Remarks on p-summing multipliers.

Oscar Blasco *

Departamento de Análisis Matemático, Universidad de Valencia,
Burjassot 46100, Valencia (SPAIN)

To Professor Manuel Valdivia on the occasion of his seventieth birthday.

Abstract

Let X and Y be Banach spaces and $1 \leq p < \infty$, a sequence of operators (T_n) from X into Y is called a p-summing multiplier if $(T_n(x_n))$ belongs to $\ell_p(Y)$ whenever (x_n) satisfies that $(\langle x^*, x_n\rangle)$ belongs to ℓ_p for all $x^* \in X^*$. We present several examples of p-summing multipliers and extend known results for p-summing operators to this setting. We get, using almost summing and Rademacher bounded operators, some sufficient conditions for a sequence to be a p-summing multiplier between spaces with some geometric properties.

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1. Introduction.

Let X and Y be two real or complex Banach spaces and let $E(X)$ and $F(Y)$ be two Banach spaces whose elements are defined by sequences of vectors in X and Y (containing any eventually null sequence in X or Y). A sequence of operators $(T_n) \in \mathcal{L}(X, Y)$ is called a *multiplier sequence* from $E(X)$ to $F(Y)$ if there exists a constant $C > 0$ such that

$$\|(T_j x_j)_{j=1}^n\|_{F(Y)} \leq C\|(x_j)_{j=1}^n\|_{E(X)}$$

for all finite families $x_1, \ldots, x_n$ in X. The set of all multiplier sequences is denoted by $(E(X), F(Y))$.

Given a real or complex Banach space X and $1 \leq p \leq \infty$, we denote by $\ell_p(X)$ and $\ell_p^w(X)$ the Banach spaces of sequences in X with norms $||(x_n)||_{\ell_p(X)} = ||(||x_n||)||_{\ell_p}$ and $\|(x_n)\|_{\ell_p^w(X)} = \sup_{||x^*||=1} \|(\langle x^*, x_n\rangle)\|_{\ell_p}$ respectively. $Rad_p(X)$ stands for the space of sequences $(x_n) \in X$ such that $\sup_n (\int_0^1 || \sum_{j=1}^n r_j(t) x_j ||^p dt)^{1/p} < \infty$, where $(r_j)_{j\in\mathbb{N}}$ are the Rademacher functions on $[0, 1]$ defined by $r_j(t) = sign(\sin 2^j \pi t)$.

It is easy to see that $Rad_\infty(X) = \ell_1^w(X)$. It follows from Kahane's inequalities (see [11], page 211) that $Rad_p(X) = Rad_q(X)$ with equivalent norms for all $1 \leq p, q < \infty$. This space will then be denoted $Rad(X)$, and we shall use the L^1-norm throughout the paper.

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The reader is referred to [4],[5],[6],[7] for the study of multiplier sequences in the case $E(X) = H^1(\mathbb{T}, X)$, corresponding to vector-valued Hardy spaces, and $F(Y) = \ell_p(Y)$ or $F(Y) = BMOA(\mathbb{T}, Y)$, to [3] ,[8],[17] and [28] for $E(X) = Rad(X)$ and $F(Y) = Rad(Y)$, to [2] for the particular cases $p = q$, $X = Y$ and $T_j = \alpha_j Id_X$ and to [1] for the case $E(X) = \ell_p^w(X)$ and $F(Y) = \ell_q(\mathbb{K})$.

In this article we shall consider the case of the classical sequence spaces $E(X) = \ell_p^w(X)$ and $F(Y) = \ell_p(Y)$. A sequence $(T_j)_{j\in\mathbb{N}}$ of operators in $\mathcal{L}(X, Y)$ is a *p-summing multiplier* if there exists a constant $C > 0$ such that, for any finite collection of vectors $x_1, x_2, \ldots x_n$ in X, it holds that

$$\Big(\sum_{j=1}^n ||T_j x_j||^p\Big)^{1/p} \leq C \sup_{||x^*||=1} \Big(\sum_{j=1}^n |\langle x^*, x_j\rangle|^p\Big)^{1/p}.$$

Note that a constant sequence $T_j = T$ for all $j \in \mathbb{N}$ belongs to $(\ell_p^w(X), \ell_p(Y))$ if and only if T is a p-summing operator, usually denoted $T \in \Pi_p(X, Y)$. This fact suggests the use of the notation $\ell_{\pi_p}(X, Y)$ instead of $(\ell_p^w(X), \ell_p(Y))$.

In the paper [1] J.L. Arregui and the author introduced and considered the notion of (p, q)-summing multipliers and concentrated on the case $Y = \mathbb{K}$. It was shown that some geometric properties on X can be described using $\ell_{\pi_{p,q}}(X, \mathbb{K})$ and also that classical theorems, like Grothendieck theorem and others, can be rephrased into this setting.

Let us now recall the basic notions on Banach space theory and absolutely summing operators to be used later on.

An operator $T \in \mathcal{L}(X, Y)$ is *absolutely summing* if for every unconditionally convergent series $\sum x_j$ in X it holds that $\sum Tx_j$ is absolutely convergent in Y.

For $1 \leq p < \infty$, an operator $T: X \to Y$ is *p-summing* (see [22]) if it maps sequences $(x_j) \in \ell_p^w(X)$ into sequences $(Tx_j) \in \ell_p(Y)$, equivalently, if there exists a constant C such that

$$(\sum_{j=1}^n \|Tx_j\|^p)^{1/p} \leq C \sup_{||x^*||=1} (\sum_{j=1}^n |\langle x^*, x_j\rangle|^p)^{1/p}$$

for any finite family $x_1, x_2, \ldots x_n$ of vectors in X.

The least of such constants is the p-summing norm of u, denoted by $\pi_p(T)$. The space $\Pi_p(X, Y)$ of all p-summing operators from X to Y then is a Banach space for $1 \leq p < \infty$. It is well known that the space of absolutely summing operators coincides with the space of 1-summing operators.

For $1 \leq p \leq 2$ (respect. $q \geq 2$), a Banach space X is said to have *(Rademacher) type* p (resp. *(Rademacher) cotype* q) if there exists a constant C such that

$$\int_0^1 ||\sum_{j=1}^n x_j r_j(t)||dt \leq C(\sum_{j=1}^n \|x_j\|^p)^{1/p}$$

(resp.

$$(\sum_{j=1}^n \|x_j\|^q)^{1/q} \leq C \int_0^1 ||\sum_{j=1}^n x_j r_j(t)||dt),$$

for any finite family $x_1, x_2, \ldots x_n$ of vectors in X .

A Banach space X is said to have the *Orlicz property* there exists a constant C such that

$$(\sum_{j=1}^{n} \|x_j\|^2)^{1/2} \leq C \sup_{||x^*||=1} \sum_{j=1}^{n} |\langle x_j, x^* \rangle|$$

for any finite family $x_1, x_2, \ldots x_n$ of vectors in X.

Let us recall that Grothendieck's theorem establishes, in this setting, that, for any compact set K, any measure space (Ω, Σ, μ) and any Hilbert space H,

$$\mathcal{L}(L_1(\mu), H) = \Pi_1(L_1(\mu), H). \quad (1)$$

or

$$\mathcal{L}(C(K), L^1(\mu)) = \Pi_2(C(K), L^1(\mu)). \quad (2)$$

Because of that a Banach space X is called a GT- space, i.e. X satisfies the Grothendieck theorem if (see [24], page 71)

$$\mathcal{L}(X, \ell_2) = \Pi_1(X, \ell_2). \quad (3)$$

The basic theory of p-summing operators, type and cotype can be found, for example, in the books [11], [9], [16], [26], [23], [24] or [28] .

In this paper we restrict ourselves to the case $p = q$ for simplicity, although some of the results presented here can be easily stated in the general case. The paper is divided into three sections. In the first one we shall give several examples of p-summing multipliers. In the second one we show some general results extending known facts in the study of p-summing operators to p-summing multipliers. In the last section we relate this new notion to the class of almost summing operators or Rademacher bounded sequences and find some sufficient conditions for a sequence to belong to $\ell_{\pi_p}(X, Y)$, at least for certain spaces X and Y.

Throughout the paper (e_j) denotes the canonical basis of the sequence spaces ℓ_p and c_0, $\langle x^*, x \rangle$ the duality pairing between X^* and X, p' the conjugate exponent of p, $\mathbb{K}$ stands for $\mathbb{R}$ or $\mathbb{C}$ and, as usual, C denotes a constant that may vary from line to line.

2. Definition and examples.

It is not difficult to show (see [1] Proposition 2.1) that $(\ell_p(X), \ell_p(Y)) = \ell_\infty(\mathcal{L}(X, Y))$ for any couple of Banach spaces X and Y and $1 \leq p \leq \infty$. Let us give a name to the multipliers corresponding to $(\ell_p^w(X), \ell_p(Y))$.

Definition 2.1 *(see [1]) Let X and Y be Banach spaces, and let $1 \leq p, q < \infty$. A sequence $(T_j)_{j \in \mathbb{N}}$ of operators in $\mathcal{L}(X, Y)$ is a (p, q)-summing multiplier if there exists a constant $C > 0$ such that, for any finite collection of vectors $x_1, x_2, \ldots x_n$ in X, it holds that*

$$\Big(\sum_{j=1}^{n} ||T_j x_j||^p \Big)^{1/p} \leq C \sup_{||x^*||=1} \Big(\sum_{j=1}^{n} |\langle x^*, x_j \rangle|^q \Big)^{1/q}.$$

We use $\ell_{\pi_{p,q}}(X,Y)$ to denote the set of (p,q)-summing multipliers, and $\pi_{p,q}[T_j]$ is the least constant C for which (T_j) verifies the inequality in the definition. In order to avoid ambiguities, sometimes we shall use $\pi_{p,q}[T_j;X,Y]$.

We shall only deal with the case $p=q$. The space $\ell_{\pi_{p,p}}(X,Y)$ will be denoted $\ell_{\pi_p}(X,Y)$, its norm π_p and its elements will be called p-summing multipliers. It is not difficult to show (see [1]) that if X and Y are Banach spaces and $1\le p<\infty$ then $(\ell_{\pi_p}(X,Y),\pi_p)$ is a Banach space.

Remark 2.1 *A sequence $(T_j)\in\ell_{\pi_1}(X,Y)$ if and only if it holds that for any unconditionally convergent series $\sum x_j$ in X we have $(T_j(x_j))_j\in\ell_1(Y)$ (see [1]).*

Remark 2.2 *Let $1\le p<\infty$. A sequence $(T_j)\in\ell_{\pi_1}(X,Y)$ if and only if the map $(y_j^*)\to(T_j^*(y_j^*))$ is bounded from $\ell_{p'}(Y^*)$ into $\ell_{\pi_{1,p}}(X,\mathbb{K})$.*

Moreover $\pi_p[T_n;X,Y]=\sup_{||y_n||_{\ell_{p'}(Y^*)}=1}\pi_{1,p}[T_n^*(y_n);X,\mathbb{K}]$.

Let us now mention some basic examples of p-summing multipliers in different contexts.

Example 2.1 *Let $1\le p<\infty$ and μ be a probality measure on a compact set K. Let (ϕ_n) be a sequence of continuous functions and define $T_n: C(K)\to L^p(\mu)$ by $T_n(\psi)=\phi_n\psi$.*

If $(\sum_{n=1}^\infty|\phi_n|^{p'})^{1/p'}\in L^p(\mu)$ then $T_n\in\ell_{\pi_p}(C(K),L^p(\mu))$.

Proof. Assume $p>1$ (the case p=1 is left to the reader). Let $n\in\mathbb{N}$ and $\psi_1,\psi_2,...,\psi_n$ in $C(K)$. Recalling that

$$||(\psi_n)||_{\ell_p^w(C(K))}=||(\sum_{k=1}^n|\psi_k|^p)^{1/p}||_\infty \tag{4}$$

then

$$\begin{aligned}\sum_{k=1}^n||T_k(\psi_k)||^p_{L^p(\mu)} &= \int_K\sum_{k=1}^n|\phi_k\psi_k|^p d\mu\\ &\le \int_K(\sum_{k=1}^n|\phi_k|^{p'})^{p/p'}(\sum_{k=1}^n|\psi_k|^p)d\mu\\ &\le ||(\sum_{k=1}^n|\psi_k|^p)^{1/p}||^p_\infty\int_K(\sum_{k=1}^n|\phi_k|^{p'})^{p/p'}d\mu.\end{aligned}$$

This shows that $\pi_p[T_j]\le(\int_K(\sum_{k=1}^n|\phi_k|^{p'})^{p/p'}d\mu)^{1/p}$. ■

Example 2.2 *Let $1\le p<\infty$, (Ω,Σ,μ) and (Ω',Σ',μ') be finite measure spaces. Let $(f_n)\subset L^p(\mu,L^1(\mu'))$ and consider the operators $T_n: L^\infty(\mu')\to L^p(\mu)$ given by $T_n(\phi)=\langle\phi,f_n\rangle=\int_{\Omega'}\phi(w')f_n(.,w')d\mu(w')$.*

If $\sup_n|f_n(w,w')|\in L^p(\mu,L^1(\mu'))$ then $T_n\in\ell_{\pi_p}(L^\infty(\mu'),L^p(\mu))$.

Proof. Given $n \in \mathbb{N}$ and $\phi_1, \phi_2, ..., \phi_n$ in $L^\infty(\mu')$ then

$$\begin{aligned}
\sum_{k=1}^{n} ||T_k(\phi_k)||^p_{L^p(\mu)} &= \sum_{k=1}^{n} \int_\Omega |\langle \phi_k, f_k(w)\rangle|^p d\mu(w) \\
&= \int_\Omega \sum_{k=1}^{n} |\int_{\Omega'} \phi_k(w') f_k(w,w') d\mu(w')|^p d\mu(w) \\
&\leq \int_\Omega \Big(\int_{\Omega'} (\sum_{k=1}^{n} |\phi_k(w')|^p |f_k(w,w')|^p)^{1/p} d\mu(w')\Big)^p d\mu(w) \\
&\leq \int_\Omega \Big(\int_{\Omega'} \sup_k |f_k(w,w')| (\sum_{k=1}^{n} |\phi_k(w')|^p)^{1/p} d\mu(w')\Big)^p d\mu(w) \\
&\leq ||(\sum_{k=1}^{n} |\phi_k(w')|^p)^{1/p}||^p_{L^\infty(\mu')} \int_\Omega \Big(\int_{\Omega'} \sup_k |f_k(w,w')| d\mu(w')\Big)^p d\mu(w).
\end{aligned}$$

This shows, using (4), that $\pi_p[T_j] \leq ||sup_n|f_n(w,w')|||_{L^p(\mu, L^1(\mu'))}$. ■

Example 2.3 *Let $1 \leq p < \infty$ and (A_n) be a sequence of matrices such that $T_n((\lambda_k)) = (\sum_{k=1}^\infty A_n(k,j)\lambda_k)_j$ defines bounded operators from c_0 to ℓ_p. If*

$$\sum_{k=1}^{\infty} \sup_n (\sum_{j=1}^{\infty} |A_n(k,j)|^p)^{1/p} < \infty$$

then $(T_n) \in \ell_{\pi_1}(c_0, \ell_p)$.

Proof. Note that $T_n = \sum_{k=1}^\infty e_k^* \otimes y_{n,k}$ where $(y_{n,k}) \in \ell_p$ is given by $y_{n,k} = (A_n(k,j))_j$. Hence, if $x_n = (\lambda_{n,k})_k$ then

$$\begin{aligned}
\sum_{n=1}^{\infty} ||T_n(x_n)|| &\leq \sum_{n=1}^{\infty}\sum_{k=1}^{\infty} |\langle e_k^*, x_n\rangle| ||y_{n,k}|| \\
&= \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} |\lambda_{n,k}| (\sum_{j=1}^{\infty} |A_n(k,j)|^p)^{1/p} \\
&\leq \sum_{k=1}^{\infty} \sup_n (\sum_{j=1}^{\infty} |A_n(k,j)|^p)^{1/p} \sum_{n=1}^{\infty} |\lambda_{n,k}| \\
&\leq (\sup_k \sum_{n=1}^{\infty} |\lambda_{n,k}|) \sum_{k=1}^{\infty} \sup_n (\sum_{j=1}^{\infty} |A_n(k,j)|^p)^{1/p} \\
&= ||(x_n)||_{\ell_1^w(c_0)} \sum_{k=1}^{\infty} \sup_n (\sum_{j=1}^{\infty} |A_n(k,j)|^p)^{1/p}.
\end{aligned}$$

■

Example 2.4 *Let $f \in L^1([0,1] \times [0,1])$ and measurable sets $E_n \subset [0,1]$ for $n \in \mathbb{N}$. Let $T_n : L^\infty([0,1]) \to L^1([0,1])$ be defined by $T_n(\phi)(t) = (\int_0^1 f(t,s)\phi(s)ds)\chi_{E_n}(t)$. Then $(T_n) \in \ell_{\pi_2}(L^\infty([0,1]), L^1([0,1]))$.*

Proof. First observe that f can be regarded as a function in $L^1([0,1], L^1([0,1]))$ and then $\phi \to \int_0^1 f(.,s)\phi(s)ds$ defines a bounded operator from $L^\infty([0,1])$ to $L^1([0,1])$ with norm ≤ 1.

Given $n \in \mathbb{N}$ and $\phi_1, \phi_2, ..., \phi_n$ in $L^\infty([0,1])$ we have, using 2

$$\begin{aligned} \sum_{k=1}^n ||T_k(\phi_k)||_{L^1}^2 &= \sum_{k=1}^n ||(\int_0^1 f(.,s)\phi_k(s)ds)\chi_{E_k}||_{L^1}^2 \\ &\leq \sum_{k=1}^n ||(\int_0^1 f(.,s)\phi_k(s)ds)||_{L^1}^2 \\ &\leq K_G^2 ||f||_{L^1}^2 ||(\sum_{k=1}^n |\phi_k|^2)^{1/2}||_\infty^2. \end{aligned}$$

This shows that $\pi_2[T_j] \leq K_G ||f||_{L^1}$. ■

Example 2.5 *Let $u \in h^2(\mathbb{D})$, i.e. a harmonic function on the unit disc $\mathbb{D}$ such that $\sup_{0<r<1} \int_{-\pi}^{\pi} |u_r(e^{it})|^2 \frac{dt}{2\pi} < \infty$ where $u_r(e^{it}) = u(re^{it})$. Let us fix an increasing sequence r_n converging to 1 and define $T_n : L^1(\mathbb{T}) \to L^2(\mathbb{T})$ by $T_n(\psi) = \psi * u_{r_n}$. Then $(T_n) \in \ell_{\pi_1}(L^1(\mathbb{T}), L^2(\mathbb{T}))$.*

Proof. It is well known (see [13]) that $u_r = P_r * \phi$ for some $\phi \in L^2(\mathbb{T})$ where P_r stands for the Poisson kernel. Therefore $T_n(\psi) = \psi * \phi * P_{r_n}$.

Given $n \in \mathbb{N}$ and $\psi_1, \psi_2, ..., \psi_n$ we have, using now (1) for the operator $T : L^1(\mathbb{T}) \to L^2(\mathbb{T})$ given by $T(\psi) = \psi * \phi$,

$$\begin{aligned} \sum_{k=1}^n ||T_k(\psi_k)||_{L^2} &= \sum_{k=1}^n ||\psi_k * \phi * P_{r_n}||_{L^2} \\ &\leq \sum_{k=1}^n ||\psi_k * \phi||_{L^2} \\ &\leq K_G ||(\psi_k)||_{\ell_1^w(L^1)} ||\phi||_{L^2}. \end{aligned}$$

Therefore one gets $\pi_2[T_j] \leq K_G ||\phi||_{L^2} = K_G ||u||_{h^2}$. ■

3. General facts on p-summing multipliers.

Let us start with some simple observations to get examples of p-summing multipliers. Examples 2.4 and 2.5 fall under the following general principle whose proof is left to the reader.

Proposition 3.1 *Let X, Y and Z be Banach spaces and $1 \leq p < \infty$. If $T \in \Pi_p(X, Y)$ and $(S_n) \in \ell_\infty(\mathcal{L}(Y, Z))$ then $(S_n T) \in \ell_{\pi_p}(X, Z)$.*

Moreover $\pi_p[S_n T] \leq \pi_p[T] \sup_n ||S_n||$.

Example 2.3 is also a particular case of the following:

Proposition 3.2 *Let X, Y be Banach spaces and $1 \leq p < \infty$.*
Given $(y_{n,k}) \subset \ell_\infty(\mathbb{N} \times \mathbb{N}, Y)$ and $(x_k^*) \in \ell_1^w(X^*)$ let us consider $T_n = \sum_{k=1}^\infty x_k^* \otimes y_{n,k}$.
If $\sum_{k=1}^\infty ||x_k^*||(\sup_n ||y_{n,k}||) < \infty$ then $T_n \in \ell_{\pi_1}(X, Y)$.

Proof. Notice that

$$\begin{aligned}\sum_{n=1}^\infty ||T_n(x_n)|| &\leq \sum_{n=1}^\infty \sum_{k=1}^\infty |\langle x_k^*, x_n\rangle|||y_{n,k}|| \\ &= \sum_{k=1}^\infty \sum_{n=1}^\infty |\langle \frac{x_k^*}{||x_k^*||}, x_n\rangle|||x_k^*||||y_{n,k}|| \\ &\leq (\sup_{||x^*||=1} \sum_{n=1}^\infty |\langle x^*, x_n\rangle|) \sum_{k=1}^\infty ||x_k^*||(\sup_n ||y_{n,k}||).\end{aligned}$$

■

Lemma 3.3 *Let X be a Banach space, $n \in \mathbb{N}$, $x_1, x_2, ..., x_n \in X$ and $x_1^*, x_2^*, ..., x_n^* \in X^*$. Then*

$$\sum_{k=1}^n |\langle x_k^*, x_k\rangle| \leq (\sup_{||x^*||=1} \sum_{k=1}^n |\langle x_k, x^*\rangle|) \int_0^1 ||\sum_{k=1}^n x_k^* r_k(t)||dt.$$

Proof.

$$\begin{aligned}\sum_{k=1}^n |\langle x_k^*, x_k\rangle| &= \sup_{|\alpha_k|=1} |\sum_{k=1}^n \langle x_k^*, x_k\alpha_k\rangle| \\ &= \sup_{|\alpha_k|=1} |\int_0^1 \langle \sum_{k=1}^n x_k\alpha_k r_k(t), \sum_{k=1}^n x_k^* r_k(t)\rangle dt| \\ &\leq \sup_{|\alpha_k|=1} \sup_{t\in[0,1]} ||\sum_{k=1}^n x_k\alpha_k r_k(t)|| \int_0^1 ||\sum_{k=1}^n x_k^* r_k(t)||dt \\ &\leq (\sup_{||x^*||=1} \sum_{k=1}^n |\langle x_k, x^*\rangle|) \int_0^1 ||\sum_{k=1}^n x_k^* r_k(t)||dt.\end{aligned}$$

■

Proposition 3.4 *Let X and Y be Banach spaces. If $(T_n) \subset \mathcal{L}(X, Y))$ is such that*

$$\sup_{||x||=1} \sum_{k=1}^\infty ||T_k(x)|| < \infty$$

then $T_n \in \ell_{\pi_1}(X, Y)$. Moreover $\pi_1[T_n] \leq \sup_{||x||=1} \sum_{k=1}^\infty ||T_k(x)||$.

Proof. Given $n \in \mathbb{N}$ and $x_1, x_2, ..., x_n \in X$ we have, using Lemma 3.3,

$$\begin{aligned}
\sum_{k=1}^{n} ||T_k(x_k)|| &= \sup_{||y_k^*||=1} \sum_{k=1}^{n} |\langle T_k(x_k), y_k^*\rangle| \\
&= \sup_{||y_k^*||=1} \sum_{k=1}^{n} |\langle x_k, T_k^*(y_k^*)\rangle| \\
&\leq ||(x_n)||_{\ell_1^w(X)} \sup_{||y_k^*||=1} \int_0^1 ||\sum_{k=1}^{n} T_k^*(y_k^*) r_k(t)||dt \\
&\leq ||(x_n)||_{\ell_1^w(X)} \sup_{||y_k^*||=1} \sup_{t\in[0,1]} ||\sum_{k=1}^{n} T_k^*(y_k^*) r_k(t)|| \\
&\leq ||(x_n)||_{\ell_1^w(X)} \sup_{||y_k^*||=1, ||x||=1} \sum_{k=1}^{n} |\langle T_k^*(y_k^*), x\rangle| \\
&\leq ||(x_n)||_{\ell_1^w(X)} \sup_{||x||=1} \sum_{k=1}^{n} ||T_k(x)||.
\end{aligned}$$

■

Theorem 3.5 *Let X, Y and Z be Banach spaces and $1 \leq p < \infty$.*

i) If $(T_n) \in \ell_{\pi_p}(X,Y)$ and $(S_n) \in \ell_\infty(\mathcal{L}(Y,Z))$ then $(S_nT_n) \in \ell_{\pi_p}(X,Z)$. Moreover $\pi_p[S_nT_n] \leq \pi_p[T_n] \sup_n ||S_n||$.

ii) If $(S_n) \in \ell_1^w(\mathcal{L}(X,Y))$ and $(T_n) \in \ell_{\pi_p}(Y,Z)$ then $(T_nS_n) \in \ell_{\pi_p}(X,Z)$. Moreover $\pi_p[T_nS_n] \leq \pi_p[T_n]||(S_n)||_{\ell_1^w(\mathcal{L}(X,Y))}$.

iii) If $T \in \mathcal{L}(X,Y)$ and $(T_n) \in \ell_{\pi_p}(Y,Z)$ then $T_nT \in \ell_{\pi_p}(X,Z)$. Moreover $\pi_p[T_nT] \leq \pi_p[T_n]||T||$.

iv) If $T \in \Pi_2(X,Y)$ and $(T_n) \in \ell_{\pi_2}(Y,Z)$ then $T_nT \in \ell_{\pi_1}(X,Z)$. Moreover $\pi_1[T_nT] \leq \pi_2[T_n]\pi_2[T]$.

Proof. (i) Take $n \in \mathbb{N}$ and $x_1, x_2, ..., x_n \in X$. Then

$$\begin{aligned}
\sum_{k=1}^{n} ||S_kT_k(x_k)||^p &\leq \sum_{k=1}^{n} ||S_k||^p ||T_k(x_k)||^p \\
&\leq \sup_n ||S_n||^p \pi_p^p[T_n] ||(x_n)||^p_{\ell_p^w(X)}.
\end{aligned}$$

(ii) Take $n \in \mathbb{N}$ and $x_1, x_2, ..., x_n \in X$. Then

$$\begin{aligned}
(\sum_{k=1}^{n} ||T_kS_k(x_k)||^p)^{1/p} &\leq \pi_p[T_n] \sup_{||y^*||=1} (\sum_{k=1}^{n} |\langle S_k(x_k), y^*\rangle|^p)^{1/p} \\
&= \pi_p[T_n] \sup_{||y^*||=1} (\sum_{k=1}^{n} |\langle x_k, S_k^*(y^*)\rangle|^p)^{1/p}
\end{aligned}$$

$$
\begin{aligned}
&= \pi_p[T_n] \sup_{\|y^*\|=1} \sup_{\|(\alpha_n)\|_{\ell_{p'}}=1} |\sum_{k=1}^n \langle x_k\alpha_k, S_k^*(y^*)\rangle| \\
&= \pi_p[T_n] \sup_{\|y^*\|=1} \sup_{\|(\alpha_n)\|_{\ell_{p'}}=1} |\int_0^1 \langle \sum_{k=1}^n x_k\alpha_k r_k(t), \sum_{k=1}^n S_k^*(y^*) r_k(t)\rangle dt| \\
&\leq \pi_p[T_n] (\sup_{\|(\alpha'_n)\|_{\ell_{p'}}=1} \|\sum_{k=1}^n x_k\alpha'_k\|)(\sup_{\|y^*\|=1} \sup_{t\in[0,1]} \|\sum_{k=1}^n S_k^*(y^*) r_k(t)\|) \\
&= \pi_p[T_n] \|(x_k)\|_{\ell_p^w(X)} \sup_{\|y^*\|=1, \|x\|=1} \sum_{k=1}^n |\langle S_k^*(y^*), x\rangle| \\
&= \pi_p[T_n] \|(x_k)\|_{\ell_p^w(X)} \sup_{\|y^*\|=1, \|x\|=1} \sum_{k=1}^n |\langle y^*, S_k(x)\rangle| \\
&= \pi_p[T_n] \|(x_k)\|_{\ell_p^w(X)} \|(S_n)\|_{\ell_1^w(\mathcal{L}(X,Y))}.
\end{aligned}
$$

(iii) Take $n \in \mathbb{N}$ and $x_1, x_2, ..., x_n \in X$. Then

$$
\begin{aligned}
\sum_{k=1}^n \|T_k T(x_k)\|^p &\leq \pi_p^p[T_n] \sup_{\|z^*\|=1} \sum_{k=1}^n |\langle T(x_k), z^*\rangle|^p \\
&= \pi_p^p[T_n] \sup_{\|z^*\|=1} \sum_{k=1}^n |\langle x_k, T^*(z^*)\rangle|^p \\
&\leq \pi_p^p[T_n] \sup_{\|x^*\|=1} \sum_{k=1}^n |\langle x_k, x^*\rangle|^p.
\end{aligned}
$$

(iv) Given $(x_n) \in \ell_1^w(X)$ and $T \in \Pi_2(X, Y)$ then $T(x_n) = \alpha_n x'_n$ where $\alpha_n \in \ell_2$ and $x'_n \in \ell_2^w(X)$ and $\|(x'_n)\|_{\ell_2^w(X)} \leq \|(x_n)\|_{\ell_1^w(X)}^{1/2} \pi_2[T]$ and $\|(\alpha_n)\|_{\ell_2} \leq \|(x_n)\|_{\ell_1^w(X)}^{1/2}$ (see [11] page 53). Hence, for each $n \in \mathbb{N}$

$$
\begin{aligned}
\sum_{k=1}^n \|T_k(Tx_k)\| &= \sum_{k=1}^n \|T_k(x'_k)\| |\alpha_k)| \\
&\leq \pi_2[T_n] \|(x'_n)\|_{\ell_2^w(X)} \|(\alpha_n)\|_{\ell_2} \\
&\leq \pi_2[T_n] \pi_2[T] \|(x_n)\|_{\ell_1^w(X)}.
\end{aligned}
$$

■

Let us now prove the natural generalization of the fact that $T \in \Pi_p(X, Y)$ if and only if $T^{**} \in \Pi_p(X^{**}, Y^{**})$. We need the following lemma.

Lemma 3.6 *(see [1], Proposition 2.9) Let X be a Banach space, $1 \leq p < \infty$ and let (x_j^*) be a sequence in X^*. Then $(x_j^*) \in \ell_{\pi_{1,p}}(X, \mathbb{K})$ if and only if there exists $C > 0$ such that*

$$
\sum_{j=1}^n |\langle x_j^{**}, x_j^*\rangle| \leq C \sup_{\|x^*\|=1} \Big(\sum_{j=1}^n |\langle x_j^{**}, x^*\rangle|^p\Big)^{1/p}
$$

for every $x_1^{**}, \ldots, x_n^{**}$ in X^{**}.

Theorem 3.7 *Let X and Y be Banach spaces, $1 \leq p < \infty$ and let $(T_n) \in \mathcal{L}(X,Y)$. Then $(T_n) \in \ell_{\pi_p}(X,Y)$ if and only if $(T_n^{**}) \in \ell_{\pi_p}(X^{**},Y^{**})$.*

Proof. The only thing to show is that if $(T_n) \in \ell_{\pi_p}(X,Y)$ then $(T_n^{**}) \in \ell_{\pi_p}(X^{**},Y^{**})$.
We have to show that there exists $C > 0$ for which

$$\Big(\sum_{j=1}^{n} ||T_j^{**}(x_j^{**})||^p\Big)^{1/p} \leq C$$

for any $x_1^{**},\ldots,x_n^{**}$ in X^{**} such that $\sup_{||x^*||=1}\Big(\sum_{j=1}^{n} |< x_j^{**}, x^* >|^p\Big)^{1/p} = 1$.
Given $(y_j^*) \in \ell_{p'}(Y^*)$, Remark 2.2 shows that $(T_j^*(y_j^*)) \in \ell_{\pi_{1,p}}(X,\mathbb{K})$. Now Lemma 3.6 gives

$$\sum_{j=1}^{n} |\langle x_j^{**}, T_j^*(y_j^*)\rangle| = \sum_{j=1}^{n} |\langle T_j^{**}(x_j^{**}), y_j^*\rangle| \leq C.$$

Therefore the result is achieved from the duality $(\ell_{p'}(Y^*))^* = \ell_p(Y^{**})$. ■

4. Connections with other classes of operators and geometry of Banach spaces.

Regarding embeddings between the spaces, let us mention that for $1 \leq p \leq q < \infty$ one has $\ell_{\pi_p}(X,Y) \subset \ell_{\pi_q}(X,Y)$. The reader is referred to [1] for general embedding theorems. The next result generalizes the well known fact of the coincidence of the classes $\Pi_1(X,Y) = \Pi_2(X,Y)$ under the assumption of cotype 2 of X (see [11], Corollary 11.16). The following is essentially contained in Corollaries 3.12 and 3.13 in [1], but we include a proof here for completeness.

Theorem 4.1

i) If X has cotype 2 then $\ell_{\pi_1}(X,Y) = \ell_{\pi_2}(X,Y)$.

ii) If X has cotype $q > 2$ then $\ell_{\pi_1}(X,Y) = \ell_{\pi_p}(X,Y)$ for any $p < q'$.

Proof. (i) Let us take $(T_n) \in \ell_{\pi_2}(X,Y)$ and let $(x_n) \in \ell_1^w(X)$. According to the identification with $\mathcal{L}(c_0,X)$ we have that the sequence $x_n = u(e_n)$ for some $u \in \mathcal{L}(c_0,X)$. Using now the cotype 2 assumption we have $\mathcal{L}(c_0,X) = \Pi_2(c_0,X)$ (see [11],Theorem 11.14). Now, since $(e_n) \in \ell_1^w(c_0)$ and $u \in \Pi_2(c_0,X)$ then (see [11], Lemma 2.23) $u(e_n) = \alpha_n x_n'$ where $\alpha_n \in \ell_2$ and $x_n' \in \ell_2^w(X)$ and $||(x_n')||_{\ell_2^w(X)} \leq \pi_2[u]$ and $||(\alpha_n)||_{\ell_2} \leq 1$ Hence, for each $n \in \mathbb{N}$

$$\begin{aligned}
\sum_{k=1}^{n} ||T_k(x_k)|| &= \sum_{k=1}^{n} ||T_k(x_k')|||\alpha_k)| \\
&\leq \pi_2[T_n]||(x_n')||_{\ell_2^w(X)}||(\alpha_n)||_{\ell_2} \\
&\leq \pi_2[T_n]\pi_2[u] \\
&\leq K_G\pi_2[T_n]||(x_n)||_{\ell_1^w(X)}.
\end{aligned}$$

(ii) follows the same lines (using Theorem 11.14 and Lemma 2.23 in [11]) for $q > 2$. ■

Definition 4.2 *(see [11], page 234) Let X and Y be Banach spaces. A linear operator $T: X \to Y$ is said to be almost summing, to be denoted $T \in \Pi_{as}(X, Y)$, if there exists $C > 0$ such that*

$$\int_0^1 ||\sum_{j=1}^n T(x_j)r_j(t)||dt \leq C \sup_{||x^*||=1} (\sum_{j=1}^n |\langle x^*, x_j\rangle|^2)^{1/2}$$

for any finite family $x_1, x_2, \dots x_n$ of vectors in X.

The least of such constants is the as-summing norm of u, denoted by $\pi_{as}(u)$.

Let us now relate these operators with p-summing multipliers.

Theorem 4.3 *Let X and H be a Banach and a Hilbert space, respectively. If $(T_n) \subset \mathcal{L}(X, H)$ are such that $T_n^* \in \Pi_{as}(H, X^*)$ for all $n \in \mathbb{N}$ and*

$$\sup_n \int_0^1 \pi_{as}[\sum_{k=1}^n T_k^* r_k(s)]ds < \infty$$

then $T_n \in \ell_{\pi_1}(X, H)$.

Moreover $\pi_1[T_n] \leq \sup_n \int_0^1 \pi_{as}[\sum_{k=1}^n T_k^* r_k(s)]ds$.

Proof. Let $(x_n) \in \ell_1^w(X)$. Then

$$\begin{aligned}
\sum_{k=1}^n ||T_k(x_k)|| &= \sum_{k=1}^n (\sum_{j=1}^\infty |\langle T_k(x_k), e_j\rangle|^2)^{1/2} \\
&\leq C \sum_{k=1}^n \int_0^1 |\sum_{j=1}^\infty \langle T_k(x_k), e_j\rangle r_j(t)|dt \\
&\leq C \int_0^1 \sum_{k=1}^n |\langle x_k, \sum_{j=1}^\infty T_k^*(e_j) r_j(t)\rangle|dt.
\end{aligned}$$

First note that, since $(e_n) \in \ell_2^w(H)$, then $S \in \Pi_{as}(H, X^*)$ implies

$$\int_0^1 ||\sum_{j=1}^\infty S(e_j)r_j(t)||dt \leq \pi_{as}(S).$$

Now using Lemma 3.3 we get

$$\begin{aligned}
\sum_{k=1}^n ||T_k(x_k)|| &\leq C||(x_n)||_{\ell_1^w(X)} \int_0^1 (\int_0^1 ||\sum_{k=1}^n (\sum_{j=1}^\infty T_k^*(e_j)r_j(t))r_k(s)||ds)dt \\
&\leq C||(x_n)||_{\ell_1^w(X)} \int_0^1 (\int_0^1 ||(\sum_{k=1}^n T_k^* r_k(s))(\sum_{j=1}^\infty e_j r_j(t))||dt)ds \\
&= C||(x_n)||_{\ell_1^w(X)} \int_0^1 (\int_0^1 ||\sum_{j=1}^\infty (\sum_{k=1}^n T_k^* r_k(s))(e_j) r_j(t)||dt)ds \\
&\leq C||(x_n)||_{\ell_1^w(X)} \int_0^1 \pi_{as}[\sum_{k=1}^n T_k^* r_k(s)]ds.
\end{aligned}$$

■

Theorem 4.4 *Let $2 \leq q < \infty$, H be a Hilbert space and X be a Banach space with the Orlicz property (for $q = 2$) or cotype $q > 2$. If $(T_n) \subset \mathcal{L}(X, H)$ are such that $T_n^* \in \Pi_{as}(H, X^*)$ for all $n \in \mathbb{N}$ and*

$$\sup_n \int_0^1 (\sum_{k=1}^n ||T_k^*(\sum_{j=1}^\infty e_j r_j(t))||^{q'})^{1/q'} dt < \infty$$

then $(T_n) \in \ell_{\pi_1}(X, H)$.

Proof. Let $(x_n) \in \ell_1^w(X)$. Then for each $n \in \mathbb{N}$, the argument in Theorem 4.3 gives

$$\sum_{k=1}^n ||T_k(x_k)|| \leq C \int_0^1 \sum_{k=1}^n |\langle x_k, \sum_{j=1}^\infty T_k^*(e_j) r_j(t)\rangle dt|.$$

Now the assumption on X allows us to write

$$\begin{aligned} \sum_{k=1}^n ||T_k(x_k)|| &\leq C \int_0^1 \sum_{k=1}^n |\langle x_k, \sum_{j=1}^\infty T_k^*(e_j) r_j(t)\rangle dt| \\ &\leq C \int_0^1 (\sum_{k=1}^n ||x_k||^q)^{1/q} (\sum_{k=1}^n || \sum_{j=1}^\infty T_k^*(e_j) r_j(t)||^{q'})^{1/q'} dt \\ &\leq C ||(x_k)||_{\ell_1^w(X)} \int_0^1 (\sum_{k=1}^n || \sum_{j=1}^\infty T_k^*(e_j) r_j(t)||^{q'})^{1/q'} dt. \end{aligned}$$

■

Definition 4.5 *(see [3], [8]) Let X and Y be Banach spaces. A sequence $(T_j)_{j \in \mathbb{N}}$ of operators in $\mathcal{L}(X, Y)$ is called Rademacher bounded if there exists a constant $C > 0$ such that*

$$\int_0^1 || \sum_{k=1}^n T_k(x_k) r_k(t)|| dt \leq C \int_0^1 || \sum_{k=1}^n x_k r_k(t)|| dt$$

for any finite collection of vectors $x_1, x_2, \ldots x_n$ in X.

We use $Rad(X, Y)$ to denote the set of Rademacher bounded sequences, and $rad[T_j]$ is the least constant C for which (T_j) verifies the inequality in the definition.

Remark 4.1

i) If $T_n = T$ for all $n \in \mathbb{N}$ then $(T_n) \in Rad(X, Y)$.

ii) If $(T_n) \in Rad(X, Y)$ and $(x_n) \in \ell_1^w(X)$ then $(T_n(x_n)) \in \ell_2^w(X)$.

Let us mention the following simple observations whose proofs follow easily from the definitions.

Proposition 4.6 *Let X, Y be Banach spaces.*

i) If X has the Orlicz property (resp. cotype $q > 1$) then $\ell_2(\mathcal{L}(X, Y)) \subset \ell_{\pi_1}(X, Y)$ (resp. $\ell_{q'}(\mathcal{L}(X, Y)) \subset \ell_{\pi_1}(X, Y)$).

ii) If Y has type 2 then $\ell_{\pi_2}(X,Y) \subset Rad(X,Y)$.

iii) If X has cotype q, Y has type p and $1/r = (1/p) - (1/q)$ then $\ell_r(\mathcal{L}(X,Y)) \subset Rad(X,Y)$. In particular, if X has cotype 2 and Y has type 2 then $\ell_\infty(\mathcal{L}(X,Y)) = Rad(X,Y)$.

iv) If Z has cotype 2, $T \in \Pi_{as}(X,Y)$ and $(T_n) \in Rad(Y,Z)$ then $(T_nT) \in \ell_{\pi_2}(X,Z)$.

Proof. (i) Let $n \in \mathbb{N}$ and $x_1, x_2, ..., x_n$ in X. Then we have

$$\begin{aligned} \sum_{k=1}^n ||T_k(x_k)|| &\leq (\sum_{k=1}^n ||T_k||^2)^{1/2} (\sum_{k=1}^n ||x_k||^2)^{1/2} \\ &\leq C(\sum_{k=1}^n ||T_k||^2)^{1/2} ||(x_k)||_{\ell_1^w(X)}. \end{aligned}$$

Obvious modifications give the case $q > 2$.

(ii) Let $n \in \mathbb{N}$ and $x_1, x_2, ..., x_n$ in X. Then we have

$$\begin{aligned} \int_0^1 ||\sum_{k=1}^n T_k(x_k) r_k(t)||dt &\leq C(\sum_{k=1}^n ||T_k(x_k)||^2)^{1/2} \\ &\leq C\pi_2[T_n] ||(x_k)||_{\ell_2^w(X)} \\ &\leq C\pi_2[T_n] \int_0^1 ||\sum_{k=1}^n x_k r_k(t)||dt. \end{aligned}$$

(iii) Let $n \in \mathbb{N}$ and $x_1, x_2, ..., x_n$ in X. Then we have

$$\begin{aligned} \int_0^1 ||\sum_{k=1}^n T_k(x_k) r_k(t)||dt &\leq C(\sum_{k=1}^n ||T_k(x_k)||^p)^{1/p} \\ &\leq C(\sum_{k=1}^n ||T_k||^r)^{1/r} (\sum_{k=1}^n ||x_k||^q)^{1/q} \\ &\leq C(\sum_{k=1}^n ||T_k||^r)^{1/r} \int_0^1 ||\sum_{k=1}^n x_k r_k(t)||dt. \end{aligned}$$

(iv) Let $n \in \mathbb{N}$ and $x_1, x_2, ..., x_n$ in X. Then we have

$$\begin{aligned} (\sum_{k=1}^n ||T_k T(x_k)||^2)^{1/2} &\leq C \int_0^1 ||\sum_{k=1}^n T_k T(x_k) r_k(t)||dt \\ &\leq C rad[T_n] \int_0^1 ||\sum_{k=1}^n T(x_k) r_k(t)||dt \\ &\leq C rad[T_n] \pi_{as}[T] \sup_{||x^*||=1} (\sum_{k=1}^n |\langle x_k, x^* \rangle|^2)^{1/2}. \end{aligned}$$

■

We are now going to get the main results of this section. We need the following lemma.

Lemma 4.7 *(see [24], Theorem 6.6 and Corollary 6.7) If X is a GT-space of cotype 2 then there exists a constant $C > 0$ such that*

$$\sum_{k=1}^{n} |\langle x_k, x_k^* \rangle| \leq C(\int_0^1 ||\sum_{k=1}^{n} x_k r_k(t)||dt) \sup_{||x||=1} (\sum_{k=1}^{n} |\langle x_k^*, x\rangle|^2)^{1/2}. \quad (5)$$

If X^* is a GT-space of cotype 2 then there exists a constant $C > 0$ such that

$$\sum_{k=1}^{n} |\langle x_k^*, x_k \rangle| \leq C(\int_0^1 ||\sum_{k=1}^{n} x_k^* r_k(t)||dt) \sup_{||x^*||=1} (\sum_{k=1}^{n} |\langle x_k, x^*\rangle|^2)^{1/2}. \quad (6)$$

Theorem 4.8 *Let (Ω, Σ, μ) be measure space and X a Banach space. If $T_n \subset \mathcal{L}(L^1(\mu), X)$ are such that*

$$\sup_{||\phi||=1} \sum_{n=1}^{\infty} ||T_n(\phi)||^2 < \infty$$

then $(T_n) \in \ell_{\pi_1}(L^1(\mu), X)$.

Proof. Let $(\phi_n) \subset L^1(\mu)$. Since $L^1(\mu)$ is a GT-space of cotype 2, Lemma 4.7 gives

$$\begin{aligned}
\sum_{k=1}^{n} ||T_k(\phi_k)|| &= \sup_{||x_n^*||=1} \sum_{k=1}^{n} |\langle T_k(\phi_k), x_k^*\rangle| \\
&= \sup_{||x_n^*||=1} \sum_{k=1}^{n} |\langle \phi_k, T_k^*(x_k^*)\rangle| \\
&\leq C(\int_0^1 ||\sum_{k=1}^{n} \phi_k r_k(t)||dt) \sup_{||x_n^*||=1} \sup_{||x||=1} (\sum_{k=1}^{n} |\langle T_k^*(x_k^*), x\rangle|^2)^{1/2} \\
&\leq C(\int_0^1 ||\sum_{k=1}^{n} \phi_k r_k(t)||dt) \sup_{||x||=1, ||x_n^*||=1} (\sum_{k=1}^{n} |\langle x_k^*, T_k(x)\rangle|^2)^{1/2} \\
&\leq C(\sup_{t\in[0,1]} ||\sum_{k=1}^{n} \phi_k r_k(t)||) \sup_{||x||=1} (\sum_{k=1}^{n} ||T_k(x)||^2)^{1/2} \\
&\leq C||(\phi_n)||_{\ell_1^w(L^1)} \sup_{||x||=1} (\sum_{k=1}^{n} ||T_k(x)||^2)^{1/2}.
\end{aligned}$$

■

Theorem 4.9 *Let X^* be a GT-space of cotype 2 and let Y^* have type 2. If $T_n \in \mathcal{L}(X, Y)$ and $(T_n^*) \in Rad(Y^*, X^*)$ then $(T_n) \in \ell_{\pi_2}(X, Y)$.*

In particular if $T_n : c_0 \to \ell_q$ for $q \geq 2$ and $(T_n^*) \in Rad(\ell_{q'}, \ell_1)$ then $T_n \in \ell_{\pi_2}(c_0, \ell_q)$.

Proof. Let $(x_n) \in \ell_2^w(X)$. Using Lemma 4.7 for X^*, one gets

$$(\sum_{k=1}^{n} ||T_k(x_k)||^2)^{1/2} = \sup_{||(y_n^*)||_{\ell_2(Y^*)}=1} \sum_{k=1}^{n} |\langle T_k(x_k), y_k^*\rangle|$$

$$
\begin{aligned}
&= \sup_{||(y_n^*)||_{\ell_2(Y^*)}=1} \sum_{k=1}^{n} |\langle x_k, T_k^*(y_k^*)\rangle| \\
&\leq C \sup_{||(y_n^*)||_{\ell_2(Y^*)}=1} (\int_0^1 || \sum_{k=1}^{n} T_k^*(y_k^*) r_k(t) || dt) \sup_{||x^*||=1} (\sum_{k=1}^{n} |\langle x_k, x^*\rangle|^2)^{1/2} \\
&\leq C||(x_n)||_{\ell_2^w(X)} rad[T_n^*] \sup_{||(y_n^*)||_{\ell_2(Y^*)}=1} (\int_0^1 || \sum_{k=1}^{n} y_k^* r_k(t) || dt) \\
&\leq C||(x_n)||_{\ell_2^w(X)} rad[T_n^*],
\end{aligned}
$$

where the last inequality follows from the type 2 condition on Y^*. ■

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