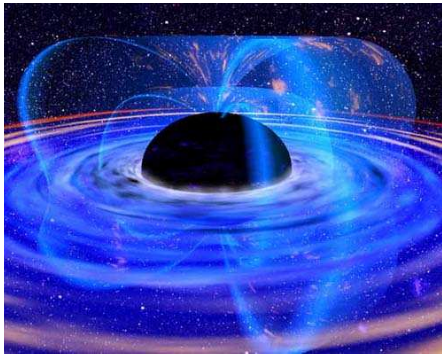

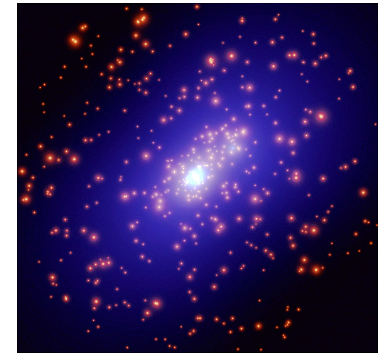

Correcciones cuánticas de agujeros negros
y
cosmología

(or Quantum correlations, black holes and cosmology)

Gonzalo J. Olmo Alba

Thesis advisor: José Navarro Salas

Universidad de Valencia

Outline

Outline

- Outline
- Quantum Correlations and BH
- Cosmology
- The end

Part I: Quantum correlations and black holes

- Subject: New approach for radiation problems in curved space.
- Structure:
  - Black hole evaporation following the standard formalism.
  - Difficult application of the standard approach when backreaction effects are considered.
  - New approach to solve the problems: correlation functions.

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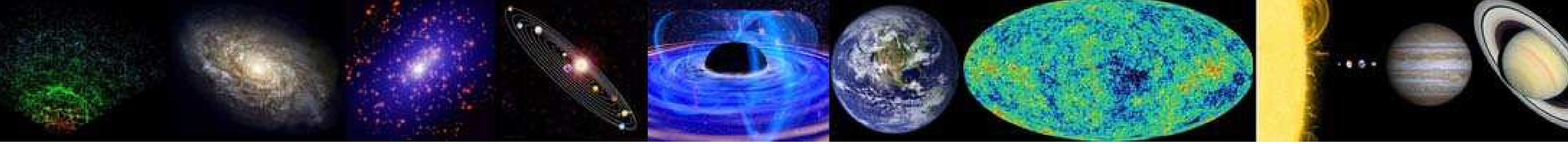
  - New approach to solve the problems: correlation functions.

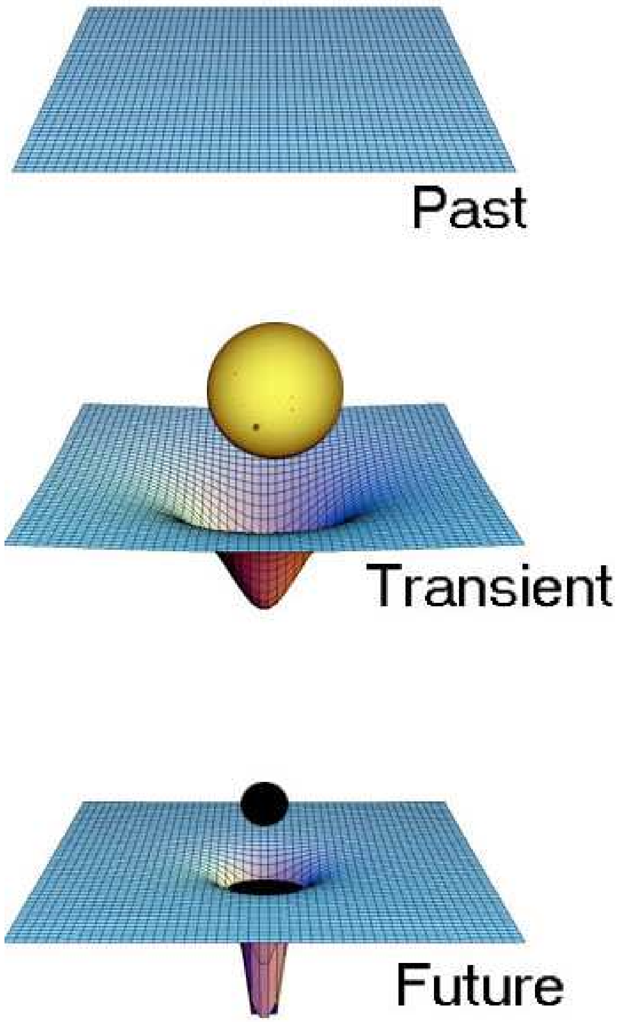
Part II: Cosmology

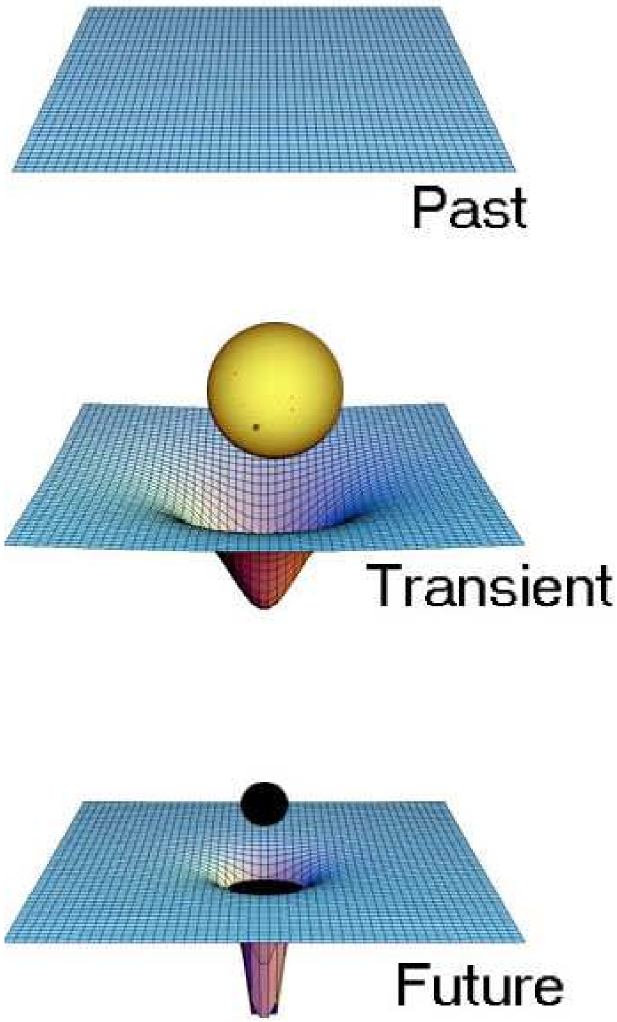
- Subject: Cosmic speed-up due to new gravitational dynamics?
- Structure:
  - Observational evidence for the cosmic accelerated expansion.
  - Possible explanations: dark energy, modified dynamics, . . .
  - Modified dynamics: $f(R)$ gravities.
  - Analyze the solar system constraints on these theories.



Part I:
Quantum correlations and black holes

Gravitational collapse and quantization

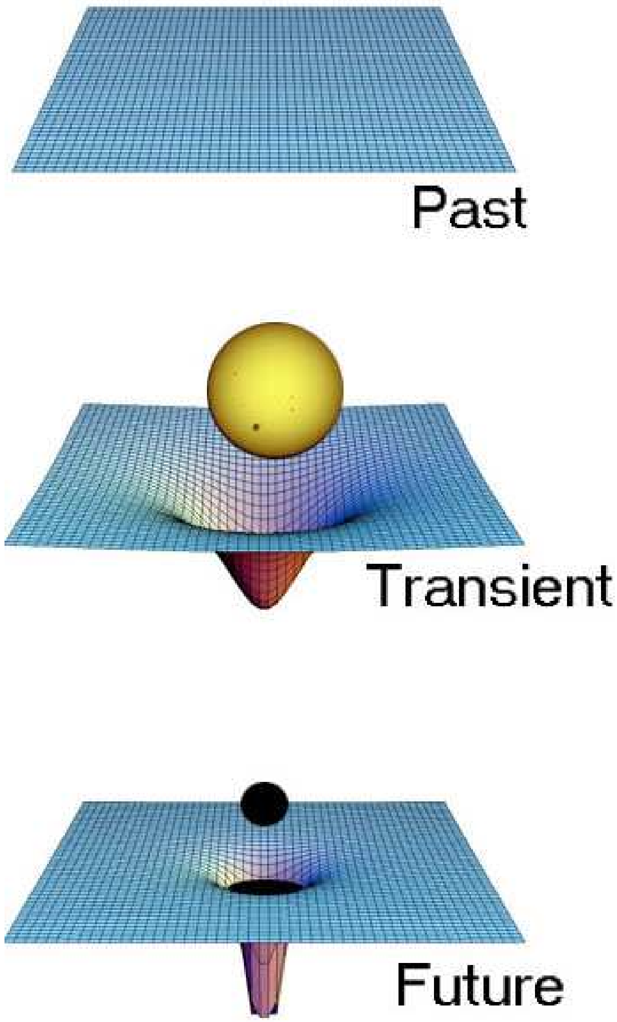




- Consider a scalar field

$$\phi(x) = \sum [a_i u_i(x) + a_i^\dagger u_i^*(x)]$$

Gravitational collapse and quantization

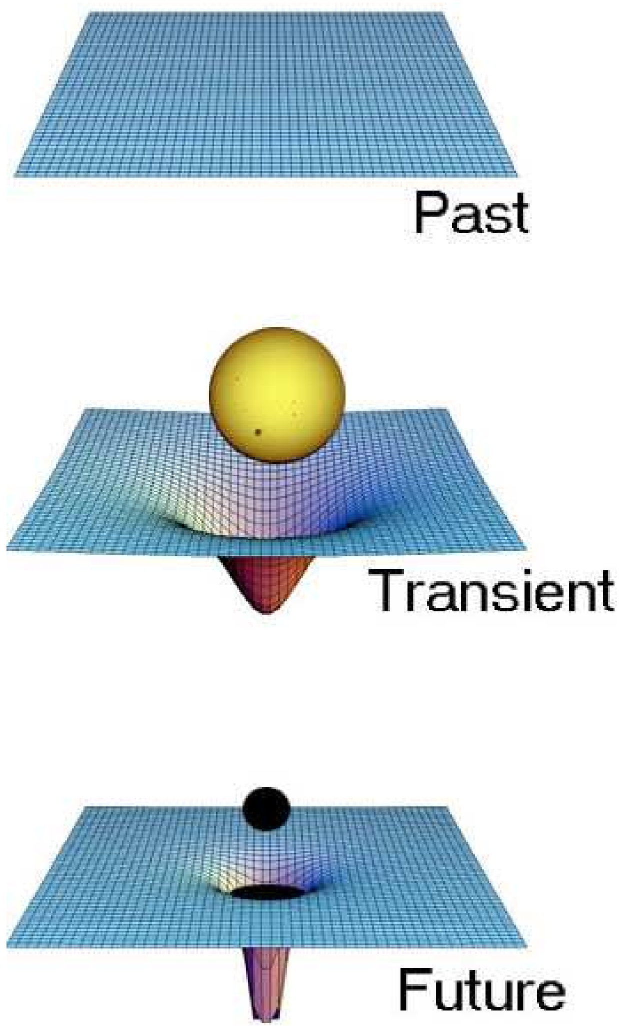


- Consider a scalar field
- "IN" expansion:

$$\phi(x) = \sum_i [a_i^{in} u_i^{in}(x) + a_i^{in\dagger} u_i^{in*}(x)]$$

Vacuum state: $a_i^{in}|0\rangle_{in} = 0$

Gravitational collapse and quantization



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$$\phi(x) = \phi_H + \phi_{I^+}$$

$$\phi_H = \sum_j [a_j^H u_j^H(x) + a_j^{H\dagger} u_j^{H*}(x)]$$

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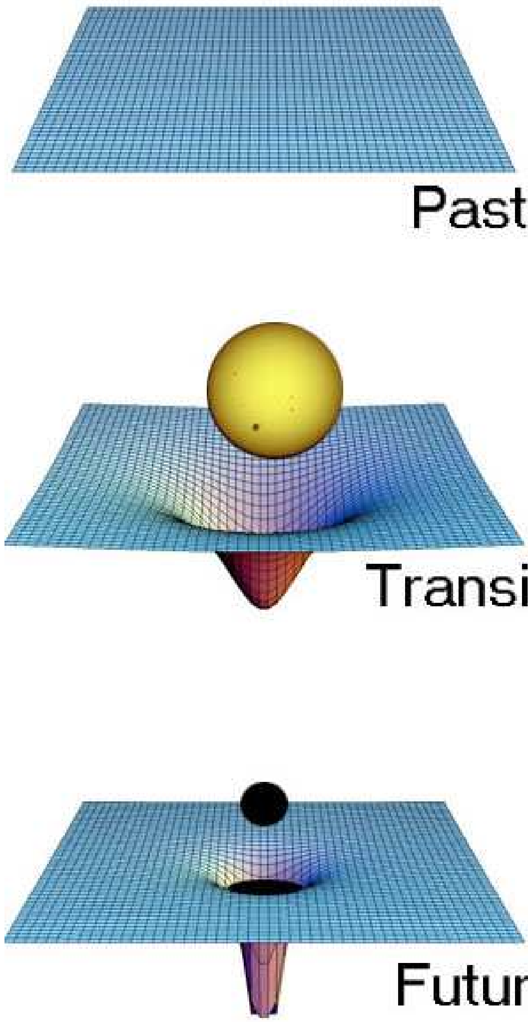

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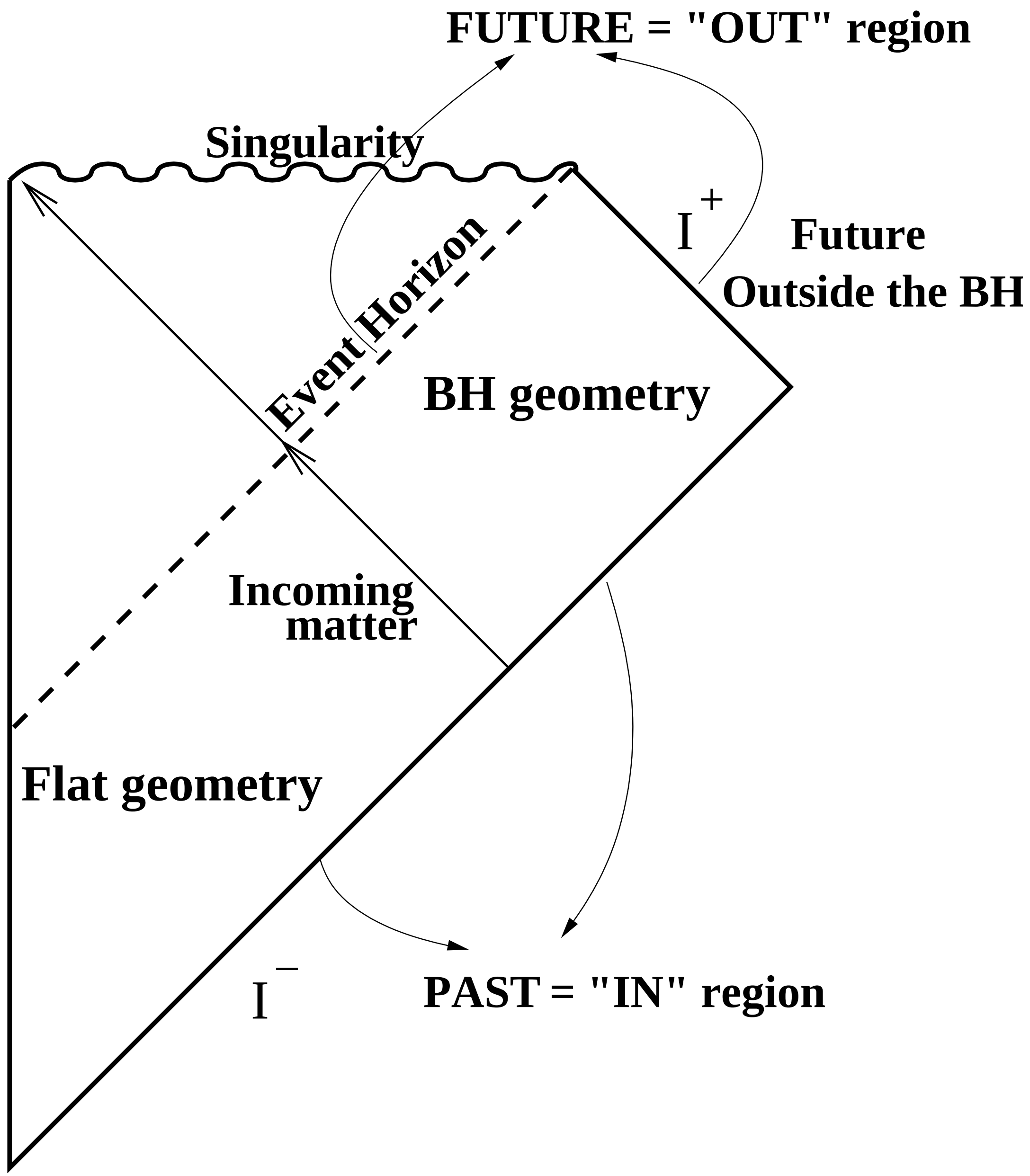
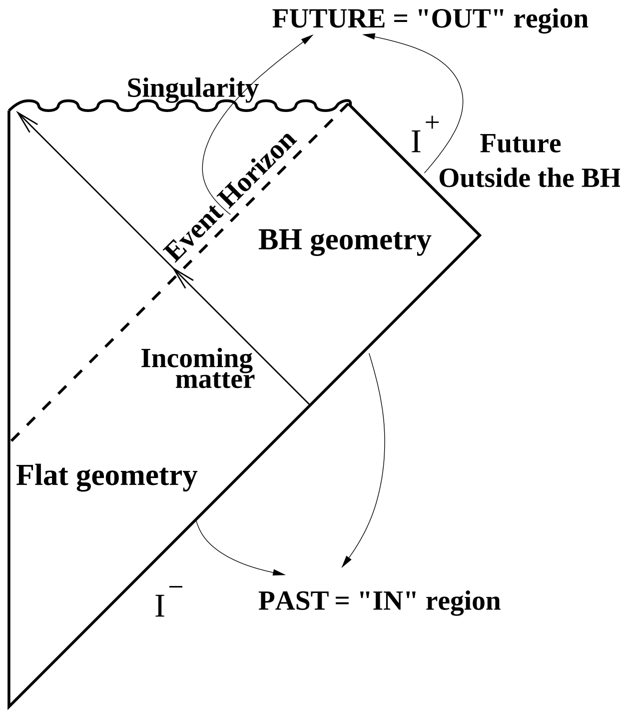
- In general $|0\rangle_{in} \neq |0\rangle_{out}$


FUTURE = "OUT" region
Singularity
I^+
Future
Outside the BH
Event Horizon
BH geometry
Incoming
matter
Flat geometry
I^-
PAST = "IN" region

Bogolubov coefficients



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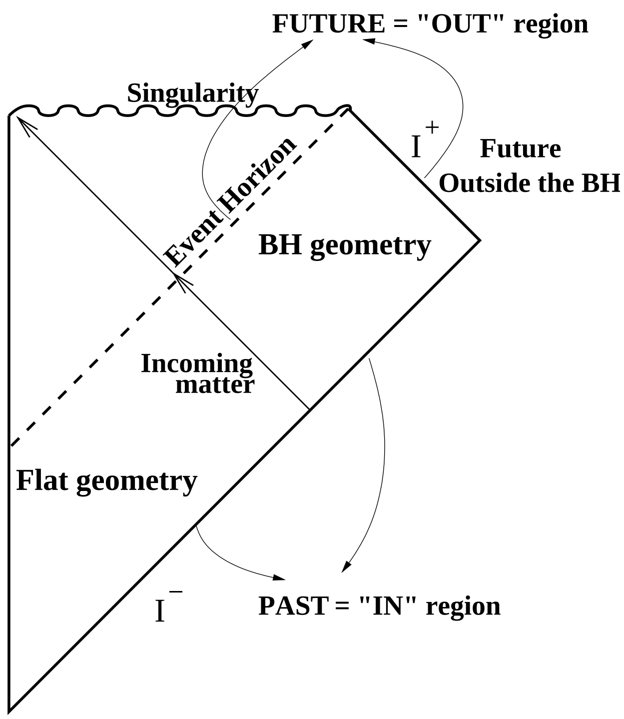

- Outside the Black Hole:

$$\begin{cases} u_j^{out} = \sum_i [\alpha_{ji} u_i^{in} + \beta_{ji} u_i^{in*}] \\ a_i^{out} = \sum_j [\alpha_{ij}^* a_j^{in} - \beta_{ij}^* a_j^{in\dagger}] \end{cases}$$

$$\alpha_{ji} = (u_j^{out}, u_i^{in}) \;,\; \beta_{ji} = -(u_j^{out}, u_i^{in*})$$

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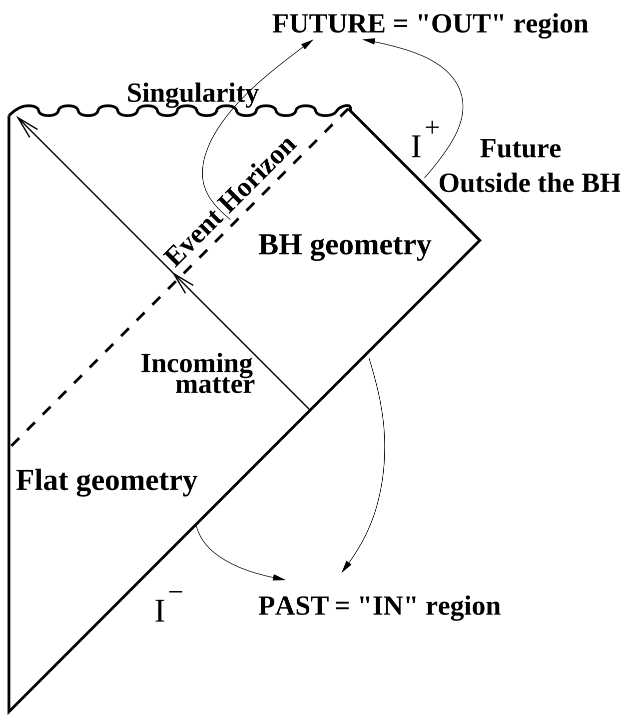

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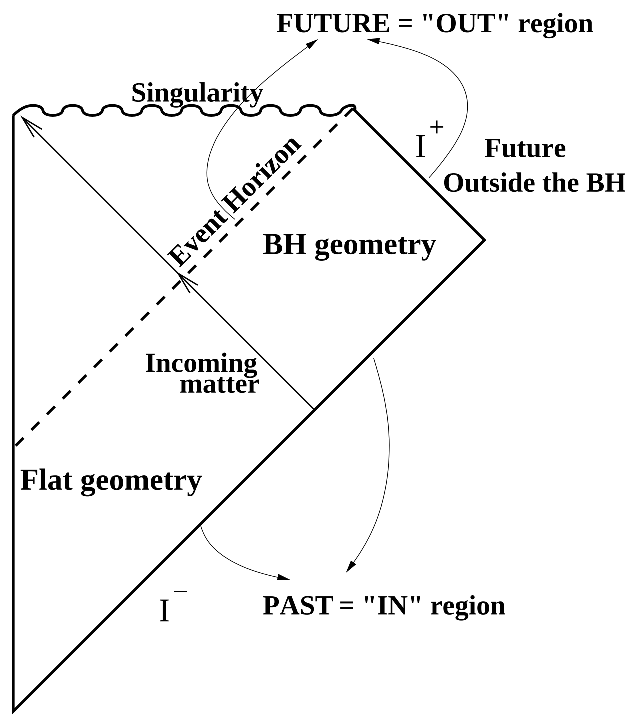

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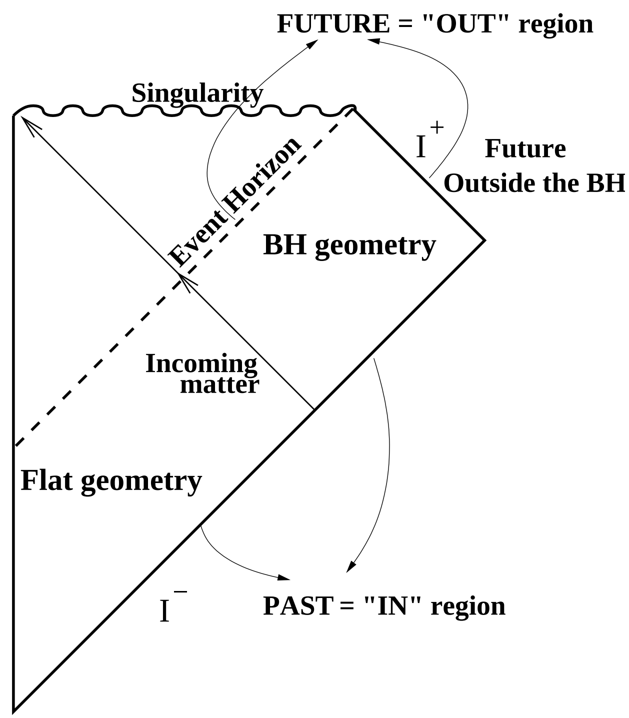
$$|0\rangle_{in} = S|0\rangle_{out}$$

$$S = S(\alpha, \beta, \gamma, \eta)$$

Bogolubov coefficients



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$$_{in}\langle 0|N_i^{out}|0\rangle_{in} = \sum_k |\beta_{ik}|^2$$

Black Holes evaporate

- Number of particles detected at I^+ (Hawking 1974):

$$ {}_{in}\langle 0 | N_i^{out} | 0 \rangle_{in} = \sum_k |\beta_{ik}|^2 = \frac{1}{e^{8\pi M \omega_i} - 1} $$

$\Rightarrow$ Planckian spectrum at $T = \frac{\hbar}{8\pi\kappa_B M}$

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- BIG PROBLEM : quantum information not radiated (Hawking 1976)

Apparent conflict between QM and GR:

Non-unitary evolution of quantum states

(Information Loss Problem)

Evaporation with backreaction

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  Strong correlations appear in the outgoing radiation:

$$C_{rel} = \frac{C_{wbr}(x_1,x_2)}{C_{nbr}(x_1,x_2)} \sim \frac{e^{2\kappa|x_1-x_2|}}{|x_1-x_2|^4}$$

where $C(x_1,x_2) \equiv {}_{in}\langle 0|\phi(x_1)\phi(x_2)|0\rangle_{in}$

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  - Unknown ${}_{in}\langle 0|N_i^{out}|0\rangle_{in} = ?$
  - Unknown density matrix, $|0\rangle_{in} \rightarrow ?$
- Moving-mirror model physically equivalent but . . .
  - Unclear computation of α and β.
  - Unclear relation between particles and energy fluxes.

Bogolubov -Vs- Correlator

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- YES!

$\Rightarrow$ We can bypass the computation of α and β !!!

New approach

- With the decomposition

$$\phi_{I^+} = \sum [a_j^{out} u_j^{out}(x) + a_j^{out\dagger} u_j^{out*}(x)]$$

We construct the normal-ordered operator

$$:\phi(x_1)\phi(x_2): \; \equiv \; \phi(x_1)\phi(x_2) - {}_{out}\langle 0|\phi(x_1)\phi(x_2)|0\rangle_{out}$$

New approach

- Outline

Quantum Correlations and BH
- Gravitational collapse
- Bogolubov coefficients
- Black Holes evaporate
- Evaporation with backreaction
- Bogolubov -Vs- Correlator
- New approach
- Example: Conformal Invariance
- Moving-mirrors and BH
- Beyond N
- Summary and Conclusions

Cosmology

The end

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- **Number of particles:**

$${}_{in}\langle 0|N_i^{out}|0\rangle_{in} = \frac{1}{\hbar} \quad d\Sigma_1^\mu d\Sigma_2^\nu [u_i^{out}(x_1)\overleftrightarrow{\partial}_\mu][u_i^{out*}(x_2)\overleftrightarrow{\partial}_\nu]\; {}_{in}\langle 0|:\phi(x_1)\phi(x_2):|0\rangle_{in}$$

Example: Conformal Invariance

- In d-dimensional Minkowski space, a conformally invariant field theory satisfies:

$$
\begin{aligned}
{}_{in}\langle 0|\phi(y_1)\phi(y_2)|0\rangle_{in} &= \frac{C}{|y_1 - y_2|^{2\Delta}} \\
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It vanishes for Conformal Transf. $\Rightarrow \; _{in}\langle 0|N_i^{out}|0\rangle_{in} = 0.$

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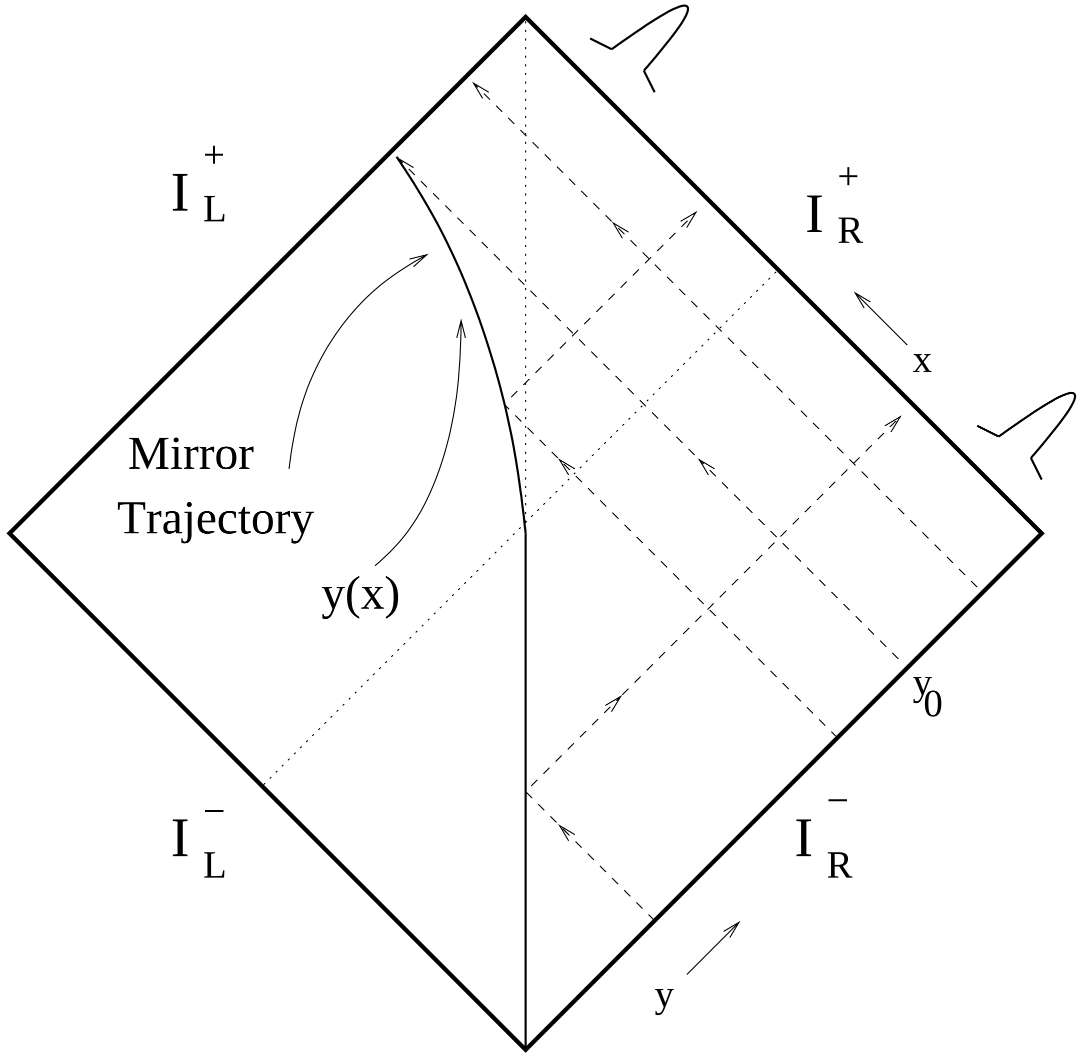
- β_{ij} should vanish for all Conformal Transf.

Moving-mirrors and black holes



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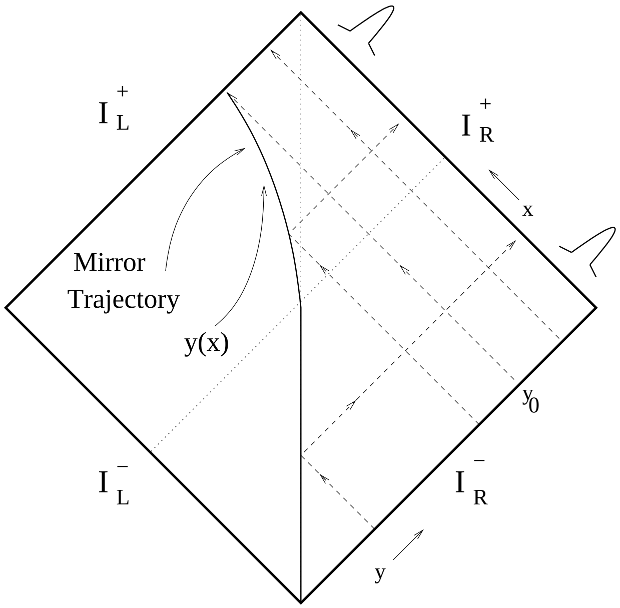

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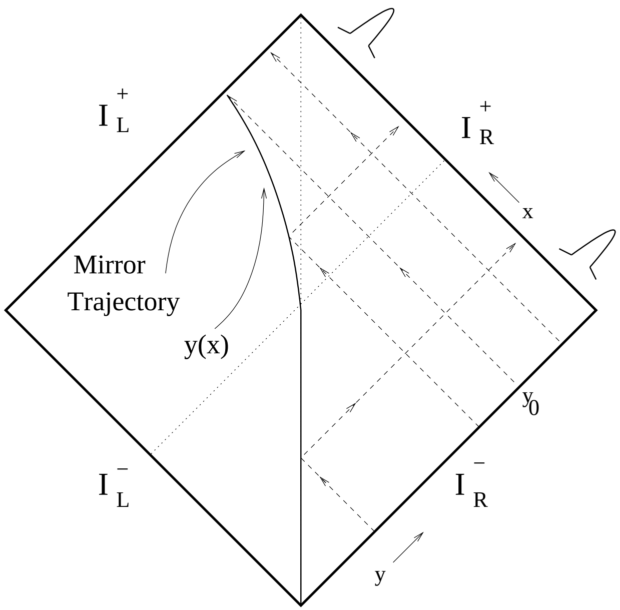

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$$ {}_{in}\langle 0|N_k^{out}|0\rangle_{in} = -\frac{1}{\pi}\int_{-\infty}^{\infty} dx_1 dx_2 u_k(x_1) u_k^*(x_2) \times $$

$$ \times \left[\frac{y'(x_1)y'(x_2)}{[y(x_1)-y(x_2)]^2} - \frac{1}{(x_1-x_2)^2} \right] $$

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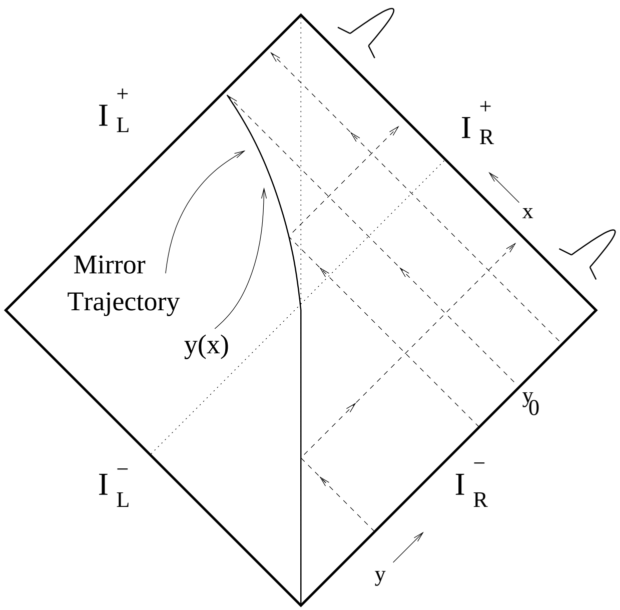

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$$ y(x) = y_0 - \frac{1}{C}e^{-Cx} \Rightarrow \text{THERMAL} $$

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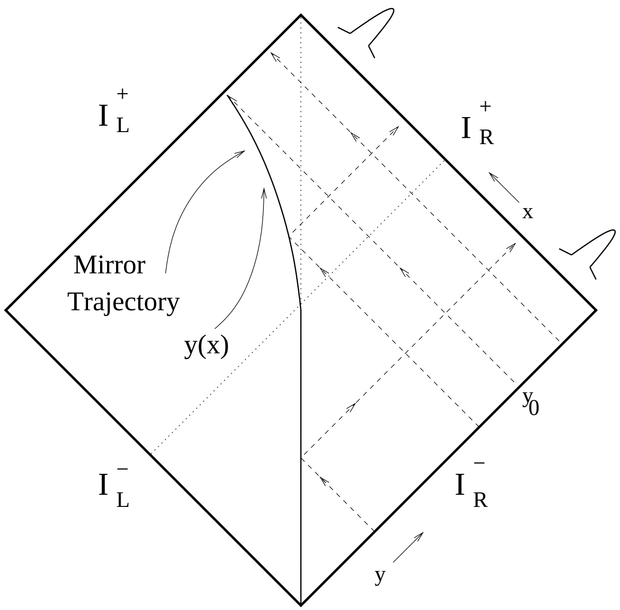

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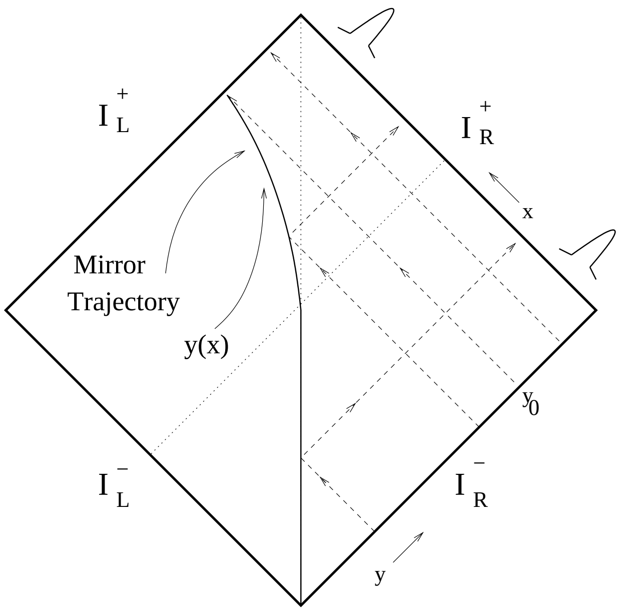

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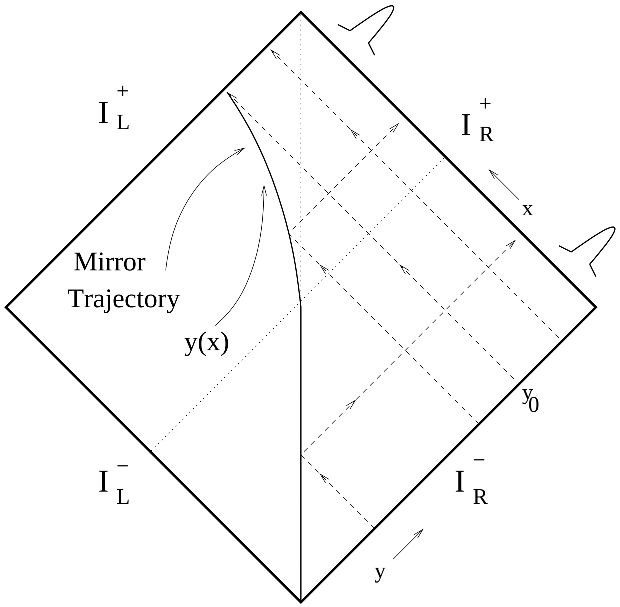

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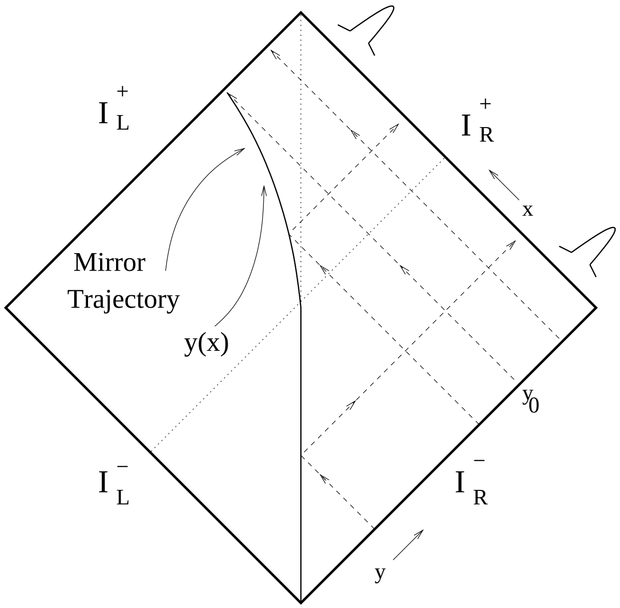

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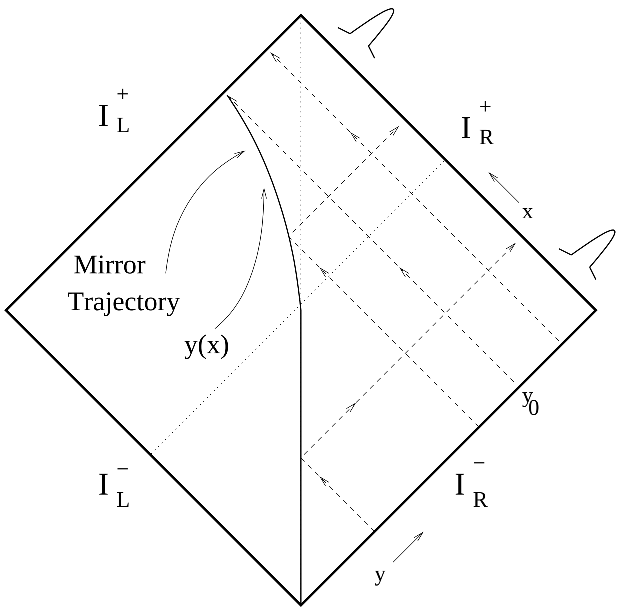

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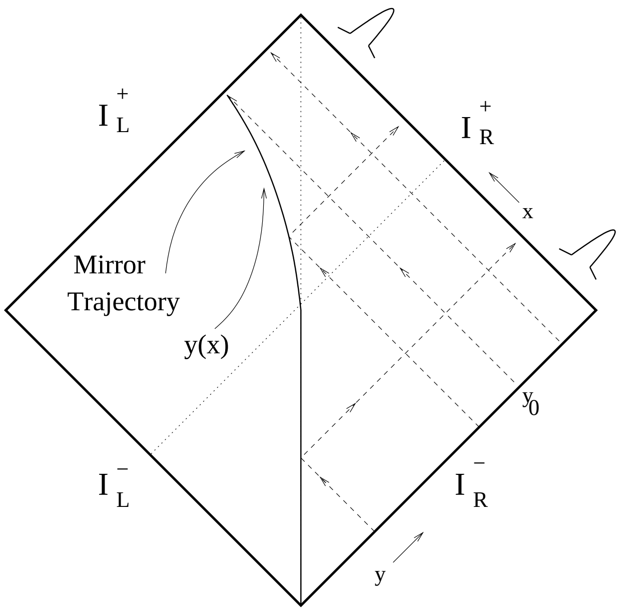

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- We are free to choose between two representations:

$\{\alpha, \beta\}$ — "IN" and "OUT" indices

$\{N, C\}$ — "OUT" indices

Summary and Conclusions

- Alternative approach to study radiation problems:

$$\{\alpha, \beta\} \text{ - Vs - } \{N, C\}$$

in terms of correlation functions: ${}_{in}\langle 0| : \phi(x_1)\phi(x_2) : |0\rangle_{in}$

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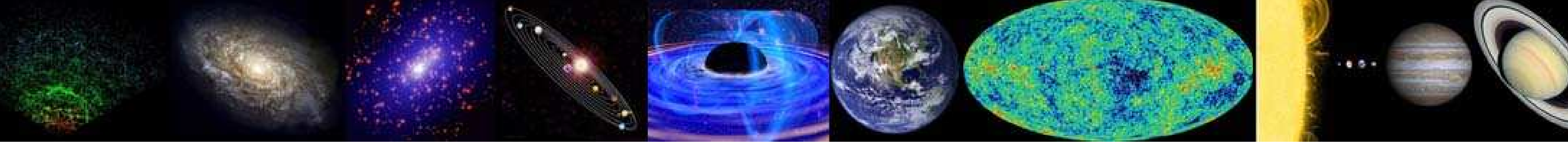

Part II: Cosmology

Standard cosmologies

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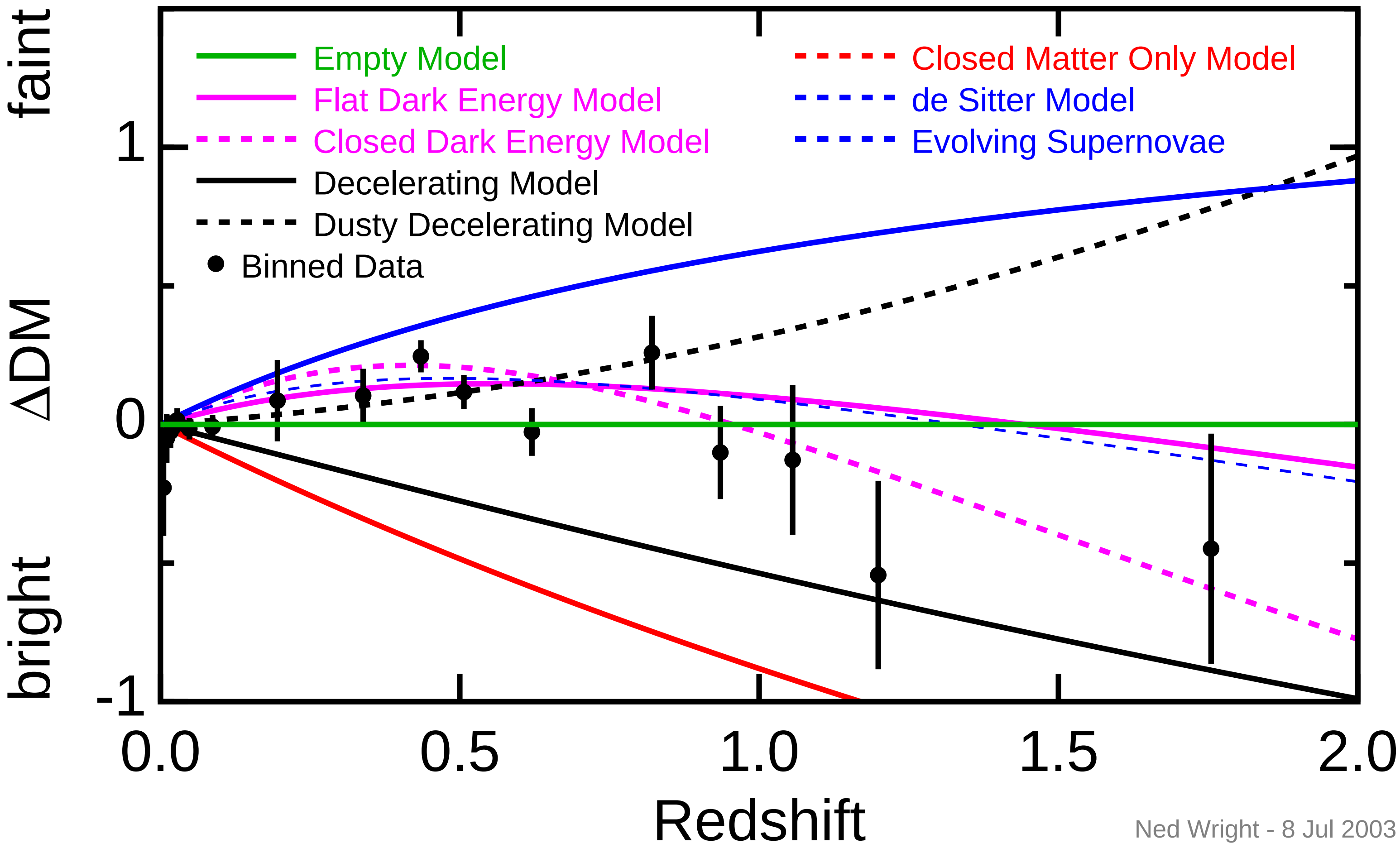
- $\ddot{a}(t_0) > 0$ was unexpected only 10 years ago.

Accelerating Universe



- Big dots represent type-Ia supernovae.
- The expansion began to accelerate some 5000 million years ago.

Mechanism for the acceleration

- Dark energy: some stuff with negative pressure.

In GR $\Rightarrow$ $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{tot} + 3P_{tot}\right)$

Matter	$\rho_m \sim 1/a^3 \, , \, P_m = 0$
Radiation	$\rho_r \sim 1/a^4 \, , \, P_r = \rho_r/3$
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- Others

f(R) gravities: motivation and examples

- "$f(R)$ gravities" stands for:

$$S = \frac{1}{2k^2} \quad d^4x\sqrt{-g}\,f(R) + S_m(g_{\mu\nu}, \psi)$$

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- GR could just be a good approximation at intermediate curvatures.

Metric and Palatini formalisms

- Scalar curvature and Ricci tensor:

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\begin{aligned}
R &= g^{\mu\nu} R_{\mu\nu} \\
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- By trial and error: (very common method)

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- Ask Nature about the admissible $f(R)$ functions.

(Method of this Thesis)

Constraining the gravity lagrangian

- Take a clean scenario to test gravity.
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- The original action $S = \frac{1}{2k^2} \quad d^4x\sqrt{-g}\, f(R) + S_m(g_{\mu\nu}, \psi)$ can be rewritten as follows:

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- This is more than a Brans-Dicke theory.
  - In B-D $V(\phi) = 0$ (or near an extremum) and ω is determined by observations $(\omega_{obs} > 40.000)$.
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PN limit I: Scalar-Tensor representation

- The original action $S = \frac{1}{2k^2} \quad d^4x\sqrt{-g}f(R) + S_m(g_{\mu\nu},\psi)$ can be rewritten as follows:

$$S = \frac{1}{2\kappa^2} \quad d^4x\sqrt{-g}\left[\phi R(g) - \frac{\omega}{\phi}(\partial_\mu\phi\partial^\mu\phi) - V(\phi)\right] + S_m$$

  - where $\phi \equiv \frac{df}{dR}$ and $V(\phi) = Rf'(R) - f(R)$
  - E.O.M. $(3+2\omega)\Box\phi + 2V - \phi\frac{dV}{d\phi} = k^2 T$
  - Metric $\Rightarrow \omega = 0 \Rightarrow$ dynamical.
  - Palatini $\Rightarrow \omega = -3/2 \Rightarrow$ non-dynamical.
- This is more than a Brans-Dicke theory.
  - In B-D $V(\phi) = 0$ (or near an extremum) and ω is determined by observations $(\omega_{obs} > 40.000)$.
  - Now ω is fixed and $V(\phi)$ is to be determined.
- We want to constraint the form of $V(\phi) \Leftrightarrow f(R)$.

PN limit II: Metric formalism or $\omega = 0$

- The metric $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$:

$h_{00}^{(2)} \approx 2G\frac{M_\odot}{r} + \frac{V_0}{6\phi_0}r^2$	$G = \frac{k^2}{8\pi\phi_0}\left[1 + \frac{e^{-m_\varphi r}}{3}\right]$	$G_{exp} = \text{const.}$
$h_{ij}^{(2)} \approx \left[2\gamma G\frac{M_\odot}{r} - \frac{V_0}{6\phi_0}r^2\right]\delta_{ij}$	$\gamma = \frac{3-e^{-m_\varphi r}}{3+e^{-m_\varphi r}}$	$\gamma_{exp} \approx 1$

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  L_S is a relatively short lengthscale.

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- Conclusion:

$$-2\Lambda \leq f(R) \leq R - 2\Lambda + \frac{l^2 R^2}{2}$$

$l^2 \ll L_S^2$ is a bound to the current range of the scalar interaction

PN limit III: Palatini formalism or $\omega = -3/2$

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$h_{00}^{(2)} \approx 2G\frac{M_\odot}{r} + \frac{V_0}{6\phi_0}r^2 + \log\left(\frac{\phi(\rho)}{\phi_0}\right)$	$G = \frac{\kappa^2}{8\pi\phi_0}\left(1 + \frac{M_V}{M_\odot}\right)$
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- **Conclusion:**

$$f(R) \leq \alpha + \frac{l^2R^2}{2} + \frac{R}{2}\sqrt{1+(l^2R)^2} + \frac{1}{2l^2}\log[l^2R + \sqrt{1+(l^2R)^2}]$$
$$f(R) \geq \alpha - \frac{l^2R^2}{2} + \frac{R}{2}\sqrt{1+(l^2R)^2} + \frac{1}{2l^2}\log[l^2R + \sqrt{1+(l^2R)^2}]$$

Expanding in l^2R we get:

$$\alpha + R - \frac{l^2R^2}{2} \leq f(R) \leq \alpha + R + \frac{l^2R^2}{2}$$

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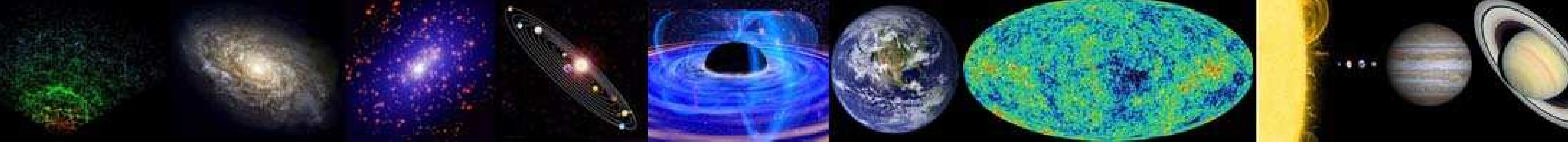
Summary and Conclusions

- Is the cosmic speed-up driven by $f(R)$ gravities?
- Solar system experiments impose severe constraints on the lagrangian $f(R)$.
- Non-linear contributions dominant at low curvatures $(1/R, \log R, \ldots)$ are ruled out by observations.
- Viable models are almost equivalent to $R - 2\Lambda$ in their late-time cosmological predictions.



Thanks!
¡Gracias!