

# Experimental Tests and Alternative Theories of Gravity

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# Motivation

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- High-precision cosmological tests have improved our view of the Universe.
- The Universe is homogeneous, isotropic, spatially flat and is undergoing a period of **accelerated expansion**.
- The description given only *ten years ago* by **General Relativity** is not compatible with the observed accelerated expansion.

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- The Universe is homogeneous, isotropic, spatially flat and is undergoing a period of **accelerated expansion**.
- The description given only *ten years ago* by **General Relativity** is not compatible with the observed accelerated expansion.
- **Something has to be done to justify the acceleration.**

# New Ingredients in the cosmic pie?

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- Introduce a new source of energy in  $T_{\mu\nu}$

# New Ingredients in the cosmic pie?

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- Introduce a new source of energy in  $T_{\mu\nu}$ 
  - Cosmological constant, long-range fields, ...
  - It has been done before and tends to work (dark matter in galaxies, neutrino in  $\beta$  decay, ...).

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To justify the current acceleration we could ...

- Introduce a new source of energy in  $T_{\mu\nu}$
- Modify Einstein's equations.
  - Because of quantum effects in curved space, string theory, higher dimensional theories, ...
  - **GR** could be the leading order of some effective gravity theory:  $R \rightarrow f(R) \approx R + \text{corrections}$  .

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- Modify Einstein's equations.

Today's choice ...

**New Physics**  
→  $f(R)$  gravities

# What are $f(R)$ gravities?

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- $f(R)$  gravities can be seen as a generalization of **GR**

$$S_{GR} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R + S_m[g_{\mu\nu}, \psi_m]$$

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■ 4D-Volume element

■ Matter action

■ Gravity Lagrangian

■ Matter fields

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- Two examples :  $f(R) = R - \frac{\mu^4}{R}$ ,  $f(R) = R + \frac{R^2}{M^2}$
- $f(R)$  can be classified as **Metric Theories of Gravity**.
- **MTG** are the only theories of gravity that can embody the **Einstein Equivalence Principle**.

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We have seen that

- The **accelerated expansion** of the Universe is currently an **unsolved problem**.
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**How do these theories work?**

- How do they change the gravitational physics?
- Do they modify elementary-particle physics?

# The Einstein Equivalence Principle

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The **EEP** states that

- Inertial and gravitational masses coincide, i.e., all bodies fall with the same acceleration → **WEP**.
- The outcome of any local **non-gravitational** experiment is independent of the velocity of the freely-falling reference frame in which it is performed → **Local Lorentz Invariance**.
- The outcome of any local **non-gravitational** experiment is independent of where and when it is performed → **Local Position Invariance**.

# Gravity as a curved-space effect

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If **EEP** is valid

- The **non-gravitational** laws of physics can be formulated by writing the laws of special relativity using the language of differential geometry:

$$\eta_{\mu\nu} \longrightarrow g_{\mu\nu}(x)$$

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- Gravitation would be described by an **MTG**

$$S_{\text{MTG}} = S_G[g_{\mu\nu}, \phi, A_\mu, B_{\mu\nu}, \Gamma_{\mu\nu}^\alpha, \dots] + S_m[g_{\mu\nu}, \psi_m]$$

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However ...

Is the **EEP** really valid ?

# Weak Equivalence Principle

- Can be tested comparing the acceleration of two bodies in an external field:  $m_I a = m_p g$
- $m_I$  and  $m_p$  are made up of rest energy, e.m. energy, weak-interaction energy,...
- If  $m_I$  and  $m_p$  have different contributions

$$m_p = m_I + \sum_i \eta^i \frac{E^i}{c^2}$$

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$$\eta \equiv \frac{2|a_1 - a_2|}{|a_1 + a_2|} = \sum_i \eta^i \left[ \frac{E_1^i}{m_1 c^2} - \frac{E_2^i}{m_2 c^2} \right]$$

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- The current bound is  $\eta = 4 \cdot 10^{-13}$

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- **LLI** would be **violated** if  $c$  would **vary** from one inertial reference frame to another.
- This violation would lead to **shifts in the energy levels** of atoms and nuclei depending on the direction of the quantization axis.

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- The current bound is  $\delta = |c^{-2} - 1| < 10^{-22}$

# Local Position Invariance

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- Can be tested by measuring the gravitational redshift of light.
- The comparison of the frequencies of two clocks at different locations boils down to the comparison of the velocities of two local Lorentz frames at rest at those positions.

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- The current bound is  $|\alpha| < 10^{-5}$ .
- It can also be tested by measuring the constancy of the **non-gravitational** constants.

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  - Testing the **EEP** we could place bounds on the strength of those interactions

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We will study now the gravitational tests of **MTG**

# Gravitational Tests

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- Post-Newtonian gravity
- Stellar systems
- Cosmology
- Gravitational waves

# Gravitational Tests

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- Post-Newtonian gravity
  - Deflection of light
  - Time delay of light
  - Perihelion Shift of Mercury
  - Tests of SEP
  - Conservation laws
  - Geodesic precession (gyroscope)
  - Gravitomagnetism

# Gravitational Tests

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- Post-Newtonian gravity
- Stellar systems
  - Gravitational wave damping of orbital period
  - Internal structure dependence
  - Strong gravity effects

# Gravitational Tests

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- Post-Newtonian gravity
- Stellar systems
- Cosmology
  - Distribution of anisotropies
  - Hubble diagram of SNIa
  - Age of the Universe

# Gravitational Tests

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- Post-Newtonian gravity
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- Cosmology
- Gravitational waves
  - Polarization
  - Speed of grav.waves

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# Parametrized P-N Formalism

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$$\begin{aligned} g_{00} = & -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 \\ & + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 \\ & - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2U - \alpha_2w^iw^jU_{ij} + (2\alpha_3 - \alpha_1)w^iV_i \\ & + O(\epsilon^3) \end{aligned}$$

$$\begin{aligned} g_{0i} = & -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i \\ & - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^iU - \alpha_2w^jU_{ij} + O(\epsilon^{5/2}) \end{aligned}$$

$$g_{ij} = (1 + 2\gamma U + O(\epsilon^2))\delta_{ij}$$

PPN parameters  $\gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4$ .

# Parametrized P-N Formalism

- In the weak-field, slow-motion limit, the metric of nearly every **MTG** has the same structure
- With the potentials given by

$$\begin{aligned}
 U &= \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|} d^3 x', & U_{ij} &= \int \frac{\rho' (x - x')_i (x - x')_j}{|\mathbf{x} - \mathbf{x}'|^3} d^3 x' \\
 \Phi_W &= \int \frac{\rho' \rho''(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \cdot \left( \frac{\mathbf{x}' - \mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x} - \mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|} \right) d^3 x' d^3 x'' \\
 \mathcal{A} &= \int \frac{\rho' [\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')]^2}{|\mathbf{x} - \mathbf{x}'|^3} d^3 x', & \Phi_1 &= \int \frac{\rho' v'^2}{|\mathbf{x} - \mathbf{x}'|} d^3 x' \\
 \Phi_2 &= \int \frac{\rho' U'}{|\mathbf{x} - \mathbf{x}'|} d^3 x', & \Phi_3 &= \int \frac{\rho' \Pi'}{|\mathbf{x} - \mathbf{x}'|} d^3 x', & \Phi_4 &= \int \frac{p'}{|\mathbf{x} - \mathbf{x}'|} d^3 x' \\
 V_i &= \int \frac{\rho' v'_i}{|\mathbf{x} - \mathbf{x}'|} d^3 x', & W_i &= \int \frac{\rho' [\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')] (x - x')_i}{|\mathbf{x} - \mathbf{x}'|^3} d^3 x'
 \end{aligned}$$

# Parametrized P-N Formalism

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- In the weak-field, slow-motion limit, the metric of nearly every **MTG** has the same structure
- It is characterized by a set of parameters:  
 $\gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4$ .
- The predictions of a particular theory depend on these **parameters**

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- The predictions of a particular theory depend on these **parameters**
- We will need to obtain the PPN parameters of  $f(R)$  gravities.

# P-N limit of $f(R)$ gravities

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- The Newtonian limit of these theories was recently discussed in *R.Dick, Gen.Rel.Grav.36* (2004)
- The correct limit could be obtained if  $|f(R_0)f''(R_0)| \ll 1$ .

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- However, these theories are much more involved due to the cosmological evolution of the boundary conditions
- I have the Newtonian limit and the parameter  $\gamma$
- ... I hope to have the remaining parameters next week or so

# Details of $f(R)$ gravities

- Defining a scalar field  $\phi = \frac{df}{dR}$
- And a potential  $V(\phi) = Rf'(R) - f(R)$
- The first corrections of the P-N limit are

$$g_{00} \approx -1 + \frac{\kappa^2}{4\pi\phi_0} \frac{M}{r} \left( 1 + \frac{e^{-m_\phi r}}{3} \right) + \frac{V_0}{6\phi_0} r^2$$
$$g_{ij} \approx \left[ 1 + \frac{\kappa^2}{4\pi\phi_0} \frac{M}{r} \left( 1 - \frac{e^{-m_\phi r}}{3} \right) - \frac{V_0}{6\phi_0} r^2 \right] \delta_{ij}$$

Where

$$m_\phi^2 = \frac{\phi_0 V''(\phi_0) - V'(\phi_0)}{3}$$

Compare

# Details of $f(R)$ gravities

- Defining a scalar field  $\phi = \frac{df}{dR}$
- And a potential  $V(\phi) = Rf'(R) - f(R)$
- The relevant parameters are

$$G_N = \frac{1}{\phi_0} \left( 1 + \frac{e^{-m_\phi r}}{3} \right)$$
$$\gamma = \frac{3 - e^{-m_\phi r}}{3 + e^{-m_\phi r}}$$

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# Examples

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$$m_\phi^2 = \frac{R_0}{6\mu^4}(R_0^2 - 3\mu^4), \quad R_0 \propto \frac{1}{t^2}$$

- Not valid in its original form
- If  $\mu^4 \rightarrow -\mu^4$ , it is valid at early times only.

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- In general,  $f(R) = R + \frac{R^n}{M^{2n+2}}$  are always viable models

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# Summary and Conclusions

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- The post-Newtonian regime is a good arena to test gravity theories
- The post-Newtonian limit of  $f(R)$  gravities is constrained by the cosmic evolution



There is nothing in this  
slide that can be of any  
interest to you

# Parametrized P-N Formalism

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PPN parameters  $\gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4$ .

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# Significance of the PPN parameters

Parameter	What it measures relative to GR	Value in GR
$\gamma$	How much space-curvature produced by unit rest mass?	1
$\beta$	How much “nonlinearity” in the superposition law for gravity?	1
$\xi$	Preferred-location effects?	0
$\alpha_i$	Preferred-frame effects?	0
$\alpha_3$	Violation of conservation of total momentum?	0
$\zeta_j$		0

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# Tests of the parameter $\gamma$

- The deflection of light

$$\delta\theta = \frac{1}{2}(1 + \gamma) \left[ -\frac{4m_{\odot}}{d} \cos \chi + \frac{4m_{\odot}}{d_r} \left( \frac{1 + \cos \Phi_r}{2} \right) \right],$$

where  $d$  and  $d_r$  are the distances of closest approach of the source and reference rays respectively,  $\Phi_r$  is the angular separation between the Sun and the reference source, and  $\chi$  is the angle between the Sun-source and the Sun-reference directions, projected on the plane of the sky.

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# Tests of the parameter $\gamma$

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## ■ The Time Delay of Light

A radar signal sent across the solar system past the Sun to a planet or satellite and returned to the Earth suffers an additional non-Newtonian delay in its round-trip travel time, given by

$$\delta t = 2(1 + \gamma)m_{\odot} \ln[(r_{\oplus} + \mathbf{x}_{\oplus} \cdot \mathbf{n})(r_e - \mathbf{x}_e \cdot \mathbf{n})/d^2]$$

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# Tests of the parameter $\gamma$

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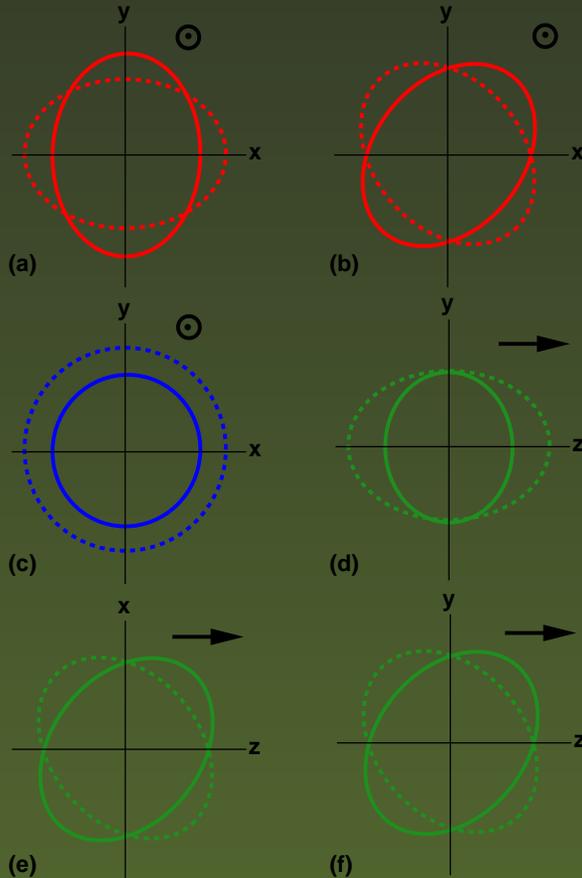
- The perihelion shift of Mercury

$$\dot{\omega} \approx 42.''98 \left[ \frac{1}{3}(2 + 2\gamma - \beta) + 3 \cdot 10^{-4}(J_2/10^{-7}) \right]$$

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# Gravitational waves polarization

- Six polarization modes permitted in any **MTG**  
**Gravitational-Wave Polarization**



- Shown displacement of each mode induced on a ring of test particles
- Propagation in  $+z$  direction
- No displacement out of the plane indicated
- In **GR** only (a) and (b)
- In scalar-tensor gravity, (c) is also possible

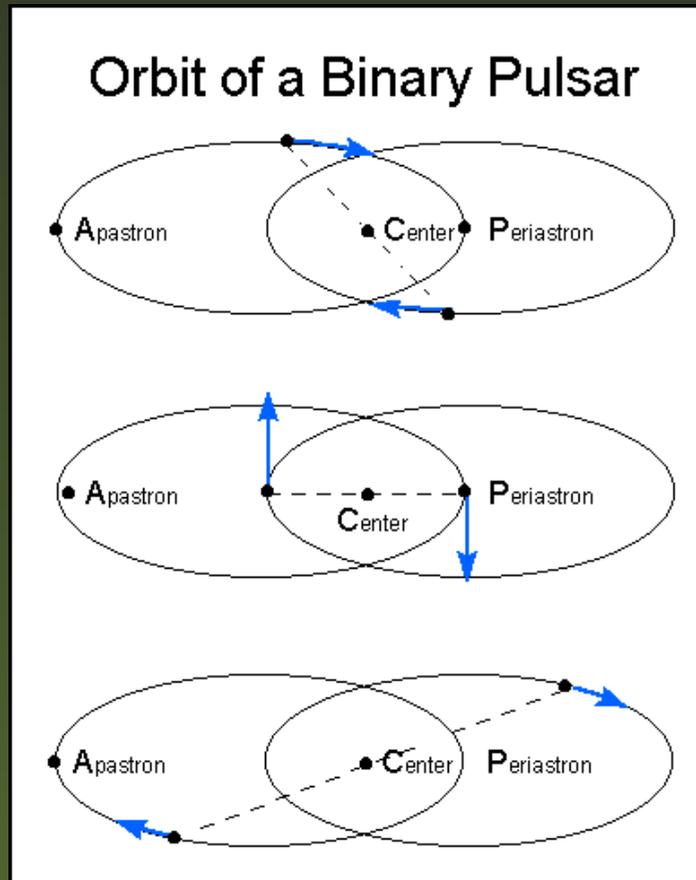
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# The binary pulsar PSR1913+16

- Magnetized NS rotating with  $T = 59$  ms

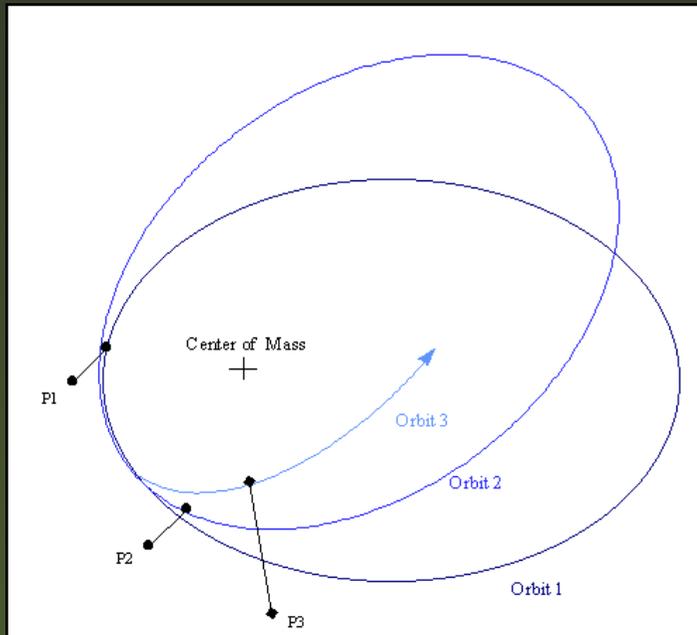
- Orbital period  $7^h 45^{min}$

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# The binary pulsar PSR1913+16

- Magnetized NS rotating with  $T = 59$  ms

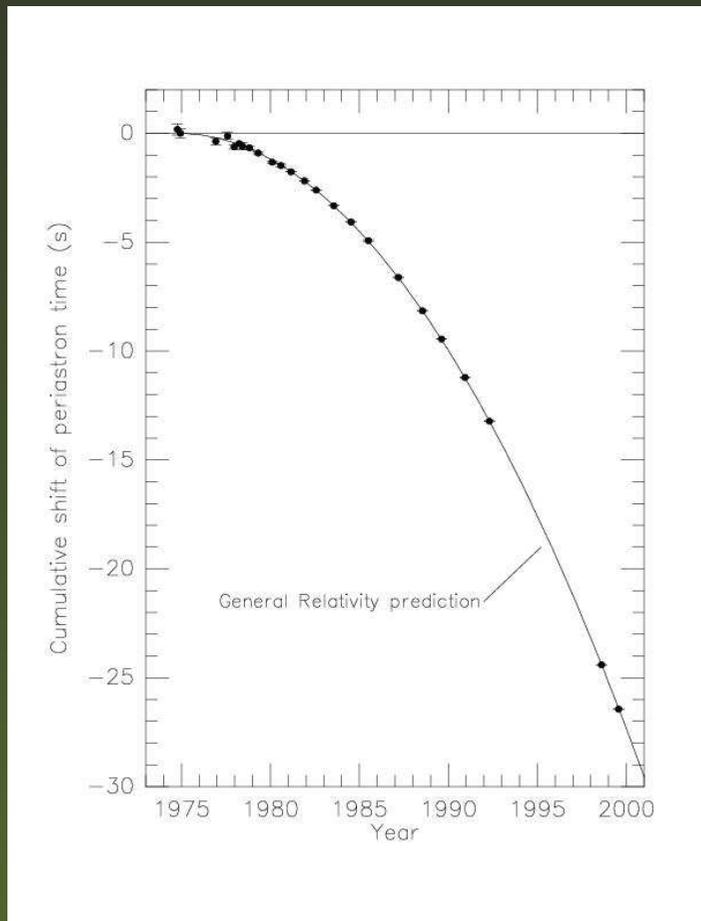


- Orbital period  $7^h 45^{min}$
- Perihelion advance in 1 day the same as Mercury in 1 century

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# The binary pulsar PSR1913+16

- Magnetized NS rotating with  $T = 59$  ms



- Orbital period  $7^h 45^{min}$
- Perihelion advance in 1 day the same as Mercury in 1 century
- The orbit is shrinking due to grav.wave radiation.

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# Ten years ago...

Ten years ago, the standard cosmology was described by

$$\underbrace{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R}_{\text{Einstein equations}} = \kappa^2 \underbrace{T_{\mu\nu}}_{\text{Visible Matter, Dark Matter, Radiation}}$$

Einstein  
equations

Visible Matter  
Dark Matter  
Radiation

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# Cosmic Microwave Brackground

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