1/p_1 + 1/p_2 = 1/p_3$, the bilinear operator B_K is bounded from $L^{p_1}(G) \times L^{p_2}(G)$ to $L^{p_3}(G)$ with "norm" $N_{p_1,p_2}(B_K)$.

We now consider the transferred operator by the formula

$$T_K(f,g)(w) = \int_G R_{-u}f(w)R_u g(w)K(u)dm(u)$$

for $f \in L^{p_1}(\mu)$ and $g \in L^{p_2}(\mu)$.

Let us present, in a particular case, a prototype result that one can produce in this setting. The assumptions can be weakened and the setting can be relaxed but we concentrate in the case for simplicity.

Theorem 2.6. *Let $G = \mathbb{R}$, (Ω, Σ, μ) a measure space, $1 \le p_1, p_2 < \infty$ and $1/p_3 = 1/p_1 + 1/p_2$. Let R be a representation of $\mathbb{R}$ on acting $L^{p_i}(\mu)$ for $i = 1, 2$ with*

$$\sup_{u\in\mathbb{R}} \|R_u\|_{L(L^{p_i},L^{p_i})} = 1$$

for $i = 1, 2$

Assume that there exists a map $u \to L(L^{p_3}(\mu), L^{p_3}(\mu))$ given by $u \to S_u$ such that S_u are invertible with $\sup_{u\in G} \|S_u^{-1}\| = 1$ and

$$S_v((R_{-u}f)(R_u g)) = (R_{v-u}f)(R_{v+u}g)$$

for $u, v \in \mathbb{R}$, $f \in L^{p_1}(\mu)$ and $g \in L^{p_2}(\mu)$.

Let K belong to $L^1(\mathbb{R})$ and be supported in $[-A.A]$. If the bilinear map B_K defined as above is bounded with norm $N_{p_1,p_2}(B_K)$ then T_K is also bounded from $L^{p_1}(\mu) \times L^{p_2}(\mu)$ to $L^{p_3}(\mu)$ and with norm bounded by $CN_{p_1,p_2}(B_K)$.

Proof. Write, for each $v \in \mathbb{R}$,

$$\begin{aligned} T_K(f,g) &= S_v^{-1}(S_v \int_{\mathbb{R}} R_{-u}fR_u gK(u)du) \\ &= S_v^{-1}(\int_{\mathbb{R}} S_v(R_{-u}fR_u g)K(u)du) \\ &= S_v^{-1}(\int_{\mathbb{R}} (R_{v-u}f)(R_{v+u}g)K(u)du) \end{aligned}$$

Hence

$$\|T_K(f,g)\|^{p_3}_{L^{p_3}(\mu)} \le \|\int_{\mathbb{R}} (R_{v-u}f)(R_{v+u}g)K(u)du\|^{p_3}_{L^{p_3}(\mu)}$$

Given $N \in \mathbb{N}$, integrating over $v \in [-N, N]$,

$$2N\|T_K(f,g)\|^{p_3}_{L^{p_3}(\mu)} \le \int_{-N}^{N} \|\int_{\mathbb{R}} (R_{v-u}f)(R_{v+u}g)K(u)du\|^{p_3}_{L^{p_3}(\mu)}dm(v).$$

Therefore

$$\begin{aligned}
2N \quad \|T_K(f,g)\|^{p_3}_{L^{p_3}(\mu)} &\leq \int_{-N}^{N}\int_\Omega |\int_{\mathbb{R}} R_{v-u}f(w)R_{v+u}g(w)K(u)du|^{p_3}d\mu(w)dv \\
&= \int_\Omega (\int_{-N}^{N}|\int_{-A}^{A} R_{v-u}f(w)R_{v+u}g(w)K(u)du|^{p_3}dv)d\mu(w) \\
&= \int_\Omega (\int_{\mathbb{R}}|\int_{\mathbb{R}} R_{v-u}f(w)\chi_{[-A-N,A+N]}(v-u)R_{v+u}g(w)\chi_{[-A-N,A+N]}(v+u)K(u)du|^{p_3}dv)d\mu(w) \\
&= \int_\Omega (\int_{\mathbb{R}} |B_K(R_uf(w)\chi_{[-A-N,A+N]}, R_ug(w)\chi_{[-A-N,A+N]})(v)|^{p_3}dv)d\mu(w) \\
&= \int_\Omega \|B_K(R_uf(w)\chi_{[-A-N,A+N]}, R_ug(w)\chi_{[-A-N,A+N]})\|^{p_3}_{L^{p_3}(\mathbb{R})}d\mu(w) \\
&\leq N_{p_1,p_2}(B_K)^{p_3}\int_\Omega \|R_uf(w)\chi_{[-A-N,A+N]}\|^{p_3}_{L^{p_1}(\mathbb{R})}\|R_ug(w)\chi_{[-A-N,A+N]}\|^{p_3}_{L^{p_2}(\mathbb{R})}d\mu(w) \\
&\leq N_{p_1,p_2}(B_K)^{p_3}(\int_\Omega \|R_uf(w)\chi_{[-A-N,A+N]}\|^{p_1}_{L^{p_1}(\mathbb{R})}d\mu(w))^{p_3/p_1} \\
&\times (\int_\Omega \|R_ug(w)\chi_{[-A-N,A+N]}\|^{p_2}_{L^{p_2}(\mathbb{R})}d\mu(w))^{p_3/p_2} \\
&= N_{p_1,p_2}(B_K)^{p_3}(\int_{-(A+N)}^{A+N} \|R_uf\|^{p_1}_{L^{p_1}(\mu)}du)^{p_3/p_1} \\
&\times (\int_{-(A+N)}^{A+N} \|R_ug\|^{p_2}_{L^{p_2}(\mu)}du)^{p_3/p_2} \\
&= N_{p_1,p_2}(B_K)^{p_3}(\int_{-(A+N)}^{A+N} \|f\|^{p_1}_{L^{p_1}(\mu)}du)^{p_3/p_1} \\
&\times (\int_{-(A+N)}^{A+N} \|g\|^{p_2}_{L^{p_2}(\mu)}du)^{p_3/p_2} \\
&\leq N_{p_1,p_2}(B_K)^{p_3}(2(A+N))\|f\|^{p_3}_{L^{p_1}(\mu)}\|g\|^{p_3}_{L^{p_2}(\mu)}.
\end{aligned}$$

Therefore

$$\|T_K(f,g\|_{L^{p_3}(\mu)} \leq (\frac{A+N}{N})^{1/p_3}N_{p_1,p_2}(B_K)\|f\|^{p_3}_{L^{p_1}(\mu)}\|g\|^{p_3}_{L^{p_2}(\mu)}.$$

■

Note that, in particular, the assumptions in the previous theorem hold for multiplicative representations, i.e. $R_u(fg) = (R_uf)(R_ug)$, selecting $S_u = R_u$.

Let us finish with an application to Ergodic theory. We state here the result for maximal version of the operators, but results in the same spirit can be seen in [2, 4]. Let (Ω, Σ, μ) be $\sigma-$finite measure space and T an invertible and bounded operator on $L^p(\mu)$. Define the"bisublinear maximal ergodic transform" by

$$M_T(f,g)(w) = \sup_{N>0}\frac{1}{2N}\sum_{n=-N}^{N} T^nf(w)T^{-n}g(w),$$

and the " bisublinear maximal ergodic Hilbert transform" by

$$H^*_T(f,g)(w) = \sup_{N>0}\sum_{0<|n|<N}\frac{T^nf(w)T^{-n}g(w)}{n}.$$

Theorem 2.7. *([2, 3]) Let $1 < p_1, p_2 < \infty$ and $1/p_1 + 1/p_2 = 1/p_3 < 3/2$, let T be an invertible operator on $L^{p_i}(\mu)$ for $i = 1, 2$ such that T and T^{-1} are power bounded. Assume that there exists an invertible operator S defined on $L(L^{p_3}(\mu), L^{p_3}(\mu))$ such that*

$$S^m(T^n f T^{-n} g) = T^{m+n} f T^{m-n} g$$

for $f \in L^{p_1}(\mu)$ and $g \in L^{p_2}(\mu)$.

Then M_T and H_T^* are bounded from $L^{p_1}(\mu) \times L^{p_2}(\mu)$ to $L^{p_3}(\mu)$.

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