RISK ANALYSIS OF DIFFERENT U.S. TREASURY BOND PORFOLIOS

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Trabajo de investigación 013/017

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1. Introduction and Objectives of the Study

The recent financial crisis has emphasized the importance of effective risk management and consequently is receiving a high attention by financial institutions. In face of growing volatility in the financial markets, more regulations, research and sophisticated risk management tools are being developed. Moreover, the increasing international cross-relations between markets have enhanced the international authorities to perform further steps in order to control the systematic risk.

Among financial risks, the market risk is considered one of the most important which should be constantly monitored and hedged against. Market risk refers to the risk of losses due to the factors that affect the overall performance of the financial markets, like adverse movements in the interest rates, commodity and equity prices, foreign exchange rates, credit spreads and others which value is set in a public market. Market risk has become even more important due to the new investment trading activities and numerous risky market positions assumed by important big investment companies and banks. Another essential reason to VaR implementation appeared due to large and various losses which took place because of deficiencies in risk management procedures, related to incorrect derivative instrument pricing and assuming excessive investment risk.

In order to manage the market risk, financial institutions employ various numerical models and techniques, between which Value at Risk (VaR) measure, despite of its drawbacks is still commonly used. Over 15 years of its existence between different risk measures, VaR has became the market standard used to quantify the market risk and as a basis for setting regulatory minimum capital standards. The popularity of this analysis is related to its simplicity. For a given portfolio, time horizon and confidence level, value at risk assigns a single value of a loss associated to a given probability. So that basically only two numbers are necessary to determine the VaR value: confidence level and time period. The calculation is based on the portfolio´s changes' assumed probability distribution over the period of the analysis and the expected loss is calculated as a given percentile – lower tail of the distribution function.

Most of the regulatory institutions recommend 99 percent confidence level, as the large losses occur in extreme tails of the return distribution with low initial probability. In case of the time period, financial institutions typically use the daily VaR performing the internal risk statistics and 10-day VaR for establishing current minimum regulatory capital. For calculation of the 10-day VaR from given 1-day VaR value, Basel Committee also recommends a factor by which the 1-day VaR should be multiplied, apart of the standard square root of number of days in the analyzed period.

It has to be mentioned that VaR simplicity can be also interpreted as a threat. The crisis highlighted frailty of many risk models which reduce the extent of market exposure into a small single value. That is why it is important to emphasis and understands the VaR measure limitations.

Hannoun (2010) describes some of the VaR shortcomings. The author first considers the VaR normality assumption as it primary pitfall. As experienced during the crisis, VaR models in some occasions underestimate the tail events and the high correlation between losses in situation of systematic stress. The market prices present skewness and kurtosis – heavy tails in the extreme quintiles of the distribution, and this aspect is not taken into the consideration in the normal probability distribution function. Therefore, the VaR model performs well in normal market conditions but often fails in extreme stress events. As a consequence the Basel Committee in the Basel III framework highlights the importance of stress testing, while the capital buffers are being determined under Pillar 2. Moreover Basel III emphasis the precaution in reliance on banks' internal risk management models and adequate supervision in order to ensure that tail events and systematic risk are properly captured in banks' stress testing and risk modeling.

Between requirements of Pillar 2 of Basel III framework, established in order to improve the systematic risk management, there can be distinguished below elements:

- "Leverage in the banking system as a whole
- Systemic capital charge on SIFIs
- Countercyclical capital charge
- Interconnectedness via OTC derivatives
- Stress testing and risk modeling addressing tail risks
- Concentration risk; and
- Large exposures"[1]

We apply two models to study and analyze the yield curve movements and calculate the Value at Risk (VaR) with those two techniques for the United States Treasury bonds market. In order to perform the study we work with the US daily yield curve data for the period from 3rd of January 2006 to 12th of June 2013 – the period of more than 7 years, what gives a total number of 1864 daily observations. The VaR will be calculated both for the entire observation period and for separated sub-periods: pre-crisis, crisis and current period. In this way we will be able to observe how the vulnerability of the markets affects the bond portfolios risks and the VaR number itself.

The bond portfolios creation will be based on active portfolio strategy including barbell, bullet and laddered portfolios. Those strategies help an investor to balance the bond portfolios in order to achieve desired outcome. For example laddered will enable the bonds investor to set up a bond re-investment

[1] *Hannoun (2010)*

strategy in steps. The barbell resembles a laddered in that bonds are purchased in the short and long end. Finally with the bullet strategy each bond will share the similar maturity date. We consider those techniques as an interesting approach of capturing the portfolios behaviour in face of different market conditions and observing the riskiness of each portfolio class by the distribution of portfolio returns and its VaR. Martellini, Priaulet and Priaulet (2003) give us an insight into the different techniques of active portfolios strategies construction and application of bullet, barbell, laddered and butterfly. Those techniques are also commonly used in the literature to capture different yield curve movements. Su and Knowles (2009) compare the risk of bullet and barbell US Treasury portfolios in terms of Treasury yield factors such as level, slope and curvature and in terms of economic factors such as inflation, business cycle and market volatility. The selection of those portfolio creation techniques will allow us to observe each active portfolio changes and the comparison between different portfolios for each methodology.

In order to analyse the bond portfolios performance we daily capture the VaR for each portfolio, using two alternative approaches: Historical Simulation and model building approach by Variance-Covariance Method. Further we perform the backtesting for each model and asses the daily VaR and number of exceptions occurred. Finally each model is assessed by the Kupiec Likehood Ratio Test.

The Historical Simulation is a methodology used to predict value at risk by constructing the cumulative distributive function of the returns over the given period of time. It is widely applicable as does not require any assumption on the stationary of the distributions of returns, neither on their volatility. Its main drawback lies in the fact that this technique takes into the account the independency and identical distribution of returns (iid) although it is known that the asset returns are clearly independent and show evidences of some patterns as volatility clustering.

The Variance Covariance Method is a model building approach which assumes that the portfolio exposures are linear, that the risk factors are normally distributed and that the correlation between risk factors and portfolio's delta is constant. The VaR is calculated on the basis of this distribution assumption. The portfolio volatility is calculated making use of a covariance matrix and a vector of assets weights. The effect of each of the portfolio's instruments returns changes on the overall portfolio value is calculated from the component's delta with respect to a particular risk factor and the risk factor's volatility. The variance covariance method is limited when applied on the non-linear risks.

The objective of our analysis is to observe the interest rates exposure on the three types of the active portfolios bullet, barbell and laddered and its measurement applying two VaR methodologies. We will observe how the interest rates changes affect our portfolios values, both over the entire observation period and sub periods of pre-crisis, crisis and "actual" period and analyze how the market conditions affect our risk exposure.

This work is divided into eight sections. In the second section we study the literature, in the third – give the introduction into the VaR concept, the three methodologies applied to calculate portfolio VaR and to the portfolio selection techniques. In the fourth section we perform the empirical analysis and give the study results, in the fifth section we conclude and give a final summary, the sixth and seventh sections are annexes and in the last final section the references are specified.

2. Literature Review

Concerning the methodology, risk modelling techniques can be separated in three categories or can be created as a combination of two or more of them. Parametric techniques require the modelling of the entire distribution of returns, the non-parametric methodology is based on a historical simulation and the most recent is the parametric modelling of the tails of the return distribution function. All of the techniques are commonly used and have their pros and cons. The final method selection decision depends on many factors like data availability, market conditions, mathematical/statistical working tools and others.

The market risk and specifically VaR is a very popular topic in the available literature. Many authors also apply VaR methodology to measure fixed bond portfolios risk exposure. One line of investigation focuses on the assumption that portfolio value changes linearly with changes in risk factors. This includes in particular the "delta-gamma" methods studied, between others, by Duffie and Pan (2001), Wilson (1999) and Rouvinez (1997). These methods define the relation between risk factors and portfolio values including linear and quadratic terms. Parametric Approach appears in many articles, often as a basis of further analysis. G. Darbha (2001) applies var-cov method on a portfolio of fixed income securities as a basis of comparison with HS and EVT applied on VaR. Ferreira and Lopez (2005) perform forecast of the covariances between national interest rates and accompanying exchange rates and provide empirical support for the VaR model based on simple covariance matrix forecast and distributional assumption.

Concerning non-parametric approaches, the advantages of Historical Simulation has been fully studied by Jackson, Maude and Perraudin (1997), Mahoney (1996) and Hendricks (1996). Sousa, Esquivel, Gaspar and Real (2012) state that the historical returns cannot be used directly to compute VaR by Historical Simulation because the maturities of interest rates implied by the historical prices are not the relevant maturities at time VaR is computed. The authors adjust bonds historical returns on price basis at the time to maturity relevant for the VaR computation and show that the obtained VaR value agree with the usual market trend of smaller time to maturity being traded with smaller interest rates, carrying smaller risk and consequently smaller VaR. Vlaar (1999) investigates the consequences of dynamics in the term structure of Dutch interest rates for the accurateness of value-at-risk models. Comparing historical simulation, variance-covariance and Monte Carlo simulation methods, the author obtains the best results for combined var-cov and MC method using a term structure model with a normal distribution and GARCH specification. Fiori and Iannotti (2006) develop a value-at-risk measure to assess Italian banks' interest rate risk exposure according to parametric and non-parametric approach. Their backtesting analysis shows that the parametric approach entails some limitations in capturing volatility when interest rates are increasing. In the presence of such asymmetric patterns of volatility, the non-parametric approach performs better for banks that are exposed to an increase in interest rates.

In our investigation we are going to compare parametric variance and covariance method with non-parametric approach – historical simulation, perform backtesting and final validity test. The main contribution of this study is the analysis of long panel of data ranging from 2006 to the 2013 what will allow us to observe the risk exposure over the calm pre-crisis period and in particular during the period of financial crisis in US. We also will be able to observe current tendency in fixed income portfolios risk exposure as our analysis covers the actual period till June 2013. The other contribution of this study is the application of active portfolios strategies to derive portfolios value-at-risk. Constructing different portfolios of bonds will allow us to answer the question on how different portfolio strategies behave in face of changing market conditions and to make a comparison between them.

3. Research Methodology

3.1 VaR General Methodology

The standard benchmark used by the financial institutions to evaluate the market risk is Value at Risk. The portfolio VaR value (V) expresses the maximal potential loss on the financial portfolio during the given period of time (N days) and under the given confidence level 100 * α%, so that V is a function dependent on two parameters: fixed time horizon N and probability 1-α. The obtained result is just one number. In particular, VaR is given by:

$$\mathrm{VaR}_t^{\alpha} = -\sup[r/P_{t-1}[R_t \leq r] \leq \alpha]$$

While calculating VaR with the objective of the capital requirements for market risk, the common parameters' values used are 10 days time horizon and 99 percent confidence level. In this case it is expected that the loss could occur with 1 percent probability over 10-day period. Then the VaR value is multiplied by an add-on factor, which use to range from three to four, depending on the historical number of the exceptions and on regulation, to obtain the required capital level. The Bank for International Settlements has set up the factor value to 3. Many companies calculate the overnight VaR for internal purposes. In accordance to changing market conditions VaR models ought to be evaluated at least once a year. Furthermore the modeling, although based on a 2-week (10 days) estimation (usually based on a historical data), VaR should be calculated on basis of the daily data.

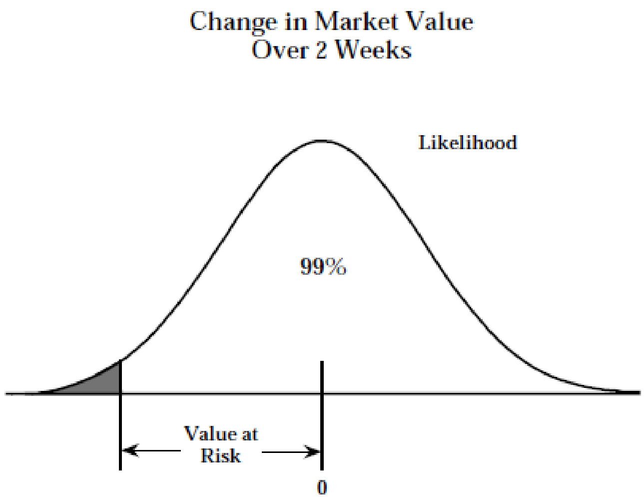

Figure 1. VaR calculation based on the confidence level ∝; from the probability distribution function of the portfolio value changes over the period of 10 days.[2]

As illustrated in Figure 1, the VaR is an expected loss over the period of N days (in this particular case N=10), expressed as the (100 – ∝)th percentile of the probability distribution function of the portfolio value changes. The above figure shows the distribution of the portfolio value returns when the normal distribution is assumed.

What is worth mentioning is that VaR measure under any circumstances should be interpreted as the firm's capital necessary to face the company's risk. It rather should be used as a benchmark for relative comparison, for instance between portfolios, projects, trades, etc. risk. The VaR number is closely related to analyzed time horizon and that is why its correct interpretation should be strongly emphasized. As stated by Duffie and Pan (1997): "Whether the VaR of a firm' portfolio of positions is a relevant measure of the risk of financial distress over a short time period depends in part on the liquidity of the portfolio of positions, and the risk of the adverse extreme net cash outflows, or of severe disruptions in market liquidity. In such adverse scenarios, the firm may suffer costs that include margins on unanticipated short-term financing, opportunity costs on forgone "profitable" trades, forced balance-sheet reductions, and the market-impact costs of initiating trades, forced balance-sheet reductions, and the market-impact costs of initiating trades at highly unfavorable spreads. Whether the net effect actually threatens the ability of the firm to continue to operate profitably depends in part on the form's net capital. Value at risk, coupled with some measure of cash-flow at risk, is relevant in this setting because it measures the extent of

[2] *D. Duffie and J. Pan (1997)*

potential forced reductions of the firm's capital over short time periods, at some confidence level. Clearly however, VaR captures only the aspect of market risk, and is too narrowly defined to be used on its own as a sufficient measure of capital adequacy."

We can distinguish between two generic models for VaR calculation. First is a model of random changes in the price of the underlying instrument, on which we are essentially going to focus in our analysis with interest rates us underlying. The second model type is based on computation of sensitivities of the analyzed financial instruments prices to the underlying prices.

3.2 VaR Computation by Historical Simulation

The oldest and still frequently used method of VaR estimation is the Historical Simulation. The VaR is computed calculating the α-th quantile of the historical returns distribution. The method does not require any assumption about the returns distribution but do assumes the *iid* of returns (identical independent distribution of returns), what is considered one of its drawback, as the real market condition often negatively verify this property. The other advantage of the method over for example Variance-Covariance Method for VaR calculation is that HS makes possible the incorporation of the non-linear positions like derivatives to the analysis. It includes in the implicit way the historical correlations and considers the fat tails aspect. On the other hand the HS methodology gives the same weights to all the observations which also are assumed independent and is limited for complex and huge portfolios. While performing the Historical Simulation we should be aware of the sensitivity of the method in face of the analyzed window size and variability of the data in the sample. The HS varies significantly depending on those to aspects, so we should adjust the analyzed period so that the results actually reflect the real position risk, considering the periods of different market variability.

The first step of the analysis is the identification of the variables that affect the portfolio position (m risk factors), in our case interest rates changes. Then the historical data of those variables should be collected for an assumed period of the analysis (n observations), as shown in the below matrix:

$$\Delta X = \overset{\text{m risk factors}}{\begin{bmatrix} \Delta X_{11} & \dots & \Delta X_{1m} \\ \dots & \Delta X_{ij} & \dots \\ \Delta X_{n1} & \dots & \Delta X_{nm} \end{bmatrix}} \ \text{n observations}$$

We use n historical data, where n is the day of today and ν_i–the value of the variable for the day i. The data for each date of the sample period is a different scenario that can reflect the possible outcome for the future position value we want to estimate. In total we have n-1 scenarios. In the first scenario we assume that the percentage change in all the variables coincide with those of the first day, in the second scenario that the percentage change in all the variables coincide with those of the second day and as so for all the other days: for the i-th scenario we suppose that the market value of the variable for the day of tomorrow (n+1) is:

$$v_{n+1} = v_n \frac{v_i}{v_{i-1}}$$

Then we calculate the daily changes in the value of the portfolio probability distribution, computing the change in the portfolio value between each two subsequent dates. The (α*N)-th worst daily change is the first percentile of the distribution, where α is the confidence level (0.01 for the VaR with assumed probability 99% and 0.05 for 95% VaR) and N is the number of the observations in the analyzed period. If we assume that the selected sample period can reflect correctly the future price movements, there is (1-α)*100% certainty that the obtained VaR is the maximal loose we can suffer. The N-day VaR can be obtained multiplying the daily VaR by square root of N.

3.3 Model Building Approach - VaR Computation by Variance-Covariance Method

The Variance-Covariance Method for VaR calculation is the main alternative to Historical Simulation. It is considered a model building approach, as we assume a model of the distribution of changes in market variables and estimate the model's parameters using the historical data. The approach is based on the Markowitz Portfolio Theory of risk-return portfolio tradeoffs and covariance matrix of the market data returns. The computation of VaR is straightforward if there is assumed a normal probability distribution of daily changes in market variables and if the 1 monetary unit change in portfolio value is lineally dependent on the percentage change in the market variables.

We have n observations in the historical data sample and m risk factors which affect those market variables. This way we can calculate the matrix of the exposition of each variable to each risk factor, as shown below:

$$W_{nxm} = \begin{bmatrix} w_{1,1} & \cdots & w_{1,m} \\ \cdots & \cdots & \cdots \\ w_{n,1} & \cdots & w_{n,m} \end{bmatrix}$$

Each column can be separated to the single vector of size m: W_{1xm}^{Tol}.

Then we calculate the variance for all the risk factors and variance-covariance matrix of historic market variables. The portfolio variance can be computed as:

$$\sigma^2[R_p] = W_{1xm}^{Tot}\,\Sigma_{mxm}\,W_{mx1}^{Tot} = [w_1^{Tot}\,w_2^{Tot}\,\dots w_m^{Tot}]\begin{bmatrix}\sigma_{1,1}^2 & \dots & \sigma_{1,m}^2\\ \dots & \dots & \dots\\ \sigma_{m,1}^2 & \dots & \sigma_{m,m}^2\end{bmatrix}\begin{bmatrix}w_1^{Tot}\\ \dots\\ w_m^{Tot}\end{bmatrix}$$

The Value at Risk can be obtained from the below formula:

$$\mathrm{VaR}_{rel} = z_c\sqrt{W_{1xm}^{Tot}\,\Sigma_{mxm}\,W_{mx1}^{Tot}}\sqrt{T}$$

where:

z_c– quantile of the distribution (for normal distribution $z_{0.95} = 1.645$ and $z_{0.99} = 2.326$)

T – time period for which we want to calculate VaR

3.4. Methodology of Portfolio Selection

The yield curve for treasury securities expresses the relationship between bonds maturities and their yields. Its shape changes constantly over time, that's why this aspect should be considered while constructing bond portfolios. If only parallel shift takes place the change in the yield curve is the same for all maturities, while the nonparallel shift indicates that the change is by different number of basic points within the entire yield curve. Between the nonparallel yield curve changes the most typically observed are the twist in the slope (a flattening and steepening in the yield curve over time) and a change in the humpedness of the yield curve, also called the butterfly shift.

The bonds selection in order to create the investment portfolio can be performed applying many different strategies. The selection of the most appropriate strategy involves picking one that is consistent with the investment objectives. There can be distinguished two basic types of the strategies: active strategies (bullet, barbell, butterfly, laddered) and passive (buy and hold and indexing). The active yield curve strategy consider portfolios capitalization on expected changes in the shape of the yield curve as a consequence of changes in future interest rates, future interest rates' volatilities or changes in future yield spreads. The objective of the active fixed income portfolio manager is to make that his/her portfolio performs superior to the benchmark index. We are going to work with the active yield curve strategy, selecting bond portfolios according to bullet, barbell and laddered strategies.

In the process of the portfolio creation, first, an investor selects a fixed income portfolio strategy according to his/her investment objectives. The decision of the active strategy selection depends on the expectations about the factors that influence the bonds performance. Then the specific bonds types should be selected to be included in the portfolio. To perform it the investor should study the evolution of the individual bonds, examining their characteristics in terms of maturity, coupon quantity, credit rating, embedded options, and those factors influence on bonds performance expectations during the entire investment period. It should be also investigated if there are any mispriced positions (undervalued or overvalued). There are some indicators that portfolio managers follow to identify misevaluations. For example the fixed income instrument can be interpreted as undervalued when its yield is higher than of similar issues with the same rating or when its price is expected to raise (yield is expected to decline), as the credit analysis says that its rating will improve. If any of those are identified, the investment will be positioned to capitalize on the positive future forecast.

Martellini, Priaulet and Priaulet (2003) distinguish between two kinds of active strategies:

1. Market timing – trading on interest rate predictions, timing bets based on:
    - no change in the yield curve
    - interest rate level
    - slope and curvature movement of the yield curve
2. Bond picking – trading on market inefficiencies:
    - the bond relative value analysis (within a given market)
    - spread and convergence trades (within different markets)

Those methodologies can be classified as scenario analysis, and for each scenario the break-even point (when losses start to occur) and the risk specifications can be defined.

We are going to concentrate on the trading on the interest rate predictions – the specific changes in the yield curve (slope and curvature movement of the yield curve), as the yield curve can be affected by many other movements that only parallel shifts. Those typically considered are level, slope and curvature movement of the yield curve. The standard strategies inside this category are barbell, bullet and laddered and between those more complicated we can distinguish the butterfly strategy.

A bullet portfolio is a portfolio that focuses investment on bonds which maturities are highly concentrated in the same point of the yield curve. The contrary strategy is performed in case of a barbell portfolio, where selected bonds' maturities are concentrated at extreme terms of the yield curve. A barbell portfolio is more convex than a bullet portfolio with the same duration. In case of a ladder portfolio, an investor selects a mixture of bonds from group of bonds with different maturities, so that the portfolio consists of instruments with diversified maturities. There can be constructed many different ladder portfolios, depending on the selected maturities.

4. Empirical Analysis

4.1 Data and Portfolios Description

The data, object of our analysis is the U.S Treasury yield data including series of zero coupon yields, par yields, and the parameters of Svensson Model (1994) – extended Nelson and Siegel Model (1987) (beta 0, beta 1, beta 2, beta 3, tau 1, tau2). The data was obtained from the official webpage of The U.S Federal Reserve[3], where the research data from 1961 is available.

The article of Gürkaynak, Sack, and Wright (2006) gave us an insight into the estimation of the U.S. Treasury yield curve using a simple and parsimonious approach. The authors work with the same data set, as we do, extending the estimated yield curve back to 1961 on a daily basis. They provide us with an interesting approach of capturing the general shape of the yield curve while smoothing through the idiosyncratic variation in the yields on individual securities. Their results are useful for understanding the general macroeconomic and other factors that have broad effects on the shape of the yield curve.

The total of our data are 1864 observations from 2006-01-03 till the last available data 2013-06-12 – what gives almost 7.5 years of analysis, which covers the period of pre-crisis, crisis and post financial crisis in U.S. The aim of the separation between free periods is the analysis of the risk present in the market and how this affects the interest rates levels and consequently bond prices and portfolios. First we analyze the market risk by different approaches of the VaR analysis for the whole period, then we separate the data set in sub-periods and compare the obtained result between different methodologies and different financial conditions.

The below sub-periods are analyzed:

- Pre-crisis period: 3rd of January 2006 – 15th of September 2008.
- Peak crisis period: 16th of September 2008 – 15th of September 2010
- "Actual" period: 16th of September 2010 – 12th of June 2013

The first period "pre-crisis period" starts from the first data of our observation period – 3rd of January 2006 and continues until the date we assume to be the moment that importantly marked the beginning of the crisis – 15th of September 2008. Although the indications of the crisis was already observed in 2007, with objective of our analysis we assumes that the moment when Lehman Brothers Holdings incorporates files for bankruptcy protection will be the event that separates the two periods.

[3] *http://www.federalreserve.gov/econresdata/researchdata.htm*

The second period "peak crisis" is computed since 15th of September 2008 to 15th of September 2010. Although and unfortunately September 2010 is not the end of the crisis and its effects will be observed long after, we expect this period to be the most volatile and the one that presents the most extreme values while calculating VaR, compared to two other periods. The third and last period we call the "actual period", includes the data which ranks from 15th of September 2010 until the last observation in our data set – 12th of June 2013. The trends in this period are assumed to be similar to the actual market conditions and potentially will sign the potential future trends. We consider the analyzed period to be an interesting period of many changes in the financial markets and financial and most of all risk measurement aspects capturing.

The data consists of 1,864 daily observations of US government bond zero-coupon yields at tenors of 3 and 6 months and from 1 to 30 years. Table 1 gives a summary description of the 6 months and 1, 2, 3, 4, 5, 7, 10, 15, 20, 25 and 30-year yields. The descriptive statistics are mean, standard deviation, minimum and maximum for each maturity and each period. First we analyze the entire period, and then we distinguish between each of the sub-periods. As it can be observed, in the first period the levels of the mean yields are comparable independently on the maturity. With the course of time the observed difference is higher until obtaining the actual values with significant difference in yields for each maturity period. Concerning the standard deviation it regularly increases over time and is significantly higher for the longer period yields.

Table 1. Zero coupon interest rates from the US Treasury bond market

Descriptive statistics (in percentage points)

1. The entire observation period (3rd of January 2006 – 12th of June 2013)

	6 months	1 year	2 years	3 years	4 years	5 years	7years	10 years	15 years	20 years	25 years	30 years
Mean	1.676	1.687	1.815	2.026	2.273	2.529	3.008	3.568	4.079	4.244	4.234	4.145
St. Dev.	1.991	1.921	1.777	1.639	1.516	1.411	1.249	1.085	0.905	0.792	0.728	0.698
Min	0.104	0.097	0.158	0.303	0.431	0.589	0.942	1.461	2.102	2.439	2.561	2.354
Max	5.350	5.303	5.253	5.190	5.146	5.128	5.167	5.295	5.436	5.453	5.322	5.099

2. Pre-crisis period: 3rd of January 2006 – 15th of September 2008

	6 months	1 year	2 years	3 years	4 years	5 years	7years	10 years	15 years	20 years	25 years	30 years
Mean	4.080	4.008	3.955	3.977	4.043	4.132	4.333	4.605	4.866	4.909	4.812	4.639
St. Dev.	1.291	1.237	1.116	0.985	0.859	0.744	0.556	0.374	0.252	0.222	0.213	0.219
Min	1.338	1.285	1.352	1.570	1.872	2.212	2.893	3.662	4.279	4.355	4.268	3.982
Max	5.350	5.303	5.253	5.190	5.146	5.128	5.167	5.295	5.436	5.453	5.322	5.099

3. Peak crisis period: 16th of September 2008 – 15th of September 2010

	6 months	1 year	2 years	3 years	4 years	5 years	7years	10 years	15 years	20 years	25 years	30 years
Mean	0.422	0.549	0.914	1.351	1.803	2.235	2.976	3.736	4.286	4.368	4.281	4.151
St. Dev.	0.382	0.308	0.285	0.324	0.364	0.395	0.433	0.447	0.420	0.422	0.489	0.588
Min	0.115	0.268	0.377	0.633	0.990	1.327	1.971	2.663	3.104	2.988	2.681	2.354
Max	1.913	1.969	2.177	2.438	2.743	3.082	3.885	4.836	5.224	5.056	5.037	5.011

4. "Actual" period: 16th of September 2010 – 12th of June 2013

	6 months	1 year	2 years	3 years	4 years	5 years	7years	10 years	15 years	20 years	25 years	30 years
Mean	0.209	0.216	0.351	0.582	0.859	1.151	1.714	2.413	3.146	3.492	3.624	3.650
St. Dev.	0.047	0.055	0.159	0.286	0.402	0.497	0.625	0.709	0.725	0.717	0.721	0.735
Min	0.104	0.097	0.158	0.303	0.431	0.589	0.942	1.461	2.102	2.439	2.561	2.561
Max	0.341	0.381	0.860	1.410	1.937	2.408	3.180	3.966	4.665	4.957	5.056	5.036

Source: FRB dataset

More precise view of the trends in the yields gives the figure 2, 3 and 4. Each graph presents the 1, 4, 5, 7, 10-year yield movements. We selected those specific yields as further in our analysis those are going to be the yields which affect our portfolios of bonds. The figure 2 runs over the period of pre-crisis (3rd of January 2006 – 15th of September 2008), figure 3 – peak crisis (16th of September 2008 – 15th of September 2010) and figure 4 –"actual" period (16th of September 2010 – 12th of June 2013).

We observe that in the beginning of the first period (pre-crisis period) the interest rates have similar values and change in the same pattern independently on the term of each spot rate. This tendency is visible over the period of one and a half year, from the begging of our data – January 2006 till the middle of 2007, when the difference between spot rates stars to be more and more significant. Short term interest rates fall down and keep much lower values than long term rates. In the subsequent periods the similar tendency is maintained. The one year spot rates remain low and less affected by the market conditions than the 4, 5, 7, 10-year interest rates. In the end of 2008 we can observe the broad decrease in the interest rates level which is compensated slowly over time. In the end of 2010 spot rates continue to increase to drop dawn again in the end of 2011. During the remaining "actual" period the deviation are lower and the rates tend to increase in the end of our observation period – 2013.

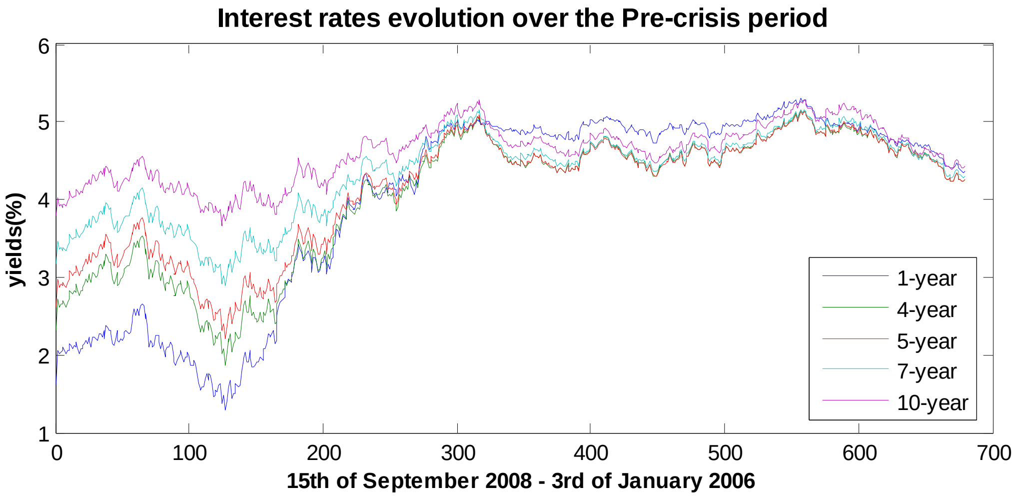

Figure 2. 1, 4, 5, 7, 10-year zero coupon interest rate changes during the pre-crisis period (3rd of January 2006 – 15th of September 2008)

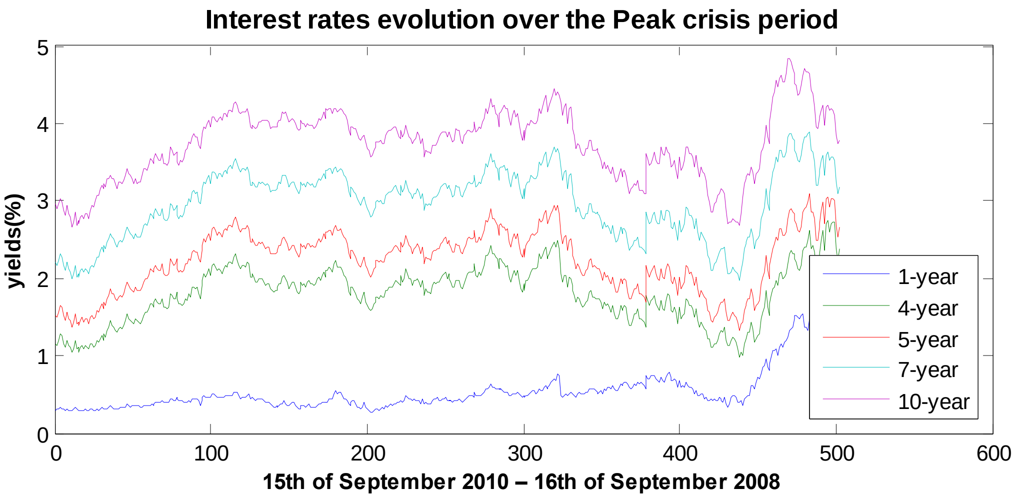

Figure 3. 1, 4, 5, 7, 10-year zero coupon interest rate changes during Peak-crisis period (16th of September 2008 – 15th of September 2010)

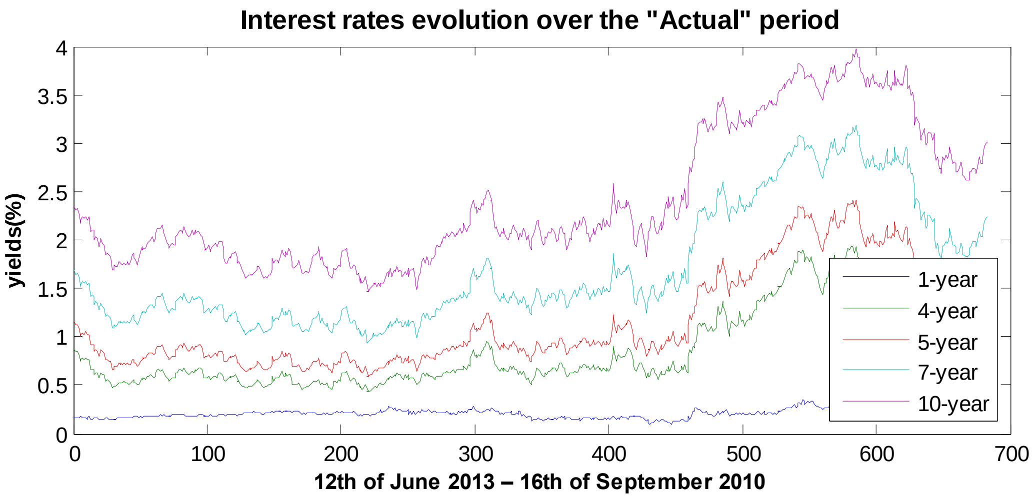

Figure 4. 1, 4, 5, 7, 10-year zero coupon interest rate changes during "actual" period (16th of September 2010 – 12th of June 2013)

The first step of our analysis is to derive the theoretical bond prices on basis of the zero coupon and par yields from our panel of data. Our calculations are based on the assumption that the U.S. Treasury authority quarterly issues new bonds for each term to maturity. These just-issued bonds are by far the most liquid assets in the secondary market. We are concerned about liquidity, thus our portfolios always include the on-the-run bonds of each maturity. To do so we quarterly rebalance the portfolios. At the beginning of each quarter, we purchase the new on-the-run bond by selling the old on-the-run or new off-the-run bond that we held in the portfolio until this time. For instance, a new 5-year bond is issued on the first observation date in our sample (3rd of January 2006). The bond pays semiannual coupon and a $1000 principal payment at maturity (3rd of January 2011). The annual coupon amount is assumed to be the average 5-year spot rate during the quarter. To price the bond we use the theoretical price obtained by discounting each semiannual coupon payment with the corresponding zero coupon interest rate provided by the yield curve at 3rd January 2006. At the 3rd of April 2006, the bond has a 4.76 remaining maturity and is sold at the market price. The just-issued bond with maturity on the 3rd April 2011 is immediately purchased. The same procedure is done for every new issued bond. Coupons are paid semiannually, the day convention is Actual/365 and the principal amount is $1000.

The zero coupon interest rates are calculate from the Svensson Model, applying given parameters beta 0, beta 1, beta 2, beta 3, tau 1, tau2 for each day of the observation period. The Svensson Yield Curve Model is an expansion of the Nielson and Siegel Model and is used to explain the Treasury yield curve changes due to the 3 factors: level, slope and curvature. Comparing to the original form, the Svensson Model gives more flexibility in yield curve estimation, as it allows for more complex shapes of the yield curves.

The Svensson Model of the continuously compounded zero-coupon rate has the following form:

$$R^c(0,\theta) = \beta_0 + \beta_1 \left[\frac{1-\exp\left(-\frac{\theta}{\tau_1}\right)}{\frac{\theta}{\tau_1}}\right] + \beta_2 \left[\frac{1-\exp\left(-\frac{\theta}{\tau_1}\right)}{\frac{\theta}{\tau_1}} - \exp\left(-\frac{\theta}{\tau_1}\right)\right] + \beta_3 \left[\frac{1-\exp\left(-\frac{\theta}{\tau_2}\right)}{\frac{\theta}{\tau_2}} - \exp\left(-\frac{\theta}{\tau_2}\right)\right]$$

$R^C(0,\theta)$ – the continuously compounded zero-coupon rate at time zero with maturity θ,

β_0 – the limit of $R^C(0,\theta)$ as θ goes to infinity (long term interest rate),

β_1 – the limit of $R^C(0,\theta) - \beta_0$ as θ goes to zero (long-to-short term spread),

β_2, β_3 – curvature parameters,

τ_1, τ_2 – scale parameters that measure the rate at which the short-term and medium-term components decay to zero.

Figure 5 illustrates the beta 0, beta 1, beta 2, beta 3, tau 1, tau2 parameters values over the observation period.

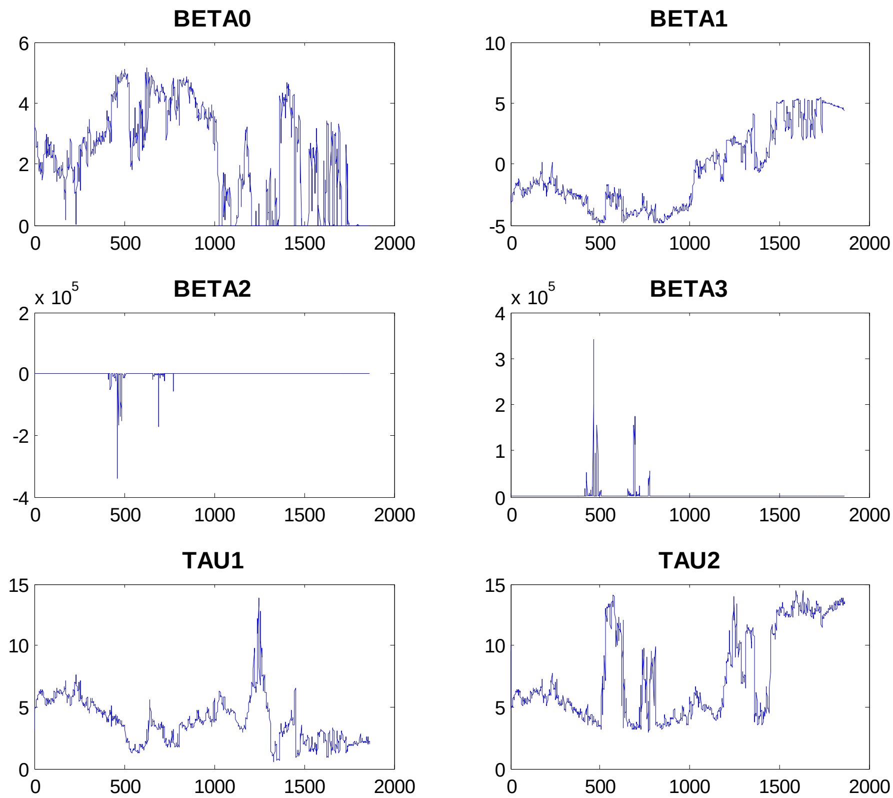

Figure 5. Svensson parameters' levels during the observation period

(3rd of January 2006 – 12th of June 2013)

With objective of our study we convert the continuously compounded zero coupon rates to discrete compounded rates applying the below conversion formula:

$$i = 100 * \left(\exp\left(\frac{R}{100} * T\right)^{1/T} - 1 \right)$$

where:

R – continuously compounded interest rate,

i – equivalent discrete compounded interest rate.

On the basis of the theoretical bond prices we construct our portfolios applying the active portfolio strategy. Thus we create three portfolios: bullet, barbell and laddered. In order to make them comparable we select the bonds in the way that portfolios duration are similar. Portfolio bullet concentrates investment in 5-year bond, bullet in 1 and 10-year bonds and laddered in 1, 4, 7, 10-year bonds. As so our investment horizon is on average a five year perspective. Portfolios are equiponderated – the weights of each bond type in the portfolio are the same and sum up one in total.

We expect the bullet portfolio to be the most risky, as it focuses investment on bonds with just one maturity, so that the risk exposure is higher - any changes in the yield curve will affect directly the portfolio value, which will not be compensated by other positions in the portfolio. In the contrary, the barbell portfolio will be affected both by the 1-year and 10-year bond prices changes. As so, this portfolio is more convex than bullet. Laddered portfolio is the most diversified as it contains four types of bonds. The expected portfolio return is lower comparing to bullet, but on the other hand this portfolio will be less affected by the negative returns.

The initial value of those three portfolios on the first day of the observation period (3rd of January 2006) is 1 million. Given the price of each bond on this day and assuming the perfect division of the bonds, n bonds are purchased (n=notional/bond price). Those positions are maintained fixed in the portfolio during the entire quarter. What changes is the portfolio value, depending on the portfolio bond prices on each day. On the first day of the next quarter, given the portfolio value on this day and the new-issued bond price, the n' new bonds are purchased and consequently for every next quarter. Figure 6 illustrates how portfolios value increase over time from initial 1 million to around 1,5 million for each portfolio strategy after 7,5 years.

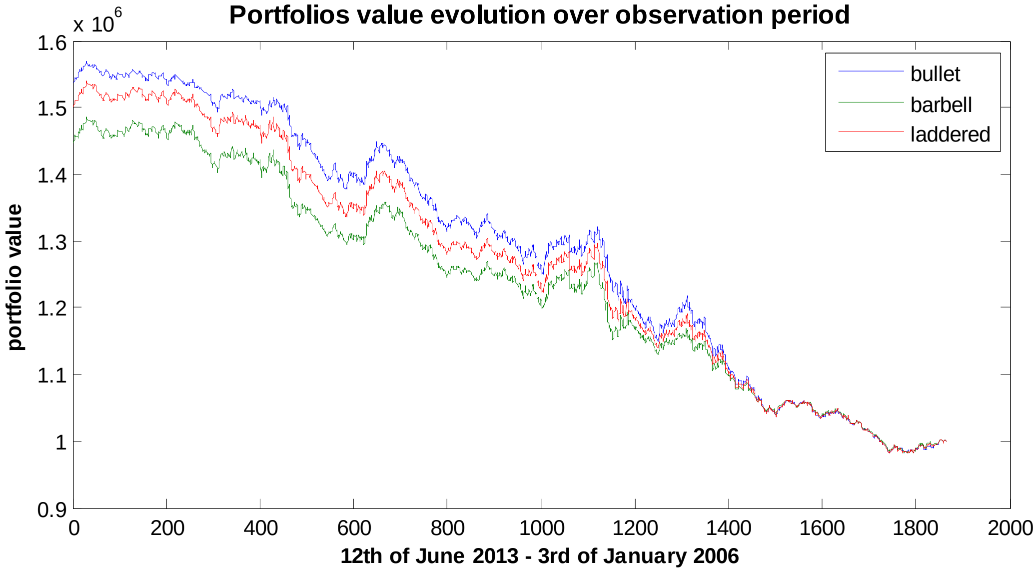

Figure 6. Bullet, barbell and laddered portfolio value evolution over the observation period (3rd of January 2006 – 12th of June 2013)

4.2 Analysis of the portfolios effective durations

The bond duration indicates how long, on average, the holder of the bond has to wait to receive cash payments, but also is a measure of a portfolio's exposure to the yield curve movements. The duration of the zero-coupon bond is iqual its time to maturity, while the coupon paying bond duration is less than its time to maturity, as some of the bond cash flows are received before bond maturity.

The bond duration can be calculate from the below formula:[4]

$$D = \sum_{i=1}^{n} t_i \left(\frac{c_i e^{-y t_i}}{B} \right) \qquad i = 1, \dots n$$

where:

c_i – i-th cash flow,

y – continuously compounded interest rate,

B – bond price,

t_i – time of the i-th cash flow payment.

[4] *Hull (2007)*

The relation bond – interest rate can be expressed as:

$$\Delta B = \frac{dB}{dy}\,\Delta y = \; -\Delta y \sum_{i=1}^{n} c_i t_i e^{-y t_i} = -\,BD\Delta y$$

Making use of this property, we calculate the bonds durations on each observation day, previously deriving internal rates of return. On basis of this durations we can calculate the entire portfolio duration, which is equal the sum of portfolio bonds durations multiplied by it weights:

$$D_P = w_1 * D_1 + \cdots + w_n * D_n$$

Table 2 illustrates the bullet, barbell and laddered portfolios durations for selected dates – dates from the beginning of pre-crisis, crisis and "actual" periods. Portfolios are equiponderated, so that we calculate the portfolios' durations multiplying single bonds durations by the same weights, what in total gives a sum of one. We selected the bonds to be included in each portfolio so that the bullet, barbell and laddered portfolios' durations are similar. We distinguish 5 key rate maturities: 1-year, 4-year, 5-year, 7-year and 10-year. In case of the hypothetical 1 percentage increase in the 5-year key rate, the bullet portfolio value will decrease by around 4.7%. Similar decline in the barbell value can be observed when 1-year key rate increase around 0.92% and 10-year key rate by 8.24%. Laddered portfolio value will fall down by 1% when 1-year key rate will increase by 0.92%, 4-year key rate by 3.75%, 7-year key rate by 6.18% and 10-year key rate by 8.24%.

15th of September 2008			
Key Rate Maturity	**Bullet**	**Barbell**	**Laddered**
1-year	0	0.92	0.92
4-year	0	0	3.73
5-year	4.59	0	0
7-year	0	0	6.18
10-year	0	8.24	8.24
Effective Portfolio Duration	**4.59**	**4.63**	**4.80**

15th of September 2010			
Key Rate Maturity	**Bullet**	**Barbell**	**Laddered**
1-year	0	0.86	0.86
4-year	0	0	3.79
5-year	4.71	0	0
7-year	0	0	6.42
10-year	0	8.67	8.67
Effective Portfolio Duration	**4.71**	**4.77**	**4.94**

12th of June 2013			
Key Rate Maturity	**Bullet**	**Barbell**	**Laddered**
1-year	0	0.83	0.83
4-year	0	0	3.79
5-year	4.74	0	0
7-year	0	0	6.54
10-year	0	8.97	8.97
Effective Portfolio Duration	**4.74**	**4.85**	**5.00**

Table 2. Bullet, barbell and laddered portfolios effective durations for 3 selected days beginning of each pre-crisis, crisis and "actual" period.

4.3 Historical Simulation VaR and its Backtesting

The first methodology we use to compute value at risk is the historical simulation. The simulation is performed for portfolio bullet, barbell and laddered. Although our data is a set of bond prices which pay semiannual coupons, for the purpose of our empirical analysis we assume that the bonds are zero-coupon bonds. Otherwise we would have to consider as many risk factors as cash flows of each bond (the interest rates corresponding to each term when coupon or/and nominal is paid).

Assuming the zero-coupon bonds, each portfolio has as many risk factors as bond types in the portfolio:
portfolio bullet: bond with 5-year maturity/5-year spot rate
portfolio barbell: bonds with 1 and 10-year maturities/1 and 10-year spot rates
portfolio laddered: bonds with 1, 4, 7 and 10-year maturities/1, 4, 7 and 10-year spot rates.

We work with n (i=1 to1864) daily historical data of bonds monetary values in USD (bond price x number of bonds of this type in the portfolio in each day) and total daily portfolio values. With the final objective of obtaining the probability distribution of portfolio daily returns, we calculate the scenarios of the future positions values on basis of our 1864 days of historical data. The last day of the observation period – 12th of June 2013 is considered the actual date of "today", respect to which we calculate the sceneries of future bond values for every bond in three portfolios. V_i is the value of the bond on i-th day. In the first scenario it is supposed that the changes in all the variables coincide with those of the first day (as of 12th of June 2013). In the second scenario it is supposed that the percentage changes are the same as of the second day of the observation period and consequently in the i-th scenario we assume that the bond market value in the next day will be:

$$v_{n+1} = v_n \frac{v_i}{v_{i-1}}$$

V_{n+1} – bond value on the n+1 scenario day,
V_n– bond value on the last observation day ("actual day") – 12th of June 2013
V_i– bond value on the i-th day of the historical observation period
V_{i-1} – bond value on the i-1th day of the historical observation period

For each scenario we calculate the daily discrete return of the portfolio value. This way we define the probability distribution of portfolio daily returns. Value at risk is obtained as a percentile of the distribution of returns with values arranged in decreasing order. The confidence levels we apply are c_1=0.95 (p_1=0.05) and c_2=0.99 (p_2=0.01).

First we calculate VaR for each portfolio for the entire observation period of seven and a half year and then separating between sub-periods of pre-crisis, crisis and "actual" period.

Our results of VaR calculation for the entire period are presented in figures 7 to 9 respectively for each portfolio and confidence level. Figure 7 illustrates the histogram of returns and VaR value for bullet portfolio at confidence level 0.95 and 0.99. Figures 8 the same for bullet portfolio and figure 9 for laddered. It can be observed that the expected maximal loss is the highest for bullet portfolio and our assumptions that this portfolio strategy implies the highest risk is confirmed. For this portfolio we obtain the VaR value of $17 213 at 0.99 confidence level. This means that the maximal expected loss on bullet portfolio which invests $1 000 000 in 5-year bonds in time horizon of 7,5-year is $17 213. The analogues VaR values at 0.99 confidence level for other portfolios is $17 029 (laddered) and $14 834 (barbell). The last portfolio – barbell presents significantly lower potential loss over the observed period. Considering the 0.95% confidence level, the similar tendency is maintained, with bullet portfolio with leading VaR value of $10 062 and the barbell can be considered the most secured investment with maximal potential loss of $9 013.

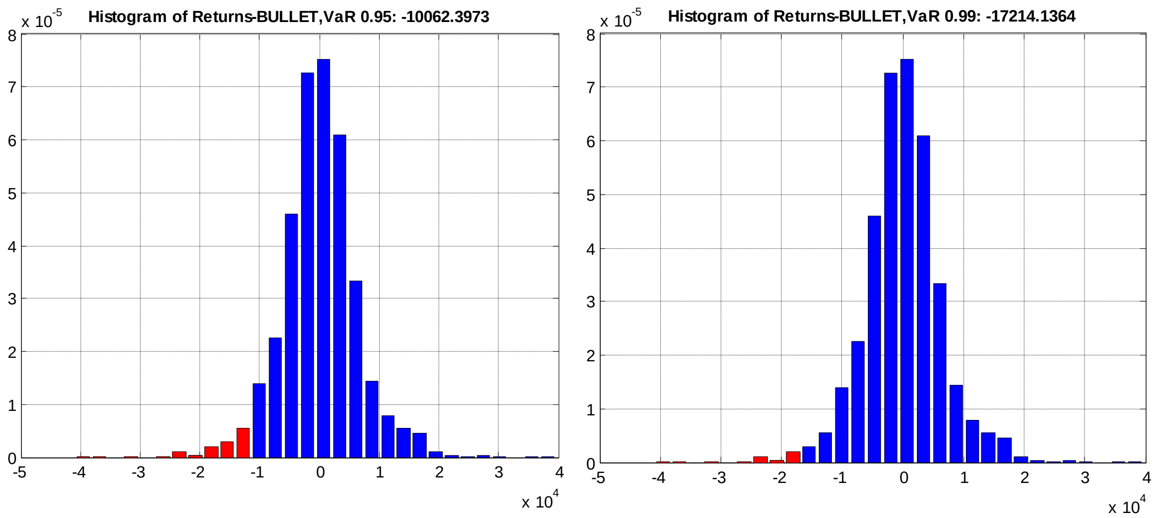

Figure 7. Histograms of returns and VaR value for bullet portfolio at 0.95 and 0.99 confidence levels

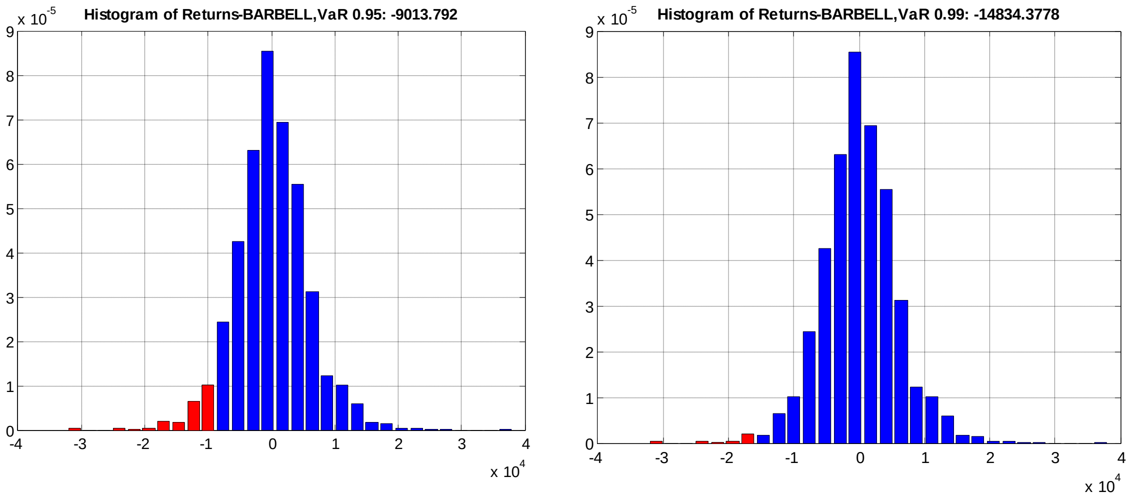

Figure 8. Histograms of returns and VaR value for barbell portfolio at 0.95 and 0.99 confidence levels

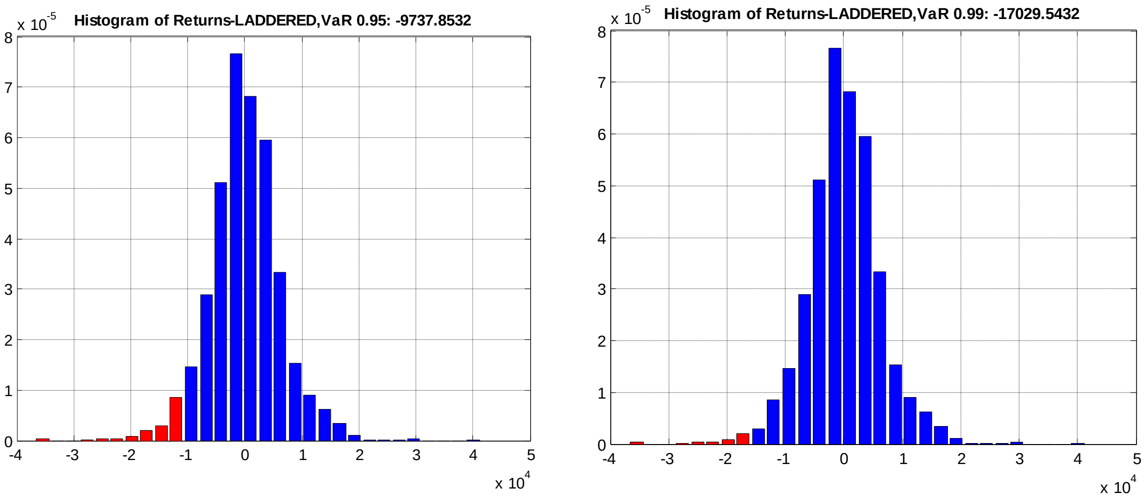

Figure 9. Histograms of returns and VaR value for laddered portfolio at 0.95 and 0.99 confidence levels

Histograms of returns and VaR value distribution for each portfolio for each sub-period (pre-crisis, crisis and "actual" period) are illustrated in figures 16 to 24, which can be find in Annex 1. Figures 16 to 18 correspond to bullet portfolio, figures 19 to 21 to barbell and figures 22 to 24 to laddered.

Table 3 summarizes the results. Comparing between portfolios, at 0.99 confidence level bullet portfolio always remains the one with the highest maximal expected loss. Value of VaR for laddered portfolio has close values to those obtained for bullet, still lower at 0.99 confidence lever, but higher in few periods at 0.95 cl. VaR value for barbell in general remains lower than VaR for other portfolios.

In pre-crisis period we can observe important differences between portfolios maximal expected losses, especially in case of lower tails of the distribution of returns – at 0.99 confidence level. While in the crisis period all three portfolios are similarly affected by negative returns and obtain similar levels of VaR. The expected maximal losses are significantly higher during the crisis and for barbell and laddered still remains higher in the actual period than initially in the first pre-crisis period.

Historic Simulation VaR						
portfolio	BULLET		BARBELL		LADDERED	
c	cl = 0.95	cl= 0.99	cl = 0.95	cl= 0.99	cl = 0.95	cl= 0.99
Total period	-10 062	-17 214	-9 014	-14 834	-9 738	-17 030
P. pre-crisis	-7 743	-13 769	-6 580	-10 552	-7 230	-13 522
P. crisis	-11 319	-17 857	-10 659	-15 835	-11 343	-17 326
P. "actual"	-7 248	-13 706	-8 155	-13 064	-8 487	-13 631

Table 3. VaR value for bullet, barbell and laddered portfolios at two confidence levels and for the entire observation period, pre-crisis, crisis and actual period.

Concluding, bullet portfolio is the most risky strategy, as it concentrates in its investment on just one risk factor – 5-year spot rate. Laddered is the most diversified portfolio affected by 4 risk factors (1-year, 4-year, 7-year and 10-year interest rates). Although its diversification effect is visible by lower looses comparing to bullet portfolio, the barbell portfolio is prevailing in the lowest portfolio losses. Barbell strategy is affected by two extreme interest rates – 1-year and 10-year interest rates. The changes in one risk factor can be compensated by another. During the period of crisis the losses on each portfolio increases significantly and diversification effect almost disappears. All three portfolios suffer high negative returns. Concerning the actual period the expected losses are still higher than before the crisis and surprisingly at 0.95 confidence level bullet is the one with the lowest negative returns. At 0.99 confidence level the losses are similar for all portfolios.

In order to verify the model quality and its precision we perform backtesting. The backtesting compares the historical losses with those predicted by VaR modeling. The standard backtesting calculates the percentage of total exceptions. Exceptions occur when the portfolio loss exceeds the expected loss – VaR value. Basel II requires the backtesting to be done on basis of the last 12 months of daily data (250 days) as a base of the supervision standards. The number of the exceptions should not exceed the expected confidence level, for 0.95 VaR – 5% and for 0.99 VaR – 1%. We calculate VaR for each date of the observation period by rolling windows of 500 observations – our backtesting period. As so we loss the 500 first observations and our VaR vector has the size of n-s=1364 (n=1864 total observations, s=500 the rolling window size). We derive the number of exceptions comparing the VaR value on day i with the portfolio loss from the day i + 1. Exception occurs when the loss on the portfolio exceeds VaR value. The total number of exceptions is a sum of the exceptions from the observation period.

Figures 10 to 12 illustrate portfolios return distribution and daily VaR values over the period from 3rd of January 2006 to 6th of June 2011 – 1364 days. Figure 10 shows portfolio bullet returns distribution and VaR value over time, figure 11 barbell portfolio return distribution and VaR values and figure 12 laddered portfolio return distribution and VaR values. All portfolios show similar pattern of the distribution of profit and losses on the portfolio. Starting from the right side (trading day 1364 – 3rd of January 2006) we can see the calm period when no exceptions occur, until around trading day 1000 (June 2007) when portfolio returns starts to be much more volatile. This period coincides with the peak of the financial crisis in USA. In our graphs the highest deviations are observed between trading days 400 and 1000: June 2007 to November 2009. In this period the number of exceptions is very high and VaR value is far too low to cover the losses. In the last period – trading days 0 to 400 (June 2011 to November 2009) the deviations decreases but still various exceptions are observed.

Table 4. summarizes the number of exceptions for each portfolio and two confidence levels. The highest number of exceptions is observed for bullet portfolio, both at 0.95 and 0.99 confidence levels. The lowest number of exceptions occurs for portfolio barbell and this way our previous observations are confirmed. Bullet portfolio value is affected by bond value of 5-year maturity which varies depending on the level of 5-year interest rate. If middle term interest rates increase, barbell value decreases proportionally. Portfolio barbell returns vary depending on the 1-year and 10-year bond values and consequently on the 1-year and 10-year interest rates. If short term rates increase, bond value with 1-year maturity decline and negatively affects portfolio return, but the total portfolio return is still compensated by the value of the bonds with long term 10-year maturity. That is why the bullet portfolio presents much higher volatility of returns and estimated loss on the portfolio is not always adequate in face of vulnerable market conditions. Who invests in bullet portfolio can expect higher returns, but also can suffer higher losses, while investing in barbell is more secure capital allocation.

Backtesting HS VaR - number of exceptions						
portfolio	**BULLET**		**BARBELL**		**LADDERED**	
c	cl = 0.95	cl= 0.99	cl = 0.95	cl= 0.99	cl = 0.95	cl= 0.99
Total period (s = 500)	87	18	58	11	72	16

Table 4. Number of exceptions for bullet, barbell and laddered portfolio at VaR 0.95 and VaR 0.99, resulting from backtesting with rolling window of 500 observations.

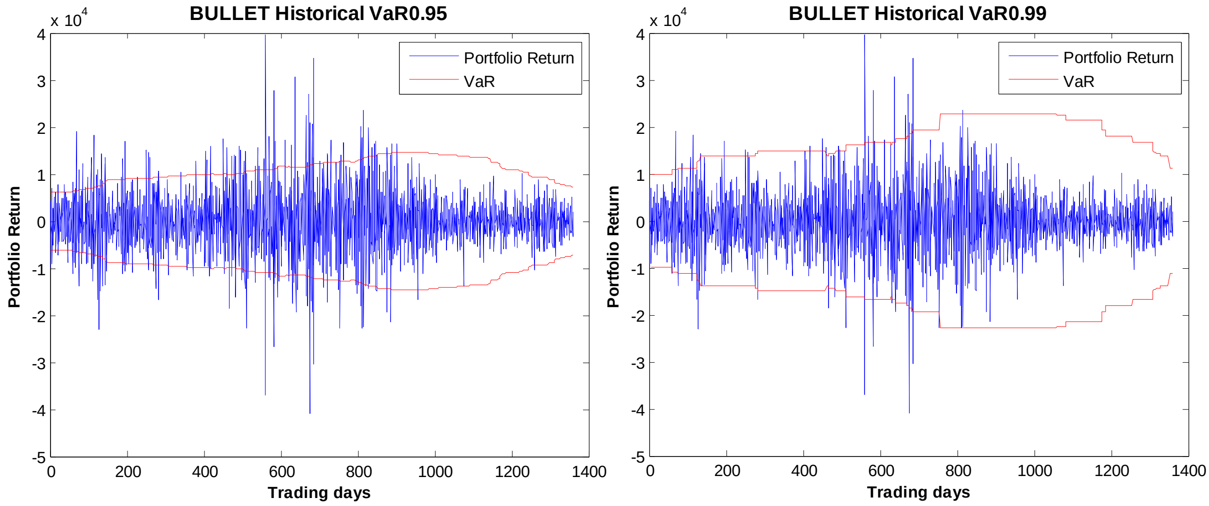

Figure 10. Portfolio returns distribution and VaR values at 0.95 and 0.99 cl. for bullet over the observation period (6^{th} of June 2011 - 3^{rd} of January 2006)

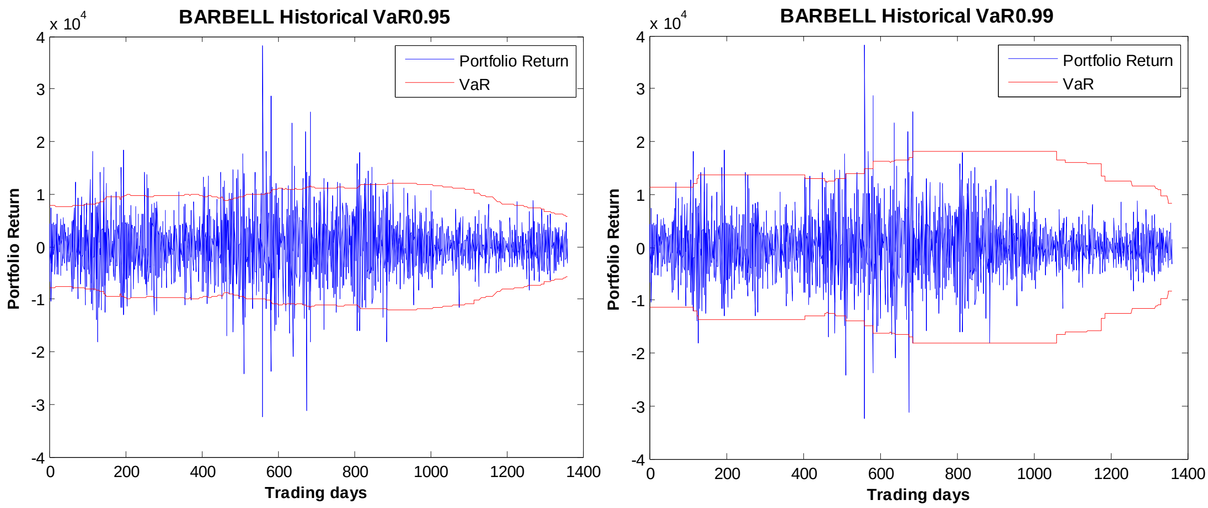

Figure 11. Portfolio returns distribution and VaR values at 0.95 and 0.99 cl for barbell over observation period (6^{th} of June 2011 - 3^{rd} of January 2006)

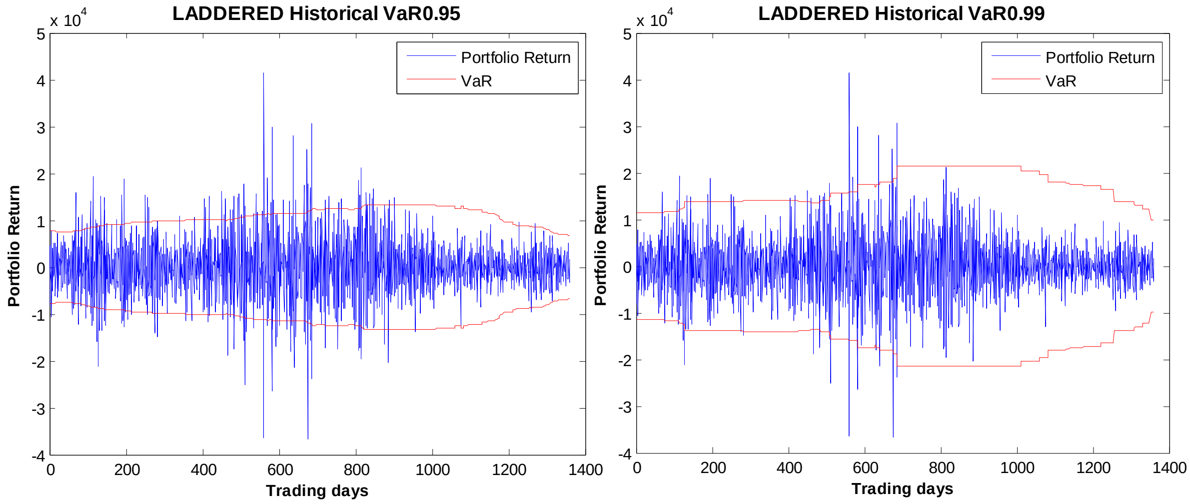

Figure 12. Portfolio returns distribution and VaR values at 0.95 and 0.99 cl. for laddered over observation period (6th of June 2011 - 3rd of January 2006)

4.4 VaR Computation by Variance-Covariance Method and its Backtesting

The second method we use to estimate the portfolio VaR, as an alternative to Historical Simulation is Variance-Covariance Approach. It is a model building approach that requires the supposition of the joint distribution of market variables. Historical data is used to estimate parameters of the model. The approach is based on the Markowitz Portfolio Theory of risk-return tradeoff in the portfolio management.

As in the previous model we work with n (i=1 to1864) daily historical data of bonds monetary values in USD (bond price x number of bonds of this type in the portfolio on each day) and total daily portfolio values. We assume that the portfolios daily returns are independent and normally distributed. For each portfolio we obtain the matrix of risk exposition. Bullet contains 5-year bond and is exposed on the risk of its price fluctuations. Consequently bullet is exposed on the risk of 1 and 10-year bonds price changes over the observation period and laddered is affected by 1, 4, 5 and 10-year bonds price deviations. Bonds are expressed in monetary values (number of bonds in the portfolio x its values).

We calculate the matrix of variances and covariance from given historical bond prices for each observation day. The total variance of the portfolio is derived as variance-covariance matrix multiplied by square of position value in the portfolio disposed to each risk factor. Value at risk is derived as a square root of the portfolio variance multiplied by the respective percentile of the normal distribution ($z_{0.95}$=1.960 and $z_{0.99}$ = 2.326). We work with discrete portfolio returns. VaR is calculated daily so that we obtain the vector of VaR values for n-s days (n=1864 – total number of historical observations, s=500 size of the rolling window). The initial period of 500 observations necessary to compute the portfolio variance is lost.

The backtesting is performed with size of the rolling window of 500 observations. Figures 13-15 illustrate the distribution of portfolio return and VaR value over the observation period for two confidence levels respectively for bullet portfolio (figure 13), barbell portfolio (figure 14) and laddered (figure 15). The similar pattern of daily portfolio returns distributions can be observed for all portfolios. The calm period is our period of pre-crisis. The second period, starting in the middle of 2007 is characterized by high volatility of returns and extreme profit and loss values not covered by VaR. In this region of the distribution of return the number of exceptions is much higher than in the two other periods. After the period of market vulnerability the number of deviations slowly decrease in the "actual" period, although still presents some extreme events.

We also run the backtesting for 3 sub periods, the results can be observed in the Annex 2. Figures 25 to 27 show the distribution of returns and VaR value over the pre-crisis (figure 25), crisis (figure 26) and actual period (figure 27) for bullet portfolio at two confidence levels. Figures28 to 30 illustrate the same for barbell and figures 31 to 33 for laddered. The results of number of exceptions for each portfolio in each period are summarized in below table 5. We can see that in the pre-crisis period the same low number of exceptions is observed for three portfolios on both VaR confidence levels. During our crisis period the number of exceptions drastically increases to 25 in case of bullet and laddered and 27 in case of barbell for 0.95 VaR. Surprisingly we observe very high number of exceptions in the current period, in few cases even higher than in the previous highly volatile period.

Backtesting Var&Cov VaR - number of exceptions						
portfolio	BULLET		BARBELL		LADDERED	
c	cl = 0.05	cl= 0.01	cl = 0.05	cl= 0.01	cl = 0.05	cl= 0.01
Total period (s = 500)	50	18	43	12	45	13
P. pre-crisis (s = 250)	5	1	5	1	5	1
P. crisis (s = 250)	25	8	27	8	25	8
P. "actual" (s = 250)	35	9	22	7	29	6

Table 5. Number of exceptions for bullet, barbell and laddered portfolio at VaR 0.95 and VaR 0.99, resulting from backtesting with rolling window of 500 observations.

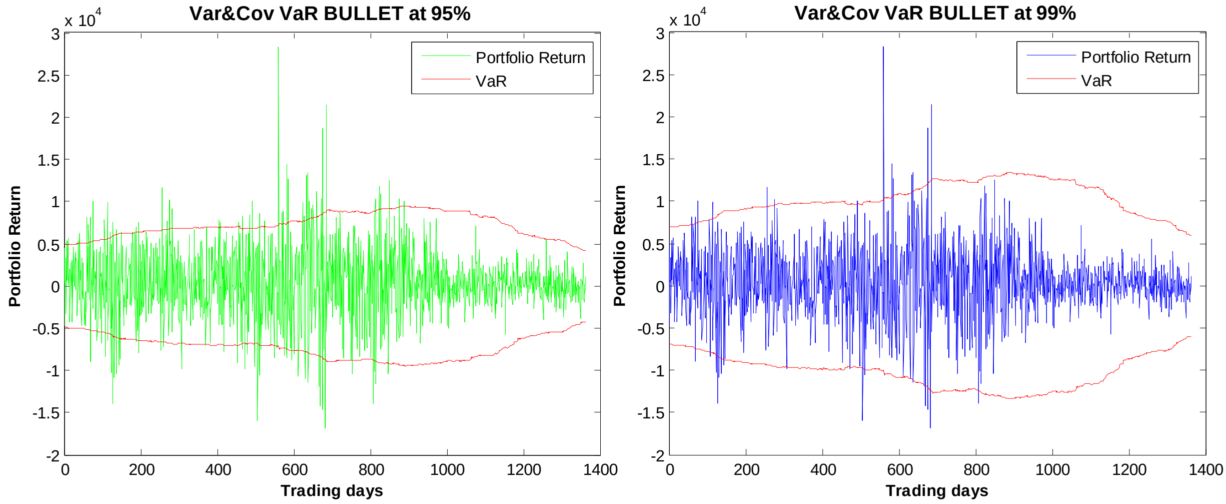

Figure 13. Portfolio returns distribution and VaR values at 0.95 and 0.99 cl. for bullet over observation period form 3rd of January 2006 to 6th of June 2011

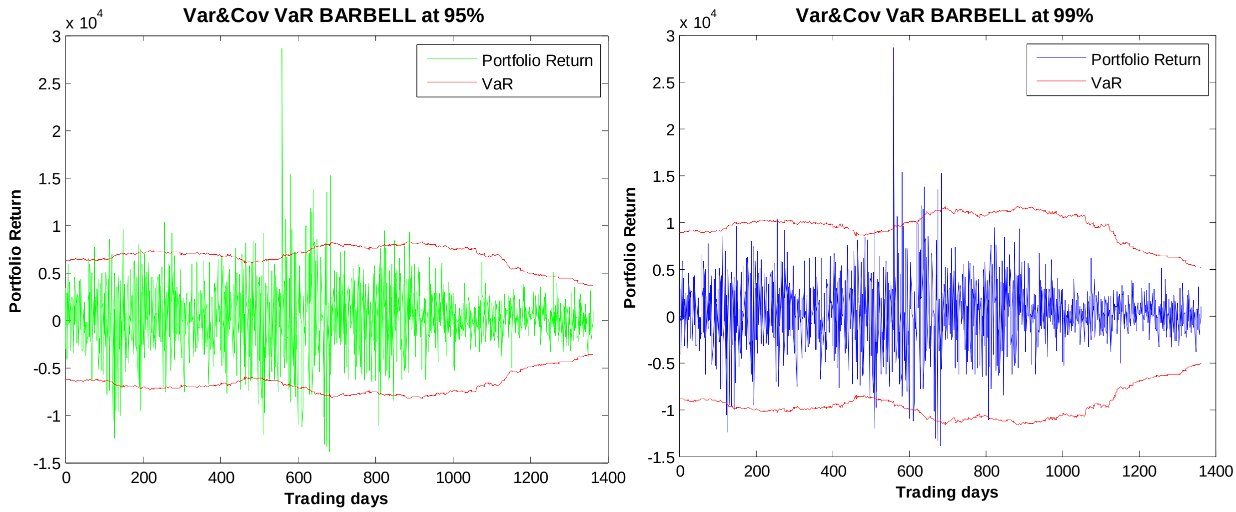

Figure 14. Portfolio returns distribution and VaR values at 0.95 and 0.99 cl. for barbell over observation period form 3rd of January 2006 to 6th of June 2011

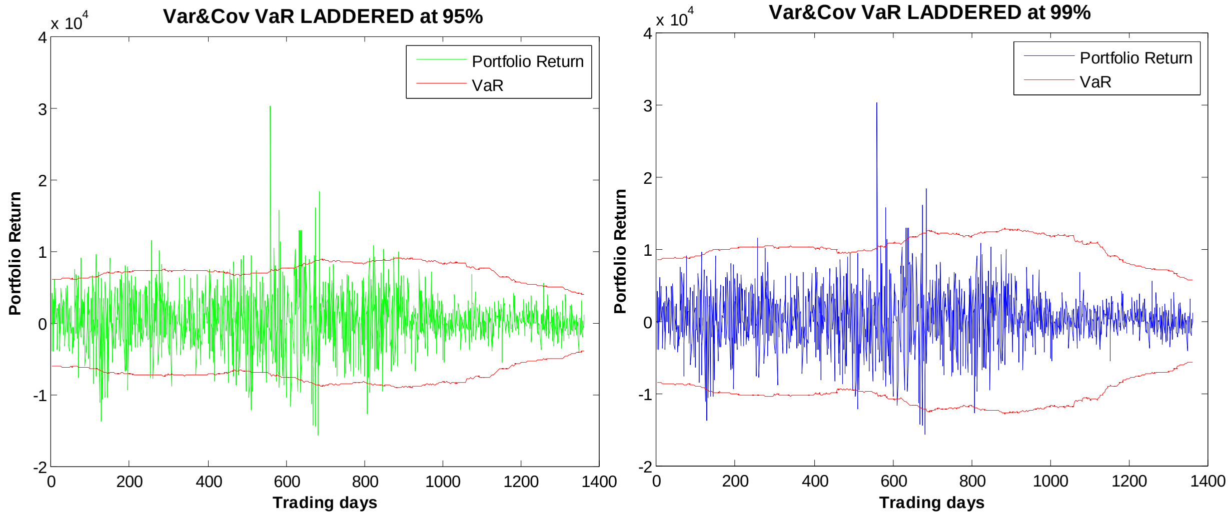

Figure 15. Portfolio returns distribution and VaR values at 0.95 and 0.99 cl. for laddered over observation period from 3rd of January 2006 to 6th of June 2011

4.5 Kupiec (1995) Likelihood Ratio Test

The Kupiec Likelihood Ratio Test – test of frequency of tail losses determines whether the observed exceptions frequency is consistent with the frequency of exceptions according to the VaR model and given confidence level. The confidence intervals are defined by below function:

$$-2\ln[(1-p)^{n-m}p^{m}] + 2\ln[\,(1-m/n)^{n-m}\,(m/n)^{m}]$$

where

p = (1-c) – probability of an exception for a given confidence level

n – number of trials (days in the observation period)

m – number of exceptions

The equation has the chi-square distribution with 1 degree of freedom down the null hypothesis that p is the correct probability. There is 5% probability that the value of chi-square with one degree of freedom will exceed the value of 3.841. If the above equation value is over 3.841, we reject the model. For the contrary, if its value is below this level, we accept the model and conclude it is correct.

In basis of the above equation, we calculate the confidence intervals for confidence levels of 0.95 and 0.99. Our observation period consists of n = 1864 days. The expected number of exceptions is 68 for c=0.95 and 14 for c=0.99. We conclude that the model will not be rejected if the number of exception ranges from 54 to 84 for 0.95 confidence level and from 8 to 21 for confidence level of 0.99. As illustrated in table 6 this is fulfilled by Variance-Covariance Model, but only at 0.99 confidence level and by Historical Simulation in almost all analyzed cases. The Historical Simulation is more efficient approach for portfolio VaR estimation under the Kupiec Test.

Var&Cov VaR			
c		0.95	0.99
p = 1 - c		0.05	0.01
expected no of exceptions		68	14
bullet	exceptions	50	18
	Kupiec test value	**5.6123**	**1.2792**
barbell	exceptions	43	12
	Kupiec test value	**11.2199**	**0.2076**
laddered	exceptions	45	12
	Kupiec test value	**9.3925**	**0.2076**
Kupiec confidence interval		**<54; 84>**	**<8; 21>**

HS VaR			
c		0.95	0.99
p = 1 - c		0.05	0.01
expected no of exceptions		68	14
bullet	exceptions	87	18
	Kupiec test value	**5.0367**	**1.2792**
barbell	exceptions	58	11
	Kupiec test value	**1.6879**	**1.2792**
laddered	exceptions	72	16
	Kupiec test value	**0.0224**	**0.1327**
Kupiec confidence interval		**<54; 84>**	**<8; 21>**

Table 6. Kupiec likehood ratio test results for Variance-Covariance and Historical Simulation Methods of VaR estimation for three portfolios and two confidence levels of 0.95 and 0.99

5. Summary and Conclusions

This study develops a Value at Risk methodology for measuring interest rate risk exposure of bond portfolios that are responsive to market conditions in terms of interest rate levels and volatility. Using 7,5 years of US Treasury interest rates daily data we derive the theoretical bond prices and create portfolios applying active portfolios bullet, barbell and laddered strategies. Bullet portfolio concentrates investment in 5-year bonds, so that the portfolio is affected just by one risk factor – 5-year spot rate. Any increase in the middle term interest rate produces the decrease in the 5-year bonds price and consequently negatively affect the bullet portfolio return. Barbell portfolio is constructed from bonds with extreme maturities of 1 and 10 years. The portfolio total return depends on short term and long term interest rates movements. Increase in the short term interest rates cause the decrease in the price of the 1-year maturity bond, what will cause partial portfolio loss. As bullet is also affected by 10-year interest rates level risk factor, assuming no change in the long term interest rates, portfolio value is partially compensated. Laddered portfolio is the portfolio creation strategy which assumes the highest diversification. Our laddered portfolio is created form 1, 4, 7 and 10-year bonds and so its value is affected by 4 risk factors: 1, 4, 7 and 10-year interest rates. Any changes in those specific interest rates proportionally affect laddered portfolio return.

The portfolios risk is evaluated through value at risk measure. The VaR is computed according to two different approaches: the parametric Variance and Covariance Approach and non-parametric technique of Historical Simulation. VaR is computed on 2 confidence levels 0.95 and 0.99 for 3 portfolios and considering 3 periods of pre-crisis, crisis and current period. The results from both approaches show evidence of significant differences in expected maximal loss levels between analyzed periods. The US financial crisis highly affected the bond portfolios returns, in some cases causing over 30% increase in the expected bond portfolios losses. In the current period we observe decadence in the VaR values respect to crisis period, which although are still maintained at relatively high level, tend to recover its values from the pre-crisis period.

Considering portfolios, bullet portfolio is the most risky strategy, as it concentrates in its investment on just one risk factor – 5-year spot rate. Laddered is the most diversified portfolio affected by 4 risk factors (1-year, 4-year, 7-year and 10-year interest rates). Although its diversification effect is visible by lower looses comparing to bullet portfolio, the barbell portfolio is prevailing in the lowest portfolio losses. Barbell strategy is affected by two extreme interest rates – 1-year and 10-year interest rates. The changes in one risk factor can be compensated by another. During the period of crisis the losses on each portfolio increases significantly and diversification effect almost disappears. All three portfolios suffer high negative returns. Concerning the actual period the expected losses are still higher than before the crisis and surprisingly at 0.95 confidence level bullet portfolio is the one with the lowest negative returns. At 0.99 confidence level the losses are similar for all portfolios.

In order to evaluate the efficiency of each method and perform comparison between them we run the backtesting, calculating VaR daily and deriving number of exceptions. We observe that the number of exceptions for both approached at 0.99 is very similar what means that both methods estimate the extreme tails of the distribution of returns with similar efficiency. But when it comes to 0.95 confidence level, the difference in the number of exceptions observed is significant. Exceptions resulting from variance-covariance method backtesting are much lower than for historical simulation at the same confidence level. The Kupiec Likehood Ratio test shows that at the 0.95 confidence level variance and covariance model is rejected in all analyzed cases while historical simulation just in one case. For 0.99 confidence level both models are accepted. Our conclusion is that variance-covariance approach overestimates the VaR at 0.95 confidence level and that the historical simulation approach is more adequate method of measuring our portfolios market risk exposure.

6. Annex 1

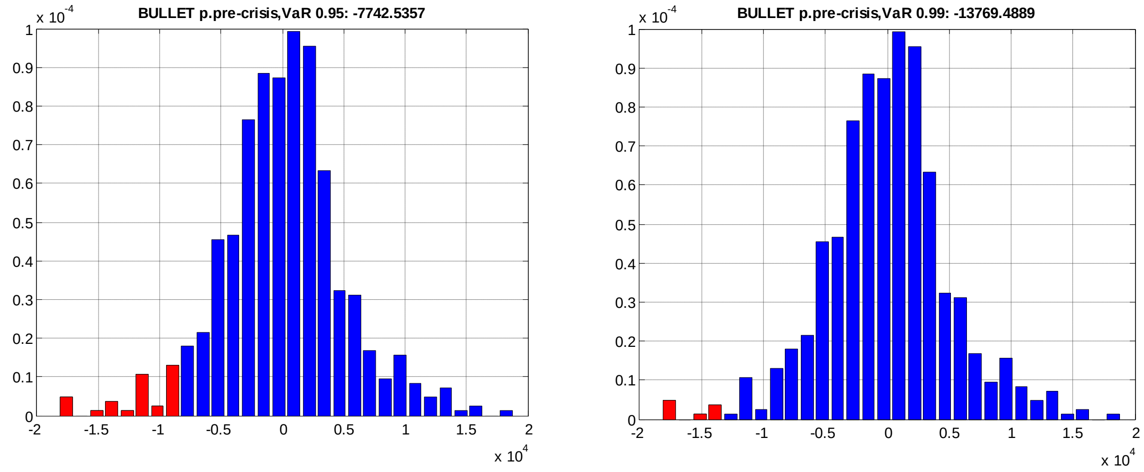

Figure 16: Histogram of returns and VaR value for bullet at 0.95 and 0.99 cl in pre-crisis period

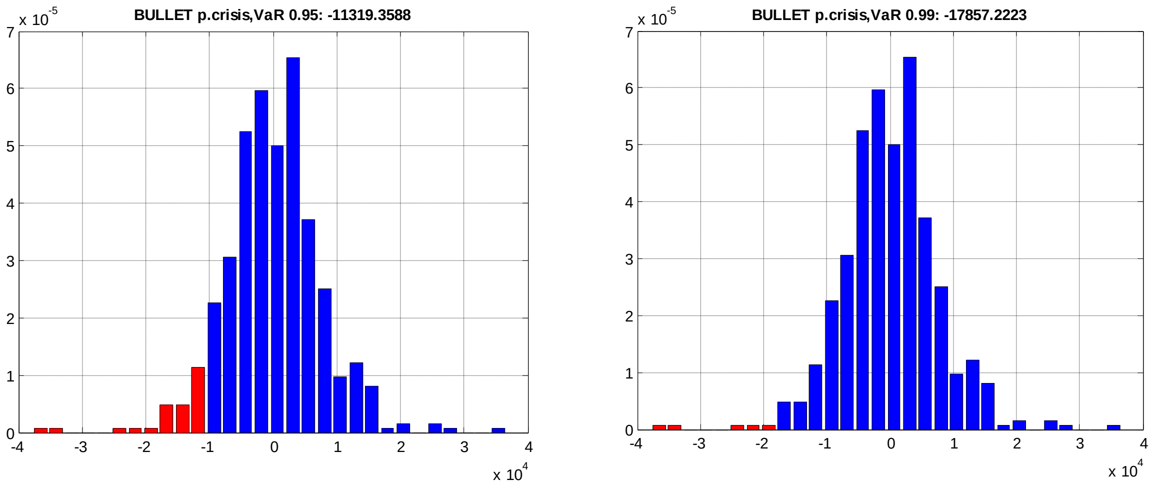

Figure 17: Histogram of returns and VaR value for bullet at 0.95 and 0.99 cl in period of crisis

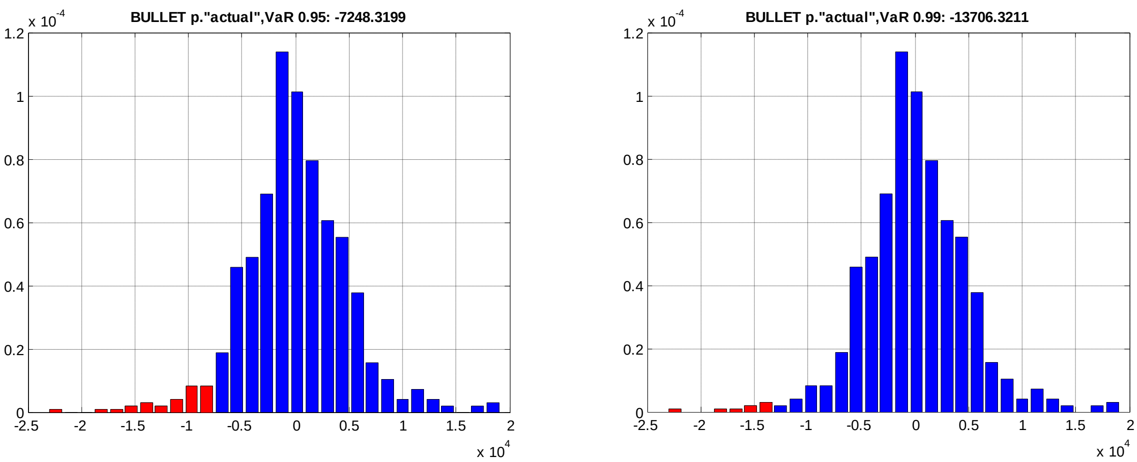

Figure 18: Histogram of returns and VaR value for bullet at 0.95 and 0.99 cl in "actual" period

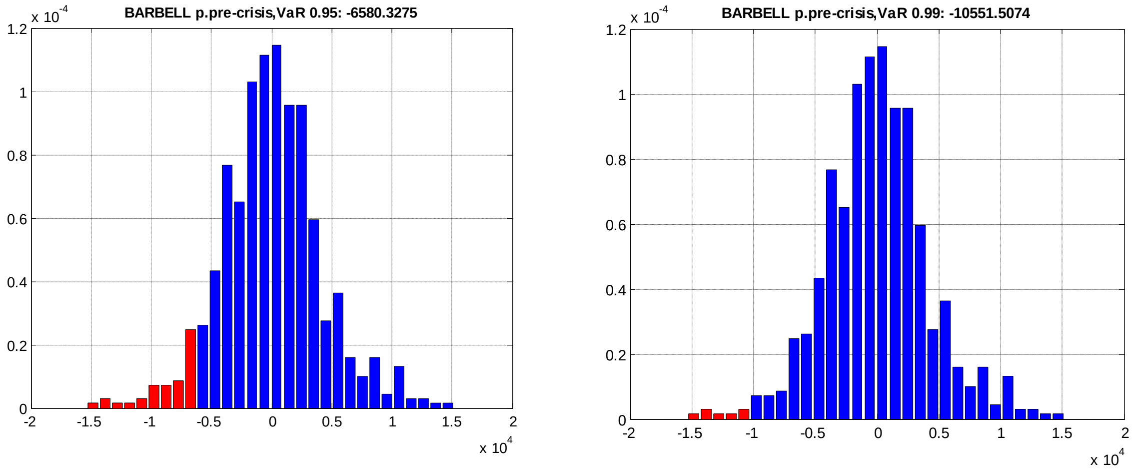

Figure 19: Histogram of returns and VaR value for barbell at 0.95 and 0.99 cl in pre-crisis period

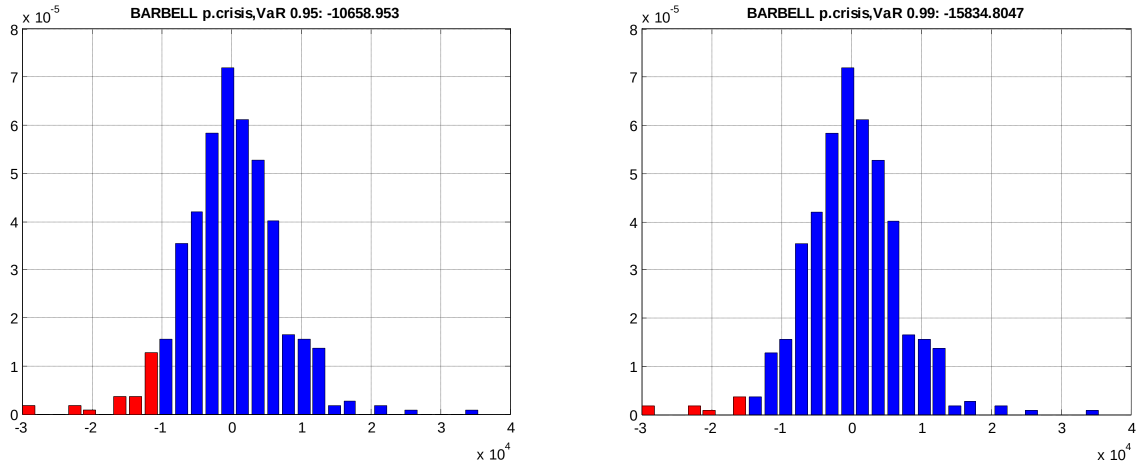

Figure 20: Histogram of returns and VaR value for barbell at 0.95 and 0.99 cl in period of crisis

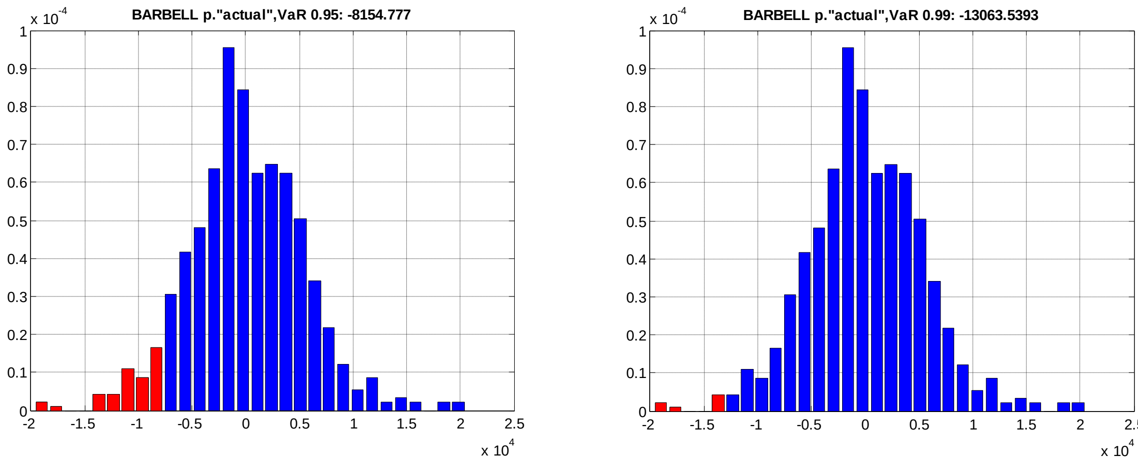

Figure 21: Histogram of returns and VaR value for barbell at 0.95 and 0.99 cl in “actual” period

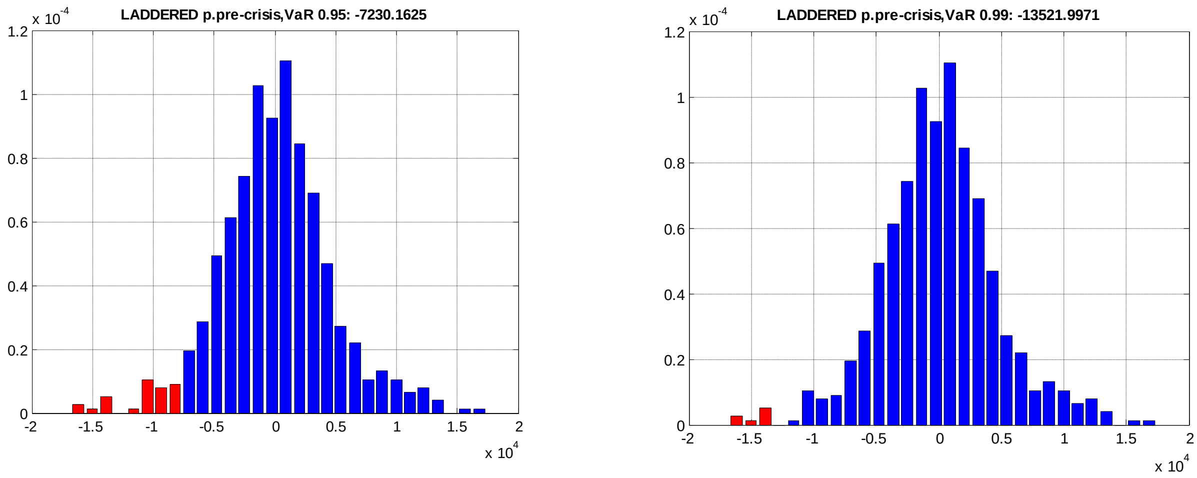

Figure 22: Histogram of returns and VaR value for laddered at 0.95 and 0.99 cl in pre-crisis period

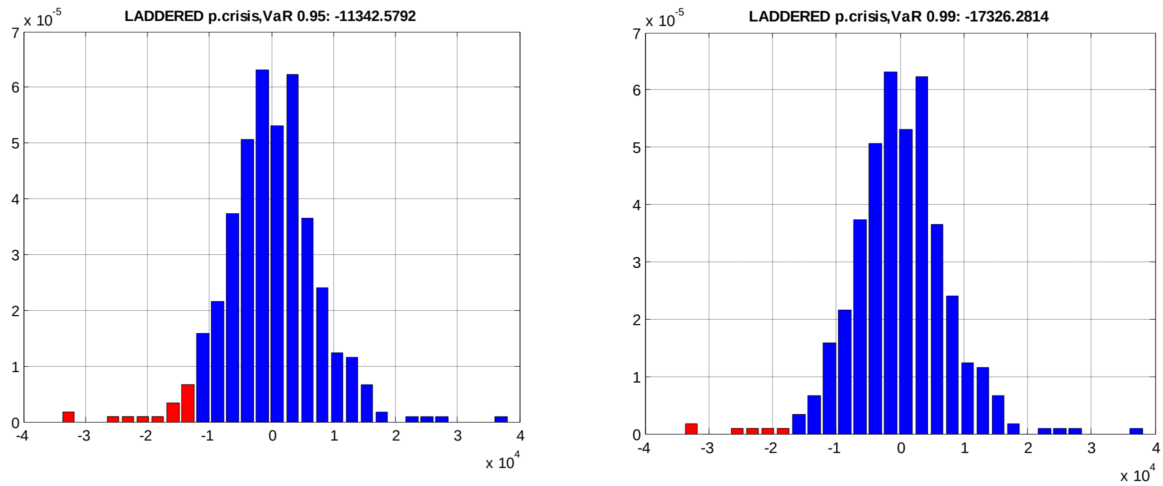

Figure 23: Histogram of returns and VaR value for laddered at 0.95 and 0.99 cl in period of crisis

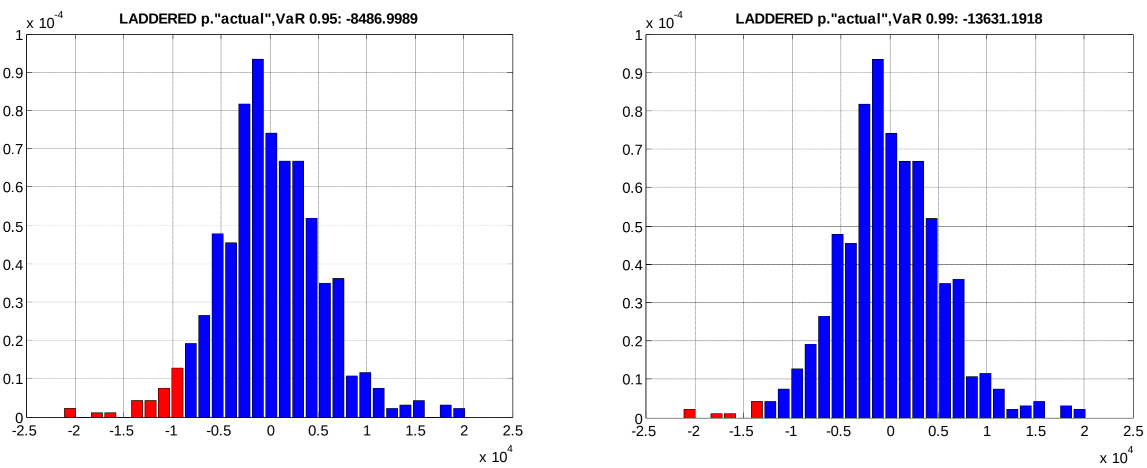

Figure 24: Histogram of returns and VaR value for laddered at 0.95 and 0.99 cl in "actual" period

7. Annex 2

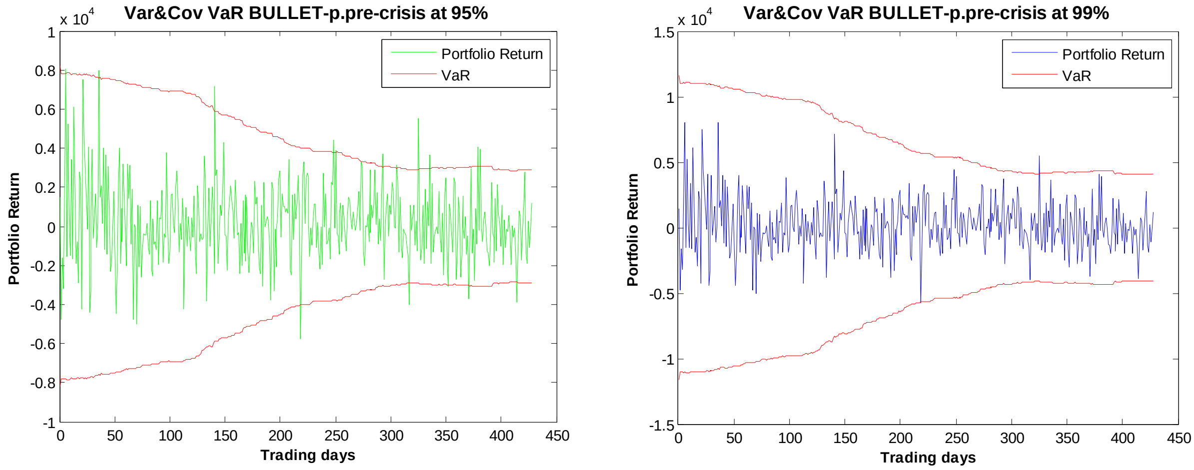

Figure 25. Portfolio returns distribution and VaR at 0.95 and 0.99 cl for bullet over pre-crisis period

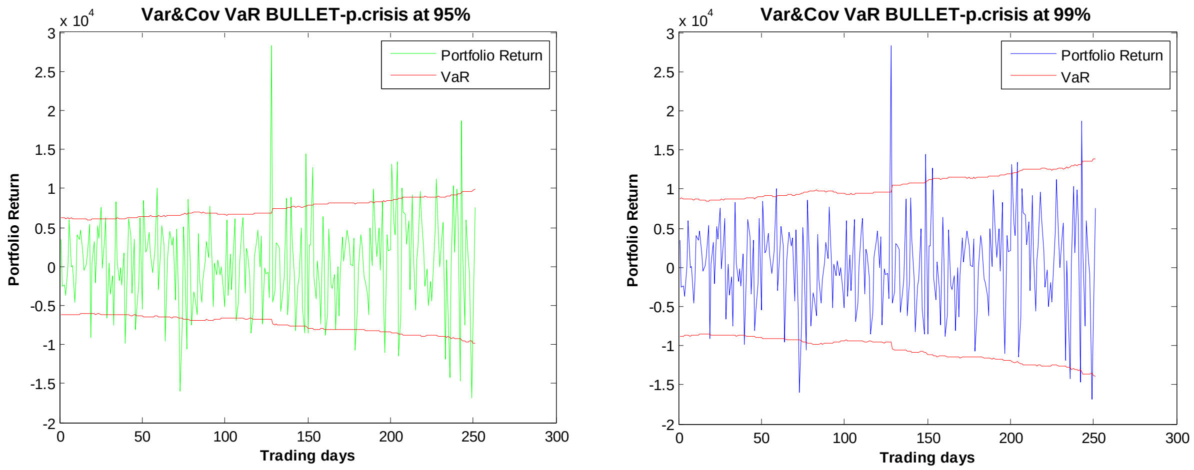

Figure 26. Portfolio returns distribution and VaR at 0.95 and 0.99 cl for bullet over the period of crisis

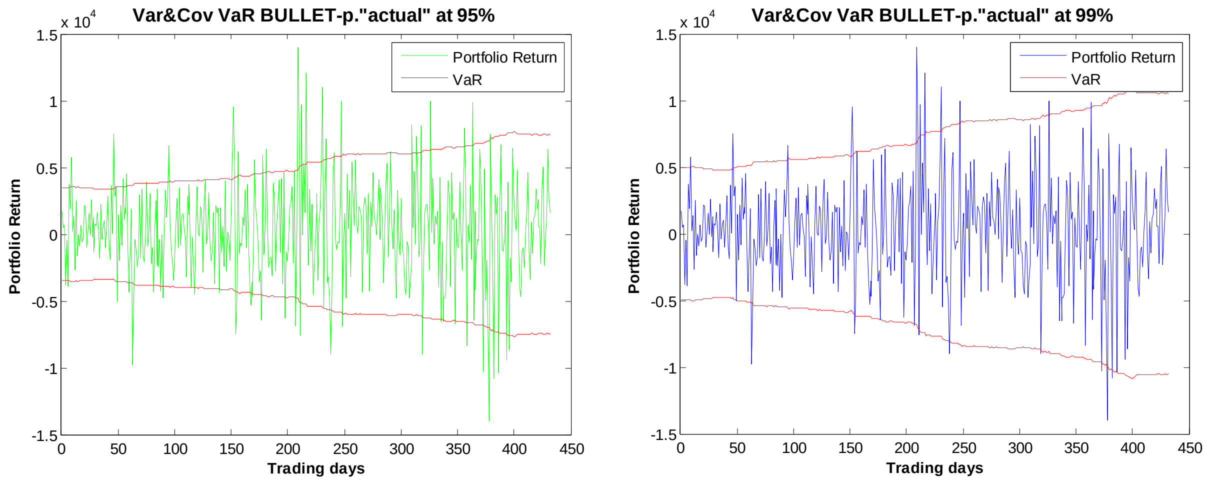

Figure 27. Portfolio returns distribution and VaR at 0.95 and 0.99 cl for bullet over the "actual" period

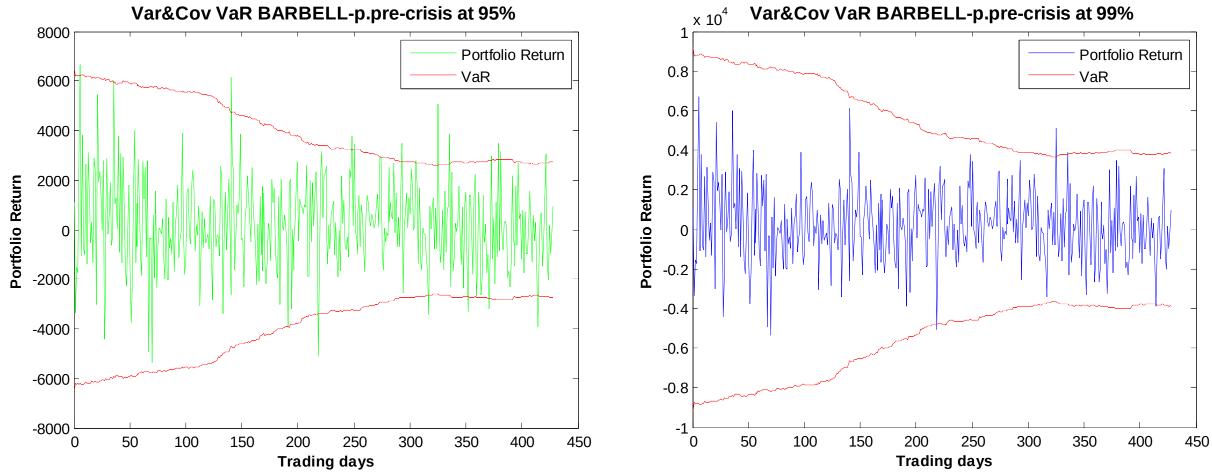

Figure 28. Portfolio returns distribution and VaR at 0.95 and 0.99 cl for barbell over the pre-crisis period

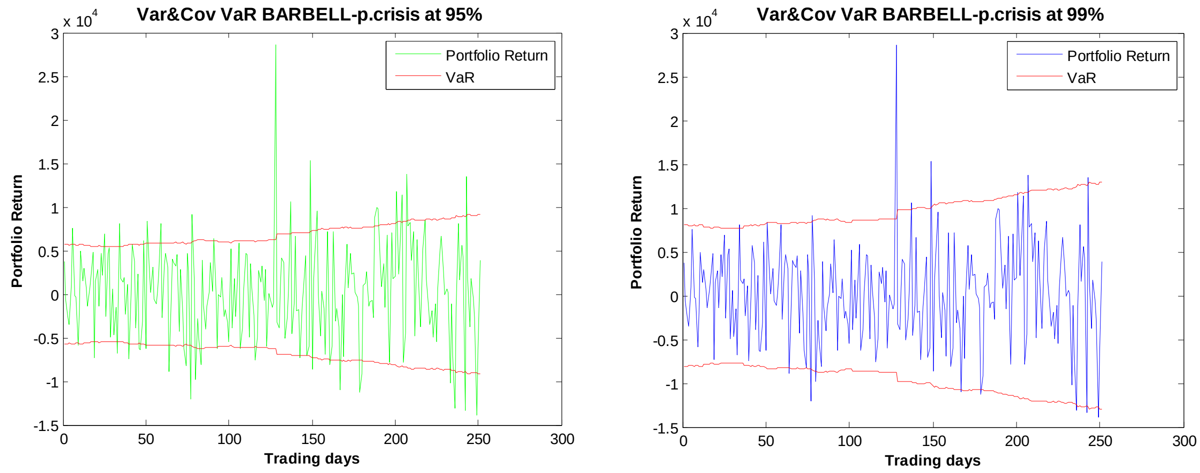

Figure 29. Portfolio returns distribution and VaR at 0.95 and 0.99 cl for barbell over the period of crisis

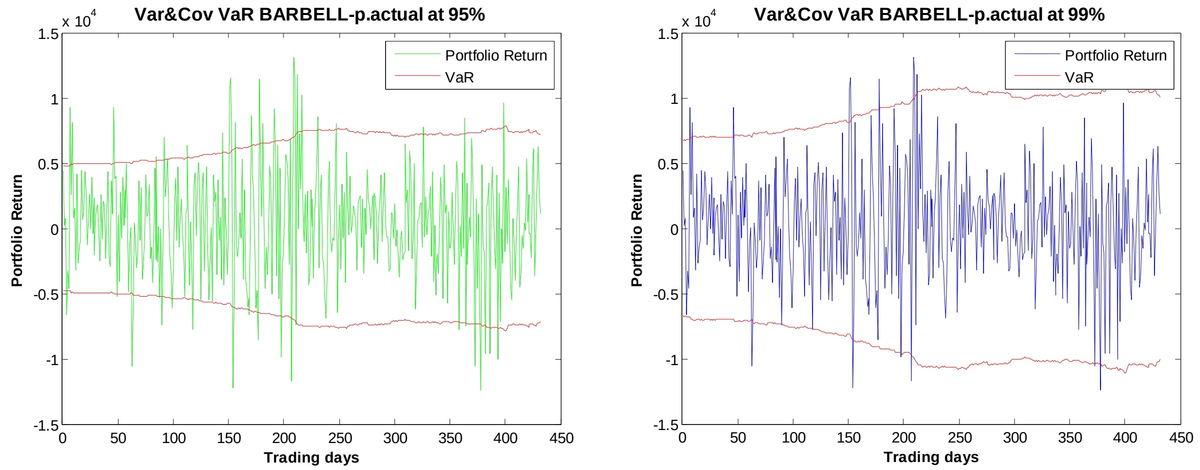

Figure 30. Portfolio returns distribution and VaR at 0.95 and 0.99 cl for bullet over the “actual” period

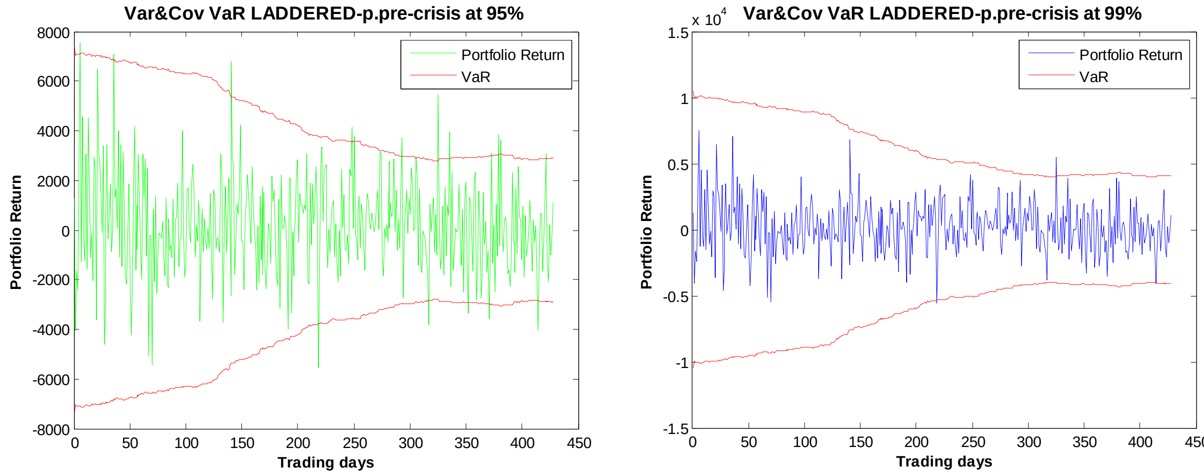

Figure 31. Portfolio returns distribution and VaR at 0.95 and 0.99 cl for laddered over the pre-crisis period

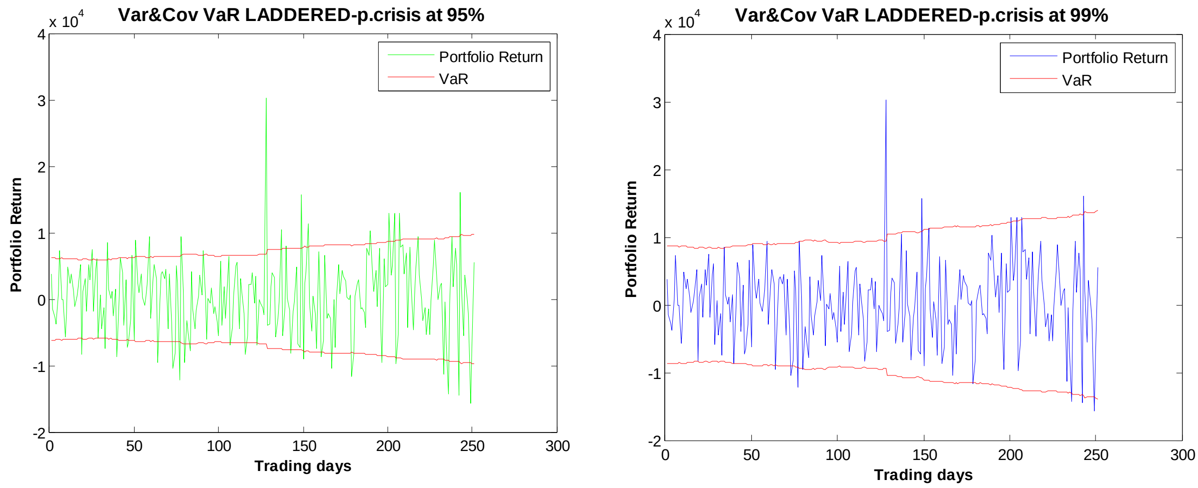

Figure 32. Portfolio returns distribution and VaR at 0.95 and 0.99 cl for laddered over the period of crisis

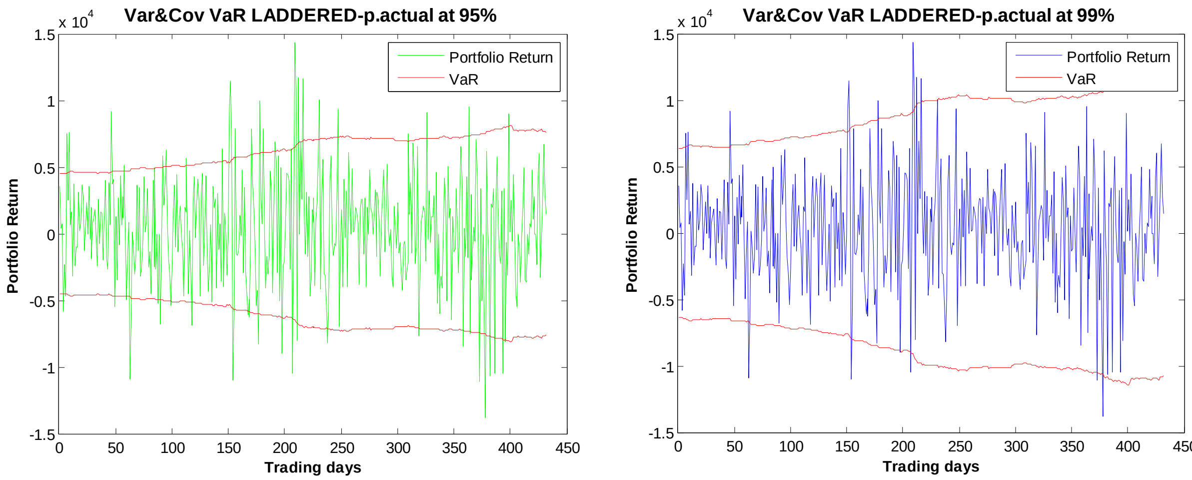

Figure 33. Portfolio returns distribution and VaR at 0.95 and 0.99 cl for laddered over the "actual" period

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