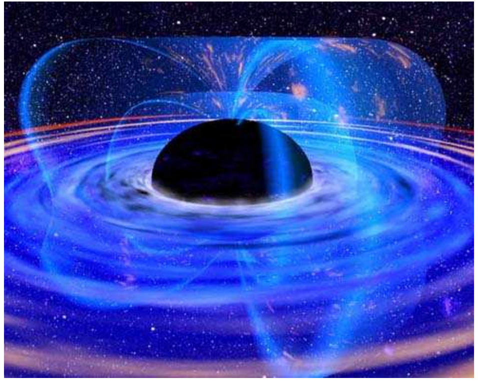

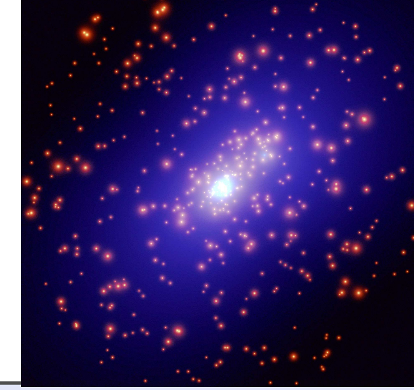

Cosmic Evolution and Local Dynamics in Modified Gravities

Gonzalo J. Olmo Alba

University of Wisconsin-Milwaukee (USA)

Universidad de Valencia (Spain)

About this talk . . .

- Motivation:

  General Relativity by itself seems unable to justify the late-time cosmic acceleration. **Modifications of the Einstein-Hilbert lagrangian relevant at very low cosmic curvatures** have been proposed as a mechanism for the late-time cosmic speed-up. Such theories **lead to cosmic acceleration**, but are they compatible with solar system observations?

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- Aim:
  - Show how the solar system dynamics in $f(R)$ gravities is affected by the evolution of the cosmic background.
  - Use elementary observational facts to constraint the form of the lagrangian $f(R)$.

$f(R)$ gravities?

- They are a family of modified gravity theories of the form

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  - They can be designed to modify the early time and/or late time cosmic dynamics keeping **GR** as an intermediate phase.

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- Defects:
  - Involved dynamical equations make difficult the analysis.
  - Though cosmic speed-up is easily verified, **supernovae luminosity curves seem unable to constraint the form of** $f(R)$.

Schwarzschild-like solutions

- The e.o.m. derived from $S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R) + S_m$ are:

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu f'(R) + g_{\mu\nu}\Box f'(R) = \kappa^2 T_{\mu\nu}$$

$$3\Box f'(R) + Rf'(R) - 2f(R) = \kappa^2 T$$

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- Schwarzschild-like solutions do exist:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2 \qquad \Lambda = \frac{R_0 f'(R_0) - f(R_0)}{2f'(R_0)}$$

$$A(r) = 1 - \frac{2C}{r} + \frac{\Lambda r^2}{3}$$

R_0 is solution of $R_0 f'(R_0) - 2f(R_0) = 0$

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- HOWEVER, $R_0 =$**const solutions do not satisfy valid boundary conditions neither at the interphase with matter nor at infinity.**
  - R is a dynamical object.
  - Right solutions must interpolate between the source region and the asymptotic cosmology.

Scalar-tensor representation

- Defining $\phi \equiv df/dR$ and $V(\phi) = Rf' - f$, the e.o.m. turn into:

$$R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = \frac{\kappa^2}{\phi}T_{\mu\nu} - \frac{1}{2\phi}g_{\mu\nu}V(\phi) + \frac{1}{\phi}\left[\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\right]$$

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- The scalar field is governed by the trace equation:

$$(3 + 2\omega_{BD})\Box\phi + 2V(\phi) - \phi \frac{dV}{d\phi} = \kappa^2 T$$

This is a Brans-Dicke theory with $\omega_{BD} = 0$ and $V(\phi) \neq 0$.

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- The Scalar-Tensor representation clarifies the interpretation of the e.o.m. and simplyfies the computation of the post-Newtonian limit:

  $g_{\mu\nu} \approx g^B_{\mu\nu} + h_{\mu\nu}$ $\Rightarrow$ Second order equations

  $\phi \approx \phi^B + \varphi$ $\Rightarrow$ Second order equations

Post-Newtonian limit

- Expanding about $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $\phi = \phi_0 + \varphi$ we find:

$$h^{(2)}_{00} \approx 2G\frac{M_\odot}{r} + \frac{V_0}{6\phi_0}r^2$$

$$h^{(2)}_{ij} \approx \delta_{ij}\left[2\gamma G\frac{M_\odot}{r} - \frac{V_0}{6\phi_0}r^2\right]$$

with

$$M_\odot = \int d^3x\rho_{sun}$$

$$G = \frac{\kappa^2}{8\pi\phi_0}\left[1 + \frac{e^{-m_\varphi r}}{3}\right]$$

$$\gamma = \frac{3-e^{-m_\varphi r}}{3+e^{-m_\varphi r}}$$

where $m^2_\varphi \equiv \frac{\phi_0 V''_0 - V'_0}{3}$ $\Leftrightarrow$ $m^2_\varphi \equiv \frac{f'_0 - R f''_0}{3 f''_0}$

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- Note: $\phi = \phi_0 + \frac{\kappa^2}{12\pi}\frac{M_\odot e^{-m_\varphi r}}{r}$ $\Leftrightarrow$ $\phi \equiv f'(R) = f'(R_0) + f''(R_0)\Delta R$

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- Elementary observational constraints:

$$\left.\begin{array}{l} G \approx \text{constant} \\ \gamma \approx 1 \end{array}\right\} \rightarrow \quad e^{-m_\varphi r} \ll 1$$

from centimeters to planetary scales

$$\left.\begin{array}{c} \text{No cosmological} \\ \text{constant effects} \end{array}\right\} \rightarrow \quad \left|\frac{V_0 r^2}{6\phi_0}\right| \ll 1$$

in solar system scales.

Constraining the $f(R)$ lagrangian

- The cosmic expansion changes the effective mass

$$\boxed{m_\varphi^2 = \frac{R_0}{3}\left[\frac{f'(R_0)}{R_0 f''(R_0)} - 1\right]}$$

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- **Viable theories must lead to constant or decreasing** l_φ.
- If $l =$ bound to today's l_φ then $l_\varphi^2 \leq l^2$ is satisfied by

$$\left[\frac{f'(R)}{Rf''(R)} - 1\right] \geq \frac{1}{l^2 R} \rightarrow \frac{d\ln f'}{dR} \leq \frac{l^2}{1+l^2 R} \rightarrow f(R) \leq A + B(R + \frac{l^2 R^2}{2})$$

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- Since $f' > 0$ and $f'' > 0$ it is also bounded from below:

$$-2\Lambda \leq f(R) \leq R - 2\Lambda + \frac{l^2R^2}{2}$$

See G.J.O. *Phys.Rev.Lett.* **95**,261102 (2005),

and G.J.O. *Phys.Rev.*D72,083505 (2005)

Summary and conclusions

- $f(R)$ gravities with nonlinear terms that grow at low curvatures lead to cosmic speed-up.

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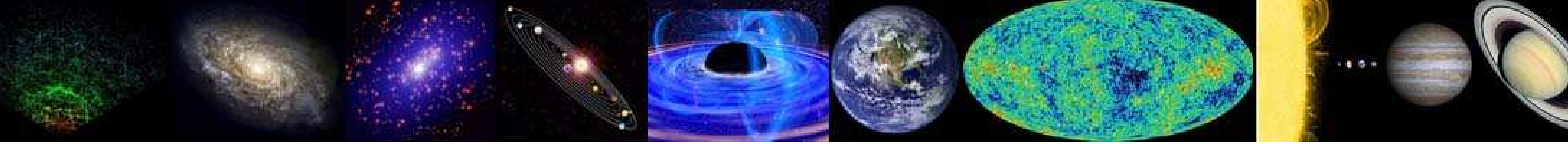
Moral

The dynamics of the **solar system, galaxies**, . . . in gravity theories with dynamical fields others than the metric $(\phi, A_\mu, \ldots)$ **might be strongly affected by the cosmic evolution**. Such effects, if they exist, should not be in conflict with consistent models of **formation and evolution**.



Thanks !!!

¡Gracias!!!