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REMARKS ON (q, p, Y)-SUMMING OPERATORS

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Abstract. An operator $T \in \Pi^Y_{q,p}(X\check{\otimes}Y, Z)$ if there exists a constant $C > 0$ such that, for any finite sequence $u_1, u_2, ..., u_N$ in $X \otimes Y$, we have

$$\Big(\sum_{k=1}^{N} \|T(u_k)\|_Z^q\Big)^{\frac{1}{q}} \leq C \sup_{x^* \in B_{X^*}} \Big\{ \Big(\sum_{k=1}^{N} \|u_k(x^*)\|_Y^p\Big)^{\frac{1}{p}} \Big\}.$$

It is shown that if $T \in \Pi^Y_p(X\check{\otimes}Y, Z)$ and $S \in \Pi_{s,t}(Z, W)$ then the operator $ST \in \Pi^Y_{r,q}(X\check{\otimes}Y, W)$ where $\frac{1}{r} = \frac{1}{p} + \frac{1}{s}$ and $\frac{1}{q} = \frac{1}{p} + \frac{1}{t}$.

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1. Introduction.

Let X, Y and Z be Banach spaces and let $1 \leq p \leq q < \infty$. An operator T from the injective tensor product $X\check{\otimes}Y$ into Z is said to be (q, p, Y)-summing if there exists a constant $C > 0$ such that, for any finite sequence $u_1, u_2, ..., u_N$ in $X \otimes Y$, we have

$$\Big(\sum_{k=1}^{N} \|T(u_k)\|_Z^q\Big)^{\frac{1}{q}} \leq C \sup_{x^* \in B_{X^*}} \Big\{ \Big(\sum_{k=1}^{N} \|u_k(x^*)\|_Y^p\Big)^{\frac{1}{p}} \Big\}$$

where $u_k(x^*) = \sum_{j=1}^{n_k} \langle x^*, x_{j,k}\rangle y_{j,k}$, for $u_k = \sum_{j=1}^{n_k} x_{j,k} \otimes y_{j,k}$, $y_{j,k} \in Y$ and $x_{j,k} \in X$.

The least of such constants is the (q, p, Y)-norm of T, denoted by $\pi^Y_{q,p}(T)$, and the space $\Pi^Y_{q,p}(X\check{\otimes}Y, Z)$ of all (q, p, Y)-summing operators is a Banach space endowed with such norm. In the case $q = p$ we simply write $\Pi^Y_p(X\check{\otimes}Y, Z)$ and $\pi^Y_p(T)$.

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Of course for $Y = \mathbb{K}$ we have $\Pi^{\mathbb{K}}_{q,p}(X\check{\otimes}\mathbb{K}, Z) = \Pi_{q,p}(X, Z)$. The reader is referred to [1], [2], [6], [10], [11] or [15] for definitions and results about these classes and their applications in Banach space theory.

The notion of (q,p,Y)-summing operator was introduced and studied by Kislyakov in [5]. Among other things he proved that (p,Y)-summing operators verify the following analogue to Pietsch's domination theorem.

Theorem 1.1. (See [5].) *Let $1 \le p < \infty$ and let X, Y and Z be Banach spaces. An operator $T : X\check{\otimes}Y \to Z$ is (p,Y)-summing if and only if there are a probability measure μ on (B_{X^*}, w^*) and a constant $C > 0$ such that for all $u \in X \otimes Y$ one has*

$$\|T(u)\|_Z^p \le C^p \int_{B_{X^*}} \|u(x^*)\|_Y^p d\mu(x^*).$$

Moreover, $\pi_p^Y(T)$ is the least of the constants verifying the previous estimate.

Recall that an operator $T : C(\Omega, X) \to Y$, where Ω is a compact Haussdorf space, is called p-dominated operator (see [3], III.19.3) if there exist a constant $C > 0$ and a probability measure μ on Ω such that

$$\|T(f)\|^p \le C \int_\Omega \|f(t)\|^p d\mu(t)$$

for all $f \in C(\Omega, X)$. For infinite dimensional Banach spaces C. Swartz (see [13]) showed that absolutely summing operators $T : C(\Omega, X) \to Y$ are always 1-dominated, but the space of 1-dominated operators from $C(\Omega, X)$ into Y coincides with $\Pi_1(C(\Omega, X), Y)$ if and only if X is finite dimensional.

Since $C(\Omega)\check{\otimes}X = C(\Omega, X)$, Theorem 1.1 implies that the class of p-dominated operators actually coincides with $\Pi_p^X(C(\Omega)\check{\otimes}X, Y)$.

Let us first point out that always we have $\Pi_{q,p}(X\check{\otimes}Y, Z) \subseteq \Pi^Y_{q,p}(X\check{\otimes}Y, Z)$.

Indeed, since, for $u_1, u_2, ..., u_N \in X\check{\otimes}Y$, we have

$$\|(u_k)\|_{\ell_p^w(X\check{\otimes}Y)} = \sup\Big\{ \Big(\sum_{k=1}^N |\langle x^* \otimes y^*, u_k\rangle|^p\Big)^{\frac{1}{p}} : \ x^* \in B_{X^*},\ y^* \in B_{Y^*}\Big\},$$

and $\langle u(x^*), y^*\rangle = \sum\langle x^*, x_j\rangle\langle y^*, y_j\rangle = \langle x^* \otimes y^*, u\rangle$ for any tensor $u = \sum x_j \otimes y_j$ in $X \otimes Y$. Hence, if $u_1, u_2, ..., u_N \in X \otimes Y$ we get

$$\|(u_k)\|_{\ell_p^w(X\check{\otimes}Y)} \le \sup_{x^* \in B_{X^*}} \Big\{ \Big(\sum_{j=1}^n \|u_j(x^*)\|_Y^p\Big)^{\frac{1}{p}}\Big\}.$$

Consequently, we have the following inclusion

$$\Pi_{q,p}(X\check{\otimes}Y, Z) \subseteq \Pi^Y_{q,p}(X\check{\otimes}Y, Z).$$

Proposition 1.2. *Let X, Y and Z be Banach spaces and let $1 \le p \le q < \infty$.*

If $\Pi^Y_{q,p}(X\check{\otimes}Y, Z) = \Pi_{q,p}(X\check{\otimes}Y, Z)$ then $\mathcal{L}(Y, Z) = \Pi_{q,p}(Y, Z)$.

In particular, $\Pi_p^Y(X\check{\otimes}Y, Y) = \Pi_p(X\check{\otimes}Y, Y)$ if and only if $dim(Y) < \infty$.

Proof. Assume $\Pi^Y_{q,p}(X\check{\otimes}Y,Z)=\Pi_{q,p}(X\check{\otimes}Y,Z)$ and let take $A\in\mathcal{L}(Y,Z)$. Fix $x_0^*\in X^*$ and consider $T_{x_0^*,A}:X\check{\otimes}Y\to Z$, given by $T_{x_0^*,A}(u)=A(\langle u,x_0^*\rangle)$.

Clearly it is $(1,Y)$-summing (in particular (q,p,Y)-summing). By assumption $T_{x_0^*,A}\in\Pi_{q,p}(X\check{\otimes}Y,Z)$.

Let $(y_j)_{j=1}^\infty\in\ell_p^w(Y)$ and $x_0\in B_X$ with $\langle x_0^*,x_0\rangle\neq 0$, then

$$T_{x_0^*,A}((y_j\otimes x_0))=A(\langle x_0^*,x_0\rangle y_j)=\langle x_0^*,x_0\rangle A(y_j)\in\ell_q(Z).$$

This shows that $A\in\Pi_{q,p}(Y,Z)$. □

COROLLARY 1.3. ([13]) *Let X and Y be Banach spaces. Then $\Pi_1^X(C(\Omega,X),Y)=\Pi_1(C(\Omega,X),Y)$ if and only if X is finite dimensional.*

As in the case of (q,p)-summing operators the following inclusion

$$\Pi^Y_{q_1,p_1}(X\check{\otimes}Y,Z)\subseteq\Pi^Y_{q_2,p_2}(X\check{\otimes}Y,Z)$$

holds if $1\leq p_2\leq p_1\leq q_1\leq q_2<\infty$, or if $p_1\leq p_2$, $q_1\leq q_2$ and $\frac{1}{p_1}-\frac{1}{q_1}\leq\frac{1}{p_2}-\frac{1}{q_2}$. The proof of this is analogous to the classical case.

Moreover, the classes coincides, at least for certain values of q_1, p_1, q_2, p_2, under some assumptions on the Banach spaces. Kislyakov proves in [5], Theorem 1.2.3, that if Y has type 2 and Z has cotype 2, then

$$\Pi^Y_p(X\check{\otimes}Y,Z)=\Pi^Y_2(X\check{\otimes}Y,Z)$$

for every $2<p<\infty$. He also prove in [5], Theorem 1.3.2, under the same assumptions, the following version of Grothendieck's theorem:

$$\Pi^Y_2(C(\Omega)\check{\otimes}Y,Z)=\mathcal{L}(C(\Omega)\check{\otimes}Y,Z).$$

Let us mention that p-summing operators acting on $X\check{\otimes}Y$ have been considered by several authors (see [9], [12]). The following map plays an important role: For each bounded operator $T:X\check{\otimes}Y\to Z$, one can consider $\Phi(T)=T^\#:Y\to\mathcal{L}(X,Z)$ defined by, $T^\#(y)(x)=T(x\otimes y)$, for $x\in X$ and $y\in Y$. This is clearly a bounded operator. So

$$\Phi(\mathcal{L}(X\check{\otimes}Y,Z))\subseteq\mathcal{L}(Y,\mathcal{L}(X,Z)).$$

A natural problem to study is the connection between the operator T and $T^\#$ for different classes of operator ideals. In [9] it was shown that if $T:X\check{\otimes}Y\to Z$ is p-summing, then $T^\#:Y\to\Pi_p(X,Z)$ is also p-summing, that is

$$\Phi(\Pi_p(X\check{\otimes}Y,Z))\subset\Pi_p(Y,\Pi_p(X,Z)).$$

When $p=1$, the reverse implication holds also true for $Y=C(K)$ (see [13] or if Y is a $\mathcal{L}_\infty$-space see [9]), that is

$$\Phi(\Pi_1(X\check{\otimes}Y,Z))=\Pi_1(Y,\Pi_1(X,Z))\text{ for }\mathcal{L}_\infty\text{-spaces }Y.$$

Next we are going to investigate the relation between T and $T^\#$ when T is (q,p,Y)-summing.

PROPOSITION 1.4. *Let X, Y and Z be Banach spaces and $1 \leq p \leq q < \infty$.*
(i) $\Phi(\Pi^Y_{q,p}(X\check{\otimes}Y,Z)) \subseteq \mathcal{L}(Y,\Pi_{q,p}(X,Z))$.
(ii) If $\Phi(\Pi^Y_{q,p}(X\check{\otimes}Y,Z)) \subseteq \Pi_{q,p}(Y,\mathcal{L}(X,Z))$ then $\mathcal{L}(Y,Z) = \Pi_{q,p}(Y,Z)$.
In particular, if $\Phi(\Pi^Y_p(X\check{\otimes}Y,Y)) \subseteq \Pi_p(Y,\mathcal{L}(X,Y))$ for some $1 \leq p < \infty$ then $dim(Y) < \infty$.

Proof. (i) Let $T : X\check{\otimes}Y \to Z$ be a (q,p,Y)-summing operator. We only have to show that $T^\#(y) \in \Pi_{q,p}(X,Z)$ for every $y \in Y$. Let $x_1,\ldots,x_n \in X$ and $y \in Y$, then

$$\begin{aligned}\Big(\sum_{j=1}^n \|T^\#(y)(x_j)\|^q_Z\Big)^{\frac{1}{q}} &= \Big(\sum_{j=1}^n \|T(x_j\otimes y)\|^q_Z\Big)^{\frac{1}{q}}\\ &\leq \pi^Y_{q,p}(T)\cdot \sup_{x^*\in B_{X^*}}\Big\{\sum_{j=1}^n \|\langle x^*,x_j\rangle y\|^p_Y\Big\}^{\frac{1}{p}}\\ &= \|y\|_Y\cdot\pi^Y_{q,p}(T)\cdot \sup_{x^*\in B_{X^*}}\Big\{\sum_{j=1}^n |\langle x^*,x_j\rangle|^p\Big\}^{\frac{1}{p}}.\end{aligned}$$

Hence $T^\#(y) \in \Pi_{q,p}(X,Z)$ with $\pi_{q,p}(T^\#(y)) \leq \|y\|_Y\cdot\pi^Y_{q,p}(T)$.

(ii) Let $x_0 \in X$ and $x^*_0 \in X^*$ such that $\|x_0\| = 1$ and $\langle x_0,x^*_0\rangle = \|x^*_0\| = 1$. For each $A \in \mathcal{L}(Y,Z)$ we consider the operator $T_{x^*_0,A} : X\check{\otimes}Y \to Y$, $T_{x^*_0,A}(u) = A(\langle u,x^*_0\rangle)$, which is $(1,Y)$-summing (and also (q,p,Y)-summing), then $T^\#_{x^*_0,A} \in \Pi_{q,p}(Y,\mathcal{L}(X,Y))$. Therefore

$$\begin{aligned}\pi^Y_{q,p}(T^\#_{x^*_0,A})\cdot\sup_{y^*\in B_{Y^*}}\Big(\sum_{j=1}^\infty |\langle y_j,y^*\rangle|^p\Big)^{\frac{1}{p}} &\geq \Big(\sum_{j=1}^\infty \|T^\#_{x^*_0,A}(y_j)\|^q_{\mathcal{L}(X,Y)}\Big)^{\frac{1}{q}}\\ &\geq \Big(\sum_{j=1}^\infty \|T_{x^*_0,A}(x_0\otimes y_j)\|^q_Y\Big)^{\frac{1}{q}}\\ &= \Big(\sum_{j=1}^\infty \|A(y_j)\|^q_Y\Big)^{\frac{1}{q}}.\end{aligned}$$

This gives the result. □

2. Composition of (p,Y)-summing operators. The classical theorem of Pietsch stated that if T is p-summing and S is q-summing then ST is r-summing, with $r = \min\{1,\frac{1}{p}+\frac{1}{q}\}$ (see [2], Theorem 2.22 or [4], Theorem 19.10.3). This result was generalized by N. Tomczak (see [14]) who proved that if S is (s,t)-summing then ST is (r,q)-summing, where $\frac{1}{r} = \frac{1}{p}+\frac{1}{s} \leq 1$, $\frac{1}{q} = \frac{1}{p}+\frac{1}{t} \leq 1$. In this section we are going to generalize Tomczak's result for (p,Y)-summing operators.

LEMMA 2.1. *Let $1 \leq p < \infty$ and let $T : X\check{\otimes}Y \longrightarrow Z$ be a (p,Y)-summing operator. There exist a probability measure μ on (B_{X^*},w^*) such that for any*

$z^* \in Z^*$ there exists a non-negative function $f_{z^*} \in L_{p'}(B_{X^*},\mu)$ verifying

$$|\langle z^*, T(u)\rangle| \leq \int_{B_{X^*}} \|\langle u, x^*\rangle\|_Y f_{z^*}(x^*) d\mu(x^*)$$

for all $u \in X\check{\otimes}Y$ and $\|f_{z^*}\|_{L_{p'}} \leq \pi_p^Y(T)\|z^*\|$.

Proof. Since T is (p,Y)-summing, from Theorem 1.1 we can find probability measure μ on (B_{X^*},w^*), a closed subspace $X_p(Y)$ of $L_p(\mu,Y)$ and an operator $\tilde{T} \in \mathcal{L}(X_p(Y),Z)$, such that $\tilde{T}j_p i_{X\check{\otimes}Y} = T$ and $\|\tilde{T}\| = \pi_p^Y(T)$, where the operator $i_{X\check{\otimes}Y} : X\check{\otimes}Y \to C(B_{X^*})\check{\otimes}Y$ is the isometric embedding defined by $i_{X\check{\otimes}Y}(\sum x_j \otimes y_j) = \sum i_x(x_j)\otimes y_j$, i_X is the natural embedding of X into $C(B_{X^*})$, and j_p is the restriction to $i_{X\check{\otimes}Y}(X\check{\otimes}Y)$ of the inclusion $j_p : C(B_{X^*},Y) \to L_p(\mu,Y)$.

Moreover, for all $u \in X\check{\otimes}Y$,

$$\begin{aligned} |\langle \tilde{T}^*(z^*), j_p i_{X\check{\otimes}Y}(u)\rangle| &= |\langle z^*, \tilde{T}j_p i_{X\check{\otimes}Y}(u)\rangle| \\ &\leq \|z^*\|_{Z^*}\|\tilde{T}\|\|j_p i_{X\check{\otimes}Y}(u)\|_{L_p(\mu,Y)} \\ &= \pi_p^Y(T)\|z^*\|_{Z^*}\|j_p i_{X\check{\otimes}Y}(u)\|_{L_p(\mu,Y)}. \end{aligned}$$

That is, $\tilde{T}^*(z^*) \in (X_p(Y))^*$ with $\|\tilde{T}^*(z^*)\| \leq \pi_q^Y(T)\|z^*\|_{Z^*}$. Then, by Hahn-Banach extension theorem and the duality $(L_p(\mu,Y))^* = V^{p'}(\mu,Y^*)$ (see for instance [3]), we can find a vector valued measure $F_{z^*} : \mathfrak{B} \to Y^*$ with p'-bounded variation such that $|F_{z^*}|_{p'} \leq \pi_p^Y(T)\|z^*\|_{Z^*}$ and $F_{z^*}|_{(X_p(Y))^*} = \tilde{T}^*(z^*)$. Then, for all $u \in X\check{\otimes}Y$, we have

$$\langle z^*, T(u)\rangle = \langle \tilde{T}^*(z^*), j_p i_{X\check{\otimes}Y}(u)\rangle = \langle F_{z^*}, j_p i_{X\check{\otimes}Y}(u)\rangle = \int_{B_{X^*}} \langle u, x^*\rangle \; dF_{z^*}(x^*)$$

and then

$$|\langle z^*, T(u)\rangle| \leq \int_{B_{X^*}} \|\langle u, x^*\rangle\|_Y d|F_{z^*}|(x^*).$$

On the other hand, it is known that there exists a non-negative function $f_{z^*} \in L_{p'}(B_{X^*},\mu)$ with

$$|F_{z^*}|(E) = \int_E f_{z^*} d\mu$$

for all $E \in \mathfrak{B}$ and $\|f_{z^*}\|_{L_{p'}} = |F_{z^*}|_{p'} \leq \pi_p^Y(T)\|z^*\|_{Z^*}$. Therefore

$$|\langle z^*, T(u)\rangle| \leq \int_{B_{X^*}} \|\langle u, x^*\rangle\|_Y f_{z^*}(x^*) d\mu(x^*).$$

This finishes the proof. □

Theorem 2.2. *Let X, Y, Z and W be Banach spaces, $T \in \Pi_p^Y(X\check{\otimes}Y,Z)$ and $S \in \Pi_{s,t}(Z,W)$. Then the operator $ST : X\check{\otimes}Y \longrightarrow W$ is (r,q,Y)-summing where*

$$\frac{1}{r} = \frac{1}{p} + \frac{1}{s} \leq 1, \quad \frac{1}{q} = \frac{1}{p} + \frac{1}{t} \leq 1$$

and $\pi_{r,q}^Y(ST) \leq \pi_{s,t}(S)\cdot \pi_p^Y(T)$.

Proof. Let $d\mu(x^*)$ be associated to T as in Lemma 2.1. Let $(u_i)_{i=1}^n$ be a finite sequence of elements in the space $X\check{\otimes}Y$ and set $u_i = \sigma_i v_i$ where $\sigma_i = (\int_{B_{X^*}} \|\langle u_i, x^*\rangle\|_Y^q d\mu(x^*))^{1/p}$. Then, by Hölder's inequality, we have

$$\begin{aligned}\Big(\sum_{i=1}^n \|ST(u_i)\|_W^r\Big)^{\frac{1}{r}} &\leq \Big(\sum_{i=1}^n |\sigma_i|^p\Big)^{\frac{1}{p}} \Big(\sum_{i=1}^n \|ST(v_i)\|_W^s\Big)^{\frac{1}{s}} \\ &\leq \pi_{s,t}(S)\|(\sigma_i)_{i=1}^n\|_{\ell_p^n} \sup_{z^*\in B_{Z^*}} \Big(\sum_{i=1}^n |\langle z^*, T(v_i)\rangle|^t\Big)^{\frac{1}{t}} \qquad (2.1)\end{aligned}$$

We have to estimate the latter expression. Observe that

$$\|(\sigma_i)\|_{\ell_p^n} = \Big(\int_{B_{X^*}} \sum_{i=1}^n \|\langle u_i, x^*\rangle\|_Y^q d\mu(x^*)\Big)^{\frac{1}{p}} \leq \sup_{x^*\in B_{X^*}} \Big(\sum_{i=1}^n \|\langle u_i, x^*\rangle\|_Y^q\Big)^{\frac{1}{p}}.$$

On the other hand, since T is (p, Y)-summing, by Lemma 2.1 we have

$$|\langle z^*, T(v_i)\rangle| \leq \int_{B_{X^*}} \|\langle v_i, x^*\rangle\|_Y f_{z^*}(x^*) d\mu(x^*).$$

Hence we observe that

$$|\langle z^*, T(v_i)\rangle| \leq \sigma_i^{-1} \int_{B_{X^*}} \|\langle u_i, x^*\rangle\|_Y^{q/p} \|\langle u_i, x^*\rangle\|_Y^{q/t} |f_{z^*}(x^*)|^{p'/t} |f_{z^*}(x^*)|^{p'/q'} d\mu(x^*)$$

and using Hölder's inequality twice

$$\begin{aligned}&\leq \sigma_i^{-1}\sigma_i\Big(\int_{B_{X^*}} (\|\langle u_i, x^*\rangle\|_Y^q |f_{z^*}(x^*)|^{p'})^{p'/t} (|f_{z^*}(x^*)|^{p'})^{p'/q'} d\mu(x^*)\Big)^{\frac{1}{p'}} \\ &\leq \Big(\int_{B_{X^*}} \|\langle u_i, x^*\rangle\|_Y^q |f_{z^*}(x^*)|^{p'} d\mu(x^*)\Big)^{\frac{1}{t}} \|f_{z^*}\|_{L_{p'}}^{\frac{p'}{q'}}.\end{aligned}$$

Then

$$|\langle z^*, T(v_i)\rangle| \leq \|f_{z^*}\|_{L_{p'}}^{\frac{p'}{q'}} \Big(\int_{B_{X^*}} \|\langle u_i, x^*\rangle\|_Y^q |f_{z^*}(x^*)|^{p'} d\mu(x^*)\Big)^{\frac{1}{t}}.$$

Summing up over $i = 1, \ldots, n$, we get

$$\begin{aligned}\Big(\sum_{i=1}^n |\langle z^*, T(v_i)\rangle|^t\Big)^{\frac{1}{t}} &\leq \|f_{z^*}\|_{L_{p'}}^{\frac{p'}{q'}} \Big(\int_{B_{X^*}} \sum_{i=1}^n \|\langle u_i, x^*\rangle\|_Y^q |f_{z^*}(x^*)|^{p'} d\mu(x^*)\Big)^{\frac{1}{t}} \\ &\leq \|f_{z^*}\|_{L_{p'}}^{\frac{p'}{q'}} \|f_{z^*}\|_{L_{p'}}^{\frac{p'}{t}} \sup_{x^*\in B_{X^*}} \Big(\sum_{i=1}^n \|\langle u_i, x^*\rangle\|^q\Big)^{\frac{1}{t}} \\ &\leq \|f_{z^*}\|_{L_{p'}} \sup_{x^*\in B_{X^*}} \Big(\sum_{i=1}^n \|\langle u_i, x^*\rangle\|^q\Big)^{\frac{1}{t}} \\ &\leq \pi_p^Y(T)\|z^*\|_{Z^*} \sup_{x^*\in B_{X^*}} \Big(\sum_{i=1}^n \|\langle u_i, x^*\rangle\|^q\Big)^{\frac{1}{t}}.\end{aligned}$$

Applying this inequality to the right hand side of (2.1), we get

$$\Big(\sum_{i=1}^{n} \|ST(u_i)\|_W^r\Big)^{\frac{1}{r}} \leq \pi_{s,t}(S)\pi_p^Y(S) \sup_{x^* \in B_{X^*}} \Big(\sum_{i=1}^{n} \|\langle u_i, x^*\rangle\|^q\Big)^{\frac{1}{q}}.$$

□

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