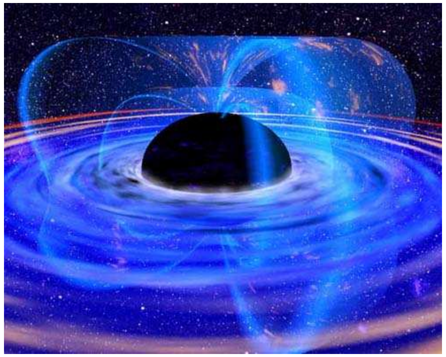

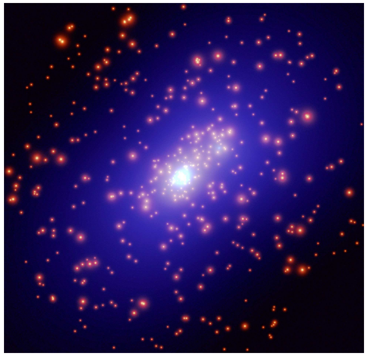

The gravity lagrangian according to solar system experiments

(or Post-Newtonian constraints on $f(R)$ cosmologies)

Gonzalo J. Olmo Alba

University of Wisconsin-Milwaukee

Universidad de Valencia

About this talk . . .

- Motivation:

  Observations indicate that the cosmic expansion is accelerating. It seems natural to ask if the cosmic speed-up could be due to new gravitational physics. In particular, modifications of General Relativity have been proposed:

  $$\boxed{f(R)=R-\frac{\mu^4}{R}} \qquad \boxed{f(R)=R+a\ln R} \qquad \boxed{f(R)=bR^n}$$

  Theories of this form lead to late-time self-acceleration and suggest that sources of dark energy could be unnecessary.

About this talk . . .

● About this talk . . .
● Outline
The cosmic speed-up
$f(R)$ gravities
The End

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  Observations indicate that the cosmic expansion is accelerating. It seems natural to ask if the cosmic speed-up could be due to new gravitational physics. In particular, modifications of General Relativity have been proposed:

  $$f(R) = R - \frac{\mu^4}{R} \qquad f(R) = R + a \ln R \qquad f(R) = bR^n$$

  Theories of this form lead to late-time self-acceleration and suggest that sources of dark energy could be unnecessary.

- Aim:
  - Study if laboratory and solar system experiments can constrain the form of the lagrangian $f(R)$.
  - Answer to:
    "May the cosmic speed-up be due to $f(R)$ gravities?"

Outline

- Part I : The cosmic speed-up.

Outline

● About this talk . . .
● Outline

The cosmic speed-up

$f(R)$ gravities

The End

- Part I : The cosmic speed-up.
  - Framework of standard cosmology.

Outline

● About this talk . . .
● Outline

The cosmic speed-up

$f(R)$ gravities

The End

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Outline

● About this talk . . .
● Outline

The cosmic speed-up

$f(R)$ gravities

The End

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Outline

● About this talk . . .
● Outline

The cosmic speed-up

$f(R)$ gravities

The End

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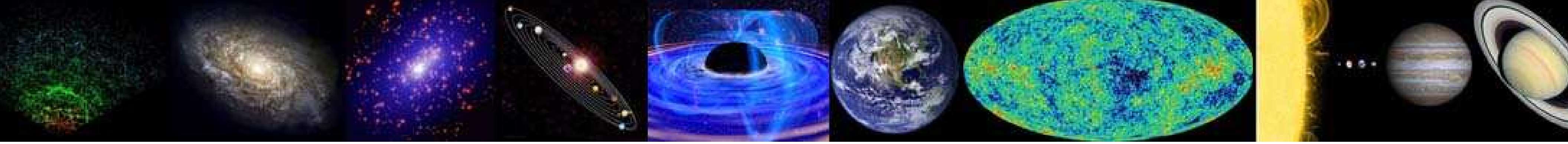

Part I:

The cosmic speed-up

Framework of standard cosmology

- Basic assumptions of standard cosmology:
  - The distribution of matter is isotropic and homogeneous at large scales (> 100 Mpc).
  - Large scale dynamics governed by gravitational interactions.

Framework of standard cosmology

- Basic assumptions of standard cosmology:
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  - Large scale dynamics governed by gravitational interactions.
- Since gravity is a geometrical phenomenon …
  - The first assumption determines de kinematics:

    $$ds^2 = dt^2 - a^2(t)d\vec{x}^2 = \frac{1}{(1+z)^2}\left[H^{-2}(z)dz^2 - d\vec{x}^2\right]$$

    where $1+z = a_0/a(t)$ and $d\vec{x}^2 = (1-kr^2)^{-1}dr^2 + r^2 d\Omega^2$.
  - The second assumption determines the dynamics of $a(t)$.

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  - The second assumption determines the dynamics of $a(t)$.
- $H(z)$ is the only non trivial function in a FRW universe.

Measuring the geometry of the universe

- From basic courses on Astronomy we learn that :

$$m(z) = \mathcal{M} + 5\log_{10}\left[\frac{H_0 d_L(z)}{c}\right]$$

Measuring the geometry of the universe

● About this talk …
● Outline

The cosmic speed-up
● Framework
● Measuring the geometry
● Observational facts
● Interpretation within GR
● Cosmological parameters I
● Cosmological parameters II

$f(R)$ gravities

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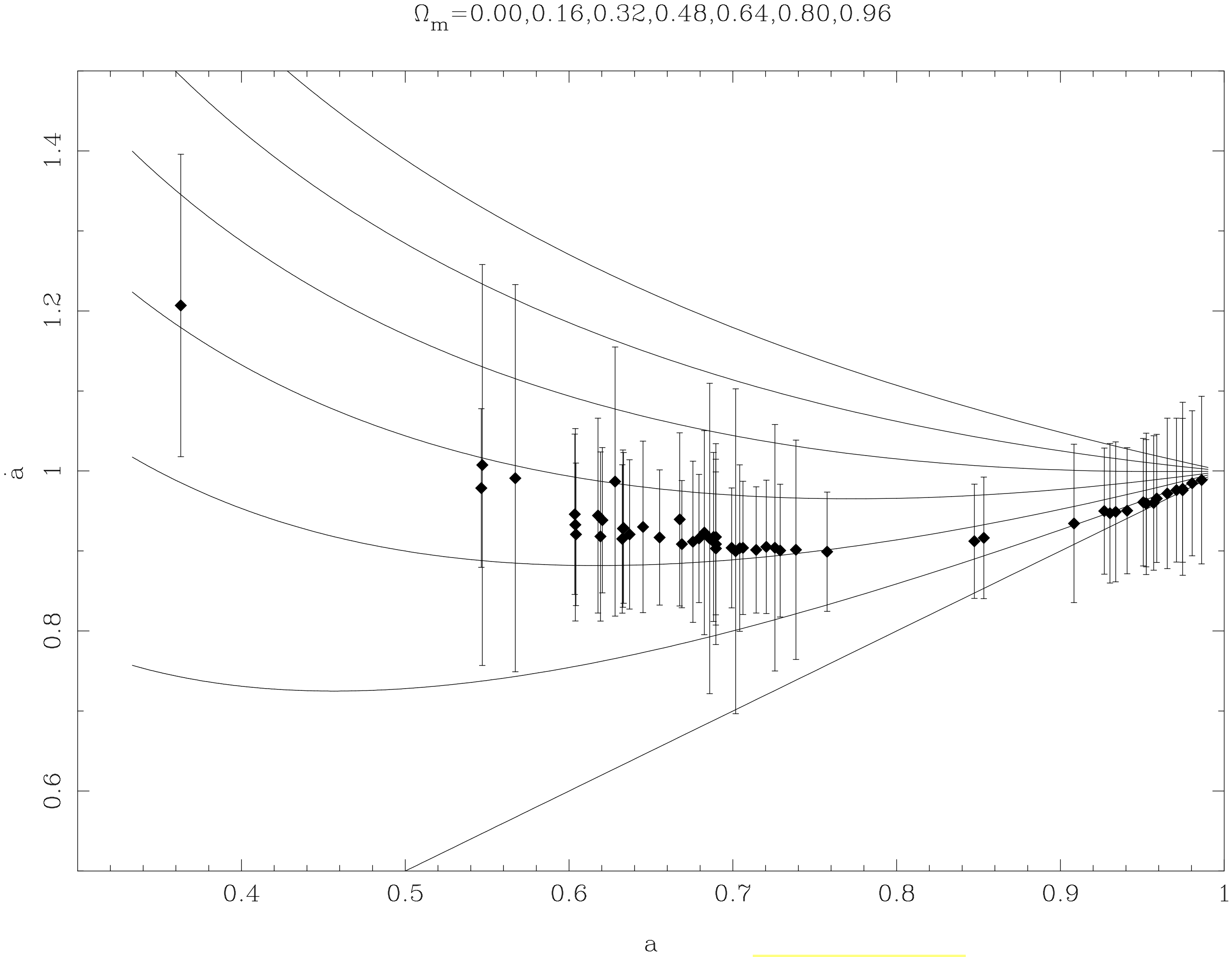
- This issue is trivial in principle, though very complicated in practice due to uncertainties in the determination of $d_L(z)$.

Observational facts



- Big dots represent type-Ia supernovae: $\dot{a}$ - Vs - a.
- The expansion began to accelerate some 5 billion years ago.

Interpretation within GR

- In **GR** the expansion factor satisfies:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{Tot} + 3P_{Tot}\right) \quad \longrightarrow$$

The composition

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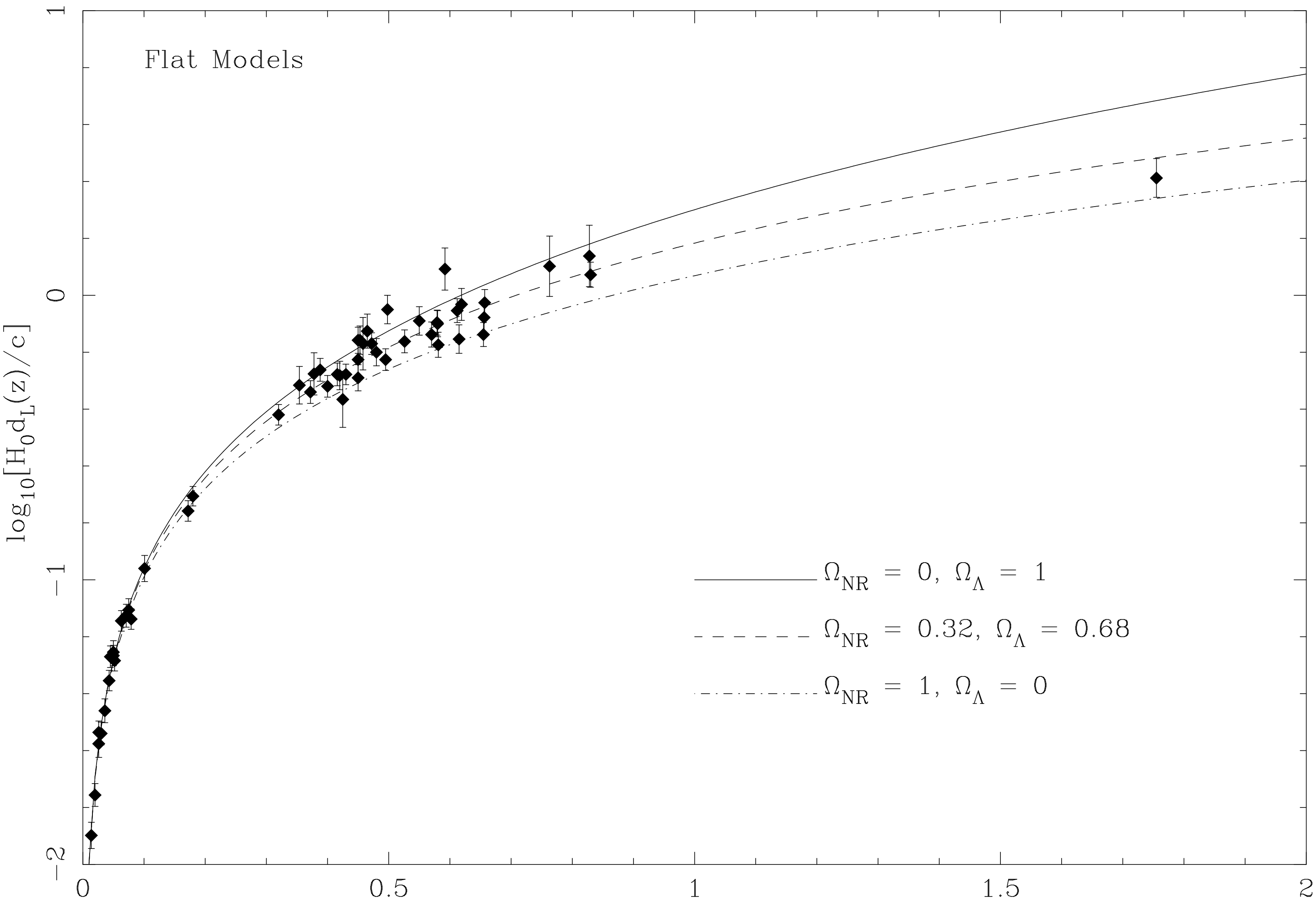
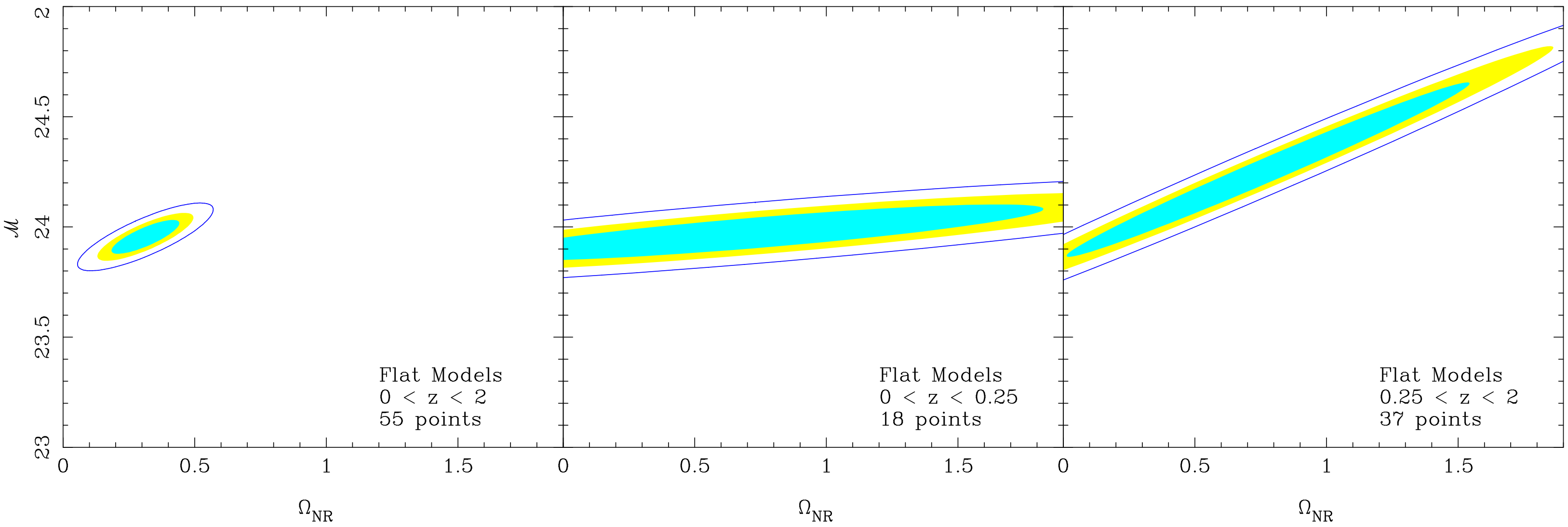
- Matter scalar fields (quintessence) are dynamical alternatives to Λ.

Cosmological parameters I



- Luminosity distance - Vs - redshift.^{z}

Cosmological parameters II



- Contour portraits of low redshift and high redshift data. From plots 2 and 3, independently, almost nothing can be stated about the preferred cosmological model.

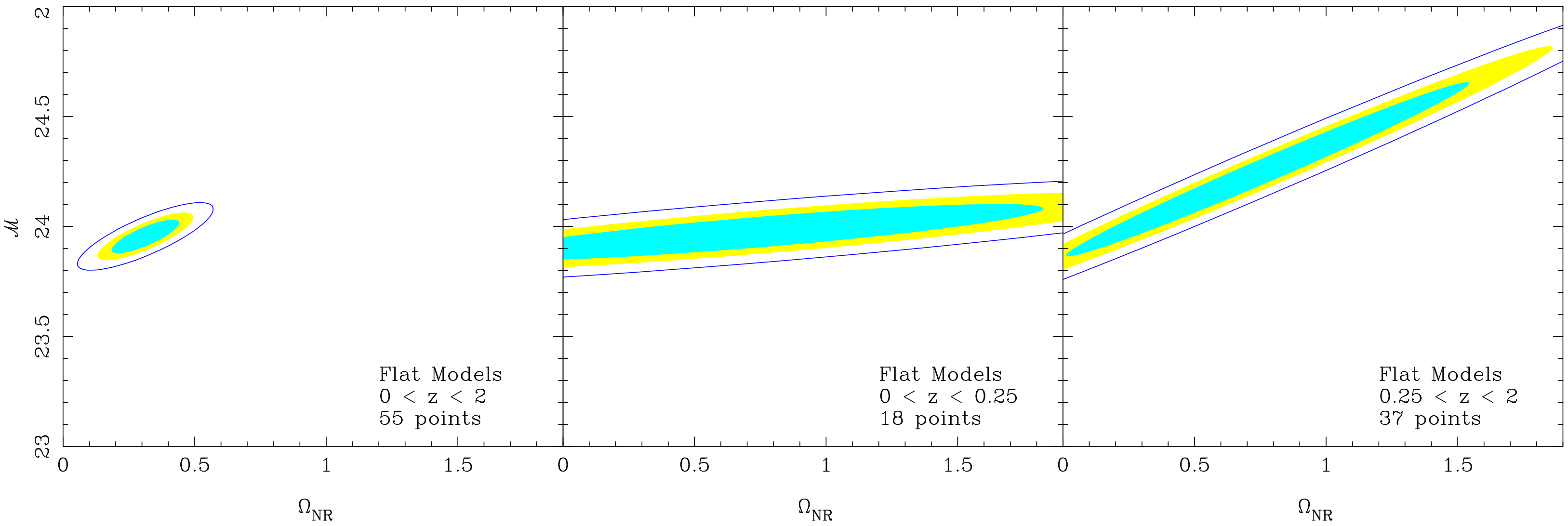
Cosmological parameters II

● About this talk . . .
● Outline

The cosmic speed-up
● Framework
● Measuring the geometry
● Observational facts
● Interpretation within GR
● Cosmological parameters I
● Cosmological parameters II

$f(R)$ gravities

The End



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● Outline

The cosmic speed-up
● Framework
● Measuring the geometry
● Observational facts
● Interpretation within GR
● Cosmological parameters I
● Cosmological parameters II

$f(R)$ gravities

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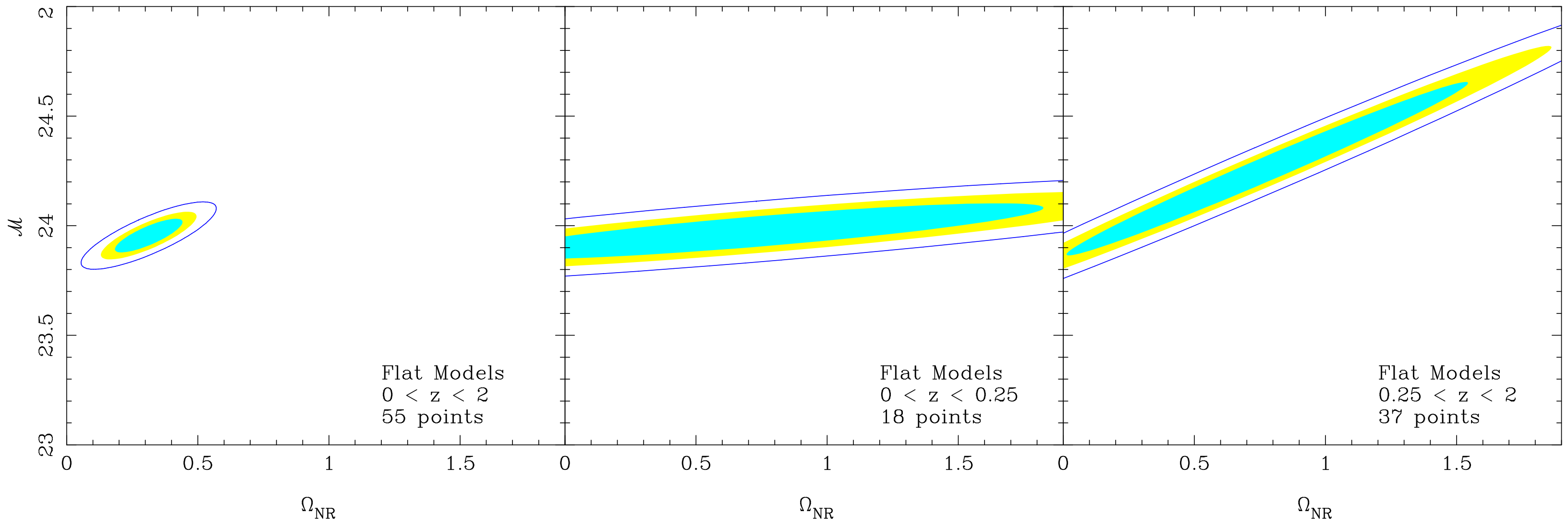
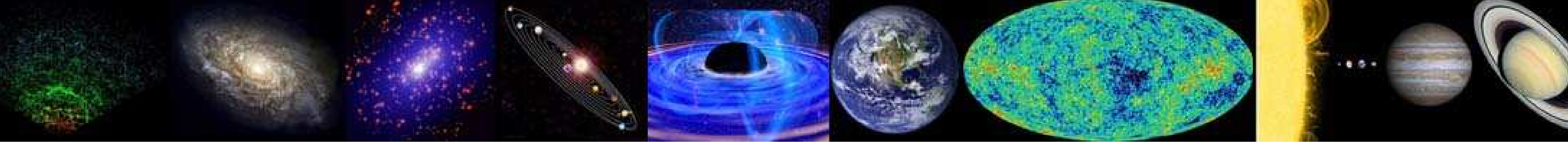


- Contour portraits of low redshift and high redshift data. From plots 2 and 3, independently, almost nothing can be stated about the preferred cosmological model.
- Only the combined set of data yields useful constraints.
- The composition is dominated by unknown sources:

Ordinary Matter:	$\sim 4\%$	Dark Matter:	$\sim 28\%$
Radiation:	$\sim 10^{-5}\%$	Dark Energy:	$\sim 68\%$



Part II:

$f(R)$ gravities

Alternative to dark energy: $f(R)$ gravities

- Early-time inflation can be justified with $\boxed{f(R) = R + \frac{R^2}{M^2}}$

Alternative to dark energy: $f(R)$ gravities

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- Could the "late-time inflation" be due to new gravitational effects?

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- The dependence of the expansion on the composition is relaxed:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3f'}\left(\rho_{Tot} + 3P_{Tot}\right)$$

with a new contribution of the form

$$\rho_f = \frac{1}{8\pi G}\left[Rf' - f - 3\frac{\dot{a}}{a}\dot{R}f''\right]$$

$$P_f = -\frac{1}{8\pi G}\left[Rf' - f - 2\frac{\dot{a}}{a}\dot{R}f'' - \ddot{R}f'' - \dot{R}^2 f'''\right]$$

governed by

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- Geometry itself may drive the expansion (*curvature quintessence*)

Two formulations of $f(R)$ gravities

Taking $R \equiv g^{\mu\nu} R_{\mu\nu}$ and

$$R_{\mu\nu} = -\partial_\mu \Gamma^\lambda_{\lambda\nu} + \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\rho\lambda} - \Gamma^\lambda_{\nu\rho}\Gamma^\rho_{\mu\lambda}$$

Two formulations of $f(R)$ gravities

- About this talk ...
- Outline

The cosmic speed-up

$f(R)$ gravities
- Alternative to dark energy
- Two formulations
- Solar system - Vs - Cosmology
- Scalar-tensor representation
- f(R) gravities as BD-like theories
- Metric formalism I
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- Palatini formalism: $g_{\mu\nu}$ and $\Gamma^\alpha_{\beta\gamma}$ are independent fields:

$$f'(\hat{R})\hat{R}_{\mu\nu}(\Gamma) - \frac{1}{2} f(\hat{R}) g_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

$$\Gamma^\alpha_{\beta\gamma} = \frac{t^{\alpha\rho}}{2}\left[\partial_\beta t_{\rho\gamma} + \partial_\gamma t_{\rho\beta} - \partial_\rho t_{\beta\gamma}\right] \quad \text{with} \quad t_{\alpha\beta} = f' g_{\alpha\beta}$$

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- Both formalisms naturally lead to cosmic speed-up.

- We must determine the right formalism and the function $f(R)$.

Solar system - Vs - Cosmology

- About this talk . . .
- Outline

The cosmic speed-up

$f(R)$ gravities

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  The solar system represents a clean scenario to test the dynamics minimizing the number of unknown variables.

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- The solar system is well described by luminous matter :

  The solar system represents a clean scenario to test the dynamics minimizing the number of unknown variables.
- Extensive amount of very precise observational data is available in the solar system regime.

Scalar-tensor representation

The e.o.m. of $f(R)$ gravities can be rewritten as follows:

$$R_{\mu\nu}(g)-\frac{1}{2}g_{\mu\nu}R(g)=\frac{\kappa^2}{\phi}T_{\mu\nu}-\frac{1}{2\phi}g_{\mu\nu}V(\phi)+$$

$$+\frac{\omega}{\phi^2}\left[\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right]+$$

$$+\frac{1}{\phi}\left[\nabla_\mu\nabla_\nu\phi-g_{\mu\nu}\Box\phi\right]$$

where $\phi\equiv df/dR$ and $V(\phi)=Rf'(R)-f(R)$.

- Metric formalism $\longrightarrow \omega=0$
- Palatini formalism $\longrightarrow \omega=-3/2$

Scalar-tensor representation

The e.o.m. of $f(R)$ gravities can be rewritten as follows:

$$\begin{aligned} R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = \frac{\kappa^2}{\phi}T_{\mu\nu} - \frac{1}{2\phi}g_{\mu\nu}V(\phi) + \\ + \frac{\omega}{\phi^2}\left[\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right] + \\ + \frac{1}{\phi}\left[\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\right] \end{aligned}$$

where $\phi \equiv df/dR$ and $V(\phi) = Rf'(R) - f(R)$.

- Metric formalism $\rightarrow \omega = 0$
- Palatini formalism $\rightarrow \omega = -3/2$

The scalar field is governed by the trace equations:

$$(3+2\omega)\Box\phi + 2V(\phi) - \phi\frac{dV}{d\phi} = \kappa^2 T$$

$f(R)$ gravities as Brans-Dicke-like theories

- The e.o.m. written above can be obtained from

$$S = \frac{1}{2\kappa^2} \quad d^4x\sqrt{-g}\left[\phi R(g) - \frac{\omega}{\phi}(\partial_\mu\phi\partial^\mu\phi) - V(\phi)\right] + S_m[g_{\mu\nu},\psi_m]$$

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- Observations must constrain the form of $V(\phi)\leftrightarrow f(R)$

Metric formalism I

The lowest-order corrections to the Minkowski metric are:

$$h_{00}^{(2)} \approx 2G\frac{M_\odot}{r} + \frac{V_0}{6\phi_0}r^2$$

$$h_{ij}^{(2)} \approx \delta_{ij}\left[2\gamma G\frac{M_\odot}{r} - \frac{V_0}{6\phi_0}r^2\right]$$

with

$$M_\odot = \quad d^3x\rho_{sun}$$

$$G = \frac{\kappa^2}{8\pi\phi_0}\left[1 + \frac{e^{-m_\varphi r}}{3}\right]$$

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Elementary observational constraints:

$$\left.\begin{array}{l} G \approx \text{constant} \\ \gamma \approx 1 \end{array}\right\} \rightarrow \quad e^{-m_\varphi r} \ll 1$$

from centimeters to planetary scales

$$\left.\begin{array}{c} \text{No cosmological} \\ \text{constant effects} \end{array}\right\} \rightarrow \quad \left|\frac{V_0 r^2}{6\phi_0}\right| \ll 1$$

in solar system scales.

Metric formalism II

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The End

- The cosmic expansion changes the effective mass

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- Since $f' > 0$ and $f'' > 0$ it is also bounded from below:

$$-2\Lambda \leq f(R) \leq R-2\Lambda+\frac{l^2R^2}{2}$$

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Elementary observational constraints:

$G \approx$ constant and universal

$\gamma \approx 1$

→ They are not universal and depend on the structure and composition of the body.

No cosmological constant effects → $\left|\frac{V_0 r^2}{6\phi_0}\right| \ll 1$ in solar system scales.

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  - $f(R)\leq\alpha+\frac{l^2R^2}{2}+\frac{R}{2}\sqrt{(f_0')^2+(l^2R)^2}+\frac{f_0'}{2l^2}\log[l^2R+\sqrt{1+(l^2R)^2}]$
  - $f(R)\geq\alpha-\frac{l^2R^2}{2}+\frac{R}{2}\sqrt{(f_0')^2+(l^2R)^2}+\frac{f_0'}{2l^2}\log[l^2R+\sqrt{1+(l^2R)^2}]$

  for $f''>0$ and $f''<0$ respectively.

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  for $f''>0$ and $f''<0$ respectively.
- In the limit $l^2R\ll 1\rightarrow$ $\alpha+R-\frac{l^2R^2}{2}\leq f(R)\leq\alpha+R+\frac{l^2R^2}{2}$

Analysis

Metric formalism:

- The cosmic-speed up, if due to the effects of nonlinear terms, would have modified $l_\varphi \sim m_\varphi^{-1}$ and affected the dynamics of the solar system, globular clusters, galaxies,…
- To prevent the growth of m_φ^{-1} with the expansion in order to satisfy the current experimental tests:

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- Conclusion: at low R the lagrangian is almost linear and bounded quadratically in R. **Nonlinear terms dominating at low R cannot be responsible for the cosmic speed-up.**

Summary and conclusions

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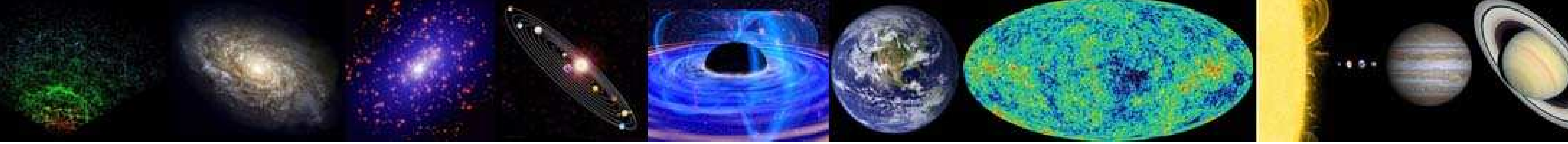
- Elementary solar system and laboratory experiments strongly constrain the dynamics of $f(R)$ gravities.
- The viable $f(R)$ theories can only affect the cosmic expansion through the local energy density of the scalar degree of freedom. They are almost equivalent to **GR** plus a cosmological constant.



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Thanks !!!

¡Gracias!!!