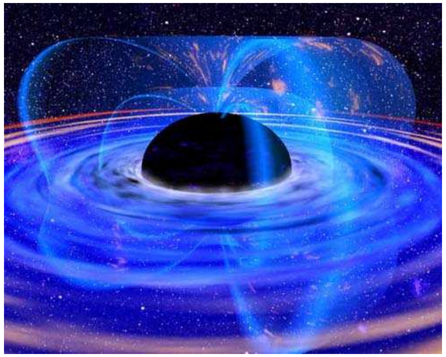

Hawking Radiation
and
Quantum Correlations

(or Analysis of Quantum Radiation by means of Correlation Functions)

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University of Wisconsin-Milwaukee

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Motivation and Outline

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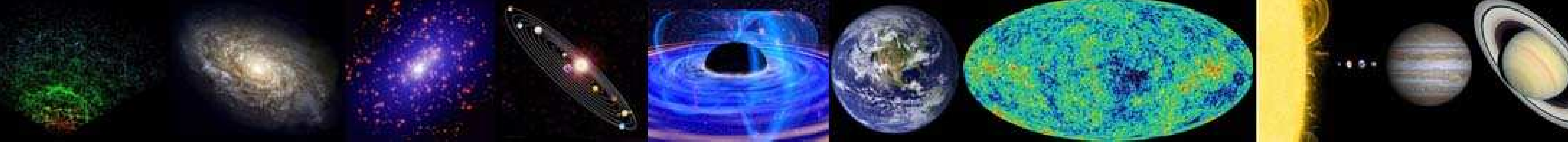
- Outline:
  - Part I . Hawking radiation: standard derivation.
    - Basics of quantization in curved spacetime.
    - Gravitational collapse in Vaidya spacetime.
    - Hawking radiation and the information loss problem.
  - Part II . New approach: correlation functions.
    - Number operator in the new approach.
    - Conformal symmetry and thermal radiation.
    - Particles, energy fluxes and thunderbolts.



Part I:

Hawking radiation: standard derivation

Canonical Quantization in curved space

- Consider a free, massless, scalar field, $\Box\phi = 0$, and its expansion
  $\phi(x) = \sum_i a_i u_i(x) + a_i^\dagger u_i^*(x)$, where $(u_i, u_j) = \delta_{ij}$, $(u_i, u_j^*) = 0$.

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- Quantize the field promoting a_i, $a_i^\dagger$ to operators :

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- In a general curved space **no natural classification of frequencies exists**.
- In stationary spacetimes a definition is possible

  $\xi^\mu \nabla_\mu u_j(x) = -i\omega_j u_j(x)$, where $\xi^\mu \nabla_\mu \equiv \partial_t \equiv$ Killing time.

Bogolubov transformations

In a non-stationary spacetime with two asymptotic stationary regions:

In the past (**IN** region):

$$\phi(x) = \sum_i a_i^{in} u_i^{in}(x) + a_i^{in\dagger} u_i^{in*}(x)$$

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$$N_j^{out} \equiv a_j^{out\dagger} a_j^{out} \quad \rightarrow \qquad {}_{in}\langle 0|N_j^{out}|0\rangle_{in} = \sum_k |\beta_{ik}|^2$$

Bogolubov transformations

Motivation and Outline

Hawking radiation
- Canonical Quantization
- Bogolubov transformations
- Gravitational collapse
- Quantization in Vaidya I
- Quantization in Vaidya II
- Particle production
- Information loss
- Backreaction effects

Correlation functions

The End

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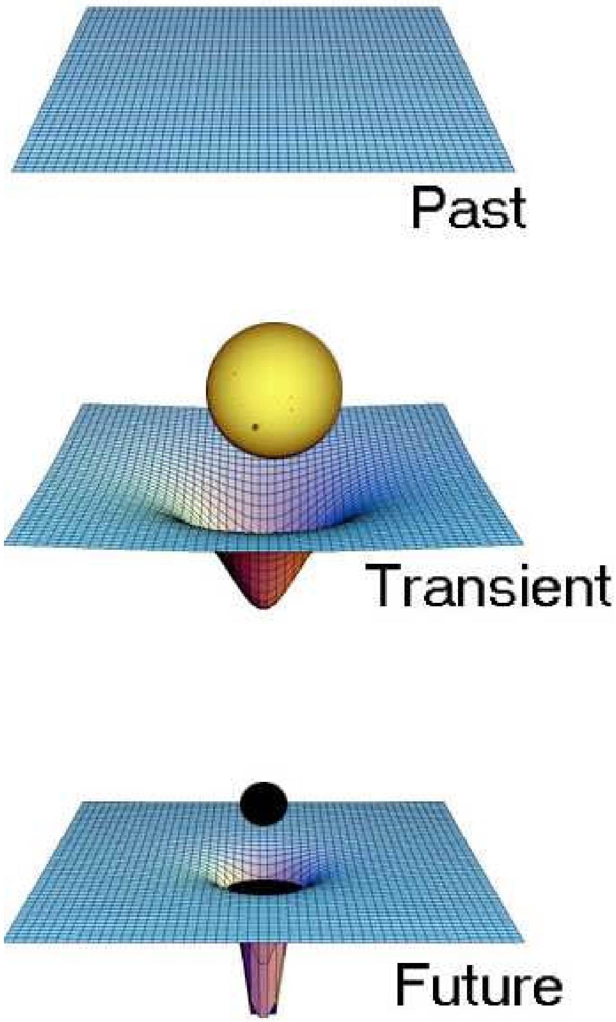
- Changes in the geometry lead to particle production.

Gravitational collapse in Vaidya spacetime



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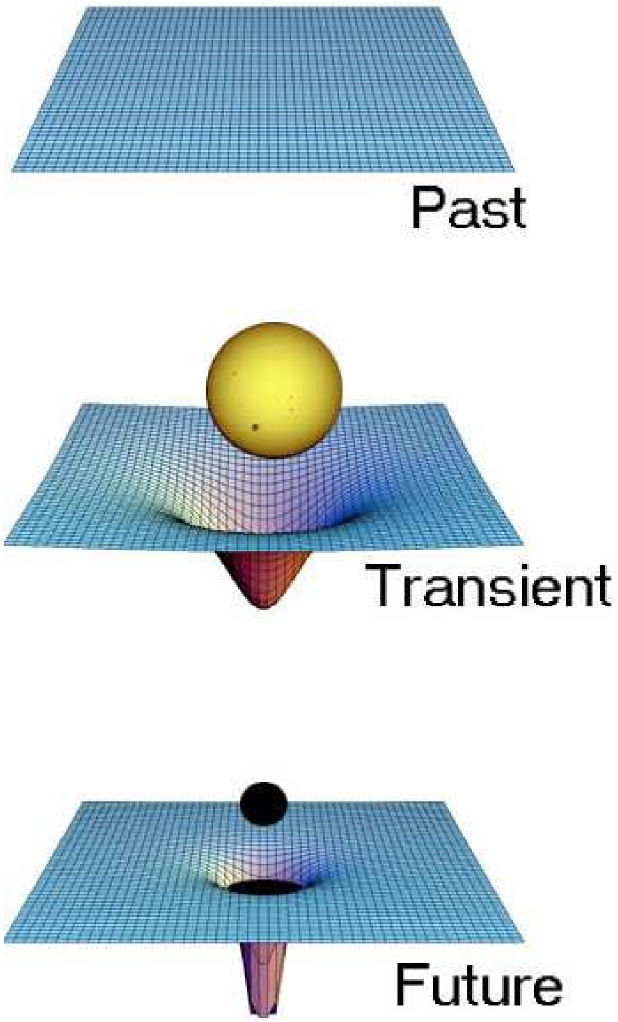
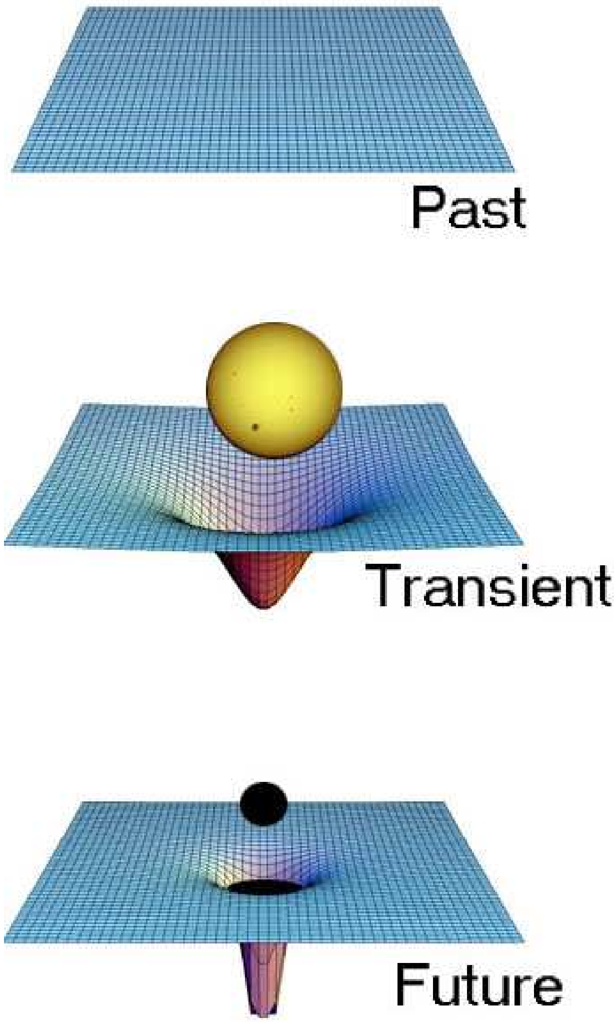


- We will assume:
  - Perfect spherical symmetry.
  - Unimportant details of the collapse.
  - Other simplifications.



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- Vaidya spacetime line element:

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

solution corresponding to $T_{vv} = \frac{dM(v)/dv}{4\pi r^2}$.

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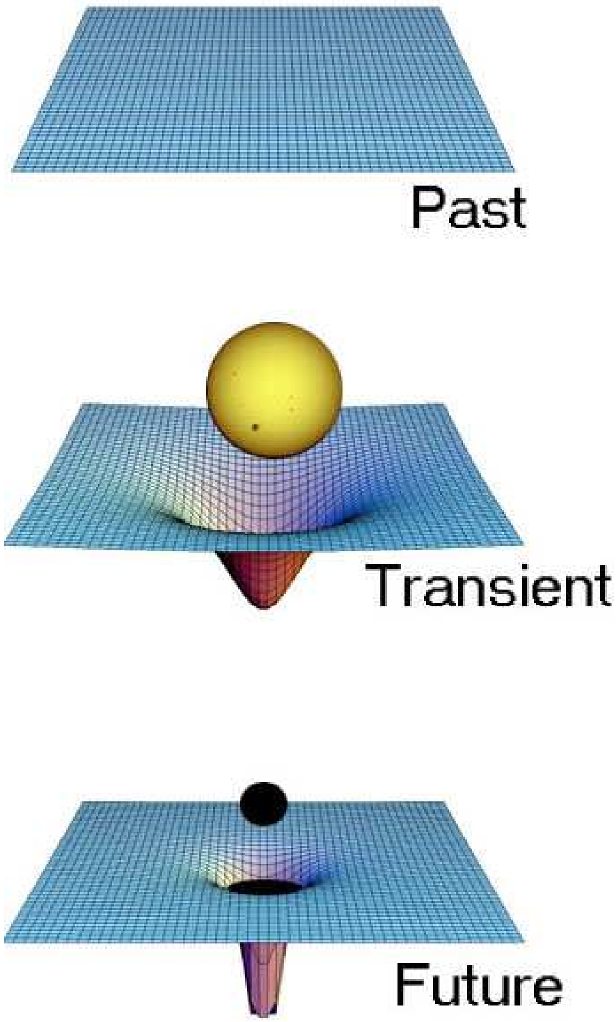

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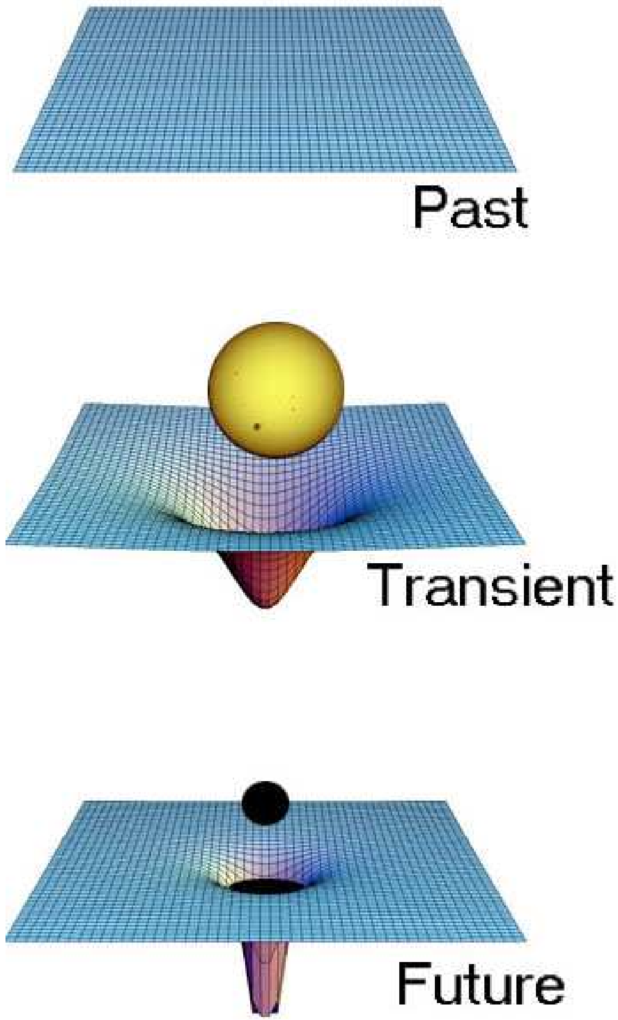

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- Only the asymptotic regions are relevant for particle production $\longrightarrow$ $T_{vv} = \frac{M_0 \delta(v - v_0)}{4\pi r^2}$
- We need to solve $\Box f = 0$ in this background.

Quantization in Vaidya spacetime I

- The line element can be written as:

$$ds^2 = \begin{cases} -dv^2 + 2dvdr + r^2 d\Omega^2 & v \leq v_0 \quad \text{Minkowski} \\ -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2 & v \geq v_0 \quad \text{Schwarzschild} \end{cases}$$

or in conformal gauge as

$$ds^2 = \begin{cases} -dvdu_{in} + r_{in}^2 d\Omega^2 & v \leq v_0 \\ -\left(1 - \frac{2M}{r}\right) dvdu_{out} + r_{out}^2 d\Omega^2 & v \geq v_0 \end{cases}$$

where $\frac{v-u_{in}}{2} = r_{in}(v, u_{in})$ and $\frac{v-u_{out}}{2} = r_{out}(v, u_{out}) + 2M \ln\left[\frac{r_{out}-2M}{2M}\right]$.

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- Expanding $f(x^\mu) = \sum_{l,m} \frac{f_l(t,r)}{r} Y_{lm}(\theta, \varphi)$ we find

$$\begin{aligned} &\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2} - \frac{l(l+1)}{r^2}\right) f_l(t,r) = 0 \quad && v \leq v_0 \\ &\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_l(r)\right) f_l(t,r) = 0 \quad && v \geq v_0 \end{aligned}$$

with $V_l(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right)$

Quantization in Vaidya spacetime II

- Neglecting $V_l(r)$ and using conformal coordinates:

$$\begin{array}{ll} \partial_v \partial_{u_{in}} f = 0 & v \leq v_0 \\ \partial_v \partial_{u_{out}} f = 0 & v \geq v_0 \end{array} \qquad \begin{array}{c} \text{since} \quad \Rightarrow \\ u_{out} = u_{out}(u_{in}) \end{array} \qquad \begin{array}{ll} \partial_v \partial_{u_{in}} f = 0 & \text{all } v \\ \partial_v \partial_{u_{out}} f = 0 & \text{all } v \end{array}$$

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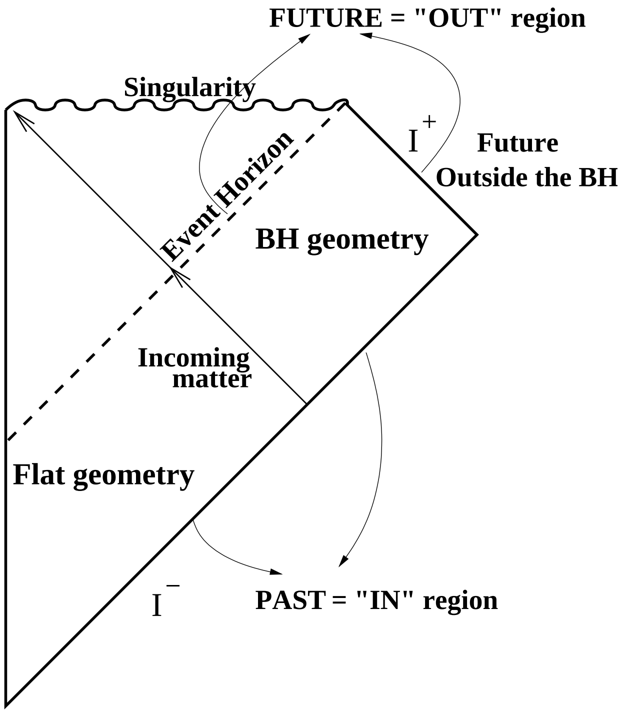
- Penrose diagram and construction of the modes:



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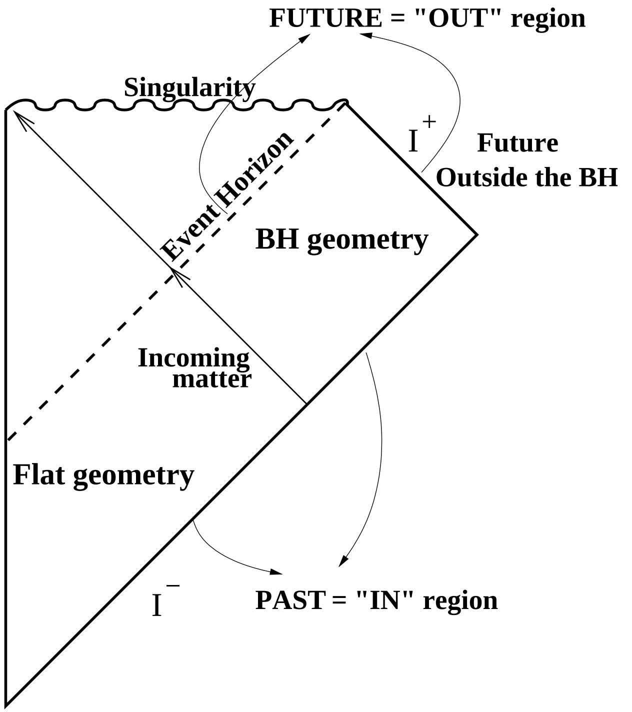

- Propagating the **IN** modes $e^{-i\omega v}$:

$$f_\omega^{in} = \frac{1}{4\pi\sqrt{\omega}} \left(e^{-i\omega v} - \theta(v_0 - v) e^{-i\omega u_{in}} \right)$$

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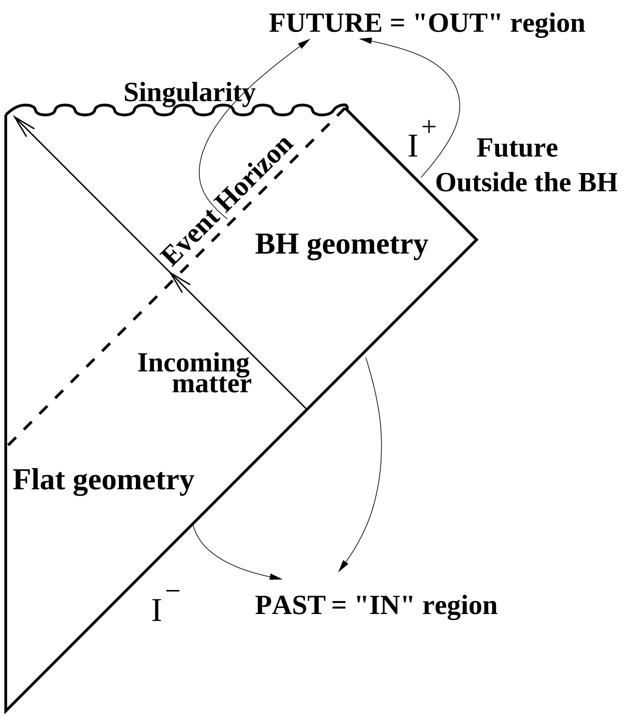

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- Propagating the **OUT** modes $e^{-i\omega u_{out}}$:

$$f_\omega^{out} = \frac{1}{4\pi\sqrt{\omega}} \left(e^{-i\omega u_{out}} - \theta(v_H - v) e^{-i\omega u_{out}(v)}\right)$$

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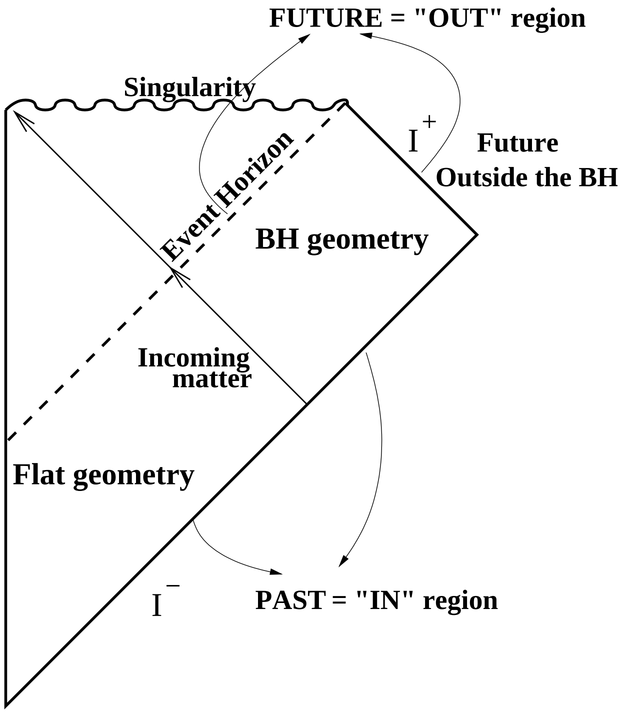

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where $u_{out}(v) = v - 4M\ln\left[\frac{v_H - v}{4M}\right]$

- Now can can compute $\alpha_{\omega\omega'}, \beta_{\omega\omega'}$

Particle production in Vaidya spacetime

- To determine the number of particles we need to compute $\beta_{\omega\omega'}$:

$$\beta_{\omega\omega'} = -(f^{out}_{\omega}, f^{in*}_{\omega'}) = -\frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{v_H} dv e^{-i\omega\left(v-4M\ln\left[\frac{v_H-v}{4M}\right]\right)-i\omega' v}$$

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- The main contribution comes from the near-horizon region $v \approx v_H \;\Leftrightarrow\; u_{out} \longrightarrow \infty$.
- Since $_{in}\langle 0|N^{out}_{\omega}|0\rangle_{in}$ diverges,we use wave packets instead of plane waves:

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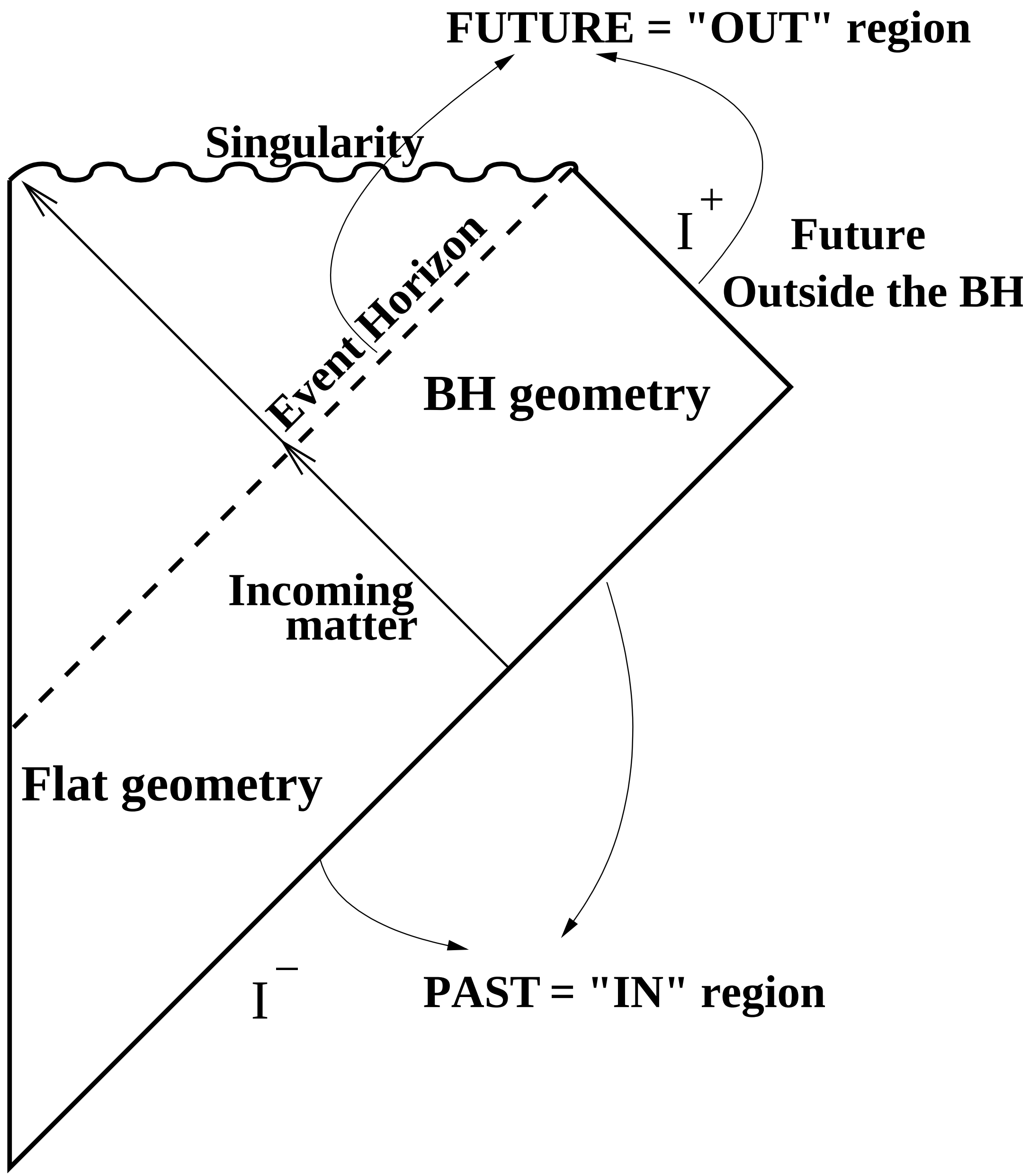
- The thermal nature of the radiation is a very robust result.

Information loss and correlations



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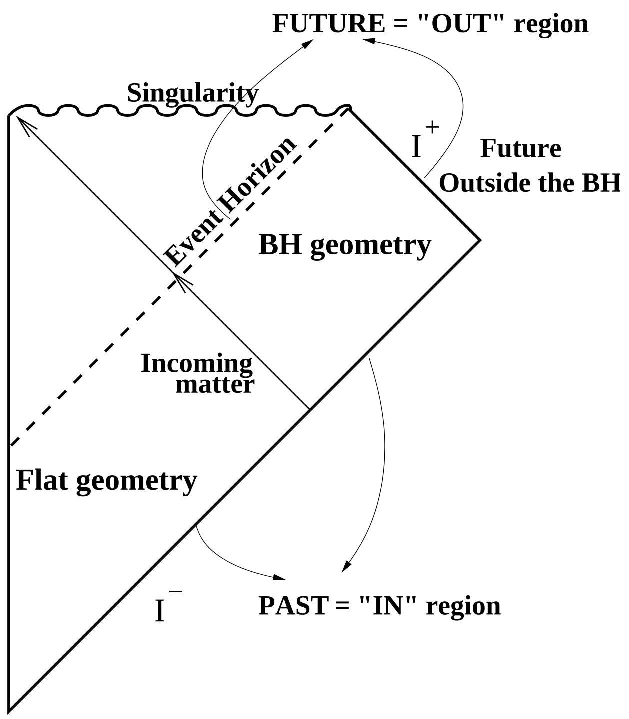

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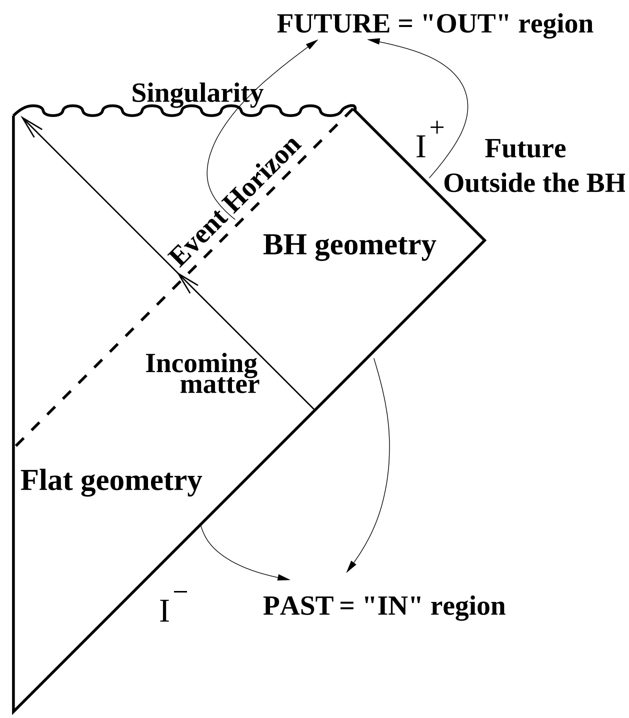

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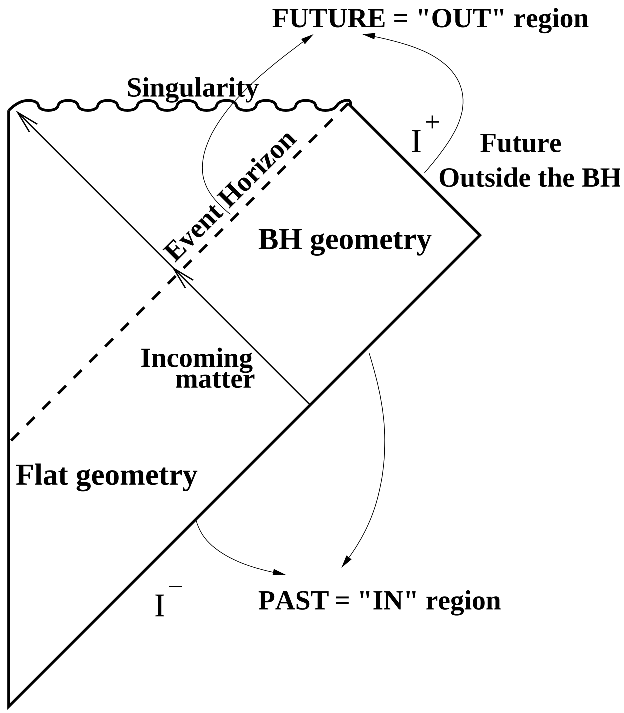

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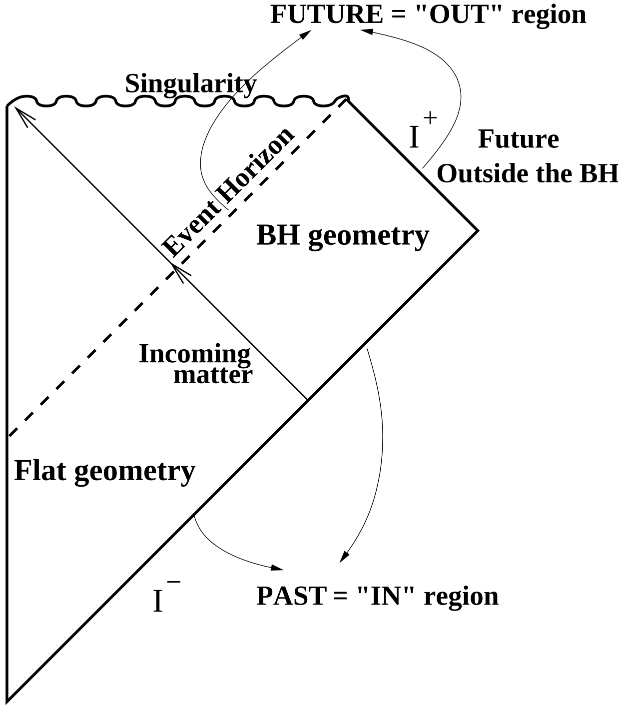

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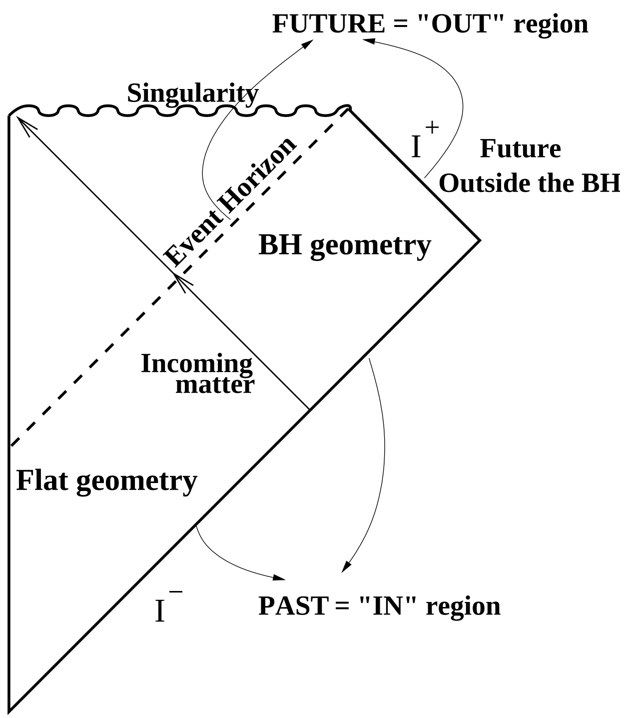

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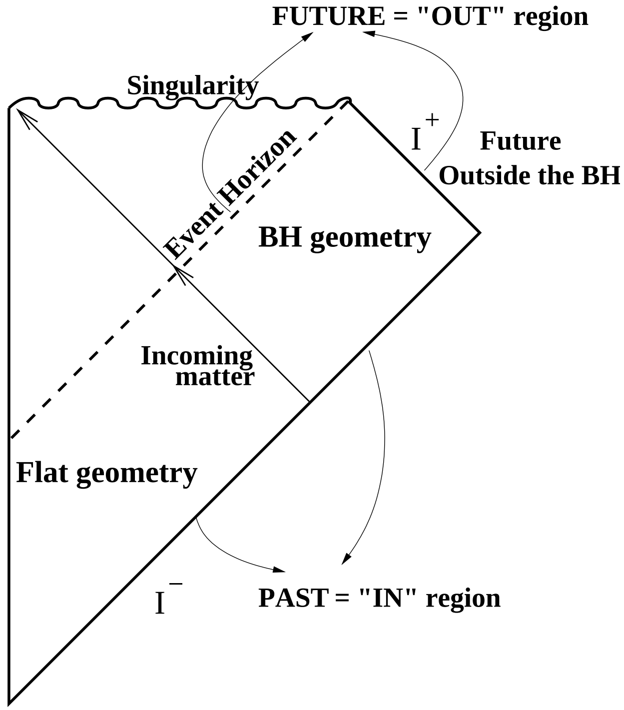

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Thermal state: $|0\rangle_{in} \approx \rho_{th}$

Backreaction effects

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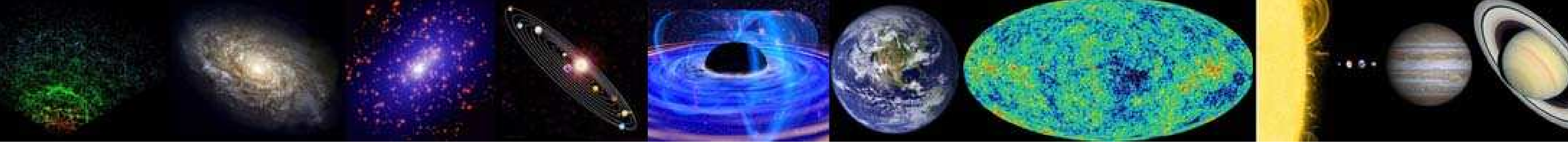
- **The analysis** of the radiation with backreaction and in moving-mirror models **using the standard approach** (Bog. coefficients) **is highly non-trivial and non-intuitive**:
  - Creation of particles without emission of energy?
  - Is $|0\rangle$ invariant under conformal transformations?
  - Is information loss related to violations of energy conservation?



Part II:

New approach: correlation functions

Bogolubov -Vs- Correlator

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- YES!

$\Rightarrow$ We can thus bypass the computation of α and β !!!

Number of particles and two-point correlator

- With the decomposition

$$\phi_{I^+} = \sum [a_j^{out} u_j^{out}(x) + a_j^{out\,\dagger} u_j^{out\,*}(x)]$$

We construct the normal-ordered operator

$$:\phi(x_1)\phi(x_2): \;\equiv\; \phi(x_1)\phi(x_2) - {}_{out}\langle 0|\phi(x_1)\phi(x_2)|0\rangle_{out}$$

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Example: Conformal Invariance

- In d-dimensional Minkowski space, a conformally invariant field theory satisfies:

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{}_{in}\langle 0|\phi(y_1)\phi(y_2)|0\rangle_{in} = \frac{C}{|y_1-y_2|^{2\Delta}}
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- Since ${}_{in}\langle 0|N_i^{out}|0\rangle_{in} = \sum_k |\beta_{ij}|^2$ then β_{ij} should vanish for all Conformal Transformations. For Special C.T. this is not trivial.

Application: thermal radiation

- In $2D$ the number of particles can be expressed as:

$$ {}_{in}\langle 0|N_k^{out}|0\rangle_{in} = -\frac{1}{\pi}\int_{-\infty}^{\infty} dx_1 dx_2 u_k(x_1) u_k^*(x_2)\left[\frac{y'(x_1)y'(x_2)}{[y(x_1)-y(x_2)]^2} - \frac{1}{(x_1-x_2)^2}\right] $$

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$$ N_{\omega_1\omega_2} = -\frac{\delta(\omega_1-\omega_2)}{2\pi\sqrt{\omega_1\omega_2}}\int_{-\infty}^{+\infty} dz^- e^{-i\Omega_+ z^-}\left[\frac{\kappa^2 e^{-\kappa z^-}}{(e^{-\kappa z^-}-1)^2} - \frac{1}{(z^-)^2}\right] $$

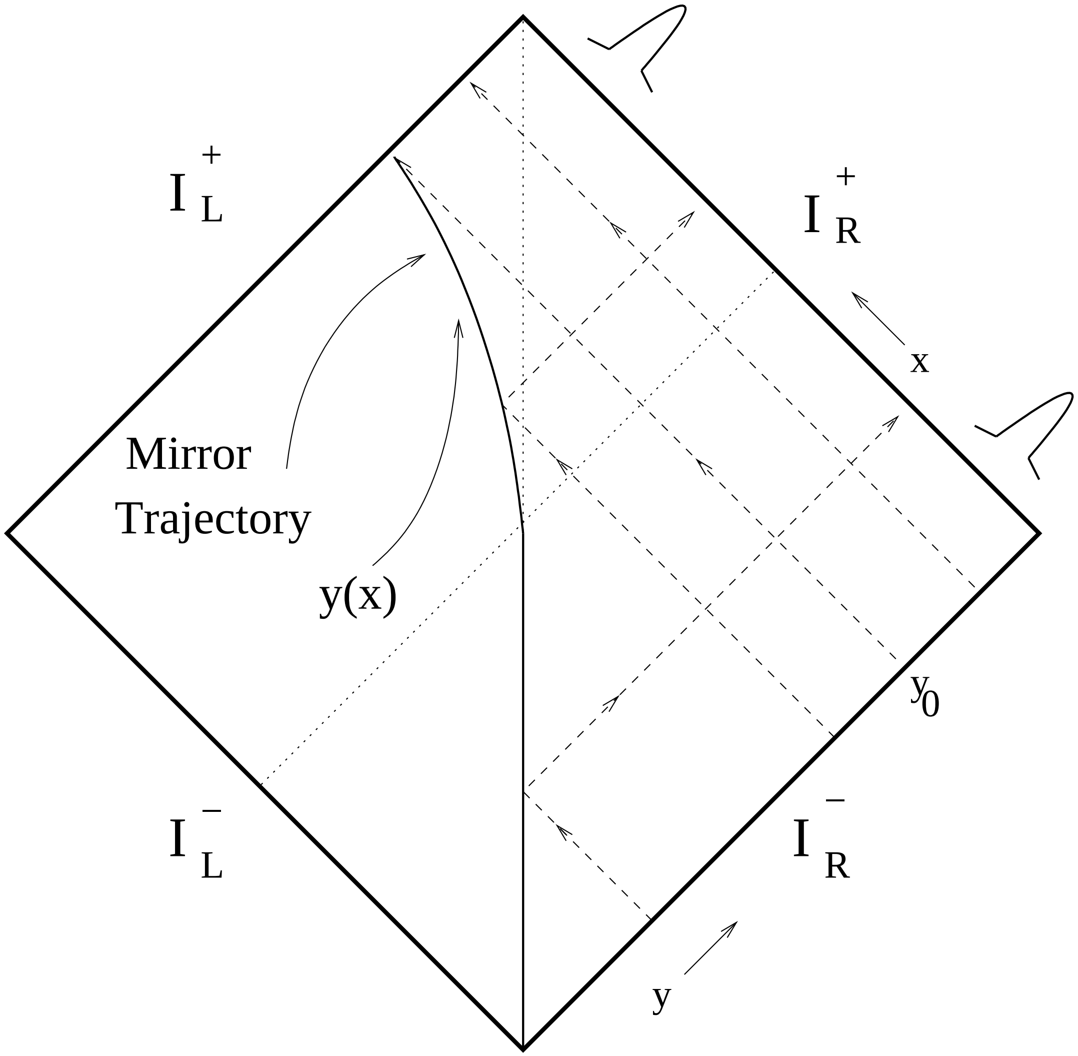
- The last integral leads to: $N_{\omega_1\omega_2} = \delta(\omega_1-\omega_2)\frac{1}{\left[e^{\frac{2\pi\omega_1}{\kappa}}-1\right]}$

- Inserting wave packets we find: $N_{ij} \approx \delta_{ij}\frac{1}{e^{\frac{2\pi\omega_i}{\kappa}}-1}$

- Other magnitudes require the use of all the correlators:

$$ |0\rangle_{in} = {}_{out}\langle 0|0\rangle_{in} \exp\left(\frac{1}{\hbar}\sum_i e^{-\frac{\pi\omega_i}{\kappa}} c_i^\dagger b_i^\dagger\right)|0\rangle_{out} $$

Moving-mirrors, particles and energy fluxes



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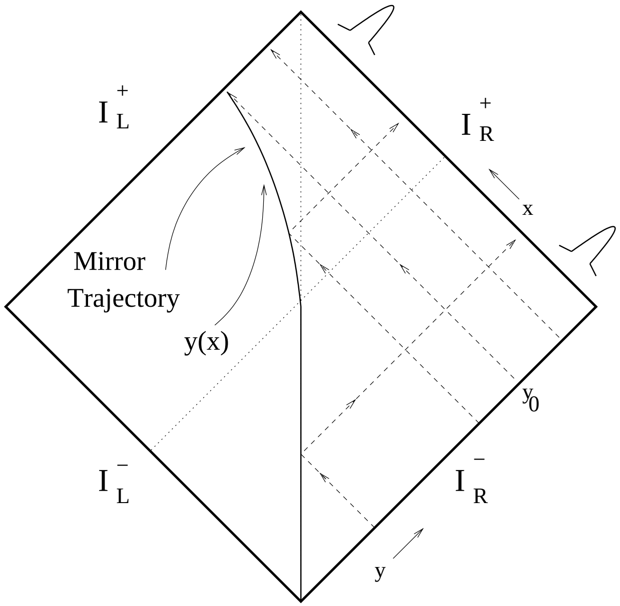

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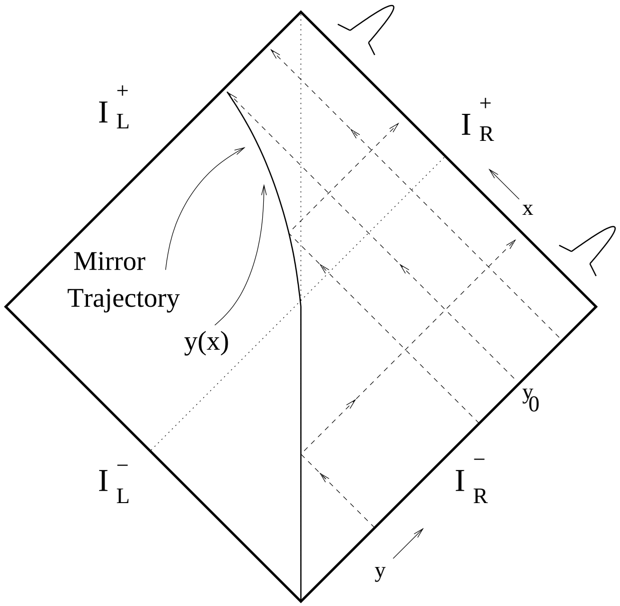

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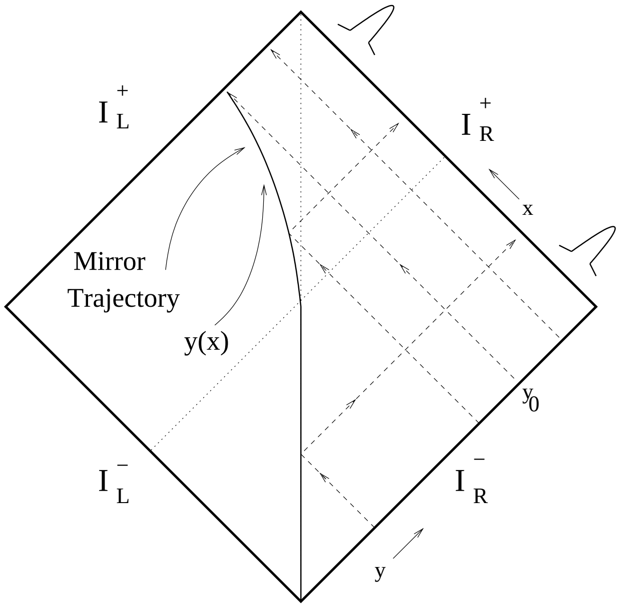

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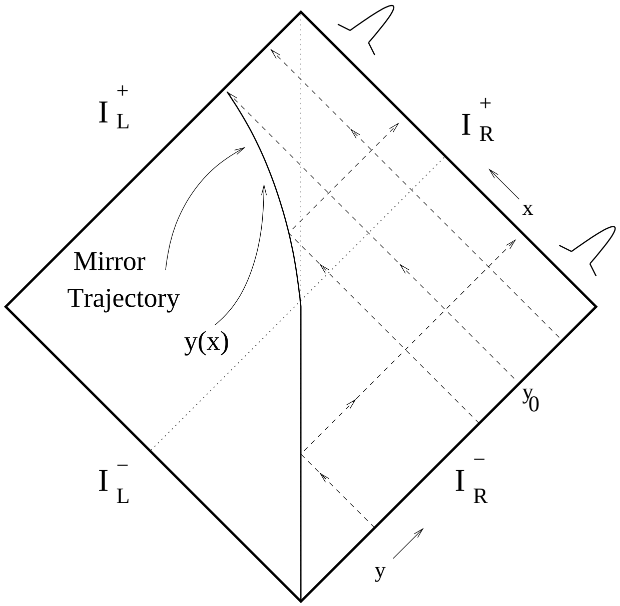

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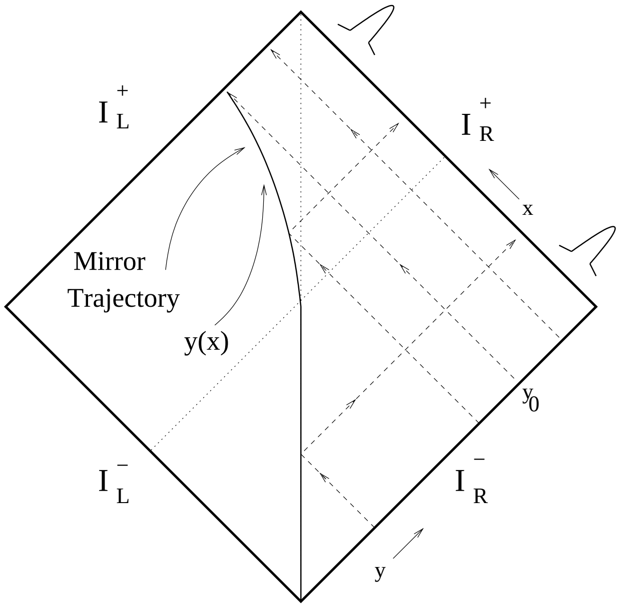

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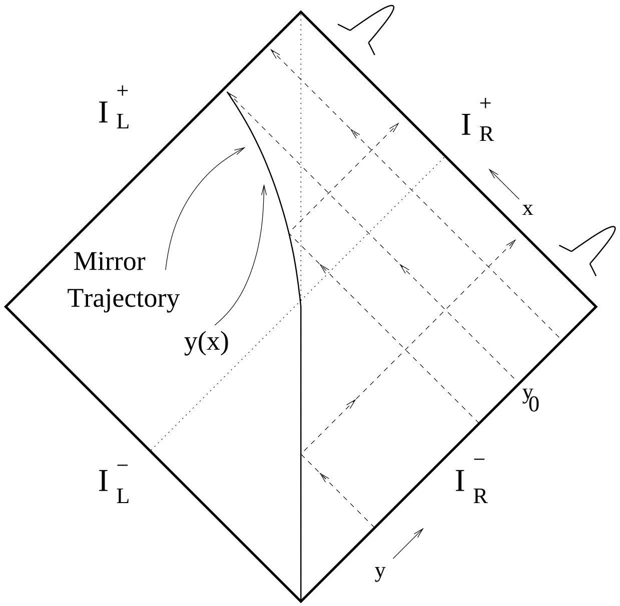

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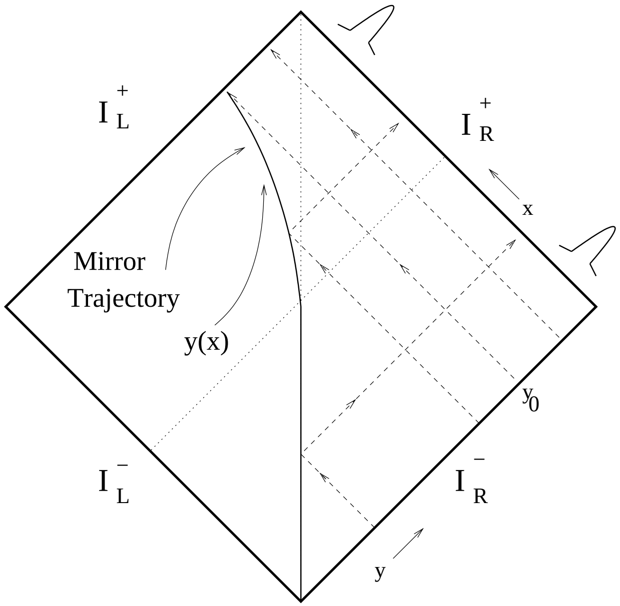

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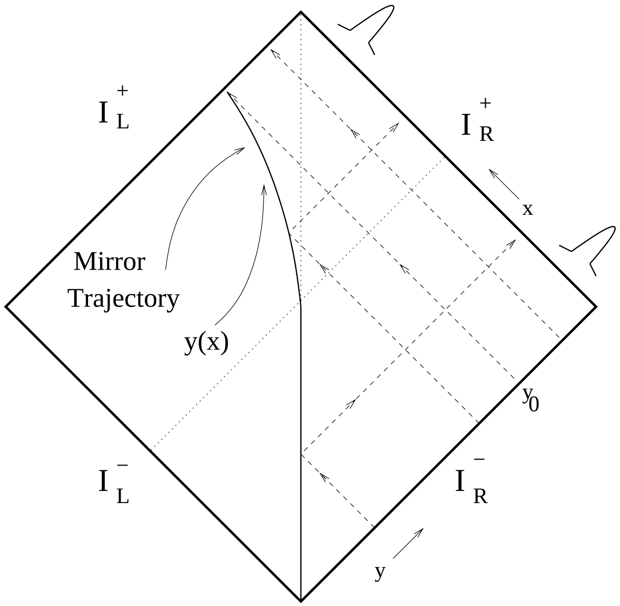

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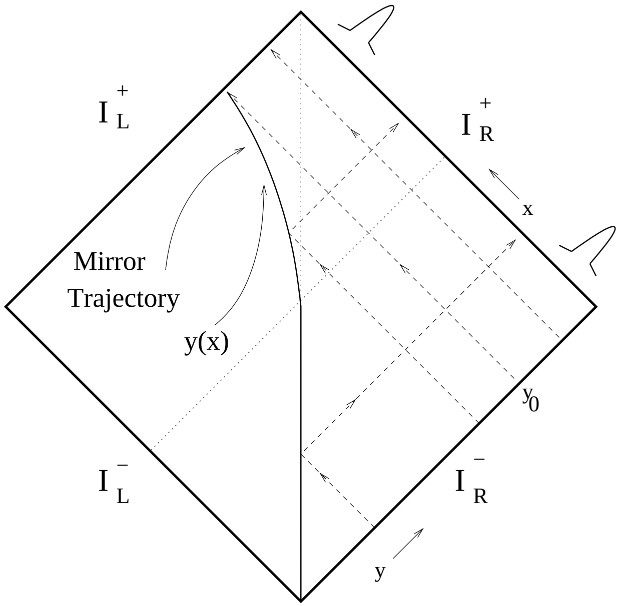

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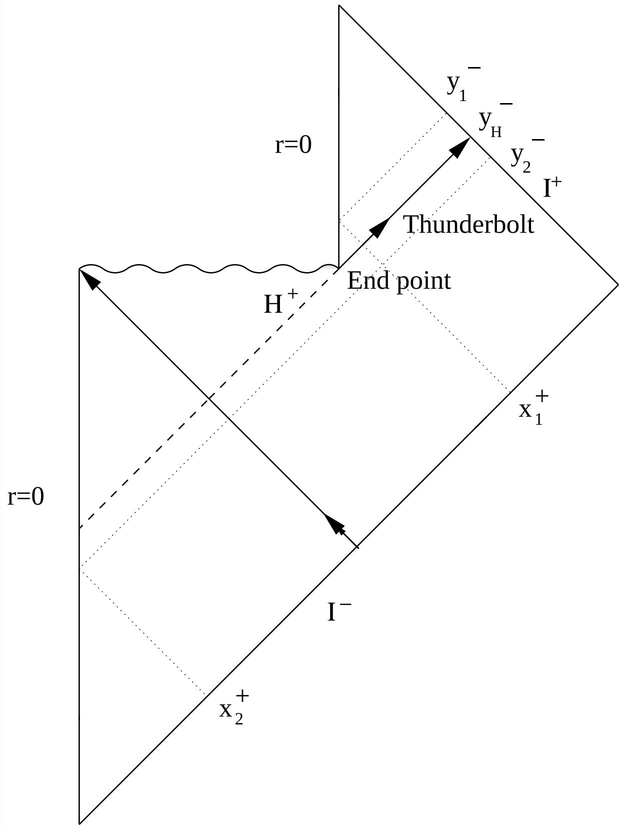
- Thunderbolt localized at $y = y_0$

BH evaporation and thunderbolts



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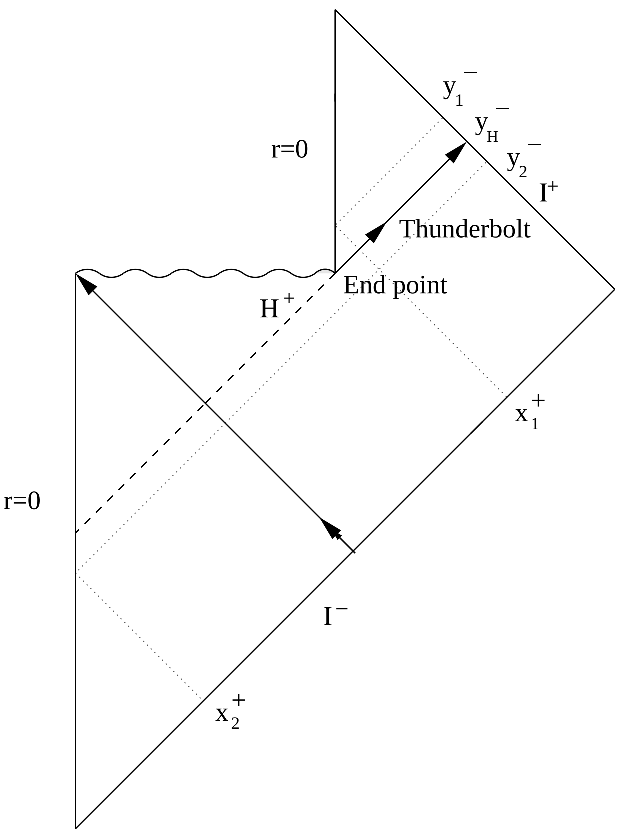

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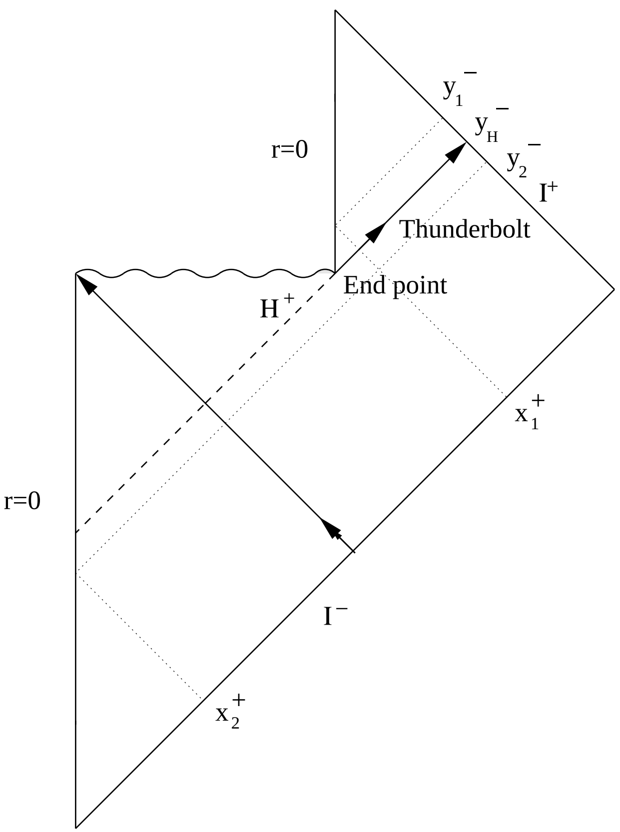

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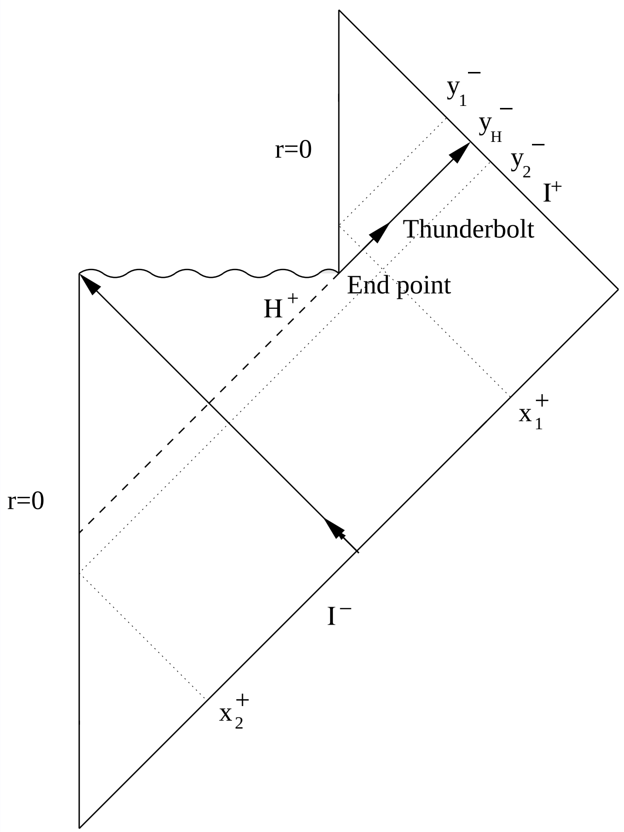
diverges as $y_1 \to y_2$ leading to a thunderbolt.



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BH evaporation and thunderbolts

Motivation and Outline

Hawking radiation

Correlation functions

Bogolubov -Vs- Correlator

Number and 2P correlator

Example: Conformal Invariance

Application: thermal radiation

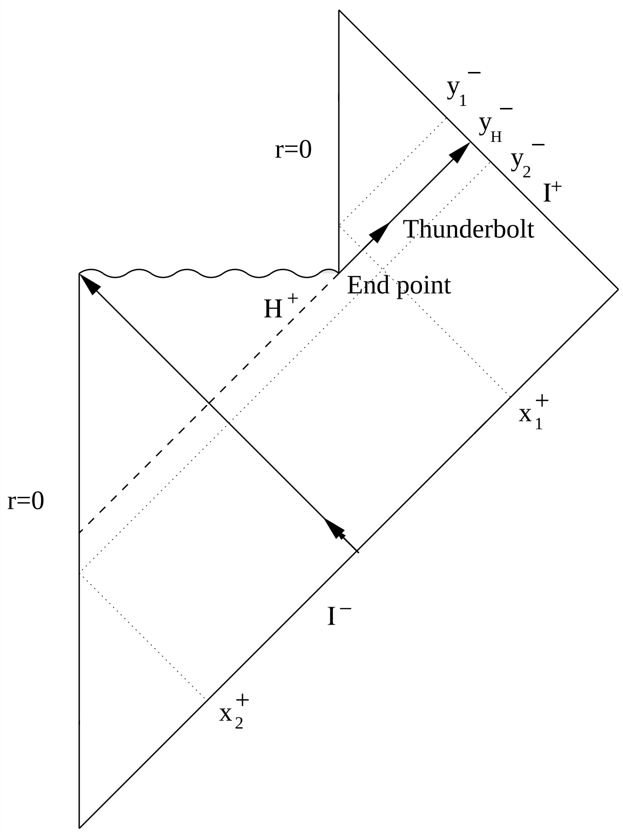
Moving-mirrors and fluxes

BH and thunderbolts

Beyond N

Summary and conclusions

The End



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- The discontinuity of the line $r=0$, due to H^+ and the singularity, causes:
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- Though the black hole contains a finite amount of energy, the emission of a thunderbolt, which is a purely topological effect, breaks the consistency of the semiclassical approach.

Beyond the number of particles

- The coeffs. $\alpha_{ik} = (u_i^{out}, u_k^{in}), \beta_{jk} = -(u_j^{out}, u_k^{in*})$ never appear alone:

$$ {}_{in}\langle 0|a_i^{out} a_j^{out}|0\rangle_{in} = -\hbar(\beta^* \alpha^\dagger)_{ij} \qquad {}_{in}\langle 0|a_i^{out\dagger} a_j^{out}|0\rangle_{in} = +\hbar(\beta\beta^\dagger)_{ij} $$

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- We are free to choose between two representations:

$\{\alpha, \beta\}$	$\{N, C\}$
"IN" and "OUT" indices	"OUT" indices

Summary and conclusions

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  - Standard formalism of Bogolubov coefficients.
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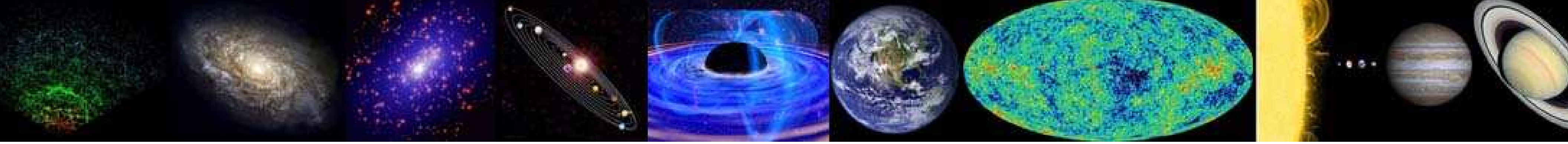
  - Difficult application and interpretation.
  - Obscure manifestation of the symmetries.
- The use of correlators
  - Simplifies some technicalities and allows for an **intuitive interpretation** of the process of particle creation and emission of energy fluxes.
  - **Clear implementation of the symmetries**.
  - Allows to detect **localized fluxes of particles and energy**.
  - Indicates that **information loss and violation of energy conservation are intimately related**.



Thanks !!!

¡Gracias!!!