Acceleration radiation and the Planck scale

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A uniformly accelerating observer perceives the Minkowski vacuum state as a thermal bath of radiation. We point out that this field-theory effect can be derived, for any dimension higher than two, without actually invoking very high energy physics. This supports the view that this phenomenon is robust against Planck-scale physics and, therefore, should be compatible with any underlying microscopic theory.

I. INTRODUCTION

The fact that the notion of particles is ambiguous in a general curved spacetime plays a crucial role to derive gravitational particle production. In cosmology this was first exploited in [1], and for black holes in [2]. The expansion of a field in two different sets of positive frequency modes, $u_j^{\text{in}}(x)$ (usually defined at past infinity) and $u_j^{\text{out}}(x)$ (defined at future infinity), leads to a relation for the corresponding creation and annihilation operators, $a_i^{\text{out}} = \sum_j(\alpha_{ij}^* a_j^{\text{in}} - \beta_{ij}^* a_j^{\text{in}\dagger})$. When the coefficients β_{ij} do not vanish the vacuum states $|\text{in}\rangle$ and $|\text{out}\rangle$ do not coincide and, therefore, the number of particles measured in the ith mode by an "out" observer in the state $|\text{in}\rangle$ is given by $\langle\text{in}|N_i^{\text{out}}|\text{in}\rangle = \sum_k|\beta_{ik}|^2$. This general framework leads to two important predictions: particle creation in an expanding universe and in a gravitational collapse.

However, even in Minkowski space, the existence of two inequivalent quantizations leading to different concepts of particles was first pointed out in [3], continued in [4], and crucially understood in terms of particle detectors in [5]. In short, the standard Minkowski vacuum state is perceived by an accelerated observer as a thermal bath of particles at the temperature $T = \frac{\hbar a}{2\pi c k_B}$, where a is the acceleration. This effect shares some physical and mathematical aspects

with the one discovered by Hawking (for a complete account see [6]). However, they are indeed distinct. For instance, in the Hawking effect there is an out-going thermal energy flux at future infinity, as perceived by an inertial observer there. In contrast, in the acceleration radiation there is no net energy flux at infinity. Only a thermal bath of radiation exists for the uniformly accelerated observer.

On the other hand, the derivation of the Hawking effect seems to invoke Planck-scale physics (see, for instance [7]). Any out-going Hawking quanta will have an exponentially increasing frequency when propagated backwards in time and measured by a free-falling observer. Accordingly, any microscopic structure of a quantum gravity theory could leave some imprint or signal in the spectrum of radiation. However, the results of string theory agree with Hawking's prediction (for low emission frequencies) [8]. The string theory microscopic description of black hole emission can also be extended to the superradiance regime of some extremal rotating black holes [9], which can be obtained at the semiclassical level as a limiting case $T \to 0$ of the general Hawking radiation formula [2]. All the above suggests that a quantum gravity theory, with new degrees of freedom at the Planck scale, should not necessarily modify the bulk of the semiclassical effects, at least for low emission frequencies. This is essentially the conclusion of the investigation of [10], which argued, within a purely field-theory framework, that ultrashort distances do not significantly contribute to the Hawking effect, if Lorentz invariance is somehow respected. Only at high emission frequencies could an

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underlying theory of quantum gravity potentially predict important deviations from semiclassical results.

In [10] we derived (gravitational) particle production rates in terms of the two-point function of the matter field. Within this new approach we have studied the role of the Planck scale in the spectrum of Hawking radiation. The aim of this paper is to extend our analysis focusing now on the acceleration radiation. We find that the acceleration radiation is more insensitive to trans-Planckian physics than is the Hawking radiation, at least in four dimensions.

In Sec. II we briefly review the basics of the acceleration radiation. In Sec. III we show how the acceleration radiation (Fulling-Davies-Unruh effect) can be obtained easily in terms of correlation functions. Section IV is devoted to a discussion of the role of Lorentz invariance in generating the Planckian spectrum. We point out that this effect can be derived, for any dimension higher than two, without actually invoking very high energy physics. This suggests that this phenomenon is robust against Planck-scale physics and, therefore, that it should persist when microscopic degrees of freedom are considered.

II. ACCELERATION RADIATION

We shall present in this section the main physical aspects of the standard derivation of the acceleration radiation. Let us consider an observer with a uniformly accelerated trajectory (from now on we take $c = 1$), known as a Rindler observer,

$$T = \frac{e^{a\xi}}{a}\sinh at, \qquad X = \frac{e^{a\xi}}{a}\cosh at,$$
$$Y = Y_0, \qquad Z = Z_0,$$
(1)

and a massless scalar field propagating in the Minkowskian background spacetime. The wave equation $\Box \phi(x) = 0$ in the coordinates of the accelerated observer becomes

$$(e^{-2a\xi}(-\partial_t^2 + \partial_\xi^2) + \partial_Y^2 + \partial_Z^2)\phi(t, \xi, Y, Z) = 0. \quad (2)$$

The Y, Z dependence can be trivially integrated using plane waves $\phi(t, \xi, Y, Z) = \phi(t, \xi)e^{ik_Y Y}e^{ik_Z Z}$. Introducing this ansatz in the equation, we find

$$[(-\partial_t^2 + \partial_\xi^2) - e^{2a\xi}(k_Y^2 + k_Z^2)]\phi(t, \xi) = 0. \quad (3)$$

This equation indicates that the free scalar field of the Minkowski observer appears like a scalar field in a repulsive potential $V(\xi) \propto e^{2a\xi}\vec{k}_\perp^2$, where $\vec{k}_\perp^2 = k_Y^2 + k_Z^2$, for the uniformly accelerated observer. The exact form of the normalized modes, with natural support on the accessible region for the accelerated observer (right-hand Rindler wedge), can be expressed as

$$u_{w,\vec{k}_\perp}^R = \frac{e^{-iwt}}{2\pi^2\sqrt{a}}\sinh^{1/2}\left(\frac{\pi w}{a}\right)K_{iw/a}\left(\frac{|\vec{k}_\perp|}{a\hbar}e^{a\xi}\right)e^{i\vec{k}_\perp\cdot\vec{X}_\perp}. \quad (4)$$

The important point is that the above positive frequency modes cannot be expanded in terms of the standard purely positive frequency modes of the inertial observer

$$u_{k_X,\vec{k}_\perp}^M = \frac{1}{\sqrt{2(2\pi)^3 k_0}}e^{-ik_0 T + i(k_X X + \vec{k}_\perp\cdot\vec{X}_\perp)}, \quad (5)$$

where $k_0 = \sqrt{k_X^2 + \vec{k}_\perp^2}$. The detailed analysis requires one to compute the corresponding Bogolubov coefficients. They are found to be [3,11]

$$\beta_{w\vec{k}_\perp, k'_X\vec{k}'_\perp} = -[2\pi a k'_0(e^{2\pi w/a} - 1)]^{-1/2}$$
$$\times \left(\frac{k'_0 + k'_X}{k'_0 - k'_X}\right)^{-iw/2a}\delta(\vec{k}_\perp - \vec{k}'_\perp). \quad (6)$$

The mean number of Rindler particles in the Minkowski vacuum is obtained as the integral

$$\int_{-\infty}^{+\infty} d\vec{k}' \beta_{w_1\vec{k}_\perp,\vec{k}'}\beta_{w_2\vec{k}_\perp,\vec{k}'}^*. \quad (7)$$

The integration in k'_X reduces to

$$\int_{-\infty}^{+\infty} dk'_X(2\pi a k'_0)^{-1}\left(\frac{k'_0 + k'_X}{k'_0 - k'_X}\right)^{-i(w_1 - w_2)/2a}$$
$$= \delta(w_1 - w_2), \quad (8)$$

and taking into account the remaining terms one easily gets

$$\int_{-\infty}^{+\infty} d\vec{k}' \beta_{w_1\vec{k}_{\perp 1},\vec{k}'}\beta_{w_2\vec{k}_{\perp 2},\vec{k}'}^* = \frac{1}{e^{2\pi w_1/a} - 1}\delta(w_1 - w_2)$$
$$\times \delta(\vec{k}_{\perp 1} - \vec{k}_{\perp 2}). \quad (9)$$

The final outcome becomes then extremely simple. A uniformly accelerated observer feels himself immersed in a thermal bath of radiation at temperature $k_B T = a\hbar/2\pi$.

This result is reinforced by Unruh's operationalism interpretation [5]. In short, the particle content of the vacuum perceived by an accelerated observer with motion $x = x(\tau)$ can be described by the response function $F(w)$ of an ideal quantum mechanical detector (see also [12])

$$F(w) = \int_{-\infty}^{+\infty} d\tau_1 \int_{-\infty}^{+\infty} d\tau_2 e^{-iw(\tau_1 - \tau_2)}\langle\phi(x(\tau_1))\phi(x(\tau_2))\rangle, \quad (10)$$

where

$$\langle\phi(x_1)\phi(x_2)\rangle = \frac{\hbar}{4\pi^2(-(T_1 - T_2 - i\epsilon)^2 + (X_1 - X_2)^2 + (Y_1 - Y_2)^2 + (Z_1 - Z_2)^2)}, \quad (11)$$

is the two-point function of the field evaluated in the Minkowski vacuum. For a uniformly accelerated trajectory the above response function, or better, the rate $\dot{F}(w)$ turns out to be

$$\dot{F}(w) = \int_{-\infty}^{+\infty} d\Delta\tau e^{-iw\Delta\tau} \langle \phi(x(\tau_1))\phi(x(\tau_2)) \rangle. \quad (12)$$

Performing the integral one obtains

$$\dot{F}(w) = \frac{1}{2\pi} \frac{\hbar w}{e^{2\pi w/a} - 1}. \quad (13)$$

It is important to note that in either approach the thermal spectrum seems to depend crucially on the validity of relativistic field theory on all scales. In the former, the intermediate integral (8) involves an unbounded integration in arbitrary large Minkowskian momentum k_X'. If one introduces an ultraviolet cutoff Λ for $|k_X'|$ in the above integral, which particularizes a given Lorentz frame, the resulting thermal spectrum is largely truncated. In the detector model approach, the role of high energy scale emerges in the evaluation of the integral (12), which crucially depends on the short-distance behavior of the Wightman function (11).

III. ACCELERATION RADIATION AND TWO-POINT FUNCTIONS

We will now present the formalism used in [10] and then will apply it to the calculation of the acceleration radiation.

A. Particle creation and two-point functions

Let us suppose that ϕ is a scalar field propagating in an arbitrary spacetime. We can rewrite the expectation values of the operator $N_i^{\text{out}} \equiv \hbar^{-1} a_i^{\text{out}\dagger} a_i^{\text{out}}$, in terms of the corresponding scalar product for the field

$$\langle \text{in}|N_i^{\text{out}}|\text{in}\rangle = \sum_k \beta_{ik}\beta_{ik}^* = -\sum_k (u_i^{\text{out}}, u_k^{\text{in}*})(u_i^{\text{out}*}, u_k^{\text{in}})$$

$$= \sum_k \left(\int_\Sigma d\Sigma_1^\mu u_i^{\text{out}}(x_1) \overset{\leftrightarrow}{\partial}_\mu u_k^{\text{in}}(x_1) \right)$$

$$\times \left(\int_\Sigma d\Sigma_2^\nu u_i^{\text{out}*}(x_2) \overset{\leftrightarrow}{\partial}_\nu u_k^{\text{in}*}(x_2) \right), \quad (14)$$

where Σ is an initial Cauchy hypersurface. If we now consider the sum of *in* modes before making the integrals of the two scalar products, and take into account that

$$\langle \text{in}|\phi(x_1)\phi(x_2)|\text{in}\rangle = \hbar \sum_k u_k^{\text{in}}(x_1) u_k^{\text{in}*}(x_2), \quad (15)$$

we obtain a simple expression for the particle production number in terms of the two-point function

$$\langle \text{in}|N_i^{\text{out}}|\text{in}\rangle = \hbar^{-1} \int_\Sigma d\Sigma_1^\mu d\Sigma_2^\nu [u_i^{\text{out}}(x_1) \overset{\leftrightarrow}{\partial}_\mu][u_i^{\text{out}*}(x_2) \overset{\leftrightarrow}{\partial}_\nu]$$

$$\times \langle \text{in}|\phi(x_1)\phi(x_2)|\text{in}\rangle. \quad (16)$$

The above expression requires one to interpret the two-point function in the distributional sense. The "$i\epsilon$ prescription" and the Hadamard condition[1] should be assumed for $\langle \text{in}|\phi(x_1)\phi(x_2)|\text{in}\rangle$, as in (11). However, taking into account the trivial identity $\langle \text{out}|a_i^{\text{out}\dagger} a_i^{\text{out}}|\text{out}\rangle = 0$ we can rewrite the above expression as

$$\langle \text{in}|N_i^{\text{out}}|\text{in}\rangle = \frac{1}{\hbar} \int_\Sigma d\Sigma_1^\mu d\Sigma_2^\nu [u_i^{\text{out}}(x_1) \overset{\leftrightarrow}{\partial}_\mu][u_i^{\text{out}*}(x_2) \overset{\leftrightarrow}{\partial}_\nu]$$

$$\times [\langle \text{in}|\phi(x_1)\phi(x_2)|\text{in}\rangle$$

$$- \langle \text{out}|\phi(x_1)\phi(x_2)|\text{out}\rangle]. \quad (17)$$

Now the Hadamard condition for both |in⟩ and |out⟩ states ensures that the difference of the above two-point distributions is a smooth function.

Intuitively the idea behind the above manipulations is simple. In the conventional analysis in terms of Bogolubov coefficients, we first perform the integration in distances and leave to the end the sum of *in* modes. In contrast, we can invert the order and perform first the sum of *in* modes, which naturally leads to introduce the two-point function of the matter field, and leave the integration in distances to the end. Despite this simple technicality, one should not underestimate the physical content of expression (17). The existence of different correlations $\langle \phi(x_1)\phi(x_2) \rangle$ between *in* and *out* observers, weighted by the form of the modes of the detected quanta, is at the root of the phenomenon of particle production. Moreover, the relevant correlations are those with support in the region where the wave-packet modes are peaked.

One of the advantages of expression (17), as compared with (16), is that it displays clearly the possible symmetries of the problem. For instance, for a conformal field theory and for *in* and *out* modes related by spacetime conformal transformations, the integrand (17) is manifestly zero. In contrast, it is the full integral (16) which vanishes.

The "$i\epsilon$-prescription" described above, when applied to a gravitational collapse turns out to be somewhat parallel to the approach of [13]. Here, as in [10], we want to put forward expression (17) to evaluate the particle production rate and to analyze the role of the Planck scale.

B. Rederiving the acceleration radiation

We shall now explain with some detail how the Fulling-Davies-Unruh effect can be derived using the two-point functions of the field. We denote the Rindler modes ("out" in the above notation) by $u_i^R = \phi_w(t, \xi)e^{-i\vec{k}_\perp \vec{x}_\perp}$, where $i \equiv (w, \vec{k}_\perp)$, and the Rindler vacuum by $|0_R\rangle$ (|out⟩ in the above notation). The Minkowski vacuum will be denoted by $|0_M\rangle$ (|in⟩ in the above notation). Now that the notation of this

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[1]The two-point distribution should have (for all physical states) a short-distance structure similar to that of the ordinary vacuum state in Minkowski space: $(2\pi)^{-2}(\sigma + 2i\epsilon t + \epsilon^2)^{-1}$, where $\sigma(x_1, x_2)$ is the squared geodesic distance [6].

section has been fixed, we will explain how to choose a suitable Cauchy hypersurface to evaluate the integrals of (17). To compute the number of particles for the accelerated observer in the Minkowski vacuum $|0_M\rangle$, one can naturally choose a hyperplane $T - \lambda X = $ constant, with $|\lambda| < 1$, as the initial Cauchy surface. However, it is convenient to consider the limiting case $\lambda = 1$ and the null plane H_0^+, defined as $U \equiv T - X = U_0 < 0$ (or the analogous null plane H_0^-, defined as $V \equiv T + X = V_0 > 0$) as our initial data hypersurface. As emphasized in [6] (Sec. 5.1), any solution of the massless Klein-Gordon equation in Minkowski space, having any dimension greater than two, is uniquely determined by its restriction to the hyperplane H_0^+ alone (or H_0^- alone). So H_0^+ (or H_0^-) is enough to characterize the field configuration[2] and can be used as the initial value surface Σ. Therefore, we convert (17) into (we introduce two indices i_1 and i_2 since we are using plane wave type modes instead of wave-packets)

$$
\begin{aligned}
\langle_M 0|N_{i_1,i_2}^R|0_M\rangle = \frac{4}{\hbar} \int_{H_0^+} &dv_1 d\vec{x}_{\perp 1} dv_2 d\vec{x}_{\perp 2} u_{i_1}^R(x_1) u_{i_2}^{R*}(x_2) \\
&\times \partial_{v_1}\partial_{v_2}[\langle_M 0|\phi(x_1)\phi(x_2)|0_M\rangle \\
&- \langle_R 0|\phi(x_1)\phi(x_2)|0_R\rangle].
\end{aligned}
\tag{18}
$$

Since the accelerated modes $u_i^R(x)$ have support on the right-handed Rindler wedge, the above integral is naturally restricted to the right wedge part of H_0^+. The relevant derivatives of the two-point functions in the Minkowski vacuum can be expressed, using the inertial null coordinates V, U, as

$$
\langle_M 0|\partial_{V_1}\phi(x_1)\partial_{V_2}\phi(x_2)|0_M\rangle = \frac{1}{4\pi^2} \int d\vec{k}_\perp G_{\vec{k}_\perp}^M(x_1,x_2) \\
\times e^{-i\vec{k}_\perp \vec{x}_\perp},
\tag{19}
$$

where, at the region H_0^+

$$
\begin{aligned}
G_{\vec{k}_\perp}^M(x_1,x_2)|_{H_0^+} &= \partial_{V_1}\partial_{V_2}\frac{\hbar}{2\pi} \\
&\times K_0(|\vec{k}_\perp|\sqrt{-(V_1-V_2)(U_1-U_2)})|_{H_0^+} \\
&= -\frac{\hbar}{4\pi}\frac{1}{(V_1-V_2)^2},
\end{aligned}
\tag{20}
$$

where $K_0(x)$ is a modified Bessel function. Therefore,

$$
\begin{aligned}
&\langle_M 0|\partial_{V_1}\phi(x_1)\partial_{V_2}\phi(x_2)|0_M\rangle|_{H_0^+} \\
&= -\frac{\hbar}{4\pi}\frac{1}{(V_1-V_2)^2}\delta(\vec{x}_{\perp 1}-\vec{x}_{\perp 2}).
\end{aligned}
\tag{21}
$$

It is now convenient to perform the calculation on the null plane H^+, obtained by the limiting case $U_0 \to 0$. As approaching to H^+ ($\xi \to -\infty$), the potential decays expo-

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[2]In two dimensions H_0^+ is not enough and we need $H_0^+ \bigcup H_0^-$ to have a proper initial surface.

nentially and the (right-) Rindler modes can be approximated as

$$
u_i^R = \frac{e^{i\gamma(w,|\vec{k}_\perp|)}}{(2\pi)^{3/2}\sqrt{2w}}(e^{-iwu} + re^{-iwv})e^{i\vec{k}_\perp \vec{x}_\perp},
\tag{22}
$$

where $v = t + \xi$, $u = t - \xi$, $e^{i\gamma(w,|\vec{k}_\perp|)} = \Gamma[1 + i\frac{w}{a}]^{-1} \times (\frac{|\vec{k}_\perp|}{2a})^{i(w/a)}$, and $r(w,\vec{k}_\perp) = e^{-i2\gamma(w,|\vec{k}_\perp|)}$ is the reflection amplitude.

Moreover, the two-point function in the accelerated Rindler vacuum can also be worked out as

$$
\begin{aligned}
&\langle_R 0|\partial_{v_1}\phi(x_1)\partial_{v_2}\phi(x_2)|0_R\rangle|_{H^+} \\
&= \frac{1}{4\pi^2}\int d\vec{k}_\perp G_{\vec{k}_\perp}^R(x_1,x_2)|_{H^+}e^{-i\vec{k}_\perp \vec{x}_\perp} \\
&= -\frac{\hbar}{4\pi}\frac{1}{(v_1-v_2)^2}\delta(\vec{x}_{\perp 1}-\vec{x}_{\perp 2}).
\end{aligned}
\tag{23}
$$

With this, Eq. (18) becomes

$$
\begin{aligned}
\langle_M 0|N_{i_1,i_2}^R|0_M\rangle = &-\frac{|r|^2}{4\pi^2\sqrt{w_1 w_2}}\int_{V_1,V_2>0} e^{-iw_1 v_1 + iw_2 v_2} \\
&\times \left[\frac{dV_1 dV_2}{(V_1-V_2)^2} - \frac{dv_1 dv_2}{(v_1-v_2)^2}\right] \\
&\times \delta(\vec{k}_{\perp 1}-\vec{k}_{\perp 2}),
\end{aligned}
\tag{24}
$$

where the transversal (Y, Z) dependence has been trivially integrated, producing the delta function. We would like to emphasize again the physical meaning of the above expression. The particle content of the Minkowski vacuum, as perceived by the accelerated observer, is displayed as an integral measuring the different vacuum correlations of inertial and accelerating observers, weighted by the form of the modes of the accelerated observer.

Taking into account that the relation between the null inertial coordinate V and the accelerated one v is $V = a^{-1}e^{av}$ we then have

$$
\begin{aligned}
\langle_M 0|N_{i_1,i_2}^R|0_M\rangle = &-\frac{|r|^2}{4\pi^2\sqrt{w_1 w_2}}\int_{-\infty}^{\infty} dv_1 dv_2 e^{-iw_1 v_1} e^{iw_2 v_2} \\
&\times \left[\frac{(a/2)^2}{\sinh^2\frac{a}{2}(v_1-v_2)} - \frac{1}{(v_1-v_2)^2}\right] \\
&\times \delta(\vec{k}_{\perp 1}-\vec{k}_{\perp 2}).
\end{aligned}
\tag{25}
$$

Note that $|r| = 1$ because the reflection amplitude r is a pure phase. Integrating over $v_1 + v_2$ we are left with (we define $\Delta v \equiv v_1 - v_2$)

$$\langle_M 0|N^R_{i_1,i_2}|0_M\rangle = -\frac{1}{4\pi^2 w_1} \int_{-\infty}^{\infty} d(\Delta v) e^{-iw_1 \Delta v}$$

$$\times \left[\frac{(a/2)^2}{\sinh^2 \frac{a}{2}\Delta v} - \frac{1}{(\Delta v)^2}\right]$$

$$\times \delta(w_1 - w_2)\delta(\vec{k}_{\perp 1} - \vec{k}_{\perp 2})$$

$$= \frac{1}{e^{2\pi w_1/a} - 1}\delta(\vec{k}_1 - \vec{k}_2). \qquad (26)$$

Alternatively, we can choose the null hypersurface H^- (defined as $V_0 = 0$), instead of H^+, as our initial hypersurface. Then one finds

$$\langle_M 0|N^R_{i_1,i_2}|0_M\rangle = -\frac{1}{4\pi^2 \sqrt{w_1 w_2}} \int_{U_1,U_2<0} e^{-iw_1 u_1 + iw_2 u_2}$$

$$\times \left[\frac{dU_1 dU_2}{(U_1 - U_2)^2} - \frac{du_1 du_2}{(u_1 - u_2)^2}\right]$$

$$\times \delta(\vec{k}_{\perp 1} - \vec{k}_{\perp 2}), \qquad (27)$$

where now $U = -a^{-1}e^{-au}$. This leads again to the same Planckian spectrum.

Note that, since the Rindler modes (22) have a reflection amplitude of unit module, we get a null net flux of radiation but a nonvanishing thermal energy density able to excite an accelerated particle detector. In contrast, because in two spacetime dimensions the left and right modes are independent and one needs $H^+ \cup H^-$ to have a complete initial surface, both expressions (24) and (27) would then be needed to properly obtain the acceleration radiation.

IV. ACCELERATION RADIATION, LORENTZ INVARIANCE AND THE PLANCK SCALE

Let us now examine with more detail the basic formulas (24) and (27) leading to the thermal spectrum. As remarked above, those formulas tell us that particles in a given mode stem from the different two-point correlations seen by inertial and accelerated observers. In this sense, one could think that the Fulling-Davies-Unruh effect seems to require the validity of special relativity for arbitrarily large and unbounded boosts. This is so because the affine distance $|V_1 - V_2|$ along the null plane H^+ in the direction of the acceleration can be made arbitrarily small

$$(V_1 - V_2)^2 \sim e^{2av_1}(v_1 - v_2)^2, \qquad (28)$$

as perceived by the accelerated observer as $v_1 \approx v_2 \rightarrow -\infty$. As a consequence, even if $|v_1 - v_2| \equiv |\Delta v|$ is well above the Planck length l_P, $|V_1 - V_2|$ involves sub-Planckian distances when $v_1 \approx v_2 \rightarrow -\infty$ due to the extreme length contraction. And this ultrahigh length contraction seems fundamental in (24) for getting the exact thermal result. We will show next, however, that the bulk of the Fulling-Davies-Unruh effect does not require the consideration of sub-Planckian lengths.

Let us first note that we work in a Lorentz invariant framework. Despite this fact, in the evaluation of (24) we explicitly used a particular inertial observer related to the accelerated observer by the change of coordinates (1) [see (25)]. In a fully Lorentz invariant framework, however, this choice of inertial observer is arbitrary, since the outcome of (24) is actually independent of the particular inertial observer chosen.[3] In such a framework, the only variable that can be naturally distinguished in (24) is the affine distance Δv, which is measured in the instantaneous rest frame of the accelerated observer. To study how short distances affect the particle spectrum seen by the accelerated observer, we restrict in (26) the integration over Δv to distances greater than $\alpha \sim l_P \ll a^{-1}$. The result is

$$-\frac{1}{4\pi^2 w} \int_{|\Delta v|>\alpha} d(\Delta v)e^{-iw\Delta v}\left[\frac{(a/2)^2}{\sinh^2 \frac{a}{2}\Delta v} - \frac{1}{(\Delta v)^2}\right]$$

$$\approx \frac{1}{e^{2\pi w/a} - 1} - \frac{\alpha a}{12\pi w/a} + O(\alpha^3 a^3), \qquad (29)$$

which shows that the spectrum is not sensitive to a microscopic (Planckian) cutoff[4] α for $|\Delta v|$.

Let us now assume that Eq. (24) is referred to a particular inertial frame. This raises a problem, since the two-point function of the inertial observer, in the region $v \rightarrow -\infty$, would involve sub-Planckian distances, as discussed above in Eq. (28). One should, therefore, consider the effect of removing from (24) the contribution of the two-point function of the inertial observer coming from sub-Planckian scales. In doing this, one sees that the particle spectrum turns out to be extremely sensitive to a microscopic cutoff for $|\Delta V|$, since then $|\Delta v|$ is macroscopic and much bigger than a^{-1}. In fact, for such Δv, the expansion (29) is no longer valid, which casts doubts on the robustness of the Planckian spectrum. However, even if a short-distance cutoff is assumed for this (arbitrary) inertial observer, there is an additional argument supporting the robustness of the acceleration radiation. Instead of H^+, one can alternatively use the H^- hypersurface for the calculation of the number of particles [see (27)]. Because of the existence of the completely reflecting potential $V(\xi) \propto e^{2a\xi}\vec{k}^2_\perp$ for the accelerated observer, the Rindler

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[3]The proof of this claim is simple. We just need to look at the relation between inertial and accelerated observers given in (1), and consider another inertial observer related to the first one by $T' = \gamma(T + \beta X)$, $X' = \gamma(X + \beta T)$. The relation between the null inertial coordinates of the new inertial observer and the accelerated observer is given by $V' \equiv T' + X' = \gamma(1 + \beta)\frac{1}{a}e^{av}$ and $U' \equiv T' - X' = -\gamma(1 - \beta)\frac{1}{a}e^{-au}$. Using these relations, we can evaluate the terms $dV_1 dV_2/(V_1 - V_2)^2$ and $dU_1 dU_2/(U_1 - U_2)^2$ of (24) and (27), respectively, and find that all β and γ factors disappear.

[4]Note that this result is still valid even if the two-point function is modified at short distances but the principle of relativity, equivalence of all inertial frames, is preserved (see the Appendix).
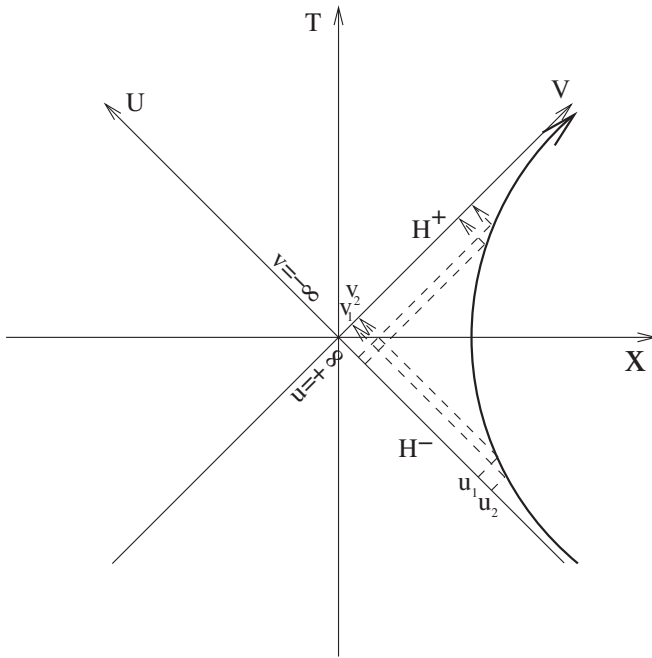
FIG. 1. A uniformly accelerating trajectory (bold line). The dotted lines represent the (Rindler) mode propagation from $t = -\infty$ and $\xi = -\infty$, its reflexion by the potential (around $\xi \sim 0$), until $t = +\infty$ and $\xi = -\infty$, as described by the accelerating observer.

(wave-packet) modes with support at $[v_1, v_2] \rightarrow -\infty$ have necessary support at $[u_1, u_2] \rightarrow -\infty$ (see Fig. 1). In this situation we have instead

$$(U_1 - U_2)^2 \sim e^{-2au_1}(u_1 - u_2)^2. \qquad (30)$$

A Planckian cutoff $|U_1 - U_2| > \alpha \sim l_P$ for the inertial affine distance in the region $[u_1, u_2] \rightarrow -\infty$ will now remain sub-Planckian for the accelerated observer. One can, therefore, restrict the integral in (27) to distances $|\Delta u| \geq \alpha$, which always imply $\Delta U > \alpha$, and get an expression identical to (29) without ever invoking sub-Planckian scales. This shows that the Fulling-Davies-Unruh effect can be derived, in dimensions greater than two, without ever invoking sub-Planckian distances (or extreme high energy scales). This strongly suggests that the acceleration radiation is indeed a low-energy phenomenon and that it should persist even if a Planck-length cutoff is introduced in the theory. Note also that this reasoning cannot be used in black holes due to the absence of a completely reflecting potential. For the Hawking radiation, extra physical inputs are needed, as argued in [10,14]. It appears, therefore, that the acceleration radiation is, in any case, more robust to trans-Planckian physics than Hawking radiation is.

It is important to note that if the above argument, namely, the interchangeable role of H^+ or H^-, were not correct, as applied to a modified theory with a Planck-length cutoff, one would find a nonvanishing net flux of

radiation. This can be seen as follows. In the full relativistic theory (without any cutoff), the accelerated observer perceives an energy flux to the right as well as an (opposite) energy flux to the left. Summing up both contributions the observer gets a null net flux of radiation but a nonvanishing energy density. On physical grounds, this can be seen as a consequence of parity symmetry in our physical scenario. If this symmetry is maintained in the presence of a microscopic cutoff only a bath of radiation seems to be physically acceptable.[5] We have seen that for the calculation of the flux to the left (integration along H^+) at $v \rightarrow -\infty$ ultrashort affine distances are required. However, for the calculation of the flux to the right (integration along H^-) when $u \rightarrow -\infty$, we do not need ultrashort affine distances to generate the thermal spectrum. So both right and left fluxes should be equal, and approximately thermal, to produce the bath of radiation. The opposite argument applies at the trajectory points $u \rightarrow +\infty$. In this case, ultrashort distances are apparently needed (for the inertial observer) in the computation of the flux to the right, but not for the computation of the flux to the left.

In conclusion, we have pointed out that the acceleration radiation effect can de rederived without actually invoking very high energy physics. This supports the view that the acceleration radiation is robust against Planck-scale physics and suggests that any theory of quantum gravity, with new microscopic degrees of freedom, should also reproduce this relativistic field-theory effect. We believe that our analysis of the acceleration radiation effect in Minkowski space can also be extended to curved spacetimes. In particular, for de Sitter space that would imply that the semiclassical Gibbons-Hawking effect [15] would remain robust against microscopic physics. In addition, the elegant kinematic method of connecting the acceleration, de Sitter, and black hole radiation given in [16] may support our view that all these semiclassical thermal effects do not depend on ultra-high-energy physics.

After completion of this work, we were informed that M. Rinaldi [17] has recently reanalyzed the Unruh effect in terms of modified dispersion relations. Conclusions similar to the present paper are also displayed.

ACKNOWLEDGMENTS

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[5]Obviously, though no net flux of radiation is allowed, there is a gradient of local temperature in the X direction due to the redshift.

APPENDIX

Let us now illustrate the discussion of the last section in terms of a particle detector. To this end we shall modify the relativistic theory by deforming[6] the two-point function *ab initio*. This has the advantage of going directly to the point of interest to us, since it is just the form of the two-point function that is relevant in the evaluation of the detector response function. Obviously one could reconstruct the underlying field theory to generate equations of motion, inner product, etc; but all that is not necessary in the analysis below.

For simplicity we shall consider the case of a massless scalar field in four dimensions. The simplest deformation of the two-point function is

$$\langle \phi(x_1)\phi(x_2) \rangle = \frac{\hbar}{4\pi^2(x_1 - x_2)^2 + l_P^2}. \quad (A1)$$

The rate $\dot{F}(w)$ can be now worked out, according to (12), as

$$\dot{F}_{l_P}(w) = -\frac{\hbar}{4\pi^2} \int_{-\infty}^{+\infty} d\Delta\tau e^{-iw\Delta\tau}$$
$$\times \frac{1}{\frac{4}{a^2}\sinh^2(\frac{a}{2}\Delta\tau) + l_P^2/4\pi^2}. \quad (A2)$$

The result is

$$\frac{\hbar}{2\pi} \frac{we^{\pi w/a}}{(e^{2\pi w/a} - 1)} \frac{\sinh[\frac{w}{a}(\theta - \pi)]}{\frac{w}{a}\sin\theta}, \quad (A3)$$

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[6]Modifications via modified dispersion relations compatible with the principle of relativity have been studied in [18].

where $\theta \equiv 2\arcsin(\frac{l_P a}{4\pi})$. This largely departs from the thermal spectrum. However it is not physically sound since, for an inertial observer $a = 0$, the response rate does not vanish, as one should expect according to the principle of relativity. To produce a meaningful expression one should subtract the naive ''inertial'' contribution, replacing (A2) by

$$\dot{F}_{l_P}(w) = -\frac{\hbar}{4\pi^2} \int_{-\infty}^{+\infty} d\Delta\tau e^{-iw\Delta\tau}$$
$$\times \left[\frac{1}{\frac{4}{a^2}\sinh^2(\frac{a}{2}\Delta\tau) + l_P^2/4\pi^2} - \frac{1}{(\Delta\tau)^2 + l_P^2/4\pi^2} \right]. \quad (A4)$$

The final result is then

$$\dot{F}_{l_P}(w) = \frac{\hbar}{2\pi} \left[\frac{we^{\pi w/a}}{(e^{2\pi w/a} - 1)} \frac{\sinh[\frac{w}{a}(\theta - \pi)]}{\frac{w}{a}\sin\theta} + \frac{\pi e^{-wl_P/2\pi}}{l_P} \right]. \quad (A5)$$

The thermal Planckian spectrum is smoothly recovered in the limit $\theta \approx l_P a/2\pi \to 0$. In fact, the rate $\dot{F}_{l_P}(w)$ can be expanded as

$$\dot{F}_{l_P}(w) \approx \frac{\hbar w}{2\pi} \left[\frac{1}{e^{2\pi w/a} - 1} - \frac{l_P a}{32\pi w/a} + O(l_P^3 a^3) \right]. \quad (A6)$$

This result is in agreement with the estimation (29). Note that the crucial ingredient to preserve the thermal spectrum is the requirement of having a vanishing detector response for all inertial observers.

Thermality is maintained until a certain frequency scale Ω, which can be estimated by requiring positivity of $\dot{F}_{l_P}(w)$ in the above approximated expressions. A simple calculation leads to the condition $32\pi\Omega e^{-2\pi\Omega/a} \approx l_P a^2$. Planck-scale effects could potentially emerge at the scale Ω, which is roughly a few orders above $T = a/2\pi$, if the acceleration is not very high.

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