Nonlinear modelling of sources of gravitational waves

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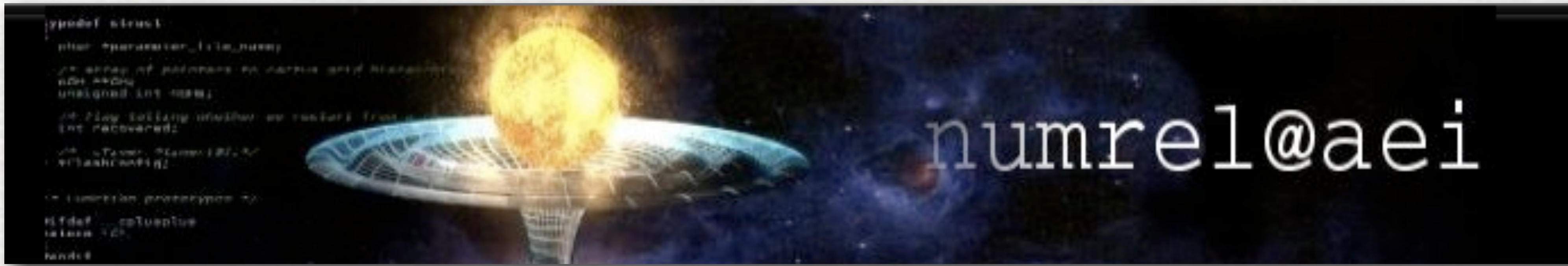


Plan of the talk

- Numerical Relativity: Why?
- Numerical Relativity: Why so hard?
- Modelling of sources of gravitational waves
  - ★ Binary black holes:
    - ✳ waveforms, recoil velocity
  - ★ Binary neutron stars:
    - ✳ waveforms, nuclear physics, gamma-ray bursts

NOTE: this talk will not be technical and cannot be exhaustive. Take it as starting point for questions!

Numerical Relativity: why?

Among other things, numerical relativity aims at:

- solve Einstein equations without approximations(!)…
- investigate the physics of gravitational collapse (singularity formation, dynamics of horizons)
- investigate structure and stability of the most relativistic astrophysical objects: neutron stars
- model the most catastrophic events in the Universe (GRBs, magnetars, etc.)
- solve the two-body problem in GR (more later on this)
- model sources of gravitational waves...

Modelling source of GWs

GSFC/NASA

It has happened over and over in the history of astronomy: as a new "window" has been opened, a "new",universe has been revealed.

The same will happen with GW-astronomy

Modelling source of GWs

A simple, back-of-the-envelope calculation in the Newtonian quadrupole approximation shows that the luminosity in gravitational waves (energy emitted in gws per unit time) is

$$L_{\rm gw} = \left(\frac{G}{c^5}\right)\left(\frac{M\langle v^2\rangle}{\tau}\right)^2 \simeq \left(\frac{G}{c^5}\right)\left(\frac{M}{R}\right)^5$$

i.e. intense sources are compact, massive and move at relativistic speeds: general relativity is indispensable.

What makes gw-astronomy challenging is

$$\left(\frac{G}{c^5}\right) \simeq 3.8 \times 10^{-60} {\rm erg\ s}^{-1}$$

i.e. even the GWs from the most intense sources will statistically reach us as very weak

Not just an academic exercise...

Several millions €s and thousands man-hours are dedicated to one among the most challenging physics experiments (GEO, LIGO, Virgo), now at design sensitivities

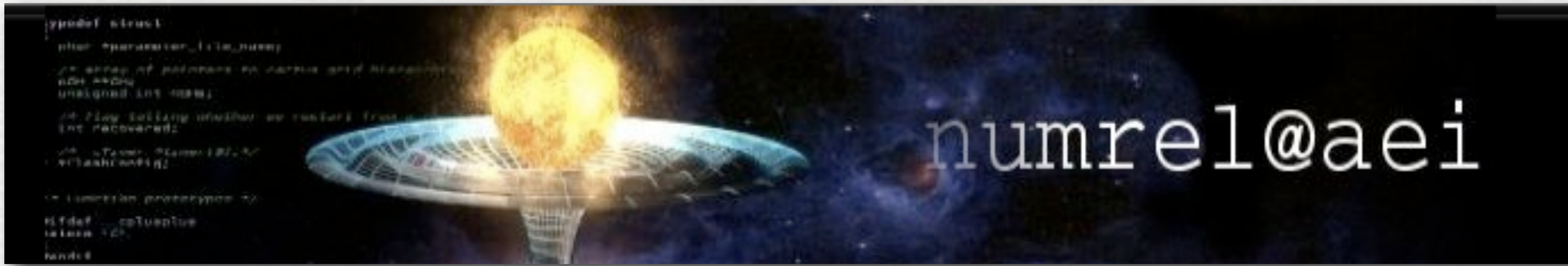
Knowledge of waveforms compensates for very small S/N (matched-filtering)

→

Enhance detection and make source-characterization possible, ie GW astronomy

Numerical Relativity: why so hard?…

- **No obviously "better" formulation of the Einstein equations**
  - ADM, conformal decomposition, first-order hyperbolic form?…
- **Coordinates (spatial and time) do not have a special meaning**
  - this gauge freedom need to be handled with care!
  - gauge conditions must avoid singularities
  - gauge conditions must counteract grid stretching
- **Einstein field equations are highly nonlinear**
  - essentially unknown in these regimes (well-posedeness not enough!...)
- **Hydrodynamics(HD)/MHD in nonlinear regimes is complex**
  - stars are less compact but are non-vacuum. Fluids tend to shock especially when moving at relativistic speeds. Special treatments are essential
- **Simply more equations to solve: stretching supercomputers resources!**
  - large turn-around times make experiments difficult (2-3 weeks/simulation)
  - implementations of AMR techniques is extremely problematic



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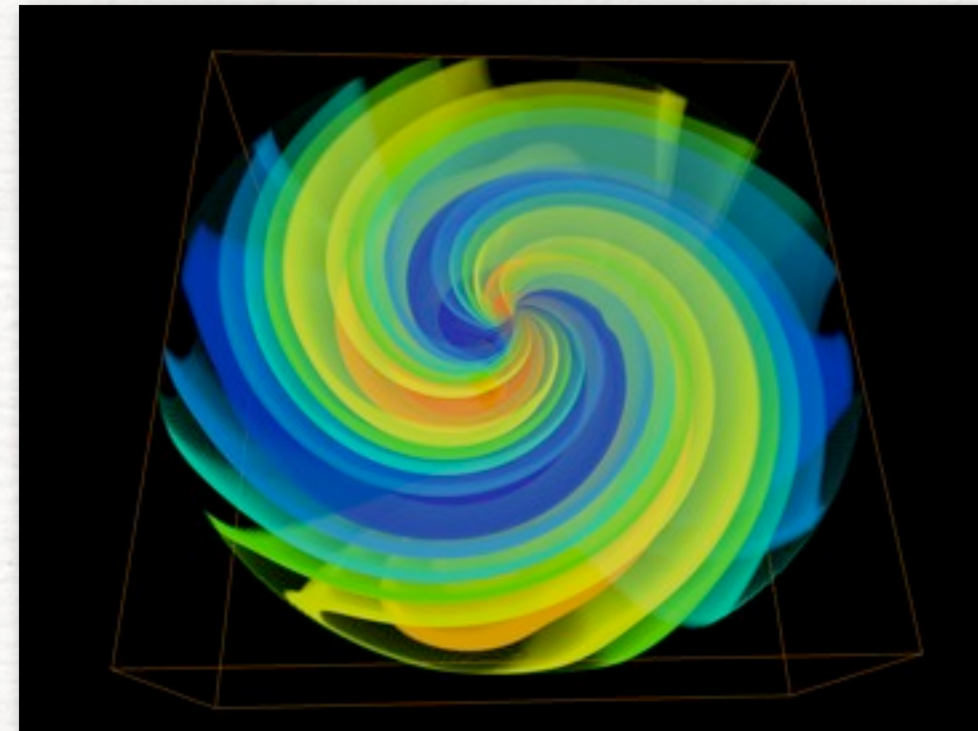

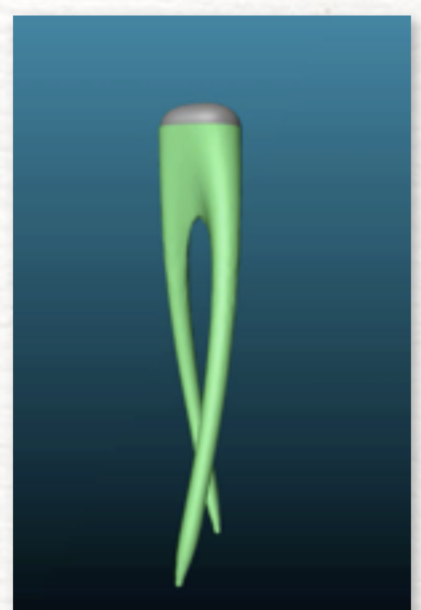

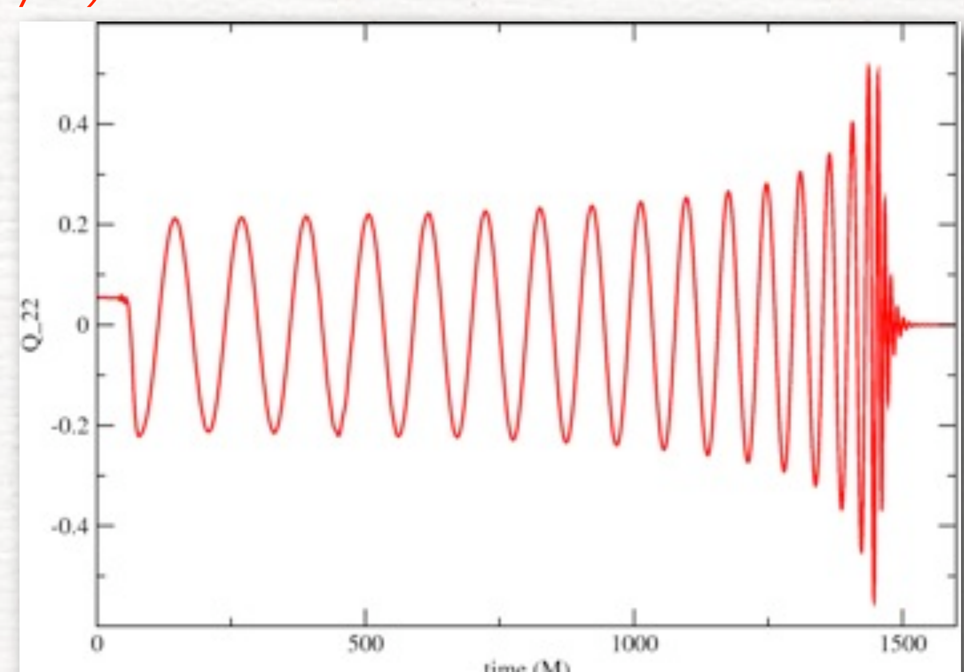

Binary black holes

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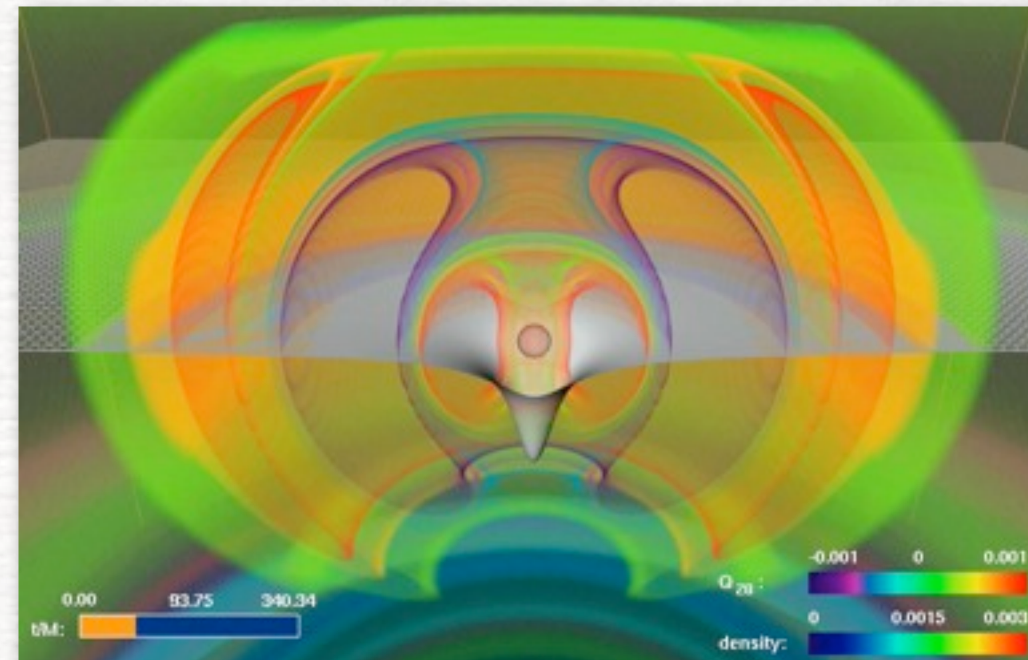
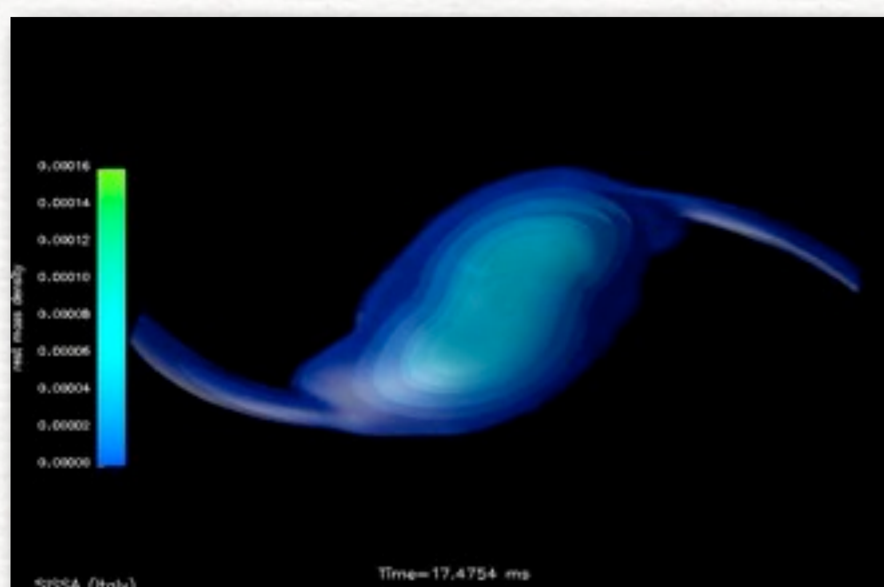
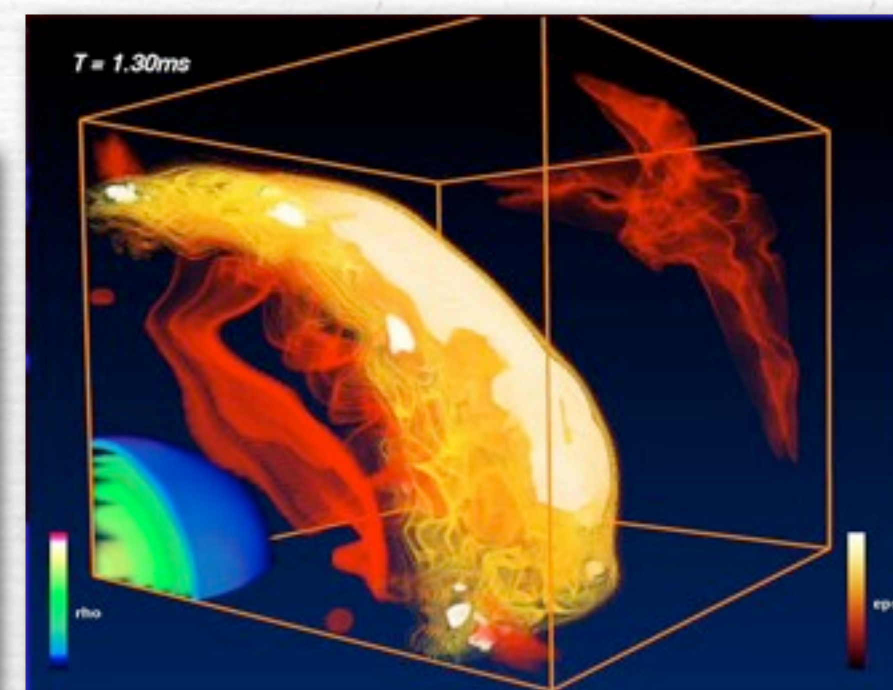


Isolated NSs, perturbation





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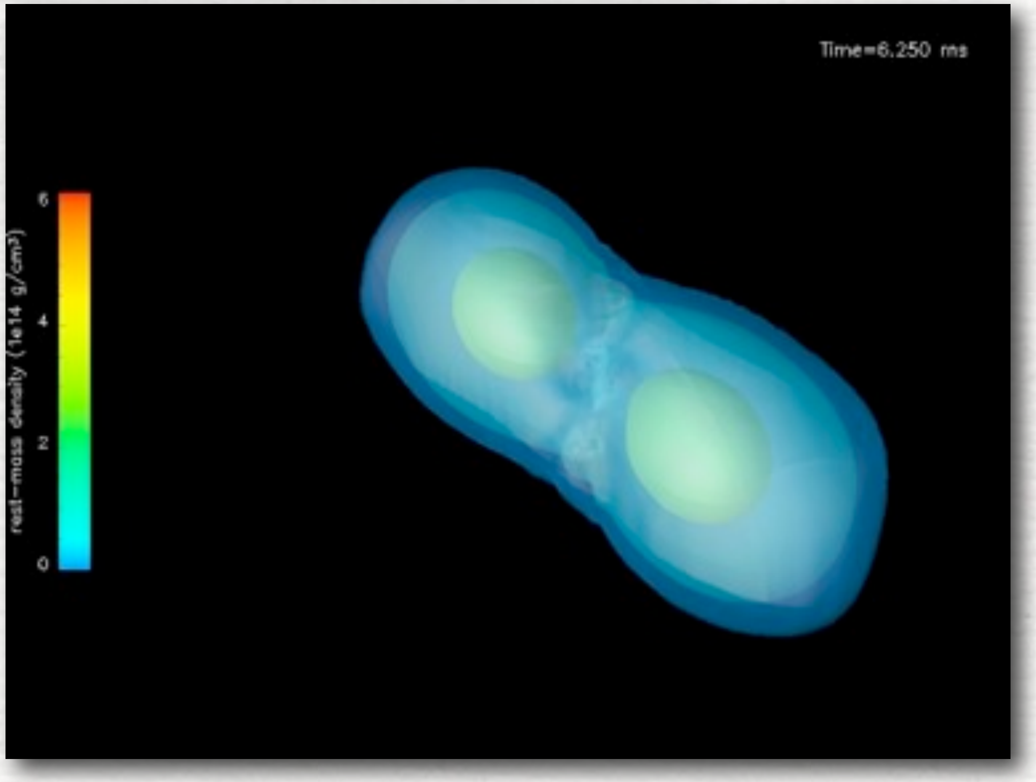
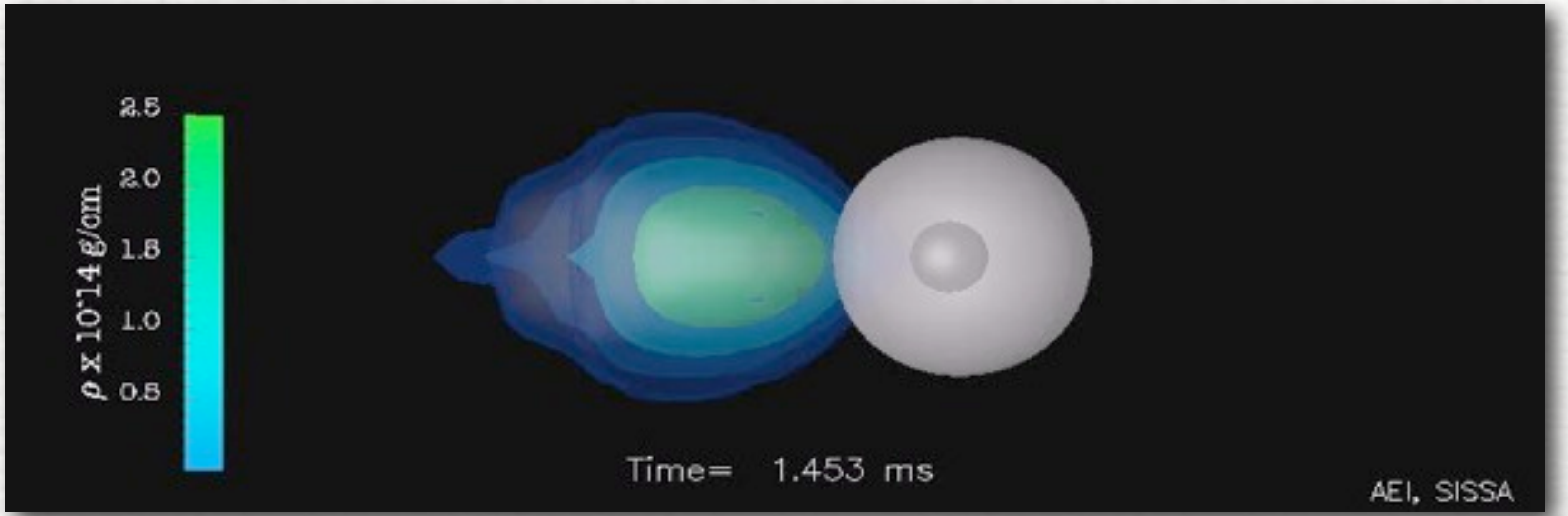
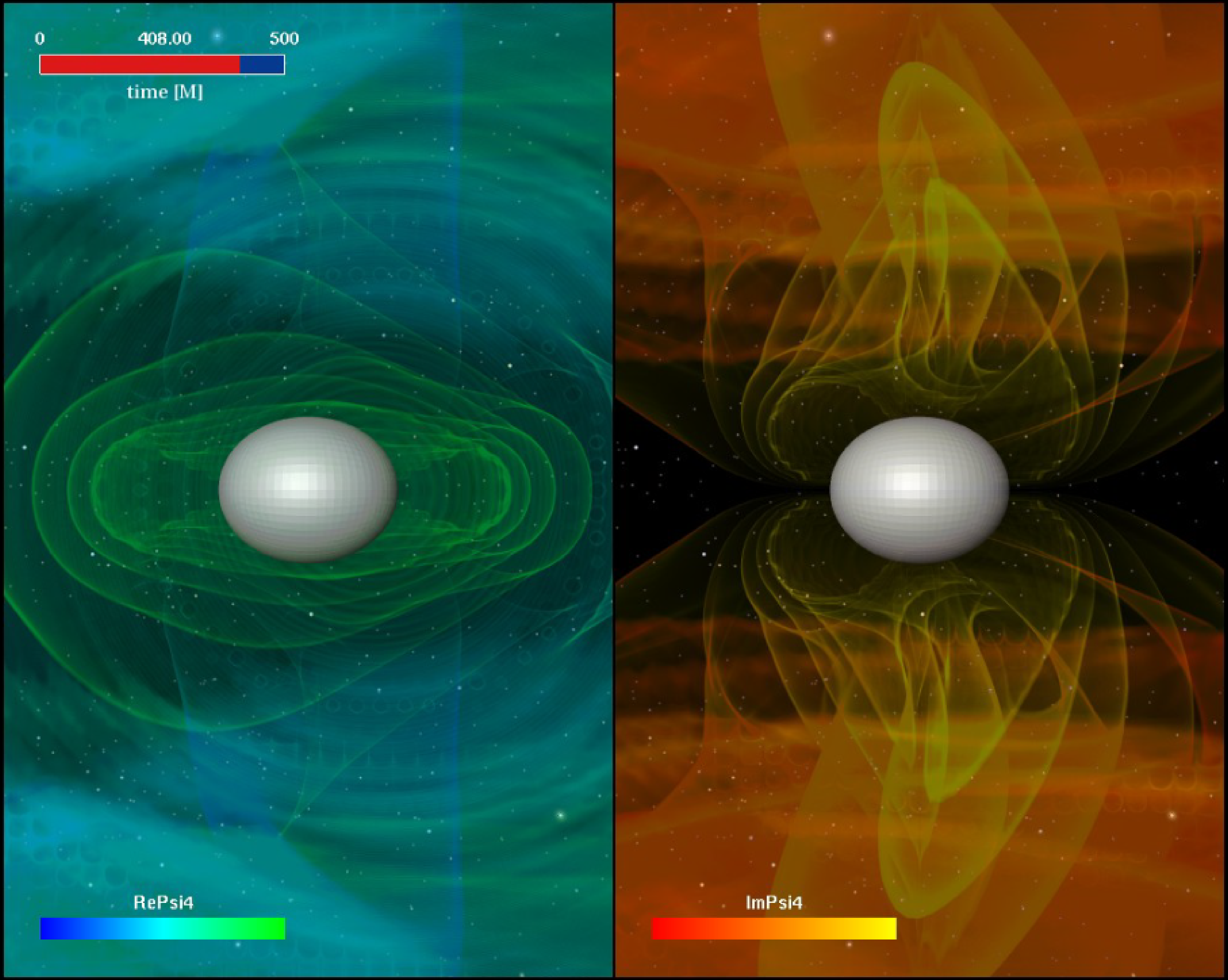


BH-NS

NS-NS binaries

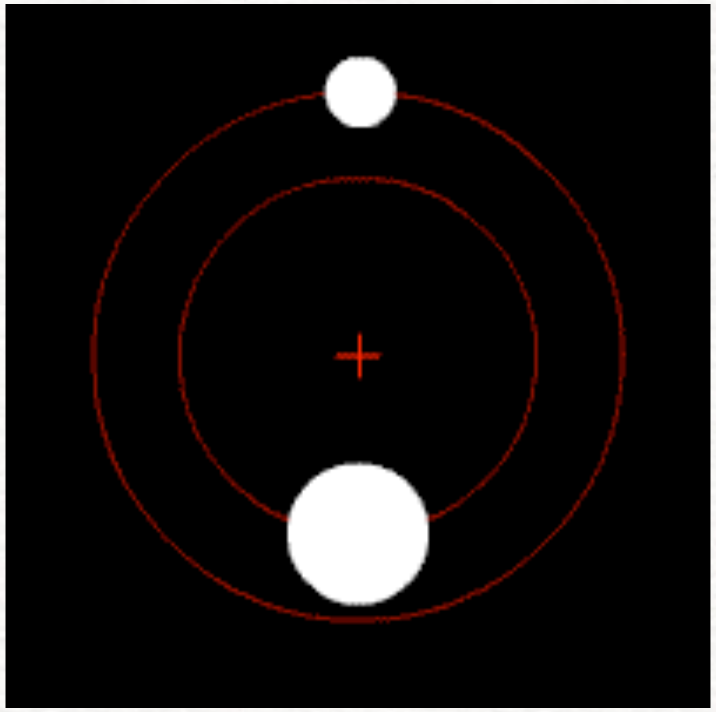


On the two-body problem



The two-body problem: Newtonian gravity

Consider two point-like bodies of mass m_1 and m_2 interacting via a (central) gravitational force: determine the equations of motion.



$$\ddot{\boldsymbol{r}} = -\frac{GM}{d_{12}^3}\boldsymbol{r}$$

where

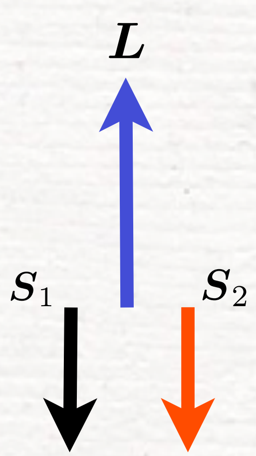
$$M \equiv m_1 + m_2\,, \boldsymbol{r} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2\,, d_{12} \equiv |\boldsymbol{r}_1 - \boldsymbol{r}_2|\,.$$

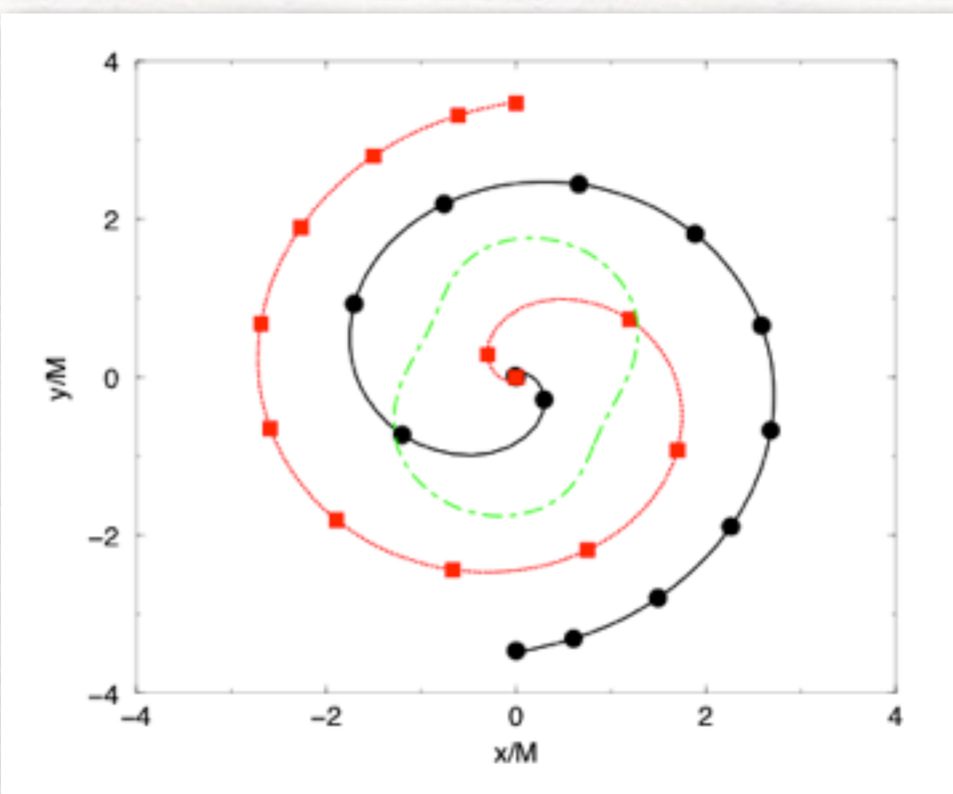
Most importantly, the system admits closed orbits (circular/elliptic). With this equation you can study to lowest order the motion of most astronomical objects.

The two-body problem: GR

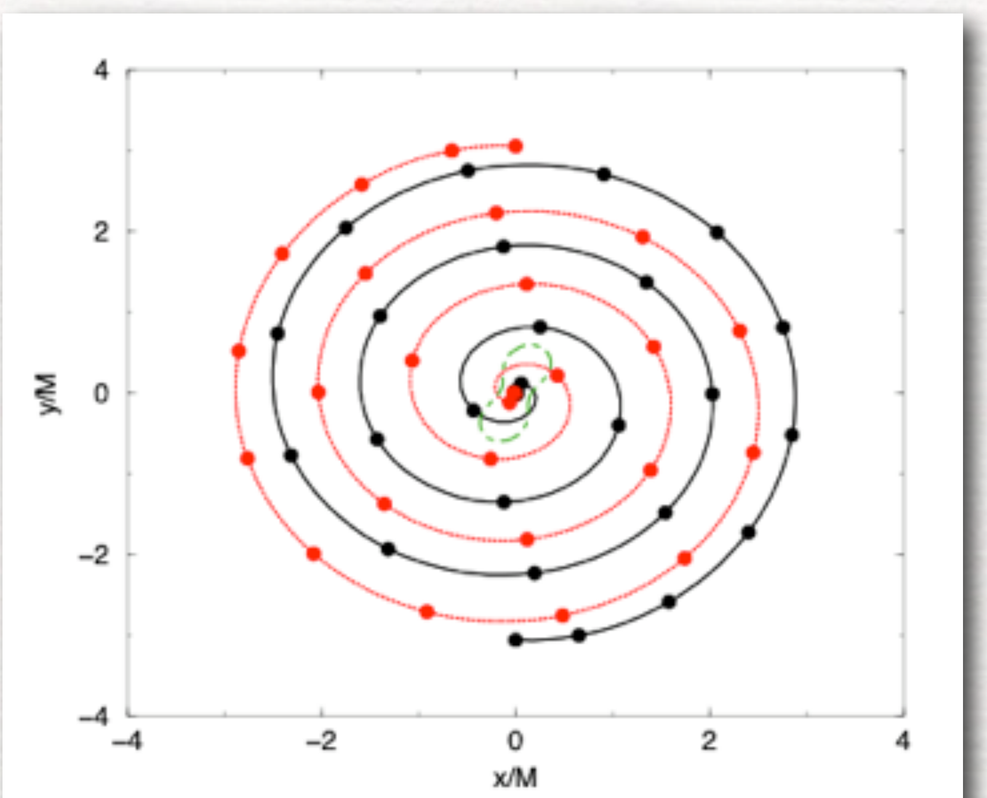
In GR the equations of motion derive directly from the Einstein equations, which need to be solved without approximations.
Most importantly: the system does not admit (realistically) closed orbits: the binary is continuously losing energy and angular momentum.
Even quasi-circular can be very complex.

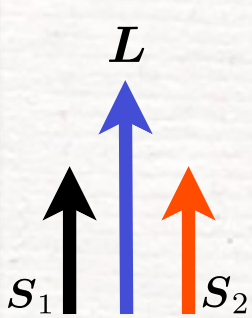
Campanelli et al (2006)





$$\hat{\boldsymbol{S}}_1 = \hat{\boldsymbol{S}}_2 = -\hat{\boldsymbol{L}}$$





$$\hat{\boldsymbol{S}}_1 = \hat{\boldsymbol{S}}_2 = \hat{\boldsymbol{L}}$$

Mathematical setup

Let's recall the equations we are dealing with:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} \qquad (\text{field eqs}: \ 6+6+3+1)$$

$$\nabla_\mu T^{\mu\nu} = 0 \, , \qquad (\text{cons. en./mom.}: \ 3+1)$$

Valencia's formulation!

$$\nabla_\mu (\rho u^\mu) = 0 \, , \qquad (\text{cons. of baryon no}: \ 1)$$

$$p = p(\rho, \epsilon, \ldots) \, . \qquad (\text{EoS}: \ 1 + \ldots)$$

$$\nabla^*_\nu F^{\mu\nu} = 0, \qquad (\text{Maxwell eqs.}: \ \text{induction, zero div.})$$

$$T_{\mu\nu} = T^{\text{fluid}}_{\mu\nu} + T^{\text{em}}_{\mu\nu} + \ldots$$

This is not yet astrophysics but our approximation to *"reality"*.

Still very crude but it can be improved: microphysics for the EOS, magnetic fields, viscosity, radiation transport,...

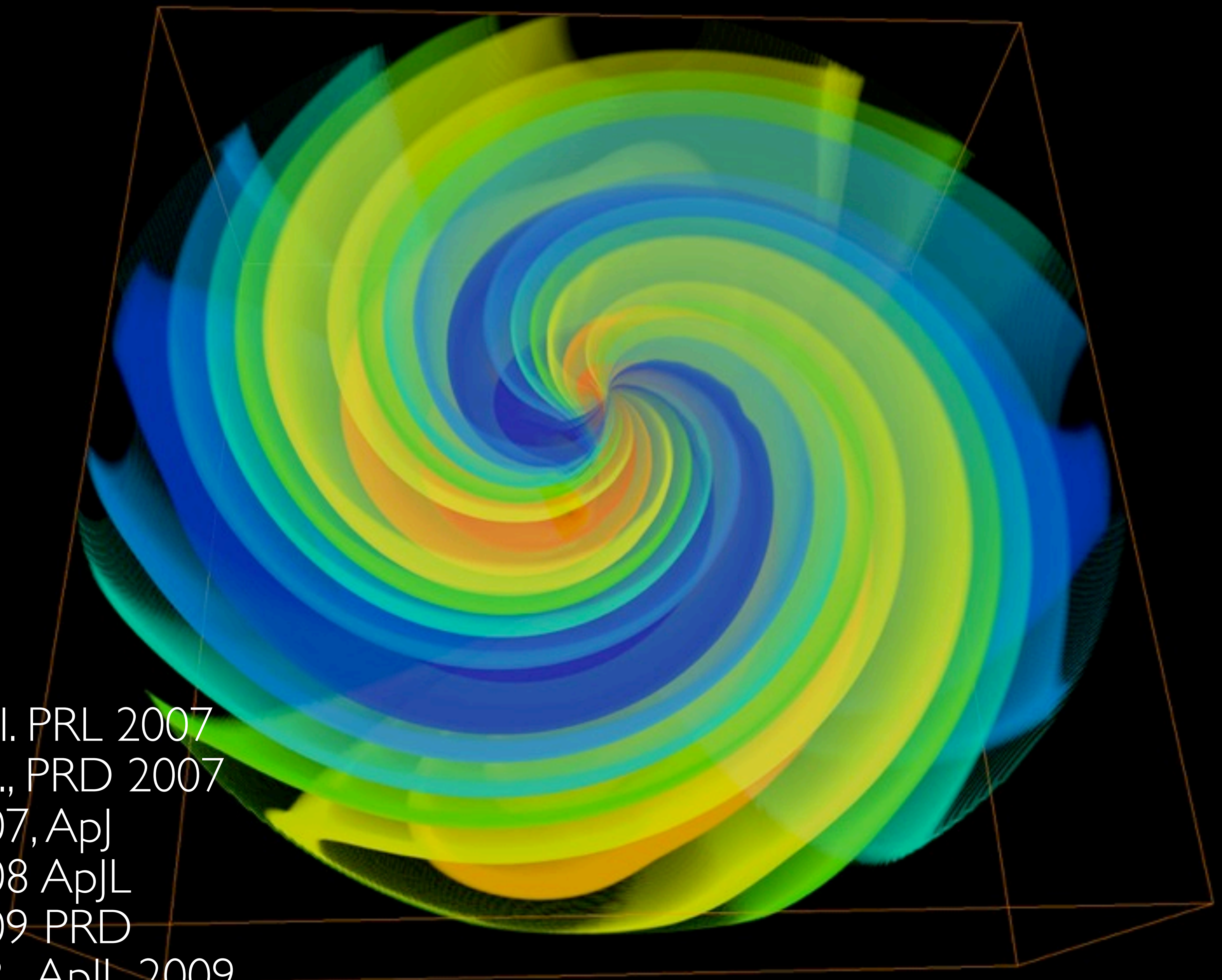
Our strengths:

- High-order (up to 8th) finite-difference techniques for the field equations.
- Flux conservative (Valencia) formulation of HD and MHD eqs; constraint transport or hyperb. divergence-cleaning for the magnetic field; HRSC methods
- Multiple options for the wave extraction (Weyl scalars, gauge-invariant pertbs)
- AMR with moving grids
- Accurate measurements of BH properties through apparent horizons (IH)
- Use excision (matter and/or fields) if needed; good gauges do most of the work

Our weaknesses:

- Idealized (analytic) EOSs (realistic EOSs are implemented but not yet used)
- Single-fluid description: no superfluids nor crusts
- Ideal-MHD: no resistive effects included (work in progress)
- Only inviscid fluid so far (not necessarily bad approximation)
- Radiation and neutrino transport totally neglected (work in progress)
- Match with astrophysical observations inexistent.
- Very coarse resolution; far from regimes where turbulence/dynamos develop

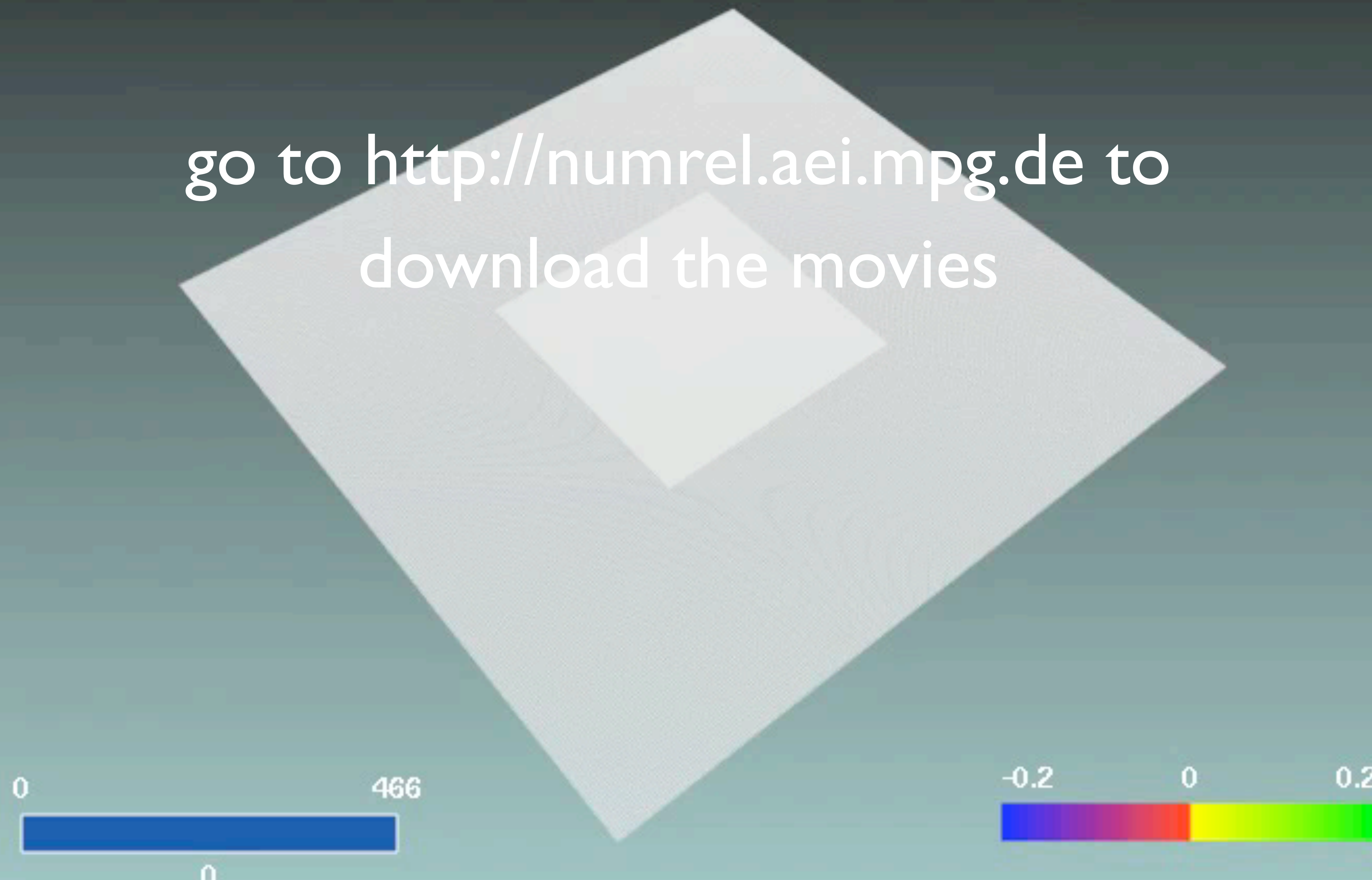
Modelling Binary Black Holes



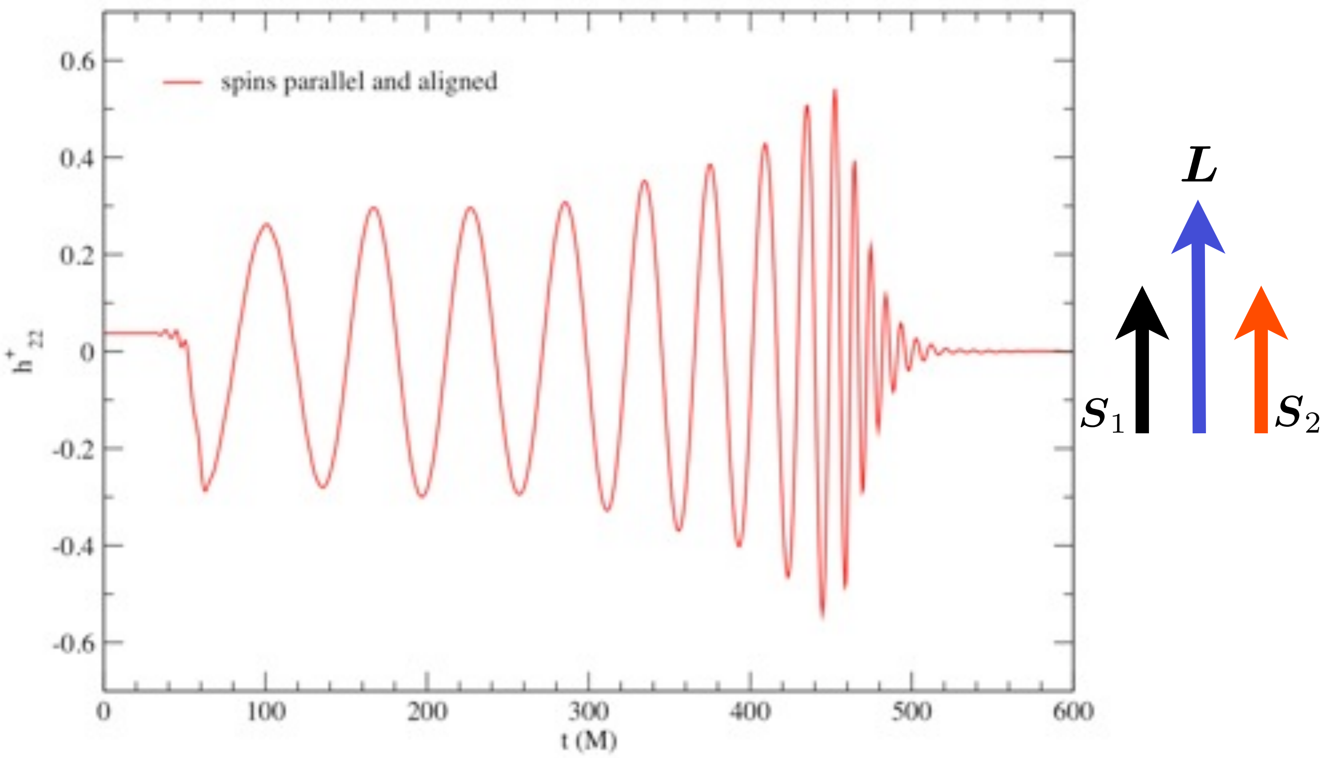
Koppitz et al. PRL 2007
Pollney et al., PRD 2007
LR et al, 2007, ApJ
LR et al, 2008 ApJL
LR et al, 2009 PRD
Barausse, LR, ApJL 2009

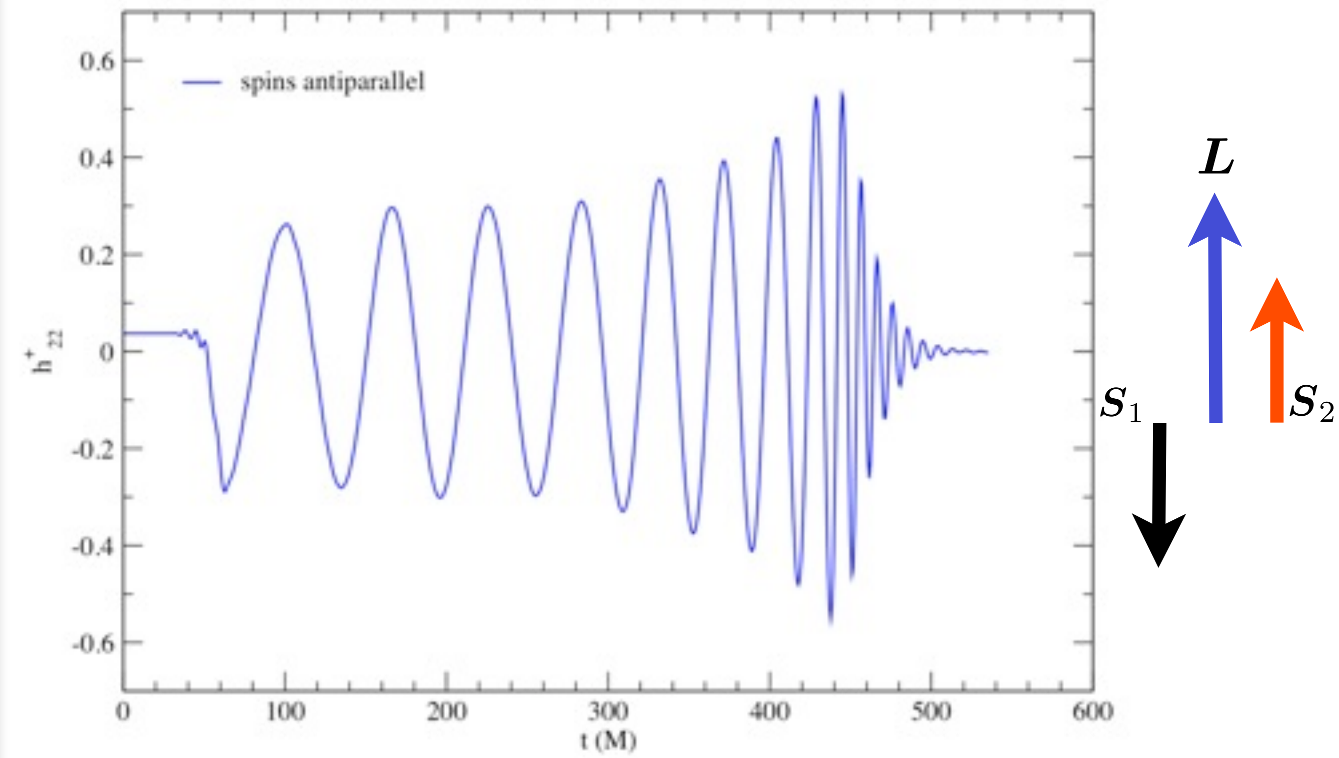
$R_{\mu\nu} = 0$ how difficult can that be?

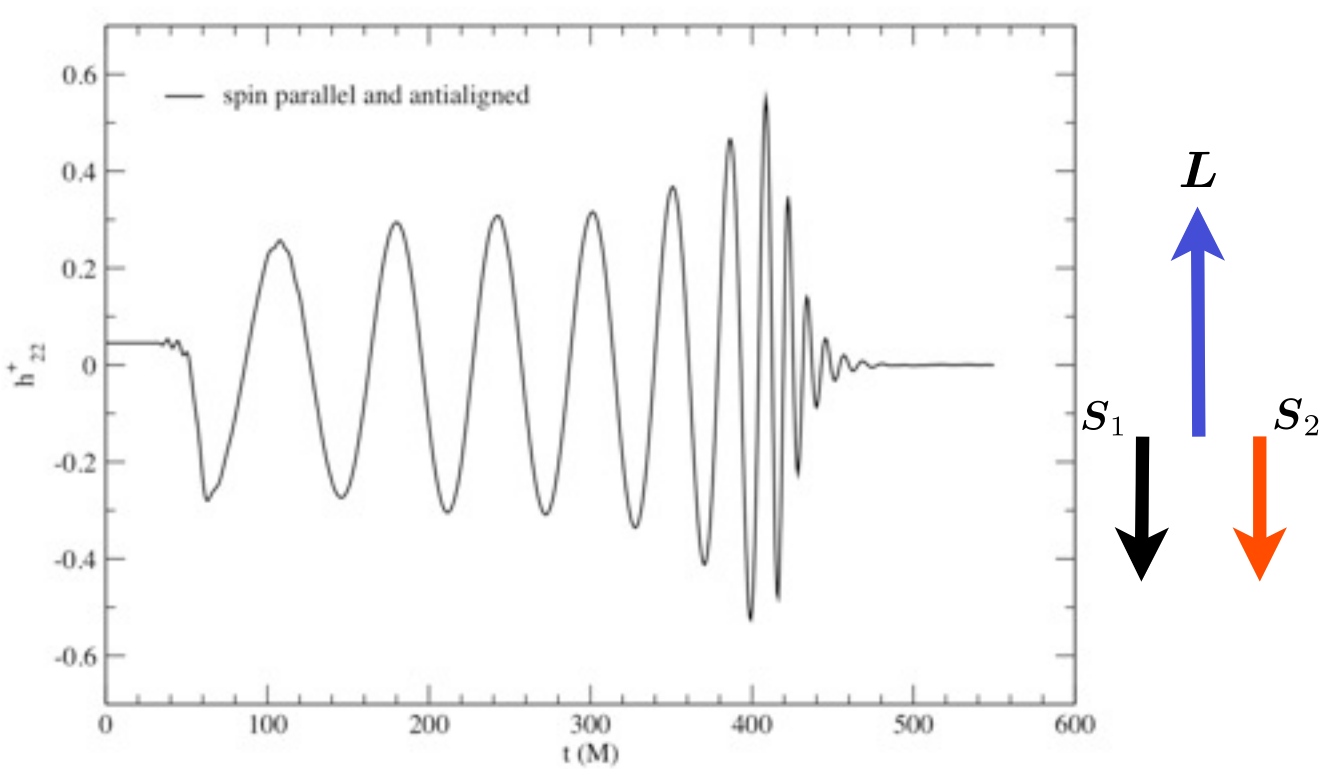
go to http://numrel.aei.mpg.de to download the movies



Animation by R. Kaehler, LR







All the information is contained in the waveforms!

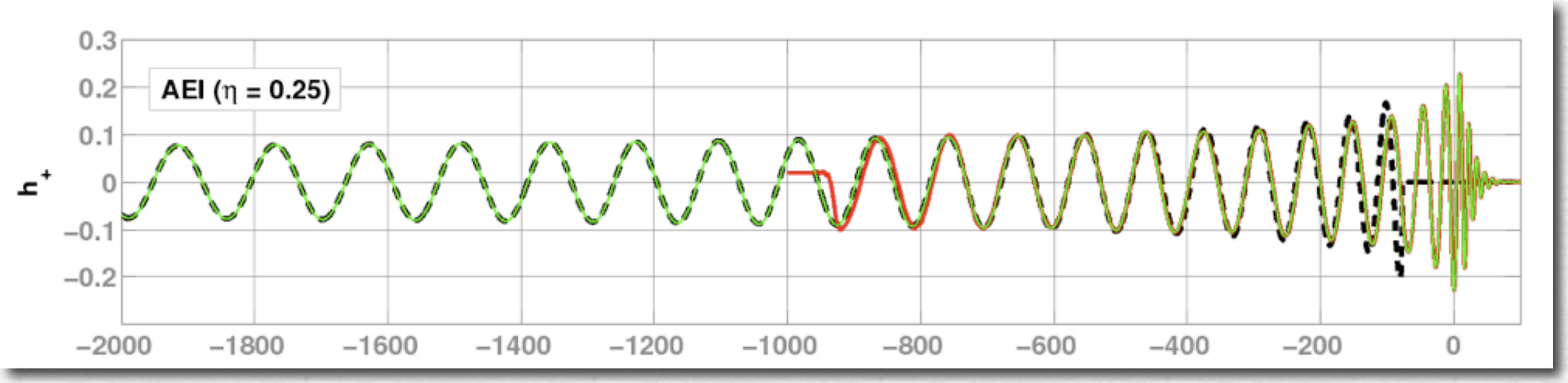
- used in matched filtering techniques (data analysis)
- compute the physical/ astrophysical properties of the merger (kick, final spin, etc.)

Phenomenological template banks

- There is no fundamental obstacle to long-term (i.e. covering ~ 10 orbits) NR calculations of the three stages of the binary evolution: inspiral, merger and ringdown
- Yet, NR are computationally expensive and building a template bank out of them is prohibitive and awkward
- Present data-analysis pipelines employ phenomenological template: analytic, fast, and collect most of the information
- In the past phenomenological templates have been built using approximations and a certain amount of heuristics
- Ideally phenomenological templates should be built upon modelling NR waveforms for the three stages.

A hybrid waveform

Ajith et al. 2007, CQG
Ajith et al. 2008, PRD



Red line is the numrel waveform

Black dashed line is the 3.5PN waveform

Green line is the hybrid waveform

Once the hybrid waveform is computed, it can be parametrized in the Fourier domain via 10 phenomenological parameters (4 for the amplitude, 6 for the phase).

The goal is to reduce them to the 2 physical ones: M mass of the binary and $\eta = M_1 M_2/(M_1+M_2)^2$ symmetric mass-ratio

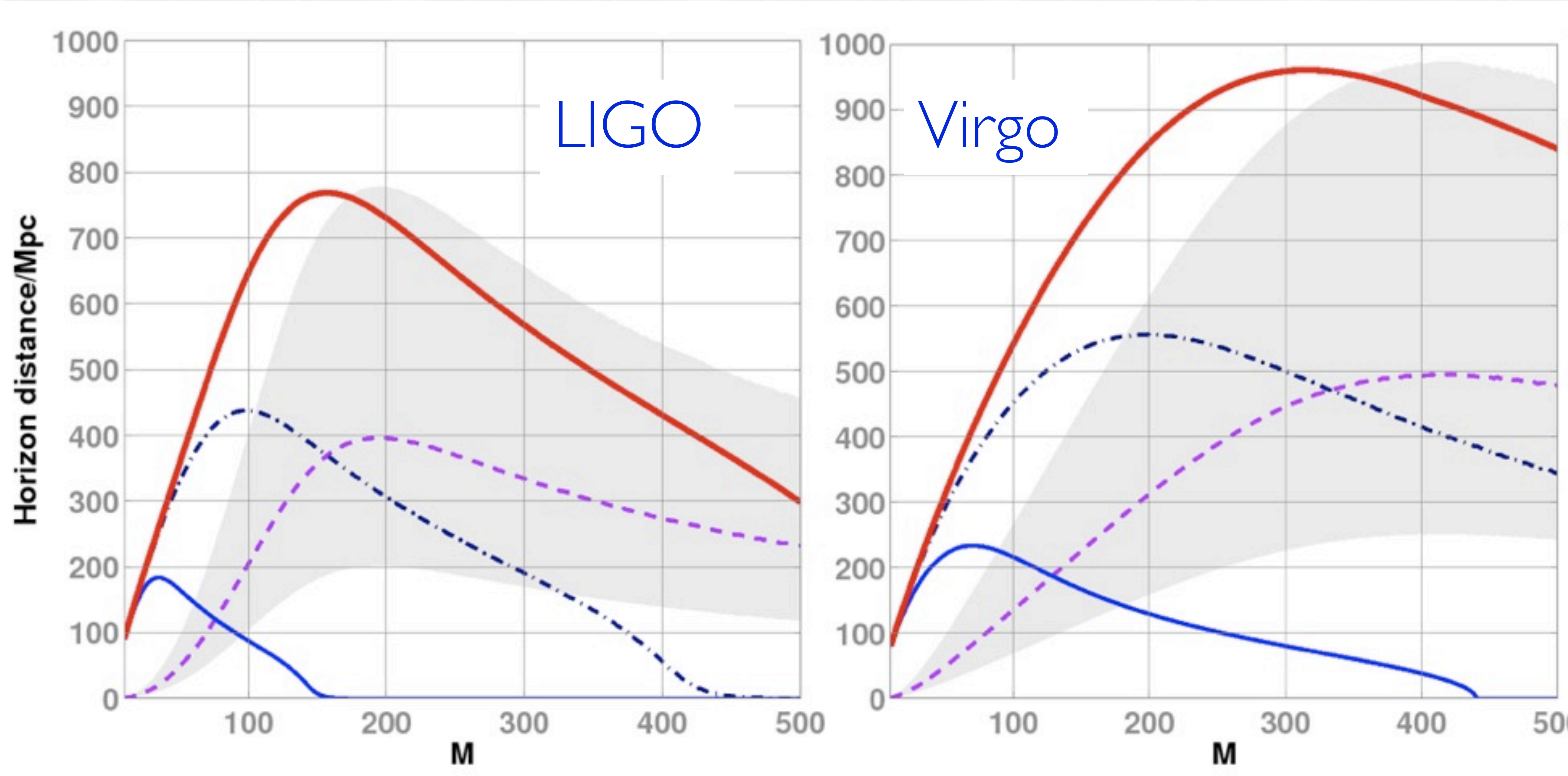
What is this good for?

Red line is the complete (inspiral, merger, ringdown) template

Blue line is the PN template truncated at ISCO

Black dot-dashed line is EOB template truncated at light-ring

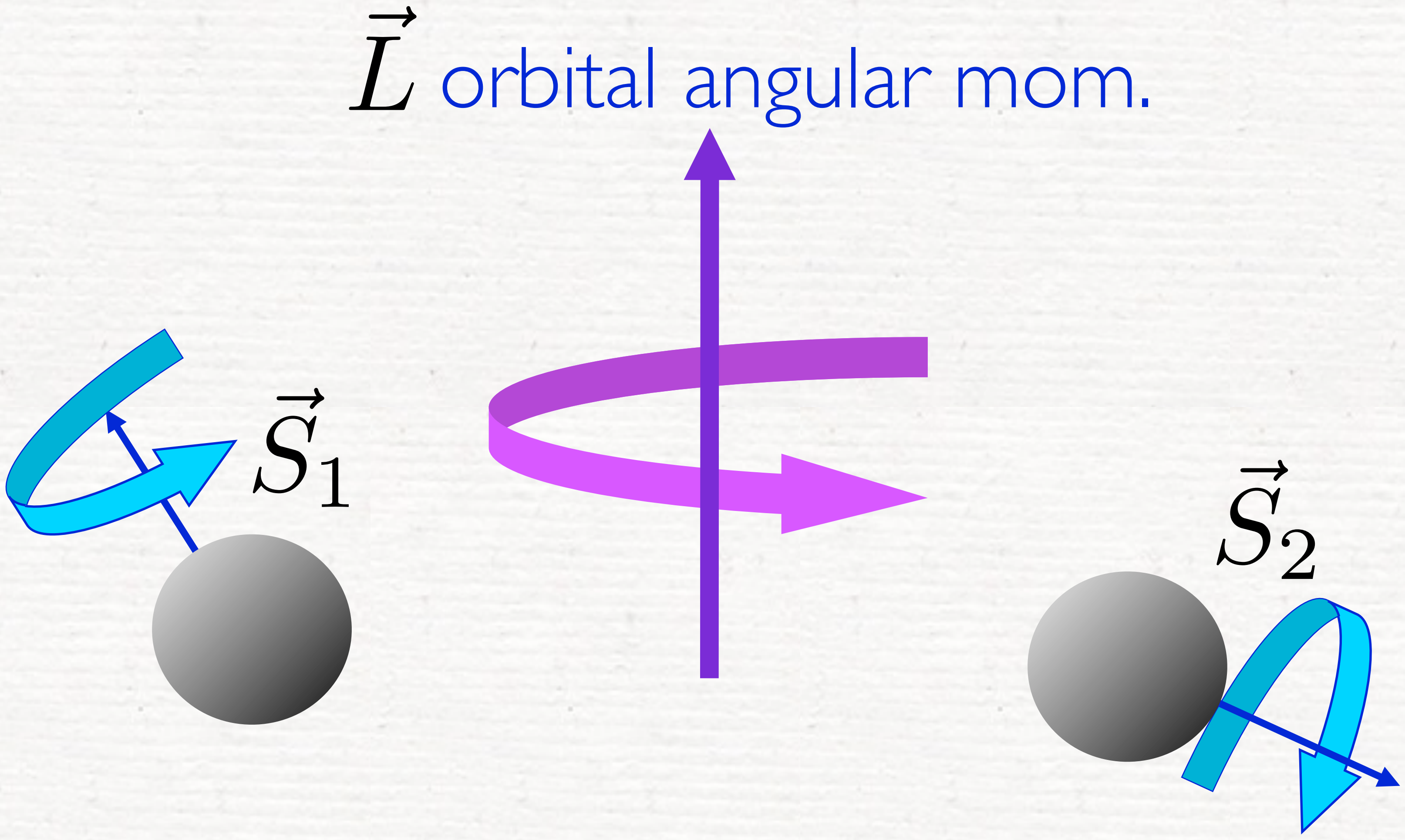
Purple dashed line is using ringdown templates



Modelling the final state

Consider BH binaries as "engines" producing a final single black hole from two distinct initial black holes

Before the merger...



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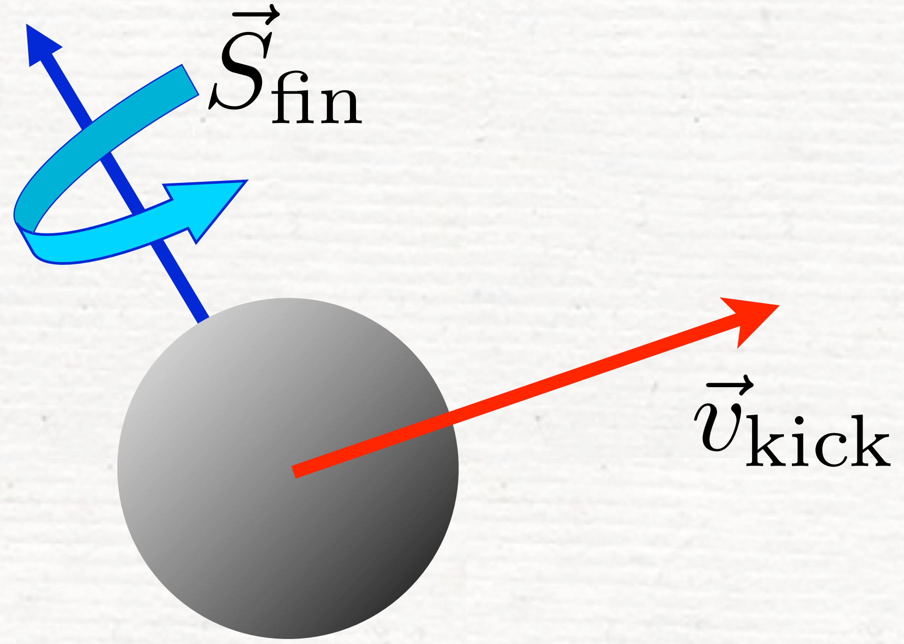
After the merger...



Can we map the initial configuration to a final one without performing a simulation?

Understanding the recoil

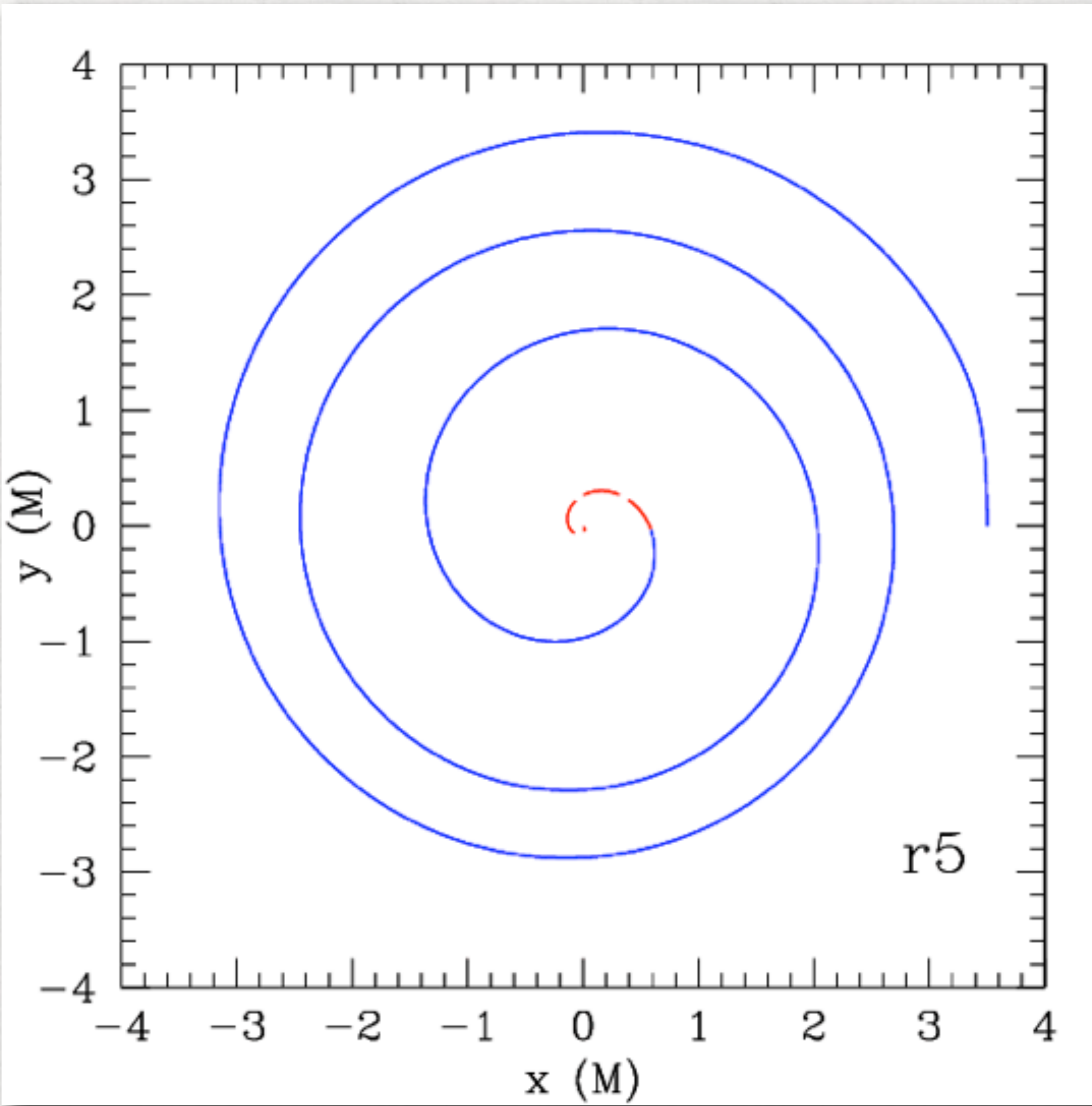
At the end of the simulation and unless the spins are equal, the final black hole will acquire a recoil velocity: aka “kick”.

A simple mechanic analogue is offered by a rotary sprinkler

The emission of gravitational waves is beamed and asymmetrical: momentum radiated at an angle will not be compensated by the momentum after one orbit.

kick!

Being sensitive to the asymmetries in the system, the recoil velocity develops very rapidly in the final stages of the inspiral: i.e. during last portion of the last orbit!



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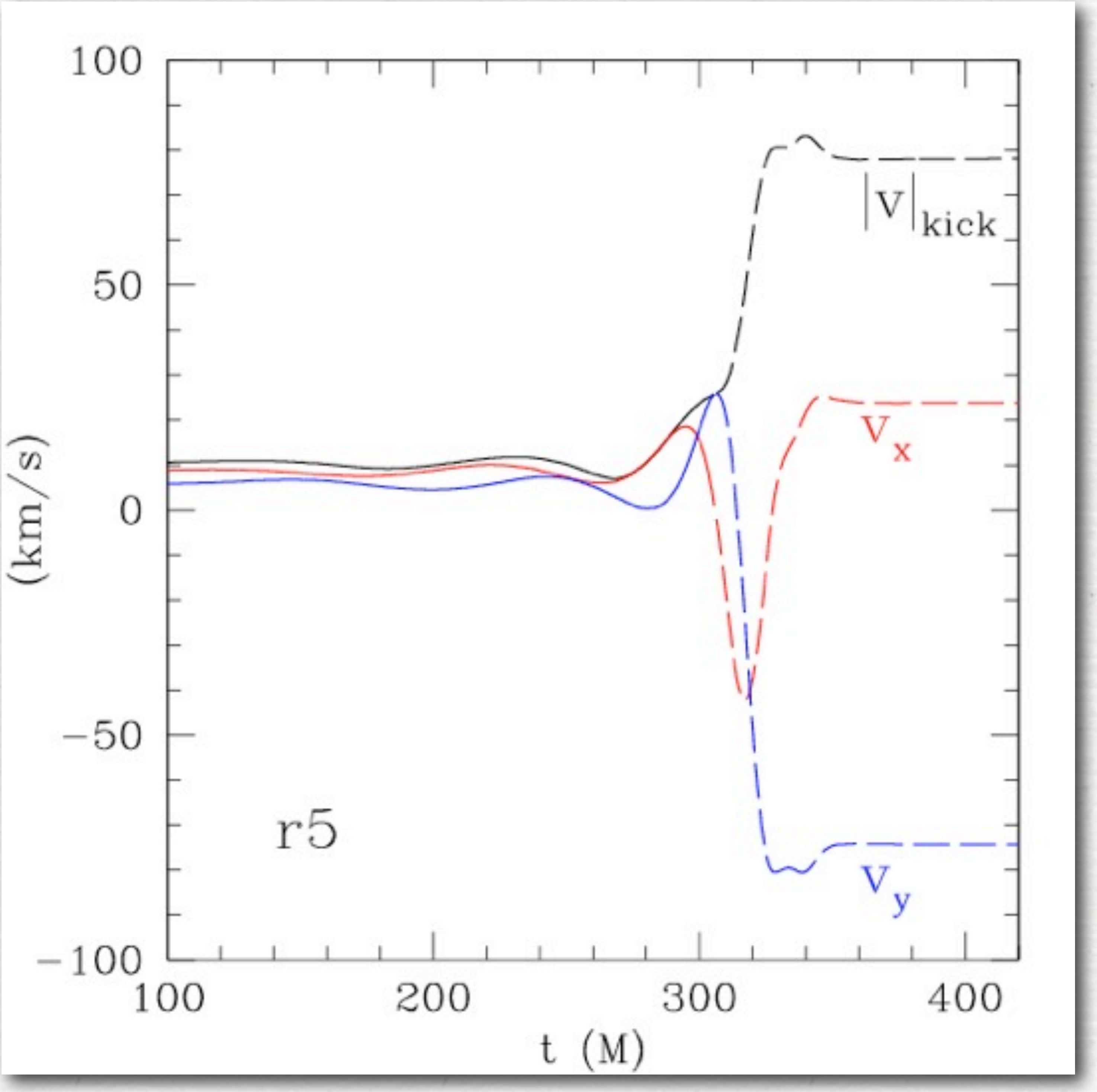


The details of the processes leading to the recoil are still, in great part, unclear.
Subtle balances in the emission of different QNMs during the ringdown are behind the final kick vector.

Sequences help investigate systematic behaviours in the recoil velocity:, eg r-series: $a_1 \in [-0.584, 0.584], a_2 = 0.584$

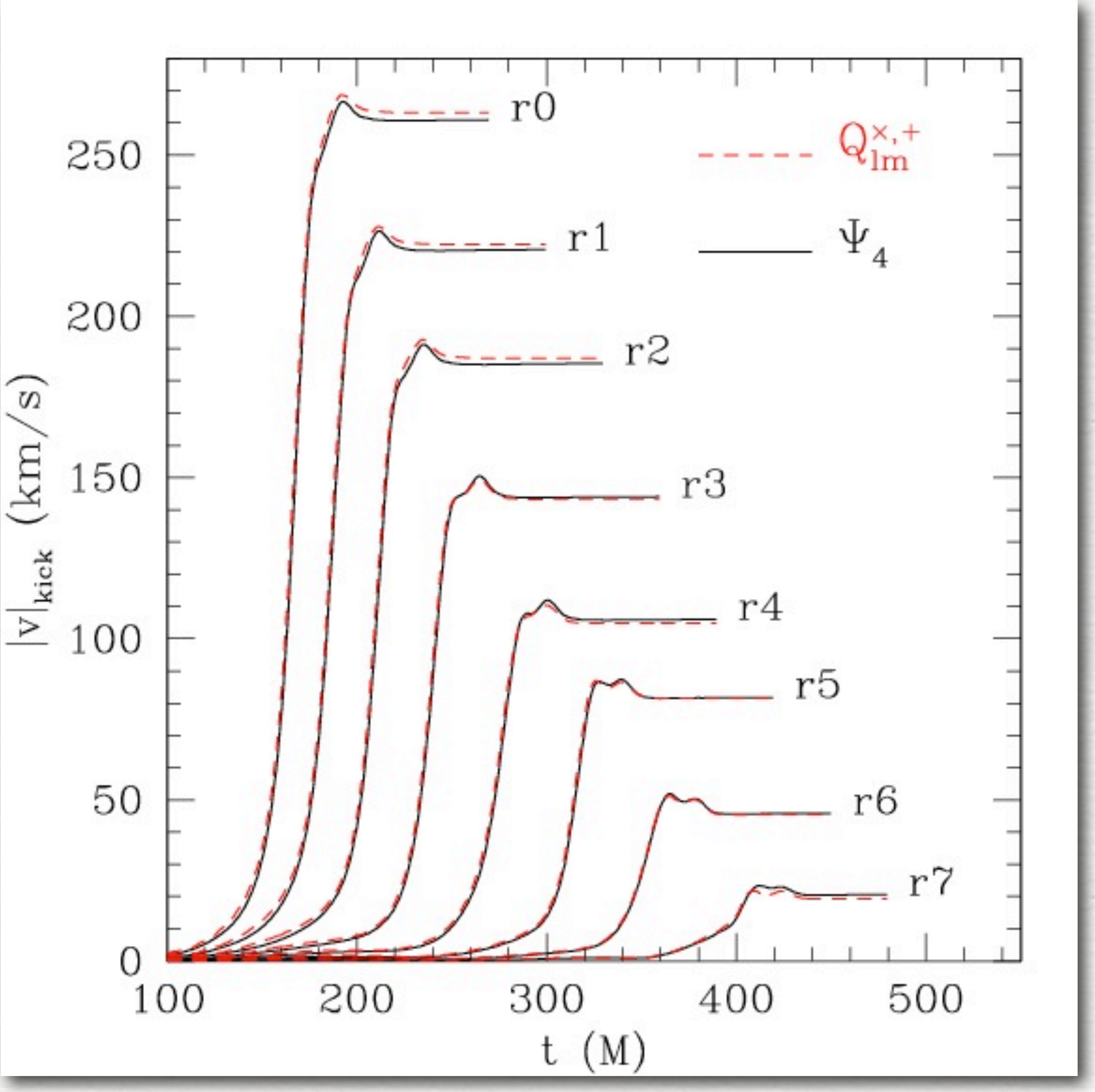
r0: ↑ ↓ (a_1/a_2=-4/4)

r2: ↑ ↓ (a_1/a_2=-2/4)

r4: ↑ . (a_1/a_2=-0/4)

r6: ↑ ↑ (a_1/a_2=2/4)

r8: ↑ ↑ (a_1/a_2=4/4)

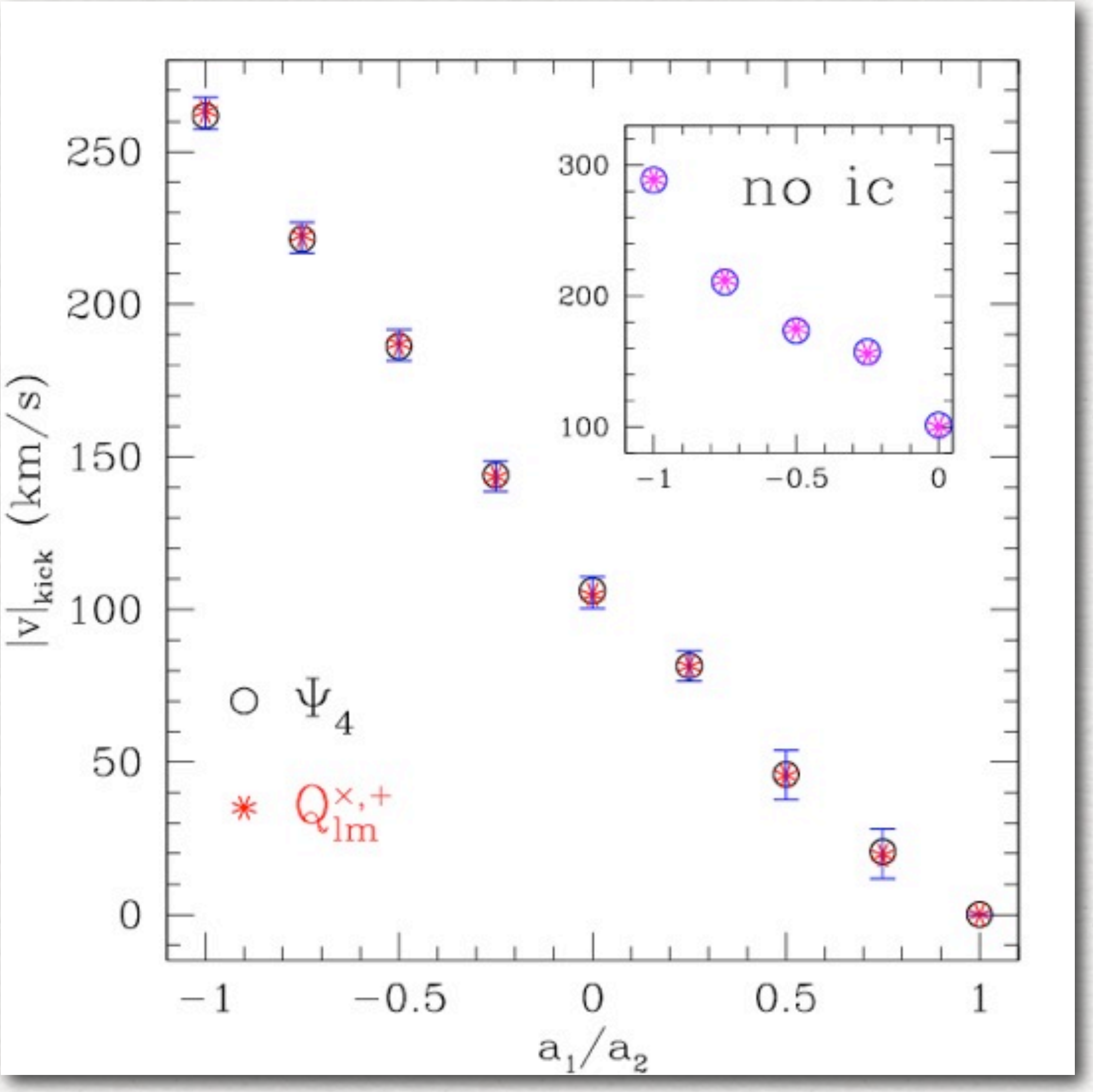


It is interesting to compare the dependence of the kick on the spin-ratio and compare it with the (2.5)PN prediction

$$|v_{\rm kick}| = c_1 \frac{q^2(1-q)}{(1+q)^5} + c_2 \frac{q^2 a_2 (1 - q a_1/a_2)}{(1+q)^5}$$

q: mass-ratio (here $q = 1$) and the constants c_1, c_2 can be calculated only in full GR

A careful study has shown there is indeed a quadratic contribution. Example that numerical relativity can provide information in regimes otherwise not accessible



What we know (now) of the kick

$$\boldsymbol{v}_{\rm kick} = v_m\,\boldsymbol{e}_1 + v_\perp(\cos(\xi)\,\boldsymbol{e}_1 + \sin(\xi)\,\boldsymbol{e}_2) + v_\parallel\,\boldsymbol{e}_3,$$

where

✓ $$v_m = A\nu^2\sqrt{1-4\nu}(1+B\nu),$$

✓ $$v_\perp = c_1\frac{\nu^2}{1+q}\left(a_2^\parallel - q a_1^\parallel\right) + c_2\left((a_2^\parallel)^2 - q^2(a_1^\parallel)^2\right),$$

$$v_\parallel = \frac{K\nu^3}{(1+q)}\left[q a_1^\perp\cos(\phi_1-\Phi_1) - a_2^\perp\cos(\phi_2-\Phi_2)\right],$$

ν^2 ?

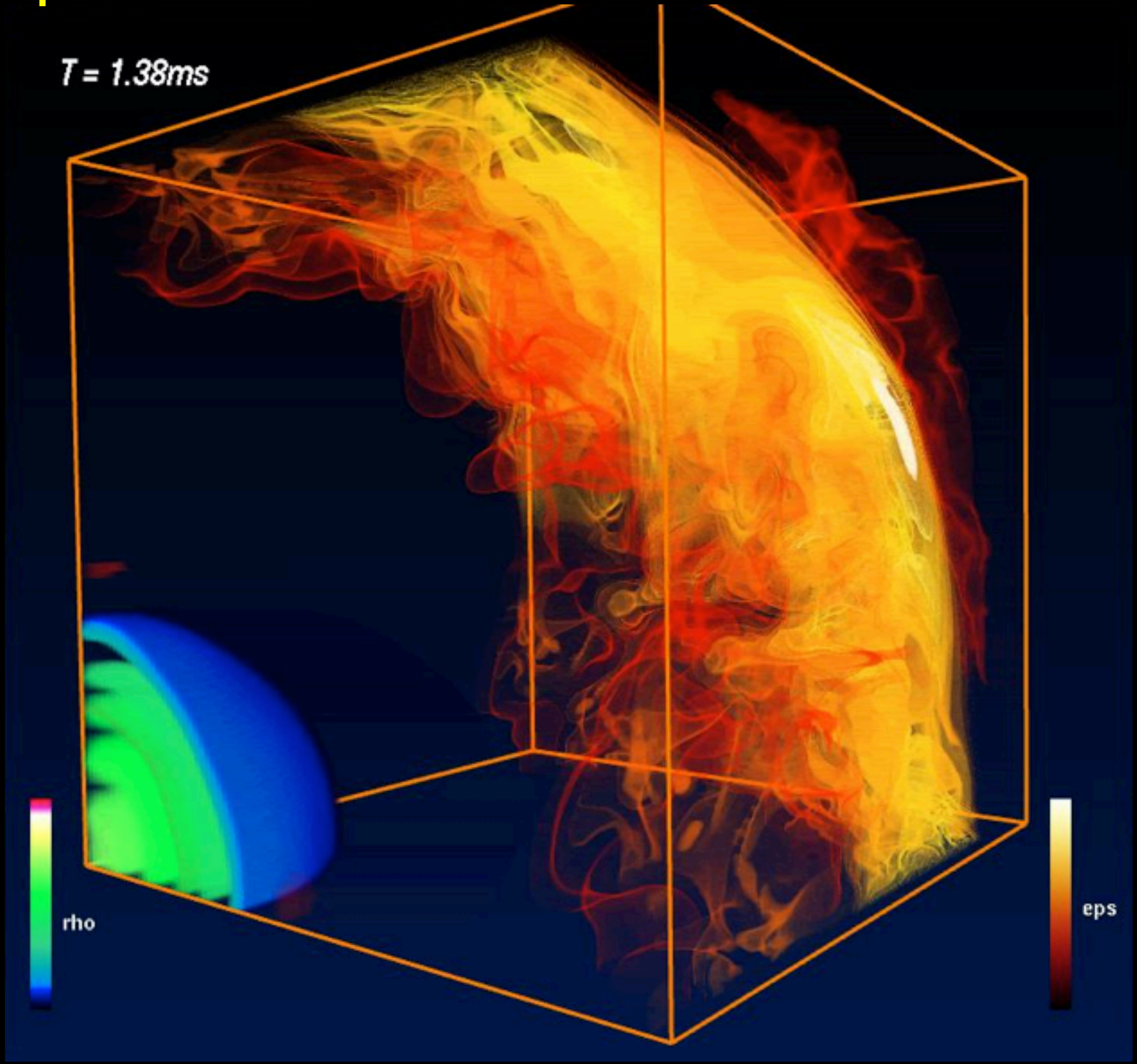
mass asymmetry $\lesssim 150\text{km/s}$

spin asymmetry; contribution **off** the plane $\lesssim 450\text{km/s}$

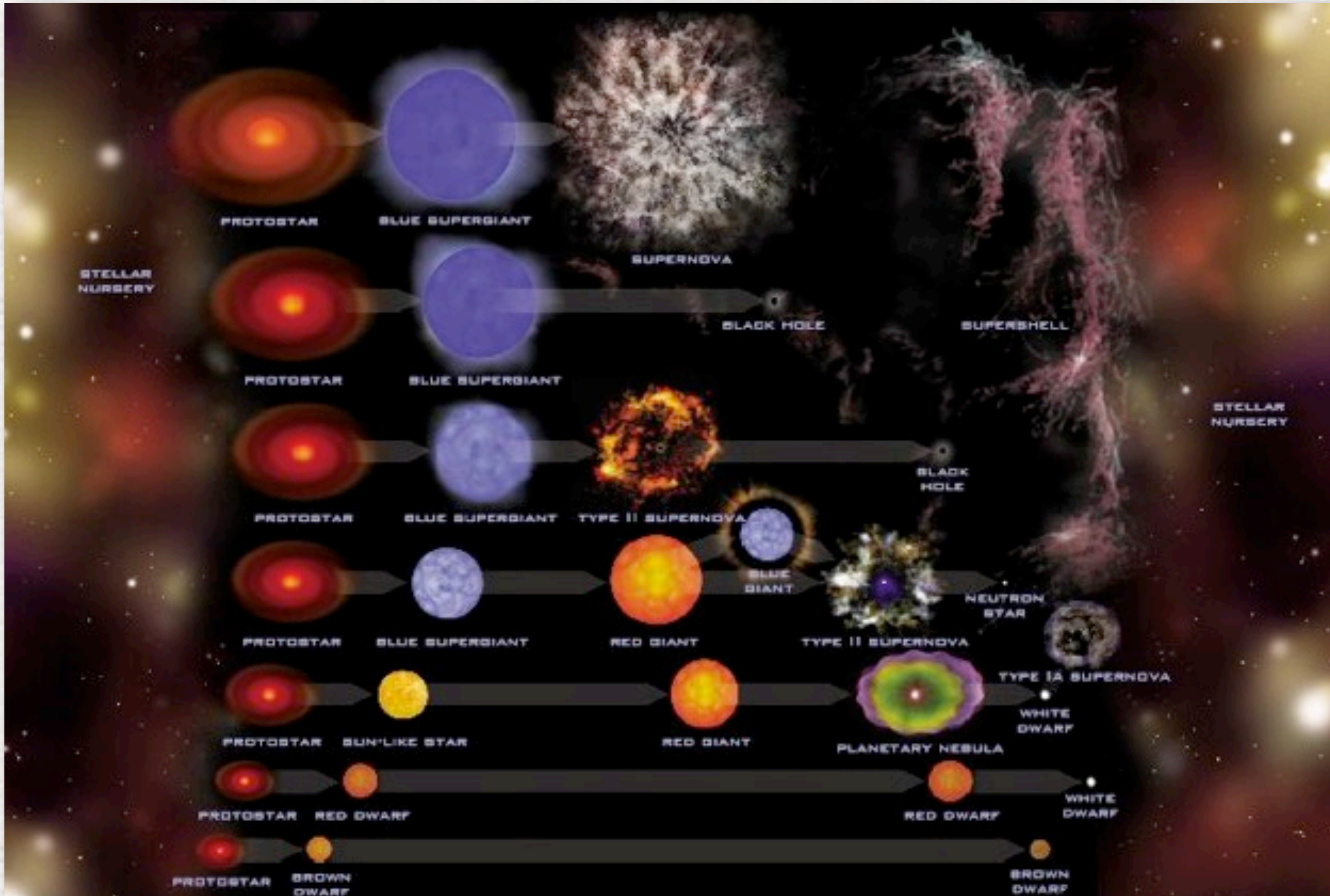
spin asymmetry; contribution **in** the plane $\lesssim 3500\text{km/s}$

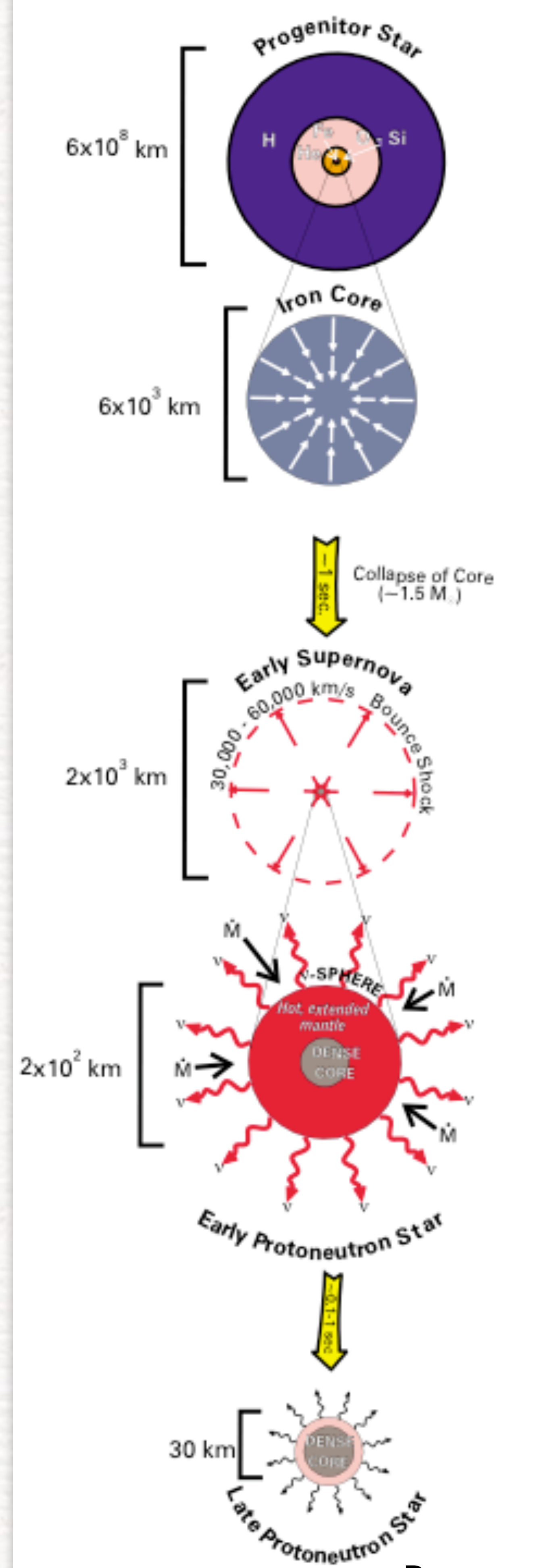
enough to eject the BH from a dwarf galaxy

Putting matter in the spacetime: neutron stars



Type II supernovae: the birth of NSs



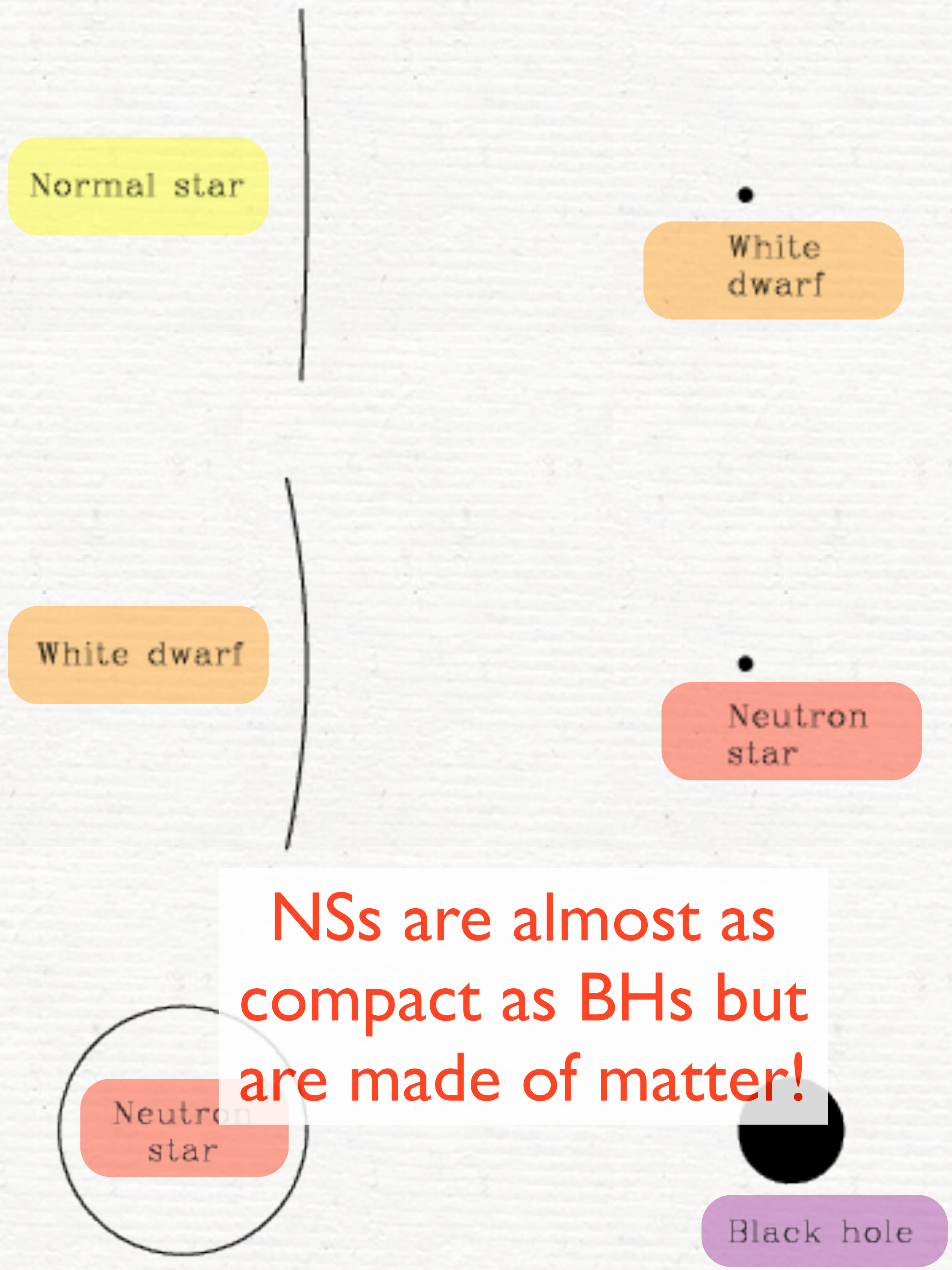


Burrows

Neutron stars are the most common end-result of the evolution of massive stars, ie stars with $10M_\odot \lesssim M \lesssim 100M_\odot$

Such stars end their evolution as supernovae

A comparative sense of the compactness



$$R_{\odot} \simeq 700000 \text{ km}$$

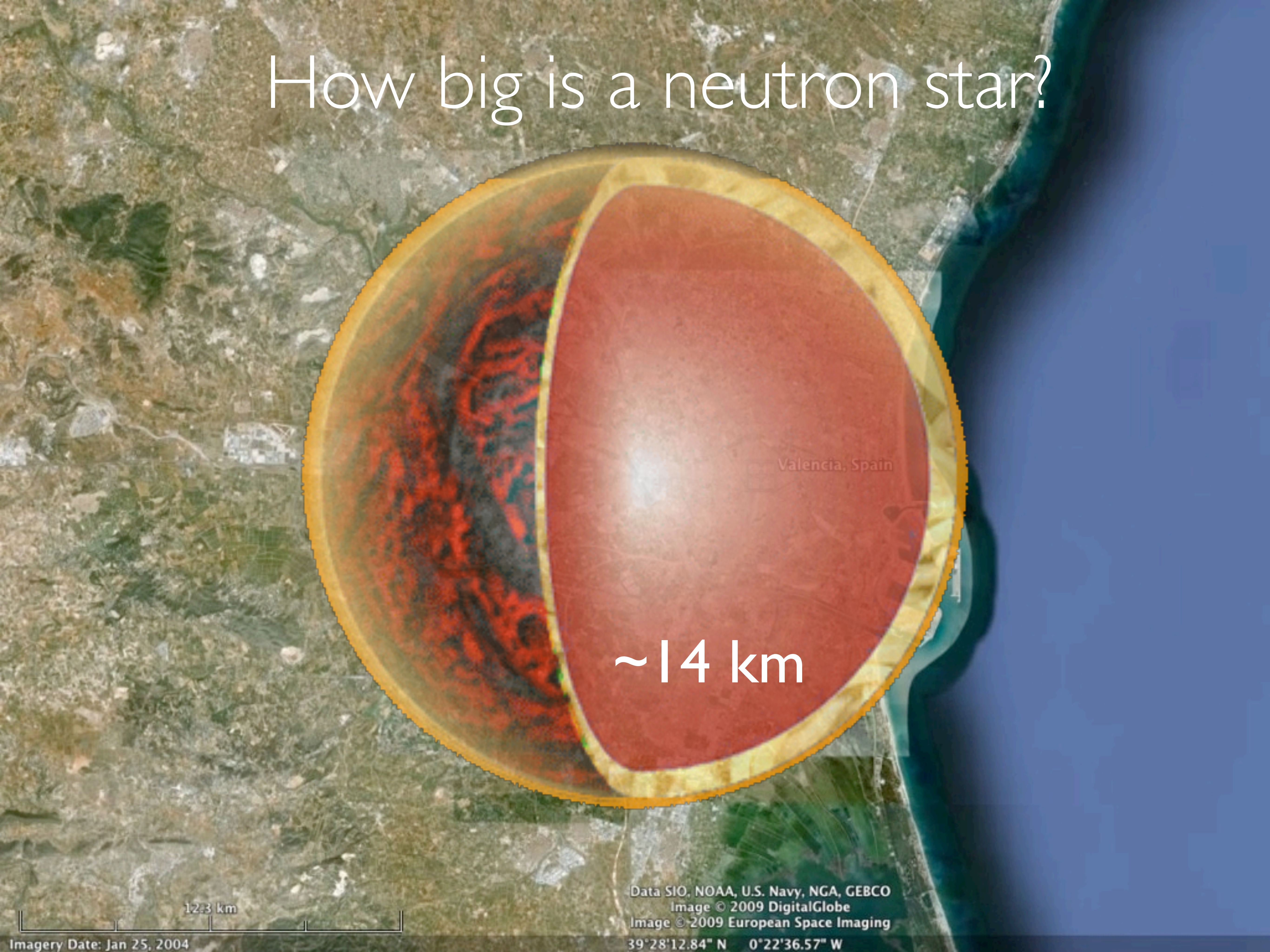
$$R_{\text{white dwarf}}\ (\mathrm{M}_{\odot}) \simeq 10000 \text{ km}$$

$$R_{\text{neutron star}} \simeq 12 \text{ km}$$

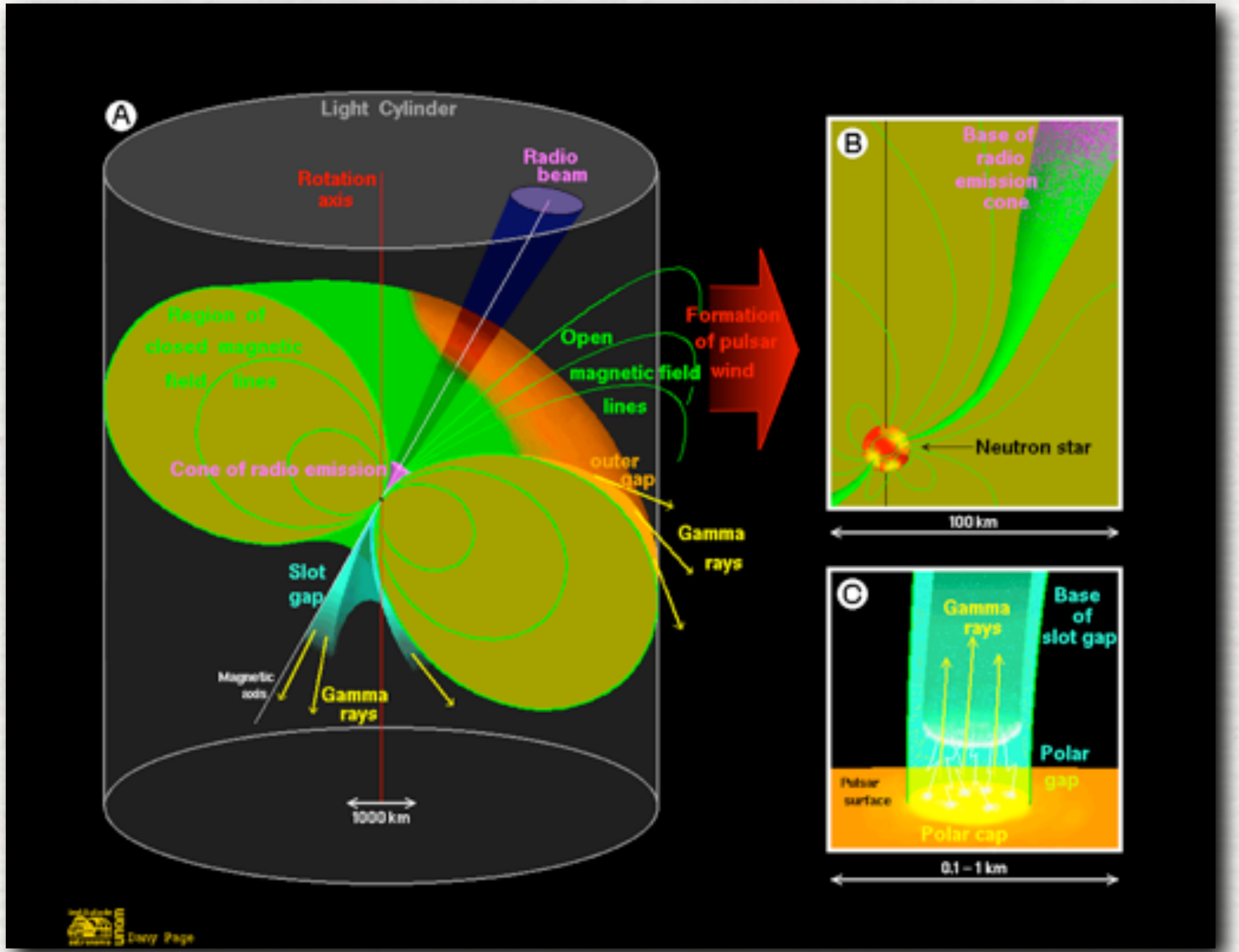
$$R_{\text{black hole}}\ (\mathrm{M}_{\odot}) \simeq 1.5 \text{ km}$$

NSs are almost as compact as BHs but are made of matter!

How big is a neutron star?



Observational evidence: pulsed radio emission



Starting from the '60 we have observations of regular pulsed radio emissions.
The small periods and the small variability

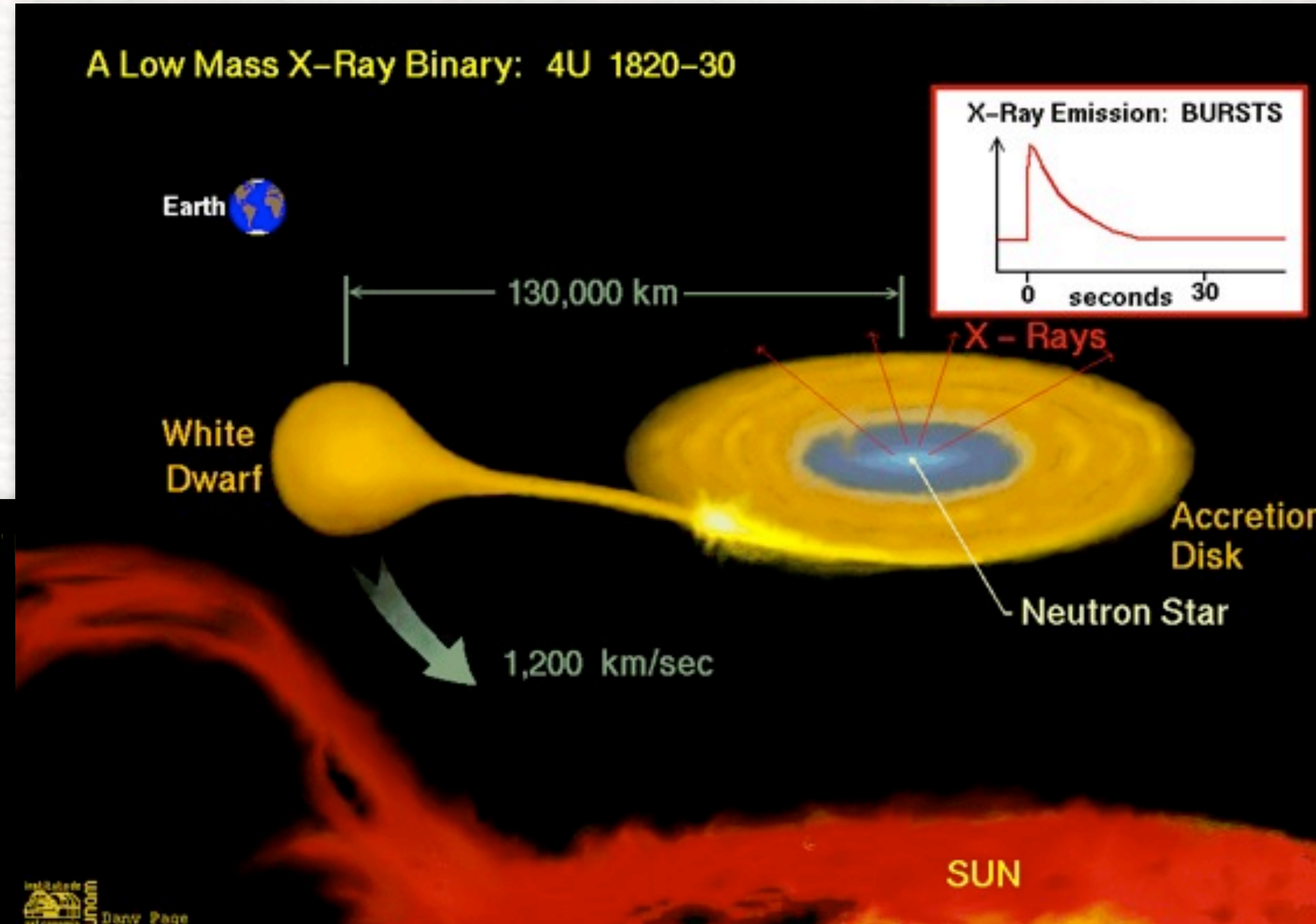
$$P \simeq 10^{-3} - 10 \text{ s}$$
$$P/\dot{P} \simeq 10^{7} \text{ yr}$$
$$\Delta P/P \simeq 10^{-15}$$

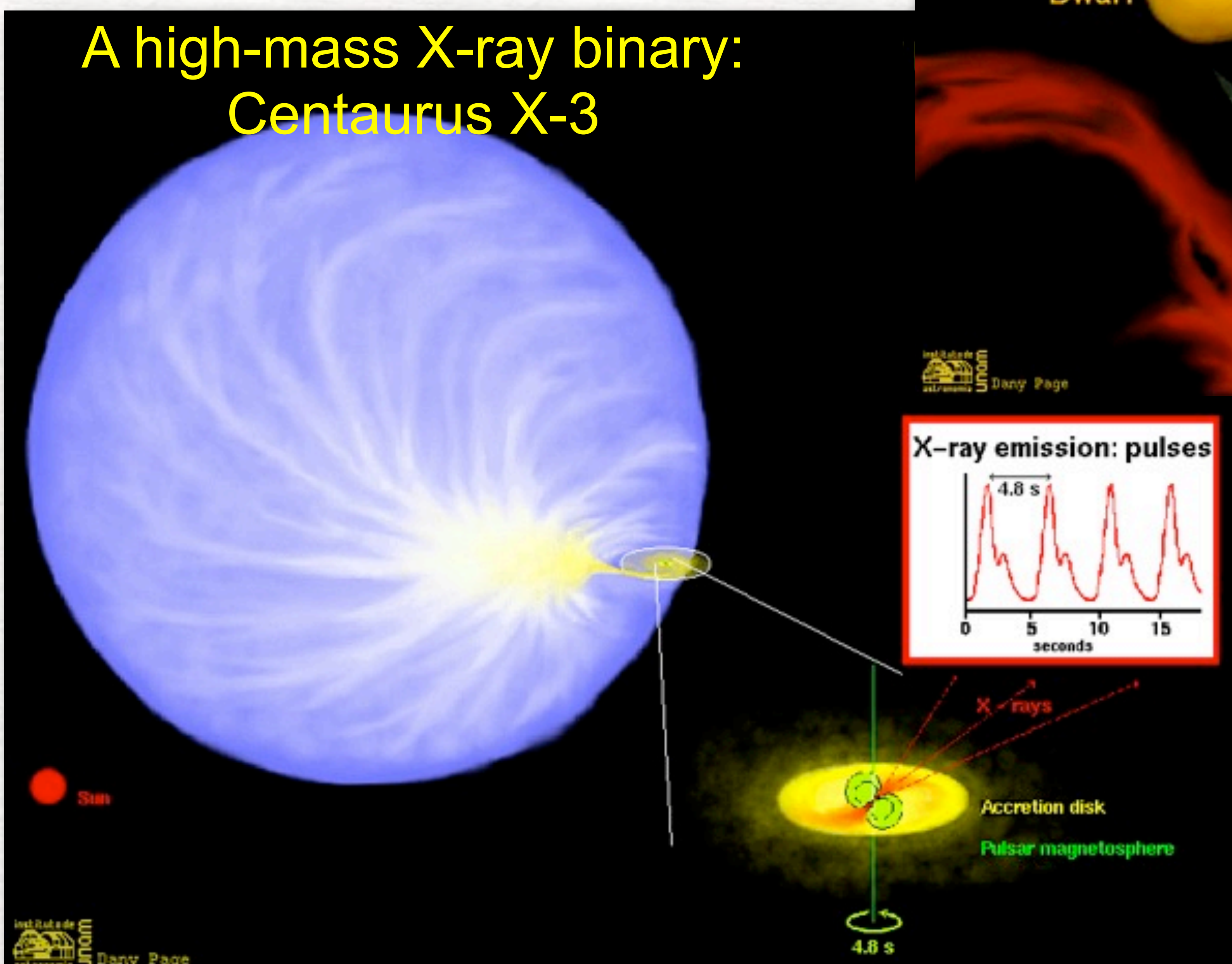
can only be explained with a rotating compact star: PULSAR

We now know they are magnetized NSs ($\mathbf{B \sim 10^{10\text{-}12}}$ **G**) emitting a beam of radiation coming from accelerated charged particles stripped from the surface.

Observational evidence: X-rays

NSs are thought to be behind the X-ray emission from some binary systems: X-ray binaries. This emission can either be in terms of bursts or quasi-periodic

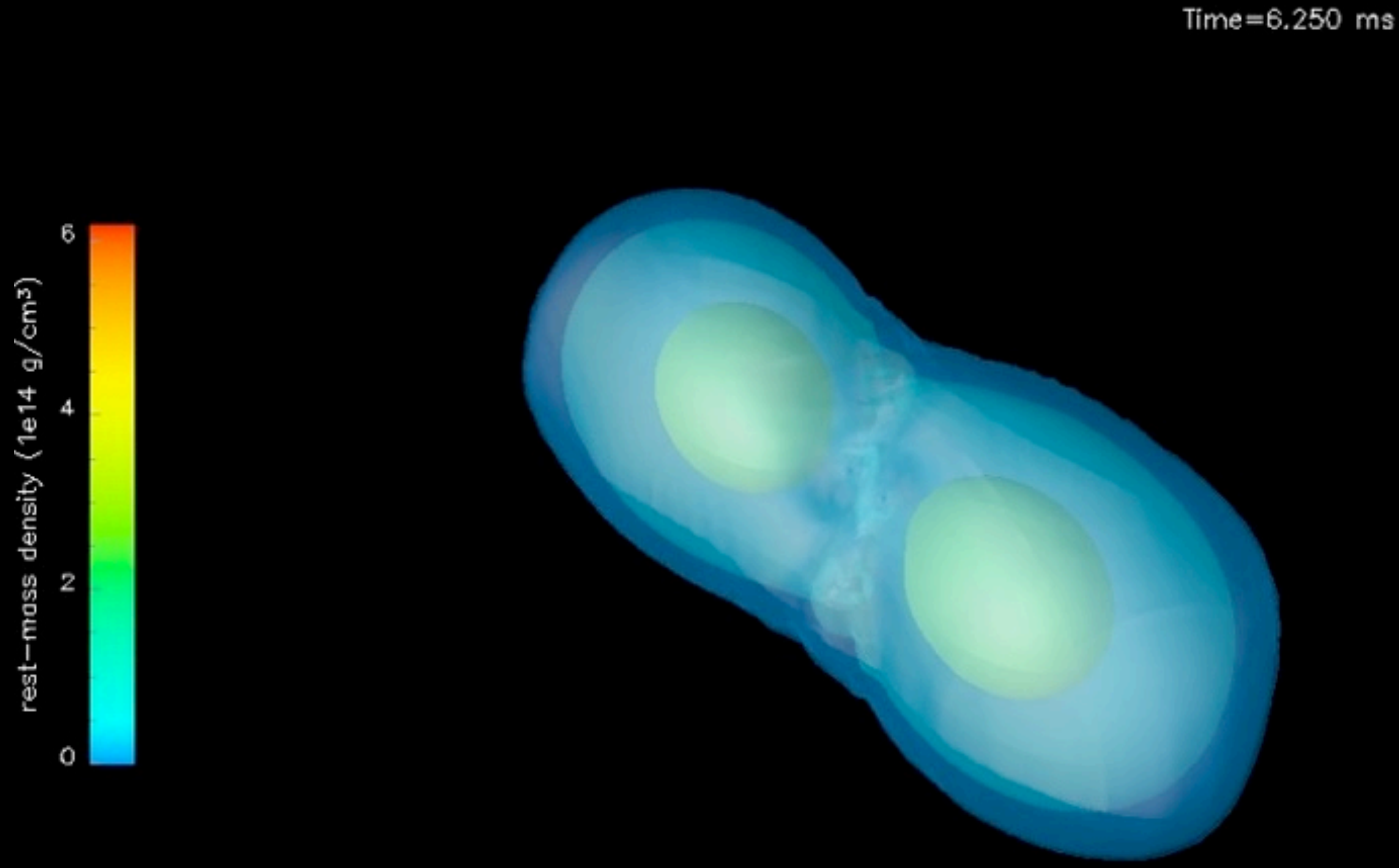




These systems represent excellent laboratories of relativistic astrophysics:

- strong curvature
- matter at supernuclear densities
- high-energy emission

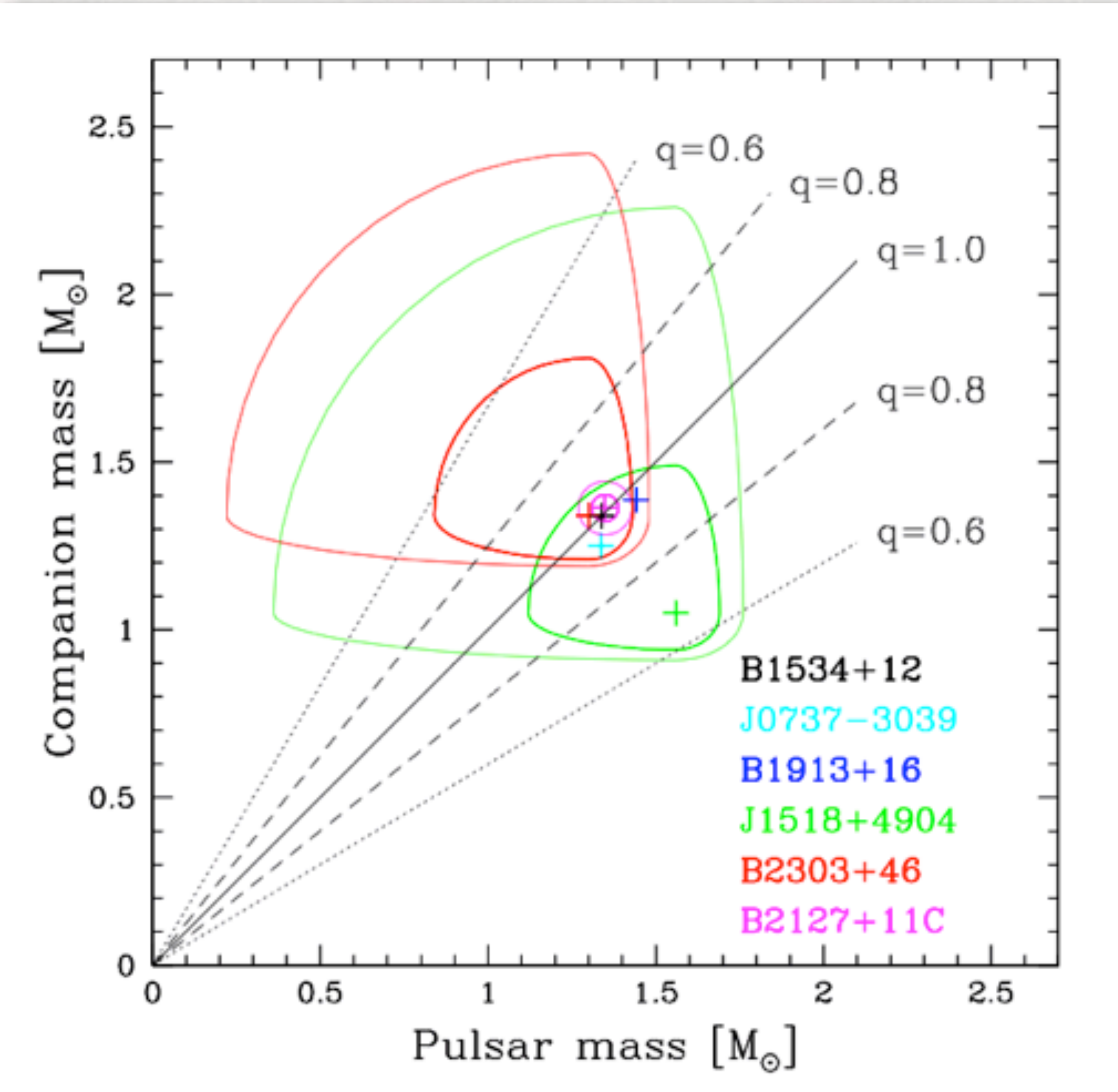
Modelling Binary Neutron Stars



Baiotti, Giacomazzo, LR, PRD, 2008; CQG 2009
Giacomazzo, LR, Baiotti, MNRASL 2009; CQG 2009
Link, LR, Baiotti, Giacomazzo, Font, CQG 2009

Do they really exist?

Direct observational evidence of BH binaries (BBHs) is still lacking, both for stellar-mass black holes and for supermassive ones. On the other hand, we see BHs in galaxies, and galaxies merge.



D. Gondek-Rosinska

Binary NS (BNSs) are instead observed and we have a dozen examples in our Galaxy

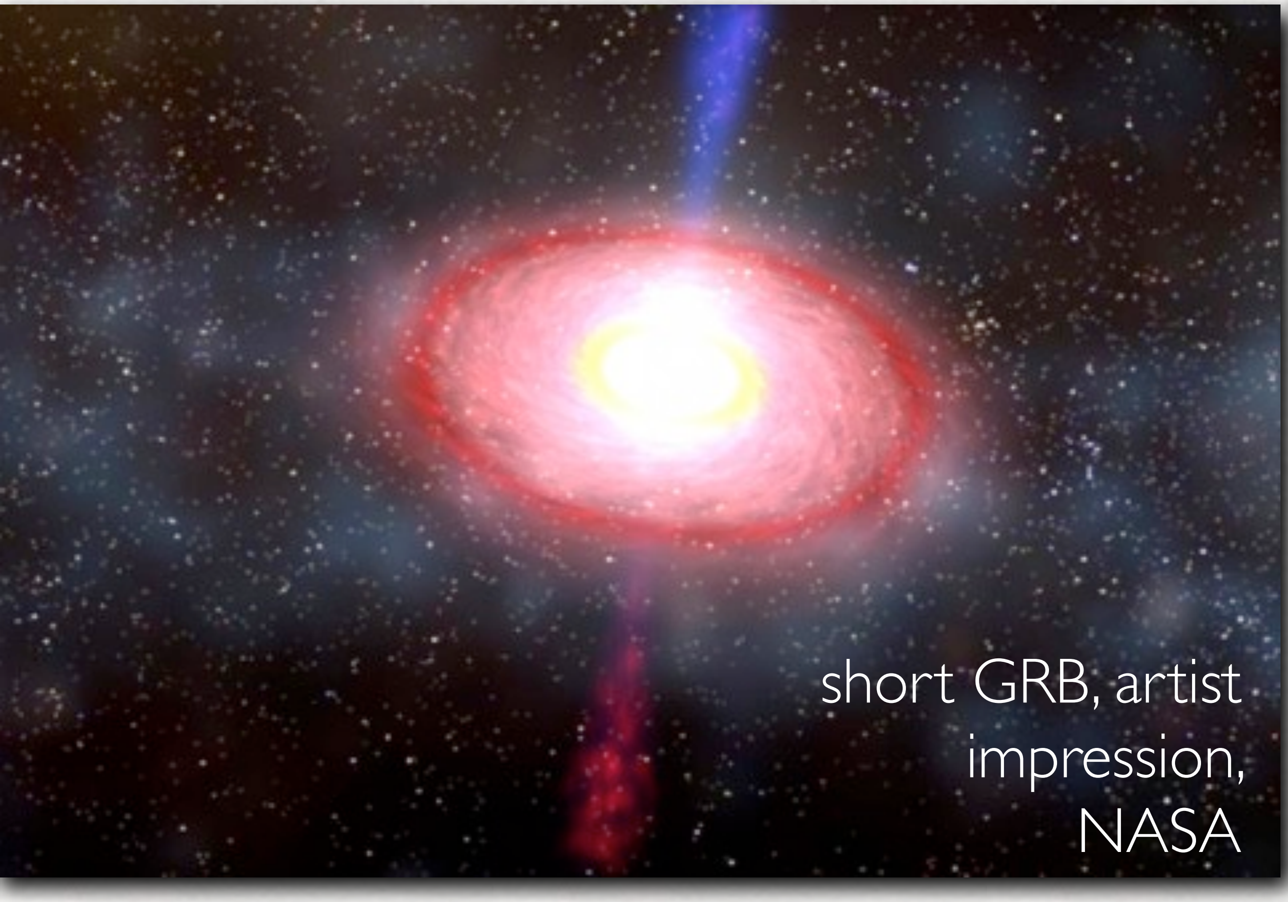
1.44	1.38	B1913+16
1.33	1.34	B1534+12
1.33	1.25	J0737-3039
1.40	1.18	J1756-2251
1.36	1.35	B2127+11C
1.35	1.26	J1906+0746
1.62	1.11	J1811-1736
1.56	1.05	J1518+4904
1.14	1.36	J1829+2456

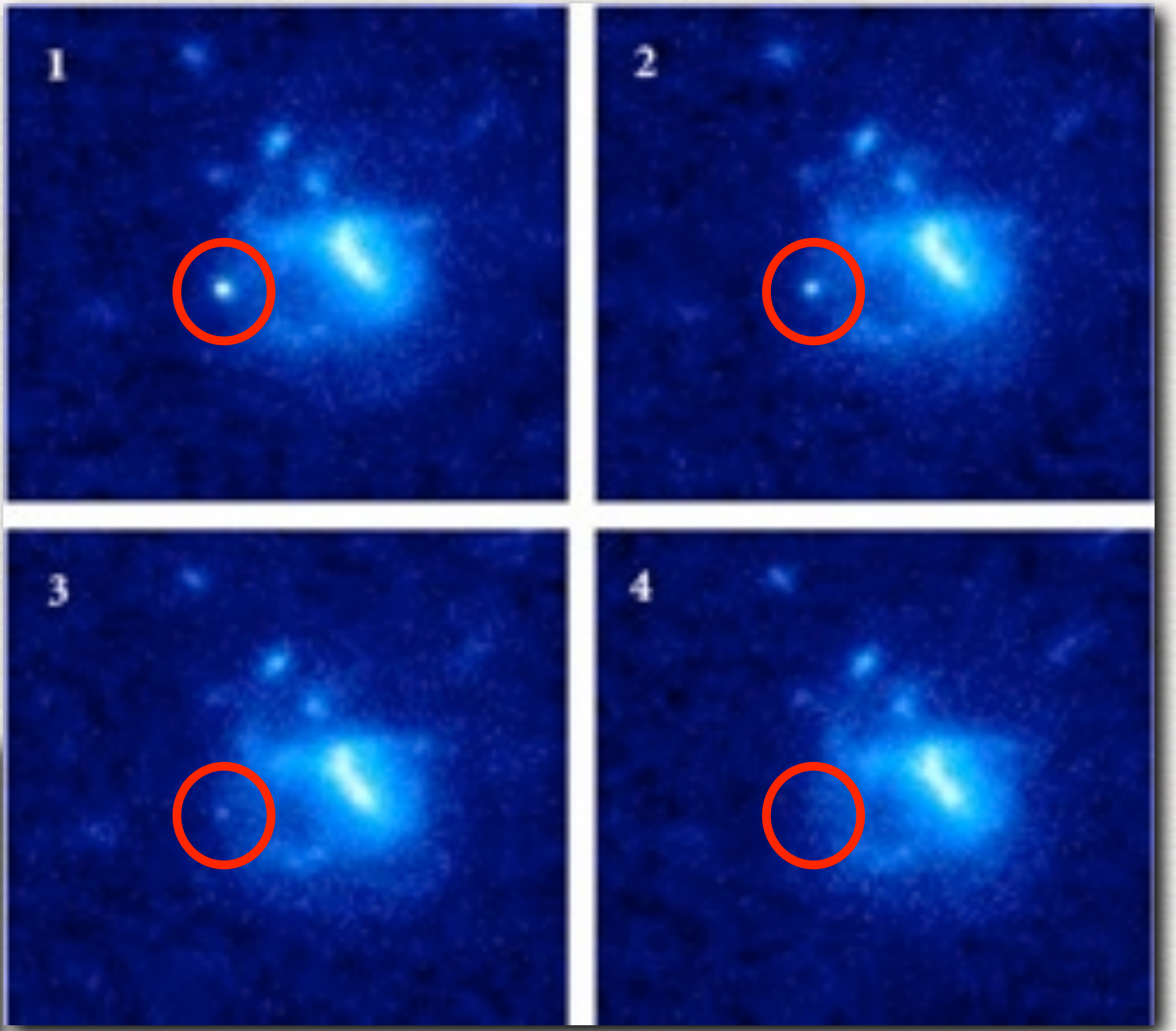
NS has small variation in mass and binaries show this by being mostly with equal-masses

The mistery of Gamma Ray Bursts

Everyday we observe intense flashes of gamma rays coming from the most distant corners of the universe.

The energies releases are huge: $10^{50\text{-}51}$ erg: this is equivalent to the luminosity of the whole Galaxy emitted over ~ 1year





Compact-object binaries are thought to be behind this emission but no consistent model has yet been produced to explain them.

Overall, we know what to expect:

"merger → HMNS → BH + torus"

This behaviour is general but only qualitatively

Gravity will prevail at the end but the timescale over which this happens depends on physics we do not fully control yet, not even with analytic EOS.

Determining this accurately is essential both for GW astronomy as well as for GRB astronomy

Cold vs Hot EOSs

Simplest example of a **"cold"** EOS is the polytropic EOS. This isentropic: internal energy (temperature) increases/decreases only by mechanical work (compression/expansion)

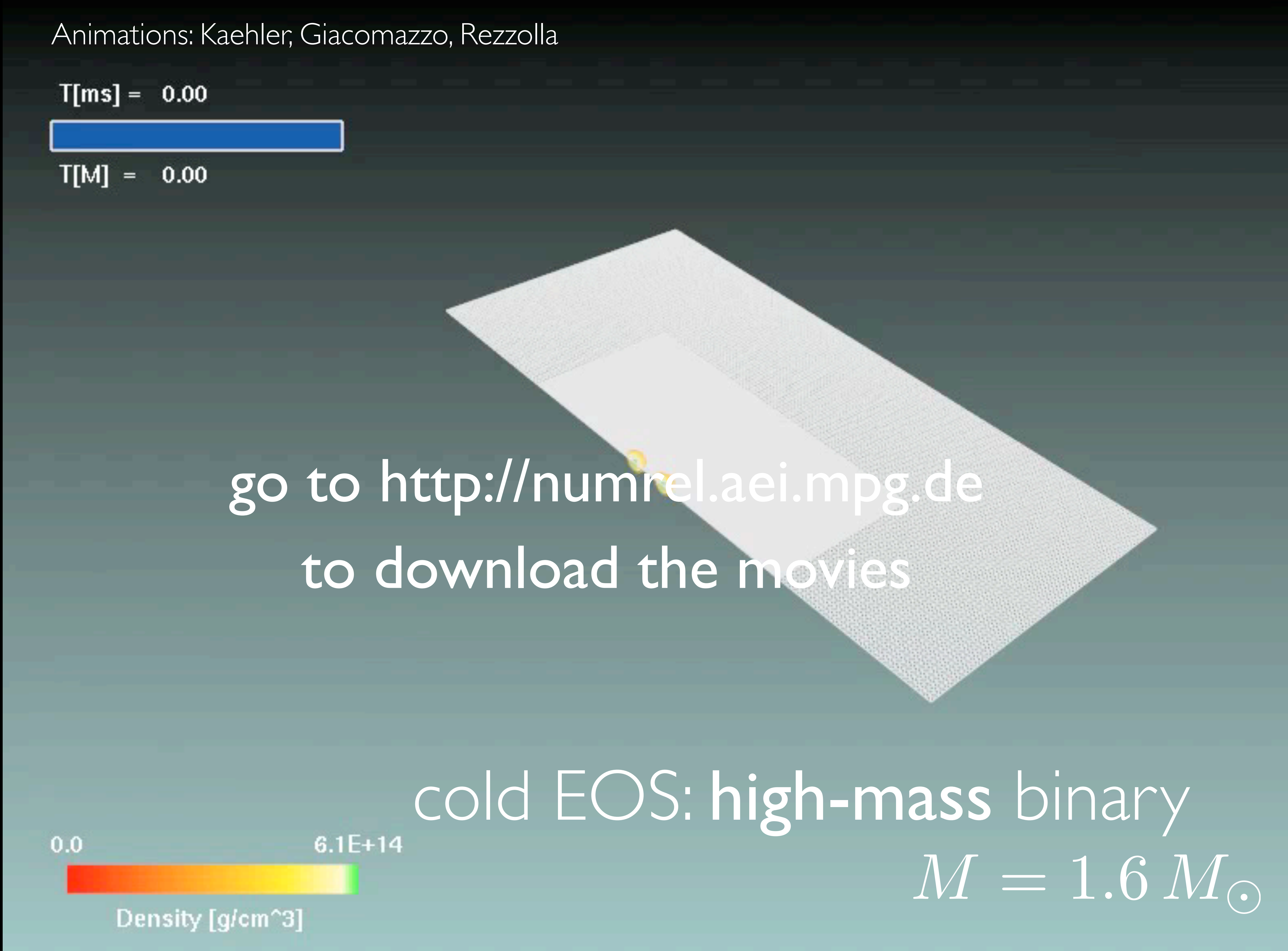
$$p = K\rho^{\Gamma}\,, \qquad \epsilon = \frac{K\rho^{\Gamma-1}}{\Gamma-1}$$

Simplest example of a **"hot"** EOS is the ideal-fluid EOS. This non-isentropic in presence of shocks: internal energy (i.e. temperature) can increase via shock heating.

$$p = \rho\epsilon(\Gamma-1)\,, \quad \partial_t \epsilon = \ldots$$

A cold EOS is optimal for the inspiral; a hot one is essential after the merger. Consider them as extremes of possible behaviours

Animations: Kaehler, Giacomazzo, Rezzolla



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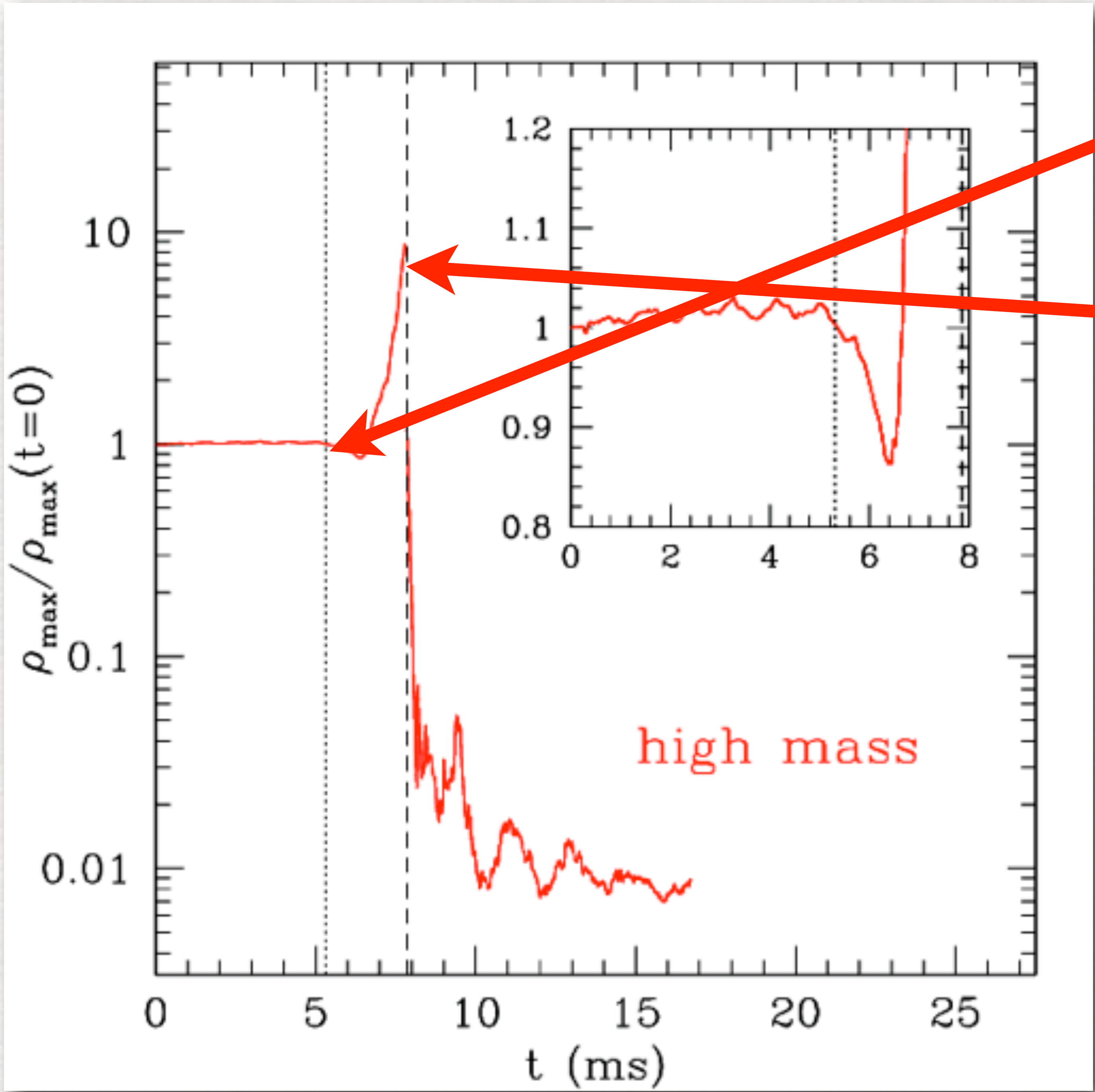
cold EOS: **high-mass** binary

$$M = 1.6\,M_{\odot}$$

Matter dynamics

high-mass binary



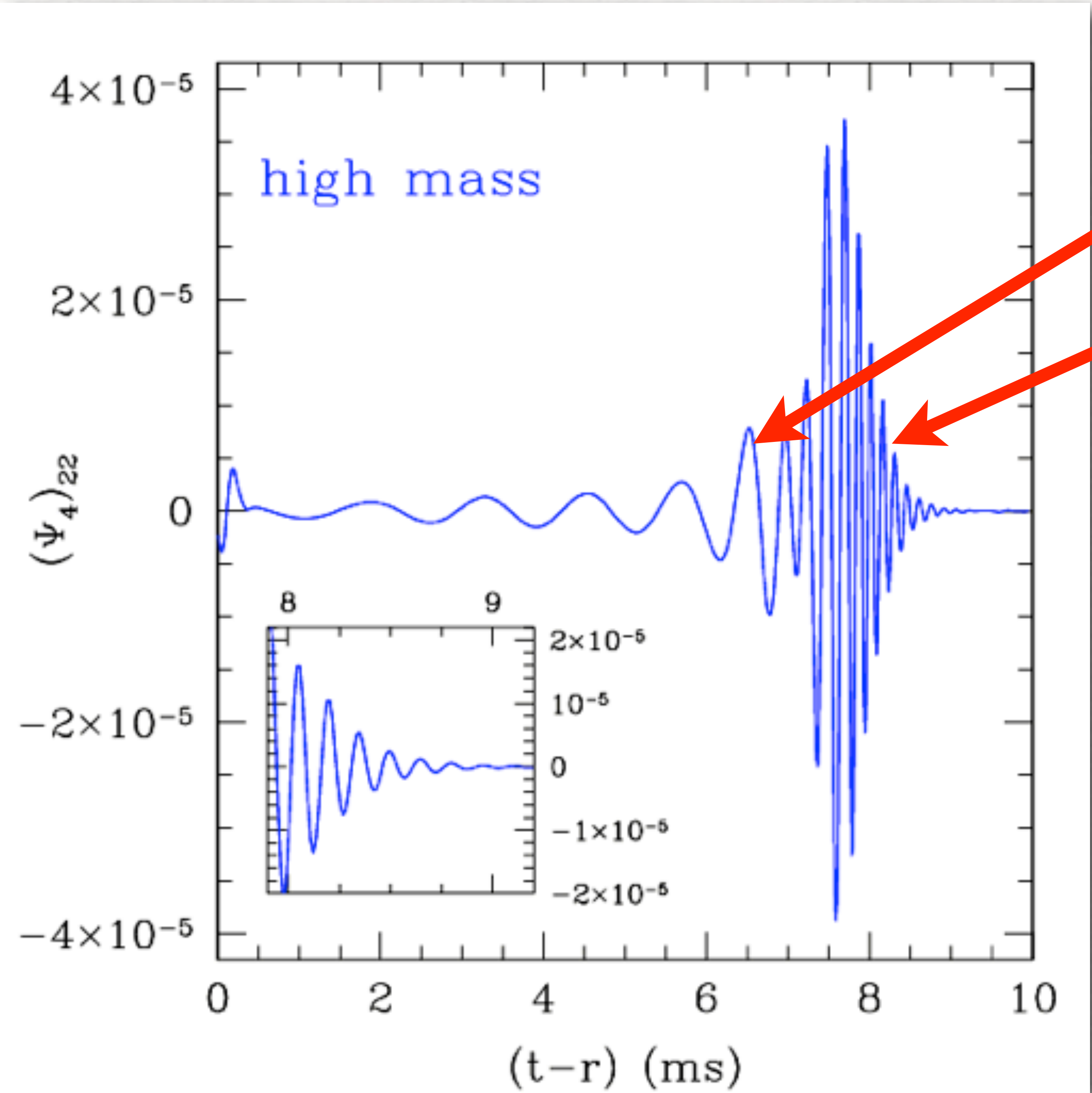
Merger

Collapse to BH

soon after the merge the torus is formed and undergoes oscillations

Waveforms: polytropic EOS

high-mass binary



Merger

Collapse to BH

first time the full signal from the formation to a bh has been computed

"merger → HMNS → BH + torus"

Quantitative differences are produced by:

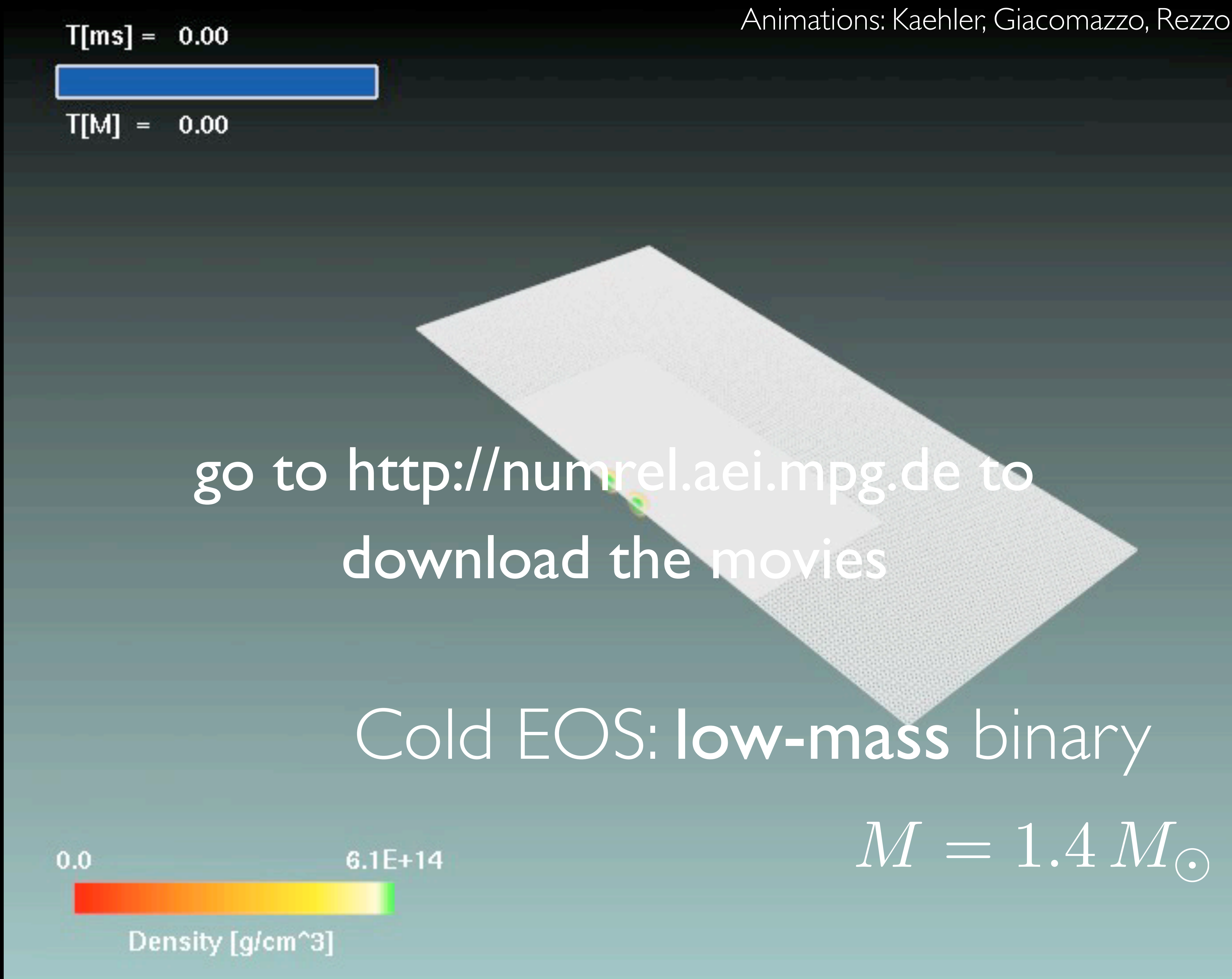
- **differences induced by the gravitational MASS:**
  a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time

- **differences induced by the EOS:**
  a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later

- **differences induced by MAGNETIC FIELDS:**
  the angular momentum redistribution via magnetic braking or MRI can increase/decrease time to collapse

- **differences induced by RADIATIVE PROCESSES:**
  radiative losses will alter the metastable equilibrium of the HMNS in certain mass ranges (work in progress)

Animations: Kaehler, Giacomazzo, Rezzolla



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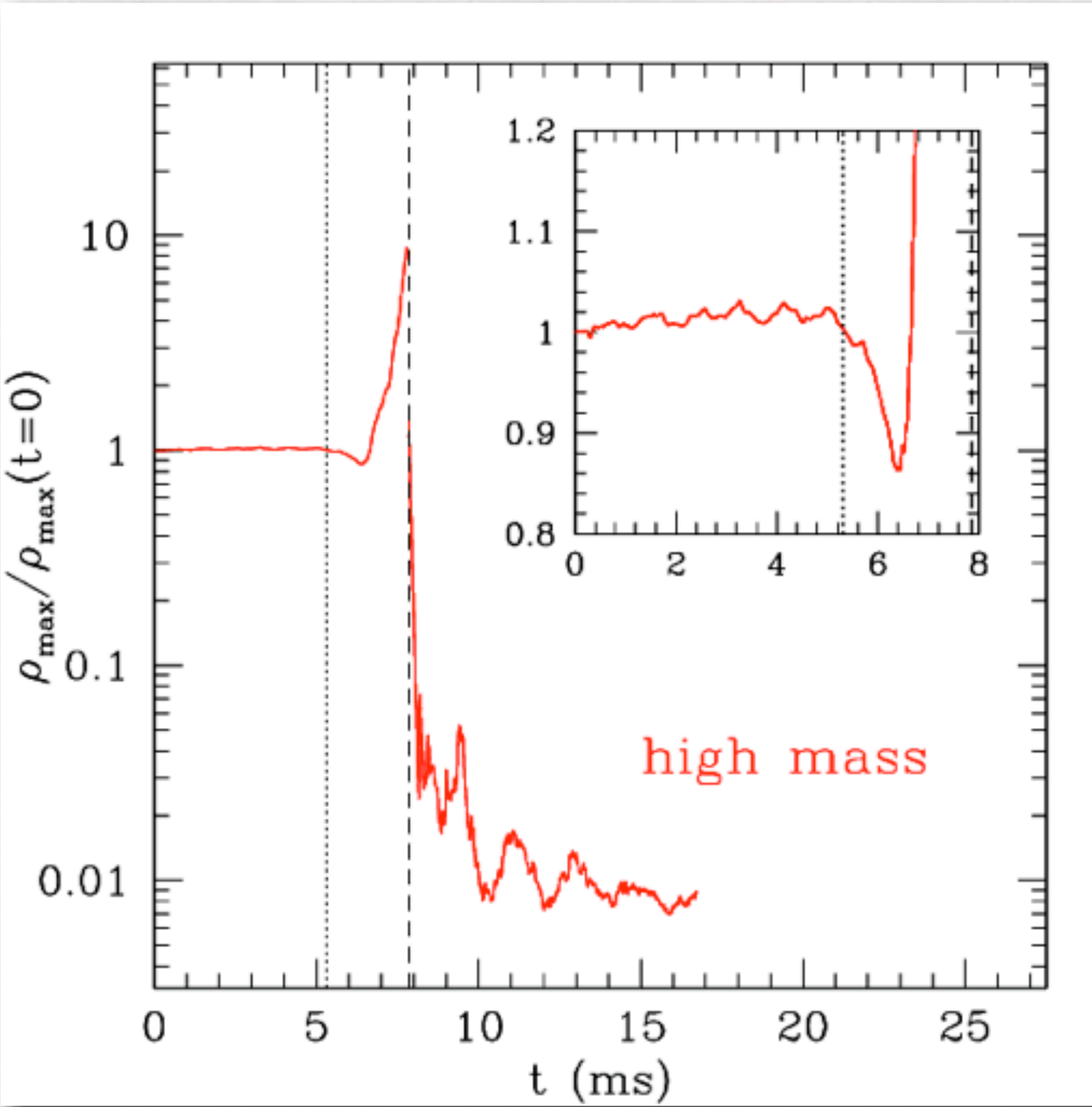
Cold EOS: **low-mass** binary

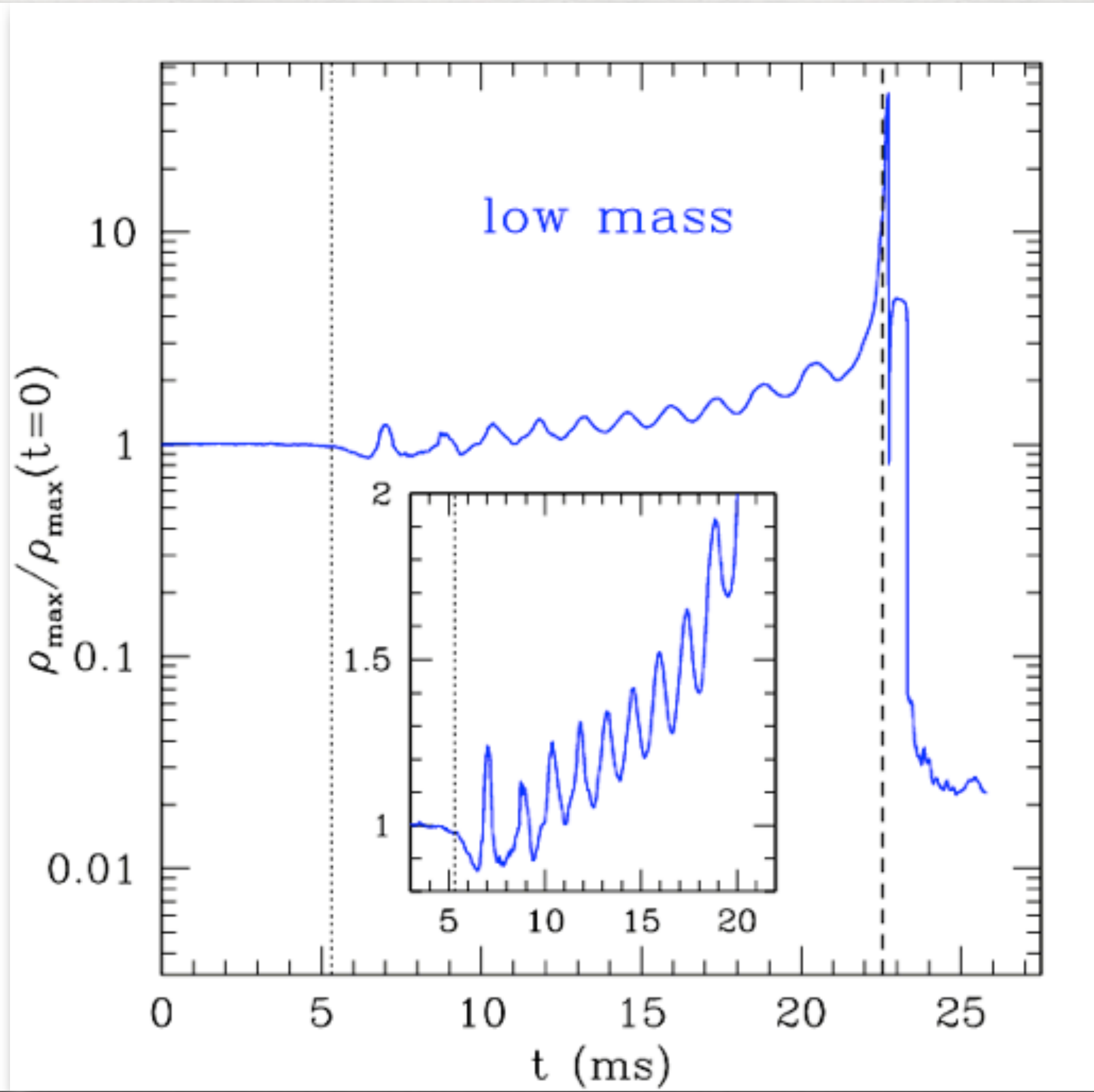
$$M = 1.4\, M_\odot$$

Matter dynamics

high-mass binary

low-mass binary



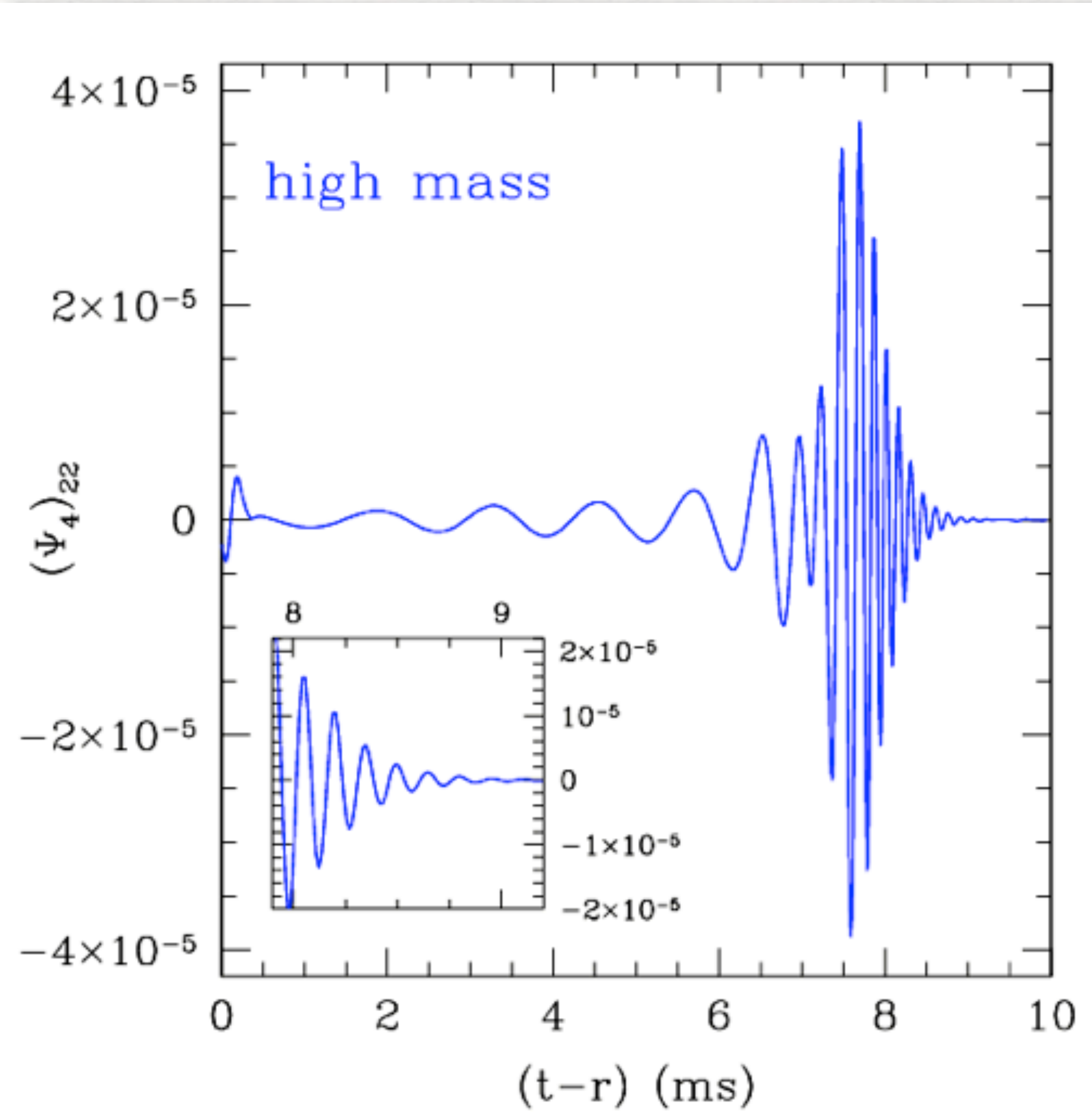


soon after the merge the torus is formed and undergoes oscillations

long after the merger a BH is formed surrounded by a torus

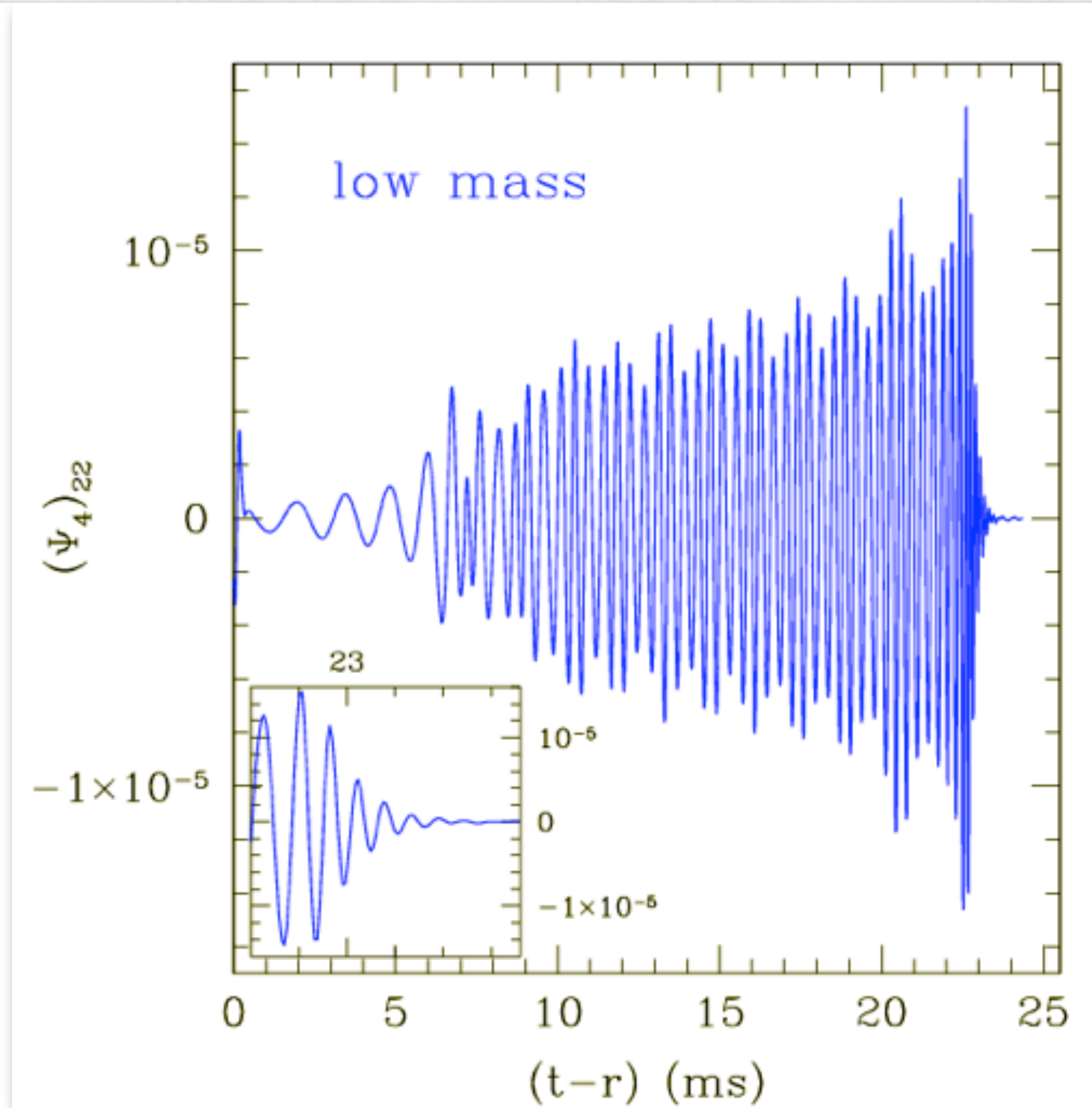
Waveforms: polytropic EOS

high-mass binary



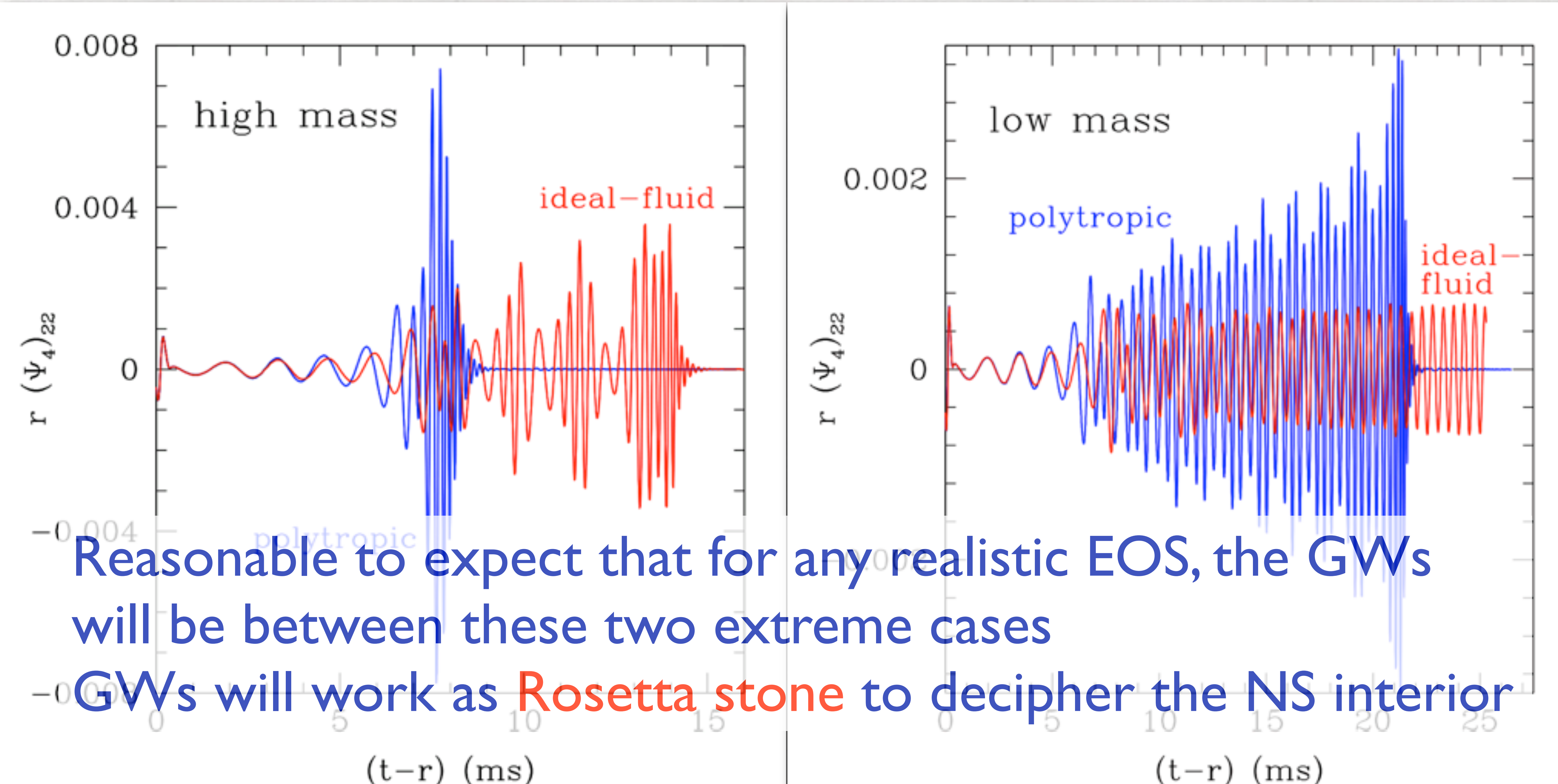
first time the full signal from the formation to a bh has been computed

low-mass binary



development of a bar-deformed NS leads to a long gw signal

Imprint of the EOS: Ideal-fluid vs polytropic



Reasonable to expect that for any realistic EOS, the GWs will be between these two extreme cases

GWs will work as Rosetta stone to decipher the NS interior

After the merger a BH is produced over a timescale comparable with the dynamical one

After the merger a BH is produced over a timescale larger or much larger than the dynamical one

Conclusions

✴ Numerical relativity has made huge progresses over the last few years; problems that were unsolved for decades are now well understood

✴ Using idealized EOSs have reached possibly the most complete description of BNSs from the inspiral, merger, collapse to BH. We can draw this picture with and without MFs

✴ GWs from BNSs are much complex/richer than from BBHs: can be the Rosetta stone to decipher the NS interior

✴ The simulation of BBHs is well understood and most interesting physics is known; higher precision is important

✴ **Much remains to be done to model realistically BNSs, both from a microphysical point of view (EOS, neutrino emission, etc) and a from a macrophysical one (instabilities, etc.)**

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