PRICING FORWARD CONTRACTS IN POWER MARKETS: A COMPARATIVE STUDY FOR THE SPANISH CASE

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A Thesis submitted by Germán Alexander Zumba Flores for the Máster in Banking and Quantitative Finance

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Abstract:

This paper examines the temporal stability of two electricity future pricing theories for the Spanish market by conducting an empirical analysis of two models. The first model, the Mean Reverting Jump Diffusion, was proposed by Cartea and Figueroa in 2005, a model which has been used widely in the energy derivative pricing theories. The second one, the Stochastic Forward Premium Model, proposed by Blanco et al., in 2014, is one of the most recent proposal where the authors model the general factor affecting the whole swap curve within each market segment (monthly, quarterly and yearly contracts).

Contents

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1 Introduction

The liberalization of the electricity sector, a phenomenon that has been occurring in many countries in recent decades, has led to the possibility of exchanging the electrical power on a market subject to competitive rules. In the case of Spain, the liberalization process has been driven by the need of adapting itself to the guidelines set by the European Union so as to continue on the path of economic integration. The aim is to consolidate a common market of goods and services.

In a more financial context, electricity is considered as a commodity, though with special characteristics. It is well known that electricity prices present high volatility. This results in an increasing interest in modelling this commodity, in order to face this market's idiosyncratic risk. Two distinctive features are present in energy markets in general, and they are very evident in electricity markets in particular: The mean reverting nature of spot prices and the existence of jumps or spikes thereof (Cartea and Figueroa, 2005). There are different approaches to modelling prices on electricity markets, that can mainly be classified in two categories: spot-based models and forward-based models.

The first one is based on specific stochastic processes of the spot price on a set of other state variables (Cartea and Figueroa, 2005). The spot prices can exhibit various features such as seasonality, spikes and mean-reversion. Since the deregulated electricity markets are still developing and growing fast, practitioners, as well as academics, have suggested several models to capture as many of these characteristics (Benth et al, 2012). Thus, the standard approach in the literature is to model the logarithmic electricity spot through a mean-reverting process (Benth et al., 2005). Notable contributions have been made by Hilliard and Reis (1998), Casassus and Collin-Dufresne (2005), German and Roncoroni (2006), Lucia and Schwartz (2002), among others. Lucia and Schwartz's (2002) extend the model developed by Schwartz[1] (1997) and derive a formula for the forward price of electricity by modelling the expected spot price during a future time period (Huisman et al., 2012). Later, Cartea and Figueroa[2] (2005) extended the model of Lucia and Schwartz (2002) by adding jumps to the process.

According to Blanco et al. (2014) these kind of models present a forward price curve which is not necessary consistent with the observable forward price. Therefore those authors defend the second category of models, based on the direct modelling of electricity forward prices' term structure. Within this framework, models are fitted

[1]The author introduces an Ornstein-Uhlenbeck type of factor model which accounts for the mean reversion of prices in 1997 (Cartea and Figueroa, 2005)

[2]Therefore, as Cartea and Figueroa (2005) state, in the process of pricing energy derivatives it is crucial that the most significant features of the spot price dynamic, and consequently the forward, are captured.

to forward prices directly, instead of modelling spot prices from which futures prices are then derived. Previous literature studies based on the dynamics of the forward curve as a whole are Heath et al.'s (1992), Cortaza and Schwartz (1994), Miltersen and Schwartz (1998), or Miltersen (2003), among others. These models consider the forward price curve as an input into the derivative pricing model (Blanco et al., 2014). On the other hand, models such as the presented by Borovkova and Gemman (2006a), Frestad, et al. (2010), Koekebakker and Ollmar (2005), Banco et al. (2014), among others are, based on the idea of modelling a given function of observed forward prices. Then, this functions' stochastic deviations are analysed and observed by means of using additional state variables.

In this paper, one outstanding model for each category has been selected to be applied to the Spanish electricity market in order to evaluate their explanatory and predictive ability for forecasting forward prices in the Spanish market area. The first one was developed by Cartea and Figueroa (2005, which was based on the Lucia and Sacharz (2002). They introduce a mean-reverting jump diffusion MRJD model for the electricity spot price, attempting to capture electricity spot prices' most important features. They also derive an expression for the forward curve in a closed-form. The second one is the model presented by Blanco et al. (2014), based on the Borovkova and Geman (2006), in which it is presented a stochastic forward premium model for the pricing of electricity derivatives. As far as we know, it is the first time that these two models are applied to the Spanish market. Then, it will very interesting to learn how these models work when applied to the same market with comparative purposes. To do so not only in-sample estimates will be applied but also out-of-sample. When applying the above mentioned models, our empirical analysis differs from the ones made in the referred papers to address some identified drawbacks. In the first model we use a GARCH-model to compute the daily volatility; and compute the forward price for each t as the average of the estimated forward prices obtained for each day included in the period when delivery will be made. Regarding the second model, the stochastic forward premium (SFP) will be estimated following Borovkova and Geman (2006).

This research project is organised as follows. Section 2 introduces the models and methodology. The data used in each model will be presented and analysed in section 3. The empirical analysis is carried out and and displayed in section 4. Section 5 addresses a comparative analysis of the two models through both in-sample and out-sample estimations. Finally, section 6 summarizes the obtained results and concludes

2 Models and Methodology

In this section the two models and their principal features, as well as their implications for pricing electricity derivatives, are commented.

2.1 Mean Reverting Jump Diffusion (MRJD) Model

The Mean Reverting Jump Diffusion (MRJD) model presents a simpler process to incorporate the observed features of the electricity market than other kind of models such as many froward-based models. Following Cartea and Figueroa (2005), two characteristics that make the MRJD model interesting are its relative simple implementation and the greater availability of the needed data. In fact, the forward based models do not usually have a wide range of data base to work with. Thereby, it is presents a one-factor mean-reversion jump diffusion model which has the capability to return a closed-form formula to the forward curve.

Let $(\Omega, \mathbb{P}, F)$ be a complete filtered probability space, with an increasingly and right-continuous filtration $[F_t]_{t\in[0,T]}$ and $T < \infty$ a fixed time horizon. Moreover the electricity spot price at time $0 \leq t \leq T$ is denoted by s(t), and following Lucía and Schwartz (2002), it is assumed that the log-price process, lnS_t, takes the form

$$lnS_t = g(t) + X_t \tag{1}$$

given that the spot price can be written as

$$S_t = e^{g(t)}e^{X_t} \tag{2}$$

let $G(t) \equiv e^{g(t)}$ be a deterministic function modelling the seasonal trend and Y_t be a stochastic process[3] whose dynamics are given by

$$dX_t = -\alpha X_t dt + \sigma(t)dZ_t + lnJdq_t \tag{3}$$

where 'α' is the speed of mean-reversion, dZ_t is the increment of the standard Brownian motion Z, $\sigma(t)$ is the time dependent volatility, J is a proportional random jump size and dq_t is a Poisson process such that

$$dq_t = \begin{cases} 1 & with \quad probability \quad ldt \\ 0 & with \quad probability \quad (1-l)dt \end{cases} \tag{4}$$

In (4) 'l' is the intensity or frequency of the process. Moreover, the following assumptions are adopted:

[3] A zero level mean-reverting jump diffusion process (MRJD).

- $lnJ \sim N(\mu_j, \sigma_j^2)$.

- J, dq_t and dZ_t are independent.

Where 'l' is the intensity or frequency of the process. Furthermore, I, d_{q_t} and dZ_t are considered independent.

The SDE for S_t is obtained from (2) and (3) as follows [4]

$$dS_t = \alpha(\rho(t) - ln(S_t))S_t dt + \sigma(t)S_t dz_t + s_t(J-1)dq_t \tag{6}$$

Where the time dependent mean reverting level is given by

$$\rho(t) = \frac{1}{\alpha}\left(\frac{dg(t)}{dt} + \frac{1}{-}\sigma^2(t)\right) + g(t) \tag{7}$$

As can be seen, the equation (5) shows a mean reverting diffusion process with an additional term $dq_t = 0$. Most of the time, this latter term has a value that is equal to zero but at random times S_t will jump from the previous jump $S_{(t-1)}$, resulting in a new value $JS_{(t-1)}$.

Regarding the jump size J, the following assumptions are made:

- J is log-Normal, i.e. $lnJ \sim N(\mu_J, \sigma_J^2)$

- $\mathbb{E}(J) = 1$

- $\mathbb{E}(lnJ) = -\frac{\sigma_J^2}{2}$

- $Var(lnJ) = sigma_J^2$

2.2 Stochastic Forward Premium (SFP) Model

In this section, the Stochastic Forward Premium (SFP) model developed by Blanco et al. (2014) is introduced, lightly modified to better explanation the seasonal factor construction. Differing from the MRJD model, in this one the forward structure is used as an input to price derivatives on electricity prices. Blanco et al. (2014) describe how to identify the factors such as the average forward price within each market segment, the deterministic seasonal factor and the stochastic changes in the forward curve shape. The stochastic factor are driven by processes which follow the Multivariate Normal Inverse Distribution (MNIG) distribution.

[4] The diffusion process followed by the spot price indicated in the equation (6) in the discrete version has the following form

$$S_t = S_{t-1} + \alpha(\rho(t) - lnS_{t-1})S_{t-1}\Delta_t + \sigma(t)S_{t-1}\sqrt{\Delta_t}n_{S,t-1} + S_{t-1}(J-1)dq_t \tag{5}$$

where, $n_{S,t-1}$ are i.i.d standard normal random variables.

The starting point must be seasonal cost-of-carry model, proposed by Borovkova and Geman (2006). This factor relates forward prices for any maturity T and delivery length period i as follows,

$$F_i(t,T) = \overline{F_i}(t)e^{(s_i(K)+\gamma_i(t,T))} \tag{8}$$

where $\overline{F_i}$ is the non-seasonal quantity defined as the average level of the swap price referred to each market segment i[5] . This component has been estimated below as the geometric average of the current swap prices

$$\overline{F_i} = \sqrt[N]{\prod_{T=1}^{N} F_i(t,T)} \tag{9}$$

or equivalently

$$ln\overline{F_i} = \frac{1}{N}\sum_{T=1}^{N} lnF_i(t,T) \tag{10}$$

$S_i(k)$ is the seasonal deterministic premia, defined as the collection of long-term average premia on swaps expiring in the calendar period, month or quarter[6], with respect to the average swap price $ln\overline{F_i}$. To estimate this deterministic factor $s_i(K)$ we have proceeded as Borovkova and Geman (2006) [7] indicate,

$$\hat{s}(T) = \frac{1}{2}\sum_{t=1}^{n}(lnF(t,T) - ln\overline{F}(t)) \tag{11}$$

where n denotes the number of days in the historical dataset. Here N is the most distant maturity. Finally the last term $\gamma_i(t,T)$ is the stochastic forward premium (SFP) for the delivery period i and expiry date T, which can be defined as

$$\gamma_i(t,T) = lnF_i(t,T) - \overline{F_i}(t) - s_i(K) \tag{12}$$

Both, the seasonal deterministic premia and the SFP are zero on average by construction.

Once the components of the model have been determined, the next step is introducing the stochastic dynamics of the state variables in terms of Multivariate Normal

[5]For instance, $i = 1 = Montly(M)$, $i = 2 = Quarterly(Q)$ and $i = 3 = Yearly(Y)$

[6]Depending on the value of i, K will take 12 (monthly) or 4 (quarterly)

[7]See Appendix D for Further information.

Inverse Gaussian MNIG. Let $L(t)$ be a d-dimensional[8] vector of MNIG. Following Blanco et al. (2014), the $L(t)$'s increments defined as $dL(t) = L(t+dt)-L(t) = X$ are standardized and MNIG distributed[9]. The MNIG distribution is included in the family of multivariate generalized hyperbolic distribution (MGH). Thus, the probability density function of the d-dimensional MNIG distribution $MNIG_d(X; \alpha, \beta, \delta, \mu, \Sigma)$ is given by (10)

$$f(X) = \frac{\delta}{2^{(d-1)/2}} \left[(\frac{\alpha}{\pi q(x)} \right]^{(d+1)/2} K_{\frac{d+1}{2}}[\alpha q(x)] e^{p(x)} \tag{13}$$

where $q(x) = \sqrt{\delta^2 + (x-\mu)'\Sigma^{-1}(x-\mu}$, $p(x) = \delta\sqrt{(\alpha^2 - \beta'\Sigma\beta)} + \beta'(x-\mu)$ and $K_{\frac{d+1}{2}(x)}$ is the modified Bessel function of the second kind with index $(d+1)$.

In addition, the mean and covariance matrix of the vector X are defined as (Oigard et al, 2005),

$$E(X) = \mu + \frac{\delta\Gamma\beta}{\sqrt{\alpha^2 - \beta'\Gamma\beta}} \tag{14}$$

$$\Sigma = \delta\sqrt{\alpha^2 - \beta'\Gamma\beta} \left[\Gamma + (\alpha^2 - \beta'\Gamma\beta)^{-1}\Gamma\beta\beta'\Gamma \right] \tag{15}$$

Taking into account the fact that the marginal distributions of the (MGH) distribution are univariate Normal Inverse Gaussian, (NIG) ,the γ_i parameter[10] can be defined as

$$\gamma_i = \sqrt{\gamma_i^2 - \beta_i^2} \tag{16}$$

Finally, the dynamics of $\overline{F_i}(t)$ and $\gamma_i(t,T)$ under the market probability measure are given by the following stochastic differential equations:

$$dln\overline{F}_i(t) = \kappa_i(\varsigma_i - ln\overline{F}_i(t))dt + \theta_{\overline{F}_i} dL_{\overline{F}_i}(t); i = 1, ..., I \tag{17}$$

$$\gamma_i(t,T) = \varpi_{i,T}\gamma_i(t,T)dt + \theta_{\gamma_i^{(T)}} dL_{\gamma_i^{(T)}}(t); i = 1, ..., I, T = 1, ..., N \tag{18}$$

Besides, Blanco et al. (2014) established that the SFP are subject to their own sources of uncertainty given by the MING, $dL_{\overline{F}_i}(T)$ and $dL_{\gamma_i}(T)$, which are assumed

[8]d=I+IxN, where I=2, due to the fact that i indicates the delivery length period given, 1(M) and 2(Q), and N=3 is the maximum liquid maturity.

[9]$L(t)$'s mean has stationary and independent increments in the sense that the distribution of $L(t)-L(s)$, $t > s \geq 0$, is only dependent on $t-s$ and not on t and s separately (Blanco et al., 2014).

[10]See Blanco et al. (2014) for a detailed proof.

to be correlated. Moreover, in the special case where the increments $L(t) - L(s)$ are normally distributed with zero mean and covariance matrix Π.

2.3 Models' evaluation

In order to evaluate the 'goodness of fit' of each model in this specific market, the Mean Square Error (MSE) is used.

$$MSE = \frac{1}{N}\sum_{t=1}^{N}(Y_t - X_t)^2, \tag{19}$$

where N indicates the total number of observations, Y denotes the market forward quotes path whereas X refers to the model-estimated forward prices.

3 Data Analysis

3.1 Spot series used in the MRJD Model

For our empirical analysis, a data set for the spot price for the Spanish electricity market has been used. Although the data series is available for a range that goes from 1/01/1998 to 1/05/2015 the first two years have been ignored with the aim of avoiding the rigidities that electricity markets usually show during the first years of liberalization. Therefore, the sample period covers from 1/01/2000 to 1/05/2015 (a total number of 5593 observations were included)

There are three key features of electricity spot prices that can be generally observed in electricity markets (Benth et al., 2005):

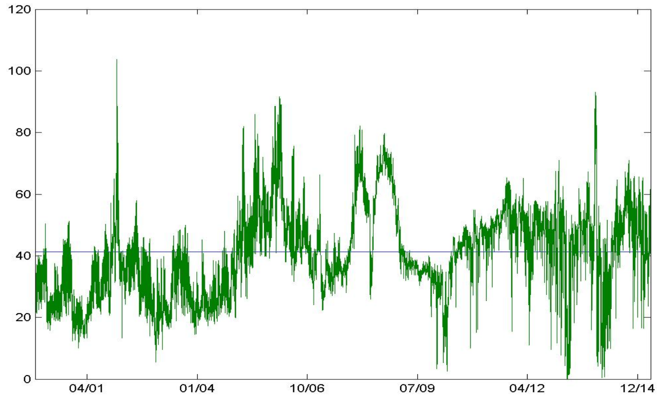
• Firstly, electricity prices typically show very sharp spikes. The consequence of this is the presence of an inelastic demand, combined with an exponentially increasing curve of marginal costs. When an abrupt change of demand or supply takes place(for instance caused by weather conditions), this results in strong jumps of electricity prices.

• Secondly, another singular feature that electricity price dynamics show is that they rather tend to quickly revert back to a mean level, which makes a mean reverting process appropriate to model spot prices.

• Finally, all magnitudes that have been included, such as the mean level, jump intensities and jump sizes, normally exhibit seasonal behaviour throughout the sample.

The peculiarities mentioned above can be checked in Figure 1. Hence, the mean reversion can be observed, together with the jumps across the series and the periods of high volatility. It can also be observed how some disturbances are going away from the mean, with the series then swiftly returning to its mean, thus revealing the mean reversion. These traits will be taken into account in the forward price modelling process in order to obtain accurate estimations.

Figure 1: Spot prices in the Spanish Electricity Market from January 01, to April 25, 2015



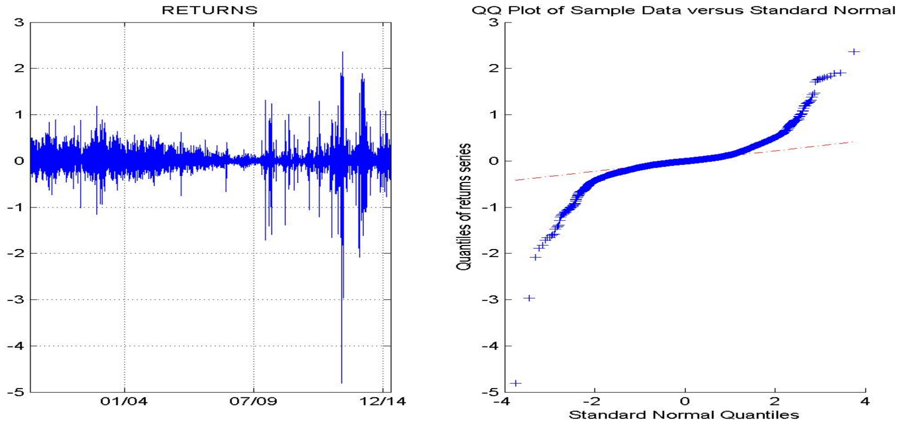
Moreover, as it can be seen in Table 1, the series presents a positive skewness. The positive skewness of electricity spot prices can be attributed to the fact that power is non-storable and to the convex shape of the power supply curve (Viehmann, 2011). Furthermore, the data shows a leptokurtic form due to the presence of significant kurtosis.

Table 1: Spot Series' Statistics

mean	0.0000
volatility	0.1118
kurtosis	42.9177
skewness	-1.1289

Taking into consideration the statistics analysed above, it is clear that the series cannot be identified as normally distributed. This fact is confirmed by representing the QQplot, shown in the Figure 2. So the possibility of assuming normality cannot be the best option to model electricity derivatives since the series present fat tails. The existence of fat tails is linked to the probability of rare events which are more frequent than the ones predicted by a Normal distribution (Cartea and Figueroa, 2005). Consequently, as can be observed in Figure 2, the returns of the electricity spot prices present a behaviour which cannot be modelled by a normal distribution. Therefore, a straight line in the QQplot, as consequence of the fat tails, cannot be seen. For instance, corresponding to a 2.12 per cent, there are returns which are higher than 0.5. This would be zero if we were working with normal distributed series. Spot prices show large spikes due to the sudden imbalances in supply and demand, for instance, when a large production utility experiences a back-out or temperatures suddenly drop (Benth et al., 2014).

Figure 2: Returns' Analysis



3.2 Forward series used in the SFP Model

With comparative purposes, daily data[11] on settlement prices has been used for monthly and quarterly baseload forward contracts traded in the Iberian Futures for the Spanish area. The data has been obtained from the *Reuters* database[12]. We have selected the most liquid contracts within each market segment, which usually are the closest to maturity. In particular, the three closest to maturity monthly (Figure 3) and quarterly (Figure 4)[13].

Figure 3: Monthly forward Price evolution from July 02,2007 to April 23,2015

[11]Ranges for each market segment: 02/07/2007-23/04/2015 for M+1 and M+2, 02/0712007-23/04/2015 for M+3, 03/07/2006-23/04/2015 for Q+1 and 02/01/2007-23/04/2015 for Q+2 and Q+3.

[12]Thomson Reuters to Launch its Industry-Leading Market Data Services in CME Group's Aurora Data Center.

[13]When there are days with no trading prices during the time of period covered by our data, prices for forward contracts from the day before have been fixed as the price of the days with missing data. The missing data percentage was 0.4030% 0.1423% for monthly and quarterly segments respectively.

Figure 4: Quarterly forward Price evolution from January 02, 2007 to April 23, 2015

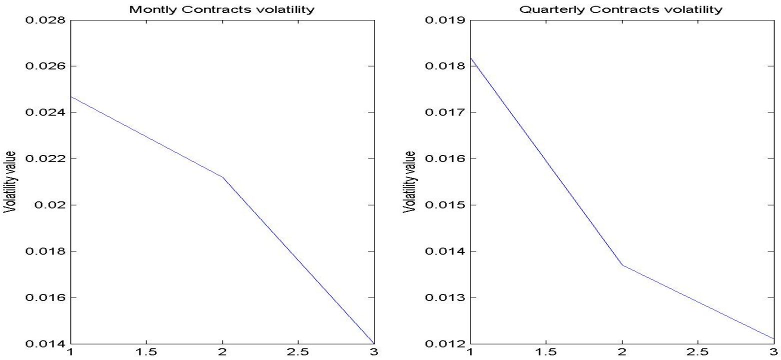
It can be observed in Figures 3 and 4 that the forward prices are much less volatile than spot prices. According to Benth et a.(2005), this is because forward contracts typically have delivery periods of fully months, quarters and years.

Table 2 displays, the statistics for each market segment (monthly, quarterly and yearly). as can be observed, the average price tends to increase with the maturity of the contract. Volatility is higher for the close-to-maturity contract (Samuelson effect), that means that short-dated forward contracts tend to be more volatile than long-dated. Additionally, the volatility presents a more complex behaviour than the mean, which shows a more stable pattern. Furthermore, volatility shows a different structure in each market segment. Thus, for the monthly contracts, the volatility has a structure similar to an inverted u-shape (Figure 5); in the case of quarterly contracts, it has a u-shape structure (Figure 5). In the case of monthly contracts, the volatility follows the expected pattern, decreasing with time to maturity, which also can be corroborated with the historical series. All the series present significant kurtosis and asymmetry, suggesting that the normality assumption is unlikely to be appropriate for these series (see Appendix A), similarly to the spot series case.

Table 2: Forward Series' Parameters

	Historical			returns		
Parameters	M1	M2	M3	M1	M2	M3
mean	48.7195	49.5505	50.0351	0.0000	0.0000	0.0000
volatility	9.7746	9.2385	7.5477	0.0247	0.0212	0.0140
kurtosis	3.015441	3.950	4.9370	56.1502	45.9101	54.9842
skewness	0.3030	0.3756	0.9589	4.1198	2.6417	1.5934
Parameters	Q1	Q2	Q3	Q1	Q2	Q3
mean	49.4563	50.1530	49.8558	0.0000	0.0000	0.0000
volatility	8.3244	7.3526	6.8061	0.0182	0.0137	0.0121
kurtosis	3.4870	5.1025	3.9969	100.1261	55.7866	52.3385
skewness	0.5326	0.9324	0.8696	4.8348	1.5871	2.9564

Figure 5: Volatility structure



4 Empirical Analysis

In this section the two considered models presented above will be applied to the Spanish electricity market. Apart from evaluating the performance of the models by means of the Mean Squared Error (MSE) concept, the estimated prices will be compared with the observed ones to check and compare their predictive ability.

4.1 Mean Reverting Jump diffusion (MRJD) model

Once the parameters[14] have been estimated, as indicated above, it is proceeded to obtain the forward price estimation. The predicted forward price of the Cartea and Figueroa (2005) model is given by[15]

$$F(t,T) = G(T)\left(\frac{S(t)}{G(t)}\right)e^{\left(\int_t^T \frac{1}{2}\sigma^2(s)e^{-2\alpha(T-s)}ds + \int_t^T \xi(\sigma_J,\alpha,T,s)l ds - l(T-s)\right)} \tag{20}$$

where $\xi(\sigma_J, \alpha, T, s) \equiv e^{-\frac{\sigma_J^2}{2}e^{-\alpha(T-s)} + \frac{\sigma_J^2}{2}e^{-2\alpha(T-s)}}$

When maturity arrives, electricity is continuously delivery for each hour for each day included in the delivery period of the corresponding forward contract, rather than on a particular day. Thereby, for instance, the January forward contracts involves the delivery of electricity from January 1st (T_1) to January 31st (T_2). Subsequently in contrast to Cartea and Figueroa (2005) that obtain an estimated forward price for a given T day, we use equation (20) to obtain an estimated forward price for each day included in the delivery period, namely the interval $[T_1, T_2]$. The average of all of these estimations will be the specific Forward price for every t. This is given as follows

[14] Annualized estimates for the standard deviation of the jumps σ_J, frequency of the jumps l and the mean reversion rateα. The 95% confidence bounds are presented in parenthesis.

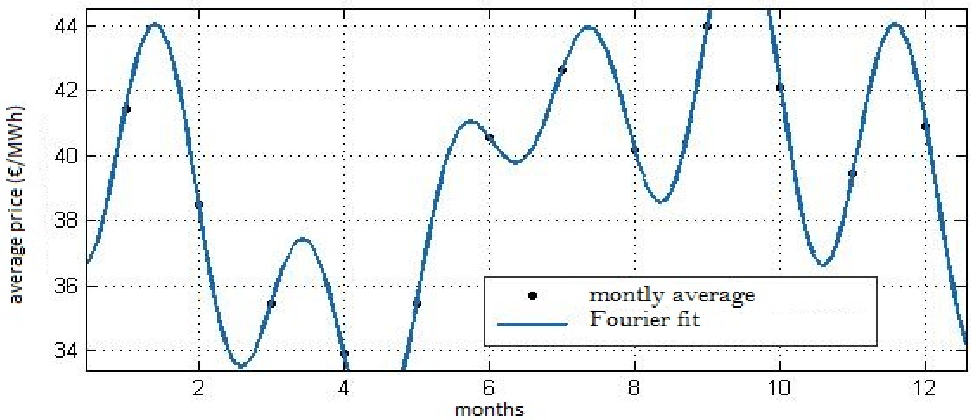
[15] Here we proceed as it is common in the literature, assuming that we are already under an equivalent measure due to the scarcity of liquidity of instruments which would enable to do the change of measure (Cartea and Figueroa, 2005).

$$F(t, T_1, T_2) = \frac{1 \sum_{T=T_1}^{T_2} F(t, T)}{T_2 - T_1} \tag{21}$$

Seasonality trend parameters estimation

As previously mentioned, electricity is a commodity with a sharp seasonal component shown in its prices (Gemman et al., 2008). Benth et al. (2005) also states that the construction of the seasonal factor modelling function is a key issue, since the specified tendency should be able to explain the market expectations for the price path for the next period (month, quarter, year). To obtain the monthly and quarterly seasonality function, we follow Cartea and Figueroa (2005). They propose a deterministic seasonality function obtained as the result of a fitting procedure where the monthly average of the spot series are fitted with a Fourier series of order 5. This procedure is applied to the monthly and quarterly averages of the historical electricity spot price for the Spanish market. Thus, it has been found that monthly averages have a goodness of fit with a Fourier series of order 5 as can be seen in figure 6.

Figure 6: Fitting Fouries series of order 5 to the monthly averages of the sample (1/01/2000-1/05/2015)



The Fourier series of order 5 equation given for this specific electricity price series is set as

$$\begin{aligned} F(x) = a_0 + a_1 cos(xw) + b_1 sin(xw) + a_2 cos(2xw) + b2sin(2xw) \\ + a_3 cos(3xw) + b_3 sin(3xw) + a_4 cos(4xw) + b_4 sin(3xw) \\ + a_5 cos(5xw) + b_5 sin(xw) \end{aligned} \tag{22}$$

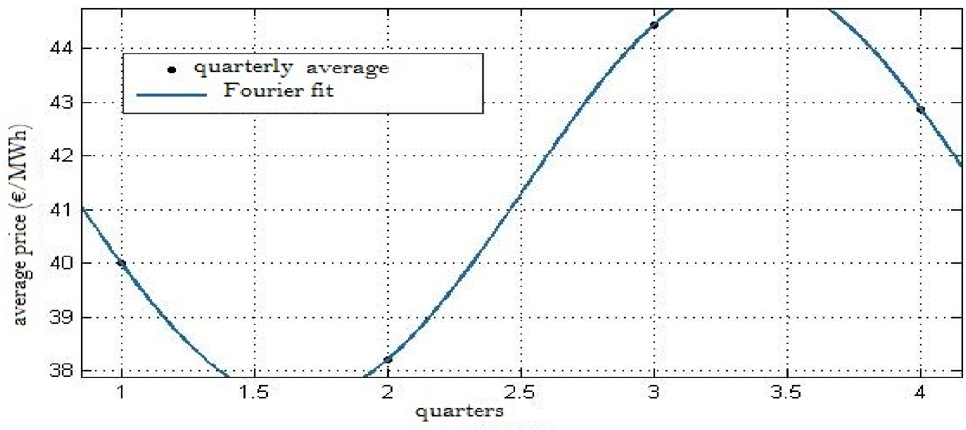
where the parameters have been obtained as the result of fitting the series to the corresponding Fourier series of order 5[16], and x takes the value corresponding to the

[16] The equation and the parameters obtained and the order of the Fourier series has been obtained through a testing procedure using the 'curve fitting' application in the Matlab program.

month under evaluation. The estimated parameters are shown in Appendix B.

On the other hand, the quarterly averages have the highest level of goodness of fit with Fourier series of order 1 as Figure 7 shows.

Figure 7: Fitting Fourier series of order 1 to the quarterly averages of the sample (1/01/2000-1/05/2015)



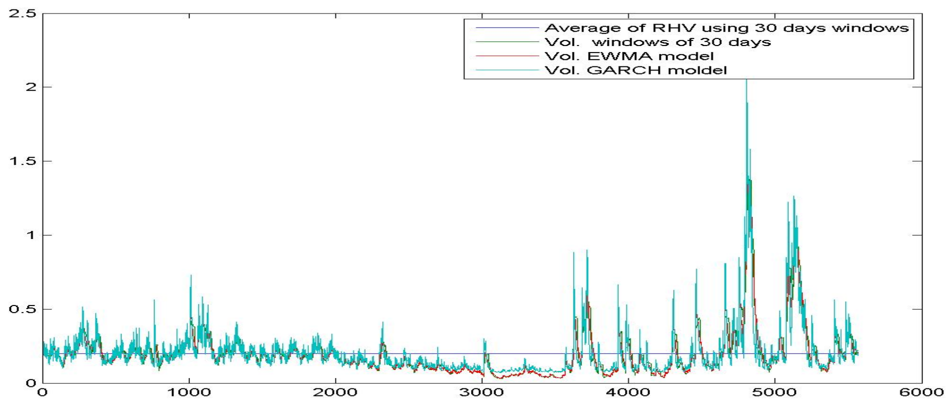
and its respective equation is given by

$$F(x) = a_0 + a_1 cos(xw) + b_1 sin(xw) + a_2 cos(2xw) \tag{23}$$

Spot price volatility '$\sigma(t)$'

As expected, spot price volatility is not constant across time. Estimating a moving historical volatility is usually considered a useful approach to check this feature (Cartea and Figueroa, 2005). Figure 8 shows the evolution of the volatility across time. In this case,a window of 30 days has been used.

Figure 8: Rolling Historical Spot Price Volatility (31/01/2000-25/04/2015)



As can be seen in Figure 8, there is not a clear seasonal pattern in the spot price volatility, which greatly contrasts with other analysed markets such as the

European Energy Exchange (EEX) studied by Benth et al. (2012). In that paper, three electricity spot price models are compared. It seems that volatility in the Spanish market presents a certain stochastic element. Volatility appears to be time dependent, which is consistent with Cartea and Figueroa (2005). Benth et al. (2012), however, set the volatility as a constant, and calculate it as the overall average of the rolling historical volatility over the period with data. In this paper, a GARCH model[17] has been used in order to obtain a volatility value for every single t.

Furthermore, the volatility obtained with the GARCH-model is more varying than the produced with windows of 30 days or through the EWMA approach (Appendix C shows a shorter sample of 200 observations). Moreover in contrast with the use of windows by using instead the EWMA or GARCH models we do not throw away any observation. If we use windows the high returns effects can does not disappear quickly affecting the following estimations. Besides, the weight of each observation included in the window is considered to be equal which is not realistic due to de different values taken by every single observation. In order to avoid the drawbacks the EWMA or GARCH model can be used. With any of them the weight parameter decreases exponentially as moving back in time. Between them, GARCH model, in contrast with the EWMA model generates a reversion to the long-run average volatility rate (Haochen 2012), which makes it specially suitable for electricity . For all of these reasons the GARCH model has been finally selected to compute spot price volatility.

Mean-Reversion Rate 'α'

To obtain the estimation for the mean-reversion rate we proceed as it was described by Cartea and Figueroa (2005). They suggest linear regression as a good alternative to estimate this parameter. In this case we regress the returns Δx_t versus the series x_t of the log-spot price.

Jump parameters 'l''

As it was tested above, the returns cannot be modelled by a normal distribution due to the presence of fat tails[18]. This suggesting that the probability of rare events occurring is higher than that predicted by a Gaussian distribution. Figure 2 showed that the Gaussian distribution cannot capture the spikes produced in this singular market. In order to obtain jumps from the data, the technique that is based on the standard deviation of the returns, applied by Cartea and Figueroa (2005), will be used. They present an iterative procedure in order to extract the jumps from the original series of returns. They do this by filtering out the returns with

[17] GARCH model is considered as a good alternative to EWMA model and rolling volatility obtained thought windows of a set number of observations. See Appendix C to further information.

[18] The high Kurtosis shown for the returns analysis gave us a primarily indication about the existence of this feature.

absolute values higher than three times the standard deviation of historical data series' returns, at the current iteration. This algorithm operates until no further returns can be filtered. The main result of this procedure is the standard deviation of jumps, σ_j and the cumulative frequency of jumps, l.
As can be observed in Figure 9, which contrasts to Figure 2, the filtered returns are closer to be normally distributed. The right graph shows the contrast between the filtered and the non-filtered returns and the differences are clear between these two series.

Figure 9: QQplot filtered serie

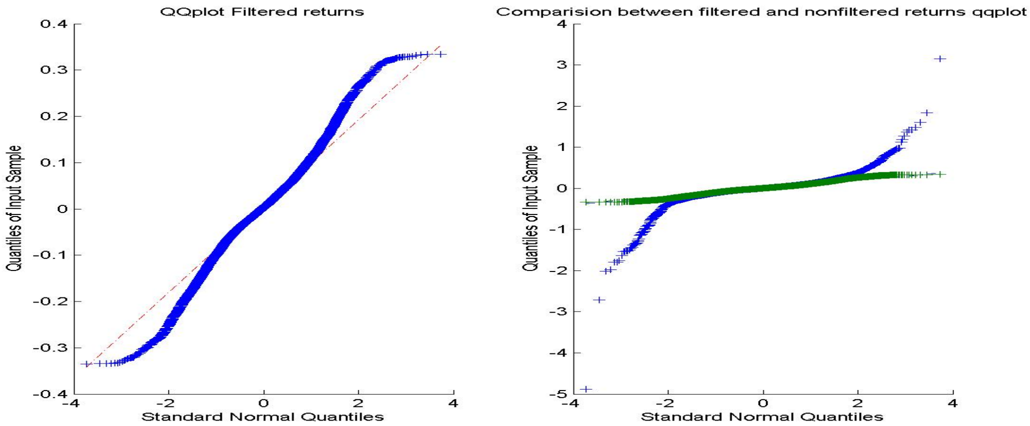


Table 3 displays the estimated parameters for the stadard deviation of the jumps, frequency of the jumps and the mean reversion rate[19] for the Spanish electricity market.

Table 3: Annualised estimates for the standard deviation of the jumps σ_j, frequency of the jumps l, and the mean reversion rate α.

Parameter	Value
σ_J	0.8004
l	21.0137
α	0.5081 (0.1271, 0.8891)

4.2 Stochastic Forward Premium (SFP) Model

Having a historical dataset of daily swaps curves $F_i(t,T)$ [20] t=1,...n, where vector T indicates the different swap contracts starting delivery dates which are available at trading date t. The starting point will be the SFP model in logarithmic version,

$$lnF_i(t,T) = ln\overline{F_i}(t) + s_i(K) + \gamma_i(t,T) \tag{24}$$

[19] At the 95% confidence level.

[20] As it was indicated above, the subscript i refers to the electricity delivery period over which is defined each curve's forward contract

where the first component's least squares optimal estimator, $ln\overline{F_i}(t)$, is the arithmetic average of log-swap prices within each market segment i, as expressed in the equation (10). The other two remaining components will be analysed below.

Seasonal Factor

In order to analyse this component[21], the procedure applied by Borovkova and Geman (2006) has been followed (See Appendix D for further indications.). Figure 10 display the seasonal factor for month-ahead contracts (left) and quarterly-ahead contracts (right).

The monthly seasonal component shows higher positive spikes in months such as January, July and November. On the other hand, there are just three negative spikes being the highest one identified in April. The explanation for these peaks can be mainly found from factors such as the season of consumption together with some working patterns. For instance, December and January are typically very cold months resulting in a higher demand for heating and, hence, higher prices. Moreover this effect is somewhat compensated in December due to the great impact of the Christmas holidays on the electricity demand by many enterprises which reduce or even stop their production or activity for this period. November is overall also a cold month and in this month the level of working patterns is higher than the remaining winter months resulting in a positive seasonal factor due to the higher demand of electricity for this period of the year. On the contrary, it is in April when the demand reaches the lowest peak, giving rise to a negative seasonal factor. This fact is directly linked to the non-storable possibilities of the electricity. The existence of inventories would help to reduce identified differences between months.

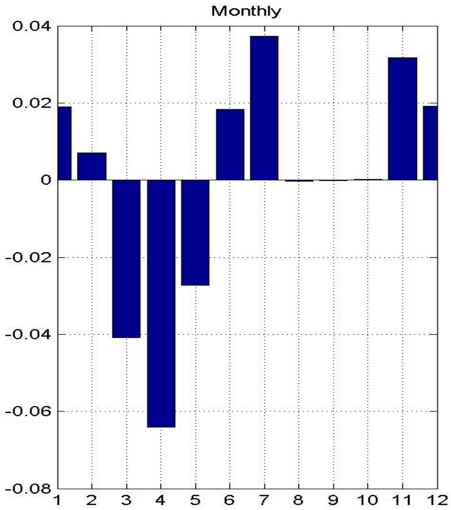
The quarterly seasonal factor (right side of Figure 10) shows the quarterly pattern of electricity pricing throughout year. Thus, it is in the first quarter where the highest value appears mainly due to the effect of usage of heating. Also, during the Summer season the demand for electricity futures rises due to the widespread use of air-conditional for this reason the third quarter display a positive value. It contrast with Spring and Fall, where the (mild) weather pushes the electricity demand down obtaining a greater evidence of such an effect during Spring, when the seasonal factor becomes negative (second quarter). This results is consistent with the obtained one for April, when analysing monthly seasonality. The fourth quarter show the lowest seasonal positive value due to the October and December effects.

[21] Is should be mentioned that the seasonal factor must be zero on average for each seasonal period (monthly or quarterly) having the following equality (Borovkova and Geman, 2006):

$$\sum_{k=1}^{K} S_i(k) = 0$$

.

Figure 10: Seasonal Factor for Monthly and Quarter segments. The quarters are organized as follow: $1^oQ = January, February\ and\ March; ...; 4^o = October, Nobember, December$



Stochastic Forward Premium SFP

To estimate this component, the equation (12) is used. Its evolution is revealed in Appendix D. Once the SFP has been estimated as well as the arithmetic mean of the logarithmic prices, the next step is obtaining the parameters of the MNIG distribution followed by the X vector (equations 13-18), being this latter vector an estimation of the vector $dL(t)$.

Thus, the procedure used will be the same as in Blanco et al. (2014) and it will be carried out in two phases as well. Firstly, all the parameters of mean reversion and volatility in discrete-time versions of equations (14) and (15) will be estimated through a Seemingly Unrelated Regression Equations (SURE)[22]. The system is described as follows,

$$\begin{bmatrix} \Delta ln\bar{F}_i(t) \\ \Delta\hat{\gamma}_i(t,T) \end{bmatrix} = \begin{bmatrix} \kappa_i\varsigma_i - \kappa_i ln\bar{F}_i(t-1) \\ \varpi_{i,T}\hat{\gamma}_i(t-1,T) \end{bmatrix} + \begin{bmatrix} \theta_{\bar{F}_i}\epsilon_{\bar{F}_i}(t) \\ \theta_{\gamma_i^{(T)}}\epsilon_{\gamma_i^{(T)}}(t) \end{bmatrix} \tag{25}$$

$$i = 1(M), 2(Q), 3(Y)$$

$$T = 1, ..., 6$$

$$t = 1, ..., n$$

There will be three systems of 4 equations for each i. The set of 12 series will be grouped in sets of series which are interrelated[23].

[22]This model has been estimated using Generalised Least Squares (GLS). Appendix E displays the estimated parameters

[23]There is 3 series for each segment market plus one $ln\bar{F}$ series for each segment market.

As suggested in Urzua (1997) the follow normalization will be used, Let the residuals from equation (25) be defined as

$$Y = \begin{bmatrix} \epsilon_{\bar{F}_i}(t) \\ \epsilon_{\gamma_i^{(T)}}(t) \end{bmatrix} \tag{26}$$

$$t = 1, ...n$$

Y is a vector with dimension of $4xn$ for each set. This vector has a mean vector of zero and covariance matrix, Ω. Obtaining the eigenvalues and eigenvectors of this latter matrix we have the following decomposition,

$$\Omega^{-1/2} = \Gamma \wedge^{-1/2} \Gamma' \tag{27}$$

where Γ denote the orthogonal matrix whose columns are the standardized eigenvectors of Γ , and Γ denote the diagonal matrix of the eigenvalues of Ω. Moreover $\Omega^{-1/2}$ refers to the inverse of the square root decomposition of Ω. Then, we obtain the X vector as

$$X = \Omega^{-1/2} Y \tag{28}$$

Once the parameters for the MNIG distribution have been obtained, the NIG distribution will be used in order to obtain the individual estimation for each singular market segment[24](Appendix F displays the MNIG parameters).

5 Model Comparison

5.1 In-sample estimation

In this section we will discuss the properties present in each estimated curve. Figures 11-22 show the in-sample simulations for the MRJD and SFP model. Firstly, and as a first 'test' for both models the corresponding simulated path has been obtained. Visually, an acceptable fit of the movements of each simulated curve with regards to the observed market prices can be observed, seeming the performance of both models quite satisfactory. However, although the series' changes obtained from the MRJD model move in the same direction as the market forward price, they present more noise than the observed series' changes as we can see in Figures 11-16 and as its volatility points out (Table 5). The principal reason of this effect is that in the first model we are using as principal component of the estimation procedure the spot price, whereas the SFP model focuses on the forward curve prices directly. In fact, the second model does not only seems to capture the movements quite satisfactory,

[24]The univariate NIG is the marginal distribution of the MNIG.

but also the estimations are very close to the observed market forward for every market segment. In contrast to the first model it can be note that the movements are no so sharply and move accordingly to the forward market price.

Table 4 shows the MSE value for each contract maturity for both models. The MRJD model display a higher MSE value for all the considered maturities in each market segment (monthly and quarterly) confirming the visuall intuition.

Table 4: MSE values to evaluate the 'goodness-of-fit' of the models. The MSE has been obtained from the returns.

	MRJD			SFP		
Parameter	M+1	M+2	M+3	M+1	M+2	M+3
MSE	0.1979	0.1923	0.1358	0.00098	0.00055	0.00054
Parameter	Q+1	Q+2	Q+3	Q+1	Q+2	Q+3
MSE	0.1766	0.11309	0.0994	0.00112	0.00102	0.00064

Table 5: Mean and Volatility of the series. The volatilities have been obtained from the returns.

	Mean			Volatility		
Model	M+1	M+2	M+3	M+1	M+2	M+3
Historical	48.72510	49.55790	50.03215	0.0247	0.0212	0.0140
MRJD model	49.4910	52.3001	54.0005	0.2922	0.2873	0.2385
SFP model	48.7236	49.58160	50.06662	0.03355	0.02723	0.02408
Parameters	Q+1	Q+2	Q+3	Q+1	Q+2	Q+3
Historical	49.45171	50.15333	49.8597	0.018285	0.0137	0.0121
MRJD model	54.2447	53.8595	51.8230	0.420604	0.3120	0.2937
SFP model	49.50246	50.23922	50.33618	0.03309	0.03137	0.02568

MRJD model estimations

Figure 11: The simulated forward vs observed forward dynamics from July 02, 2007 to April 23,2015 -MRJD model-M1

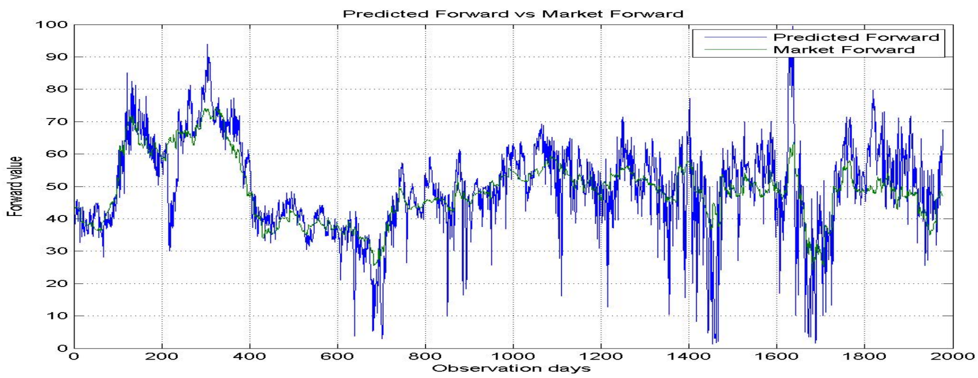


Figure 12: The simulated forward vs observed forward dynamics from July 02, 2007 to April 23,2015 -MRJD model-M2

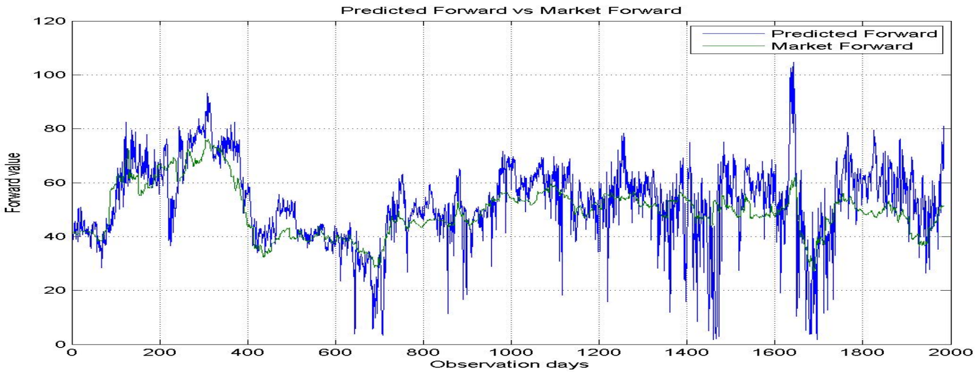


Figure 13: The simulated forward vs observed forward dynamics from January 02, 2007 to April 23,2015 -MRJD model-M3

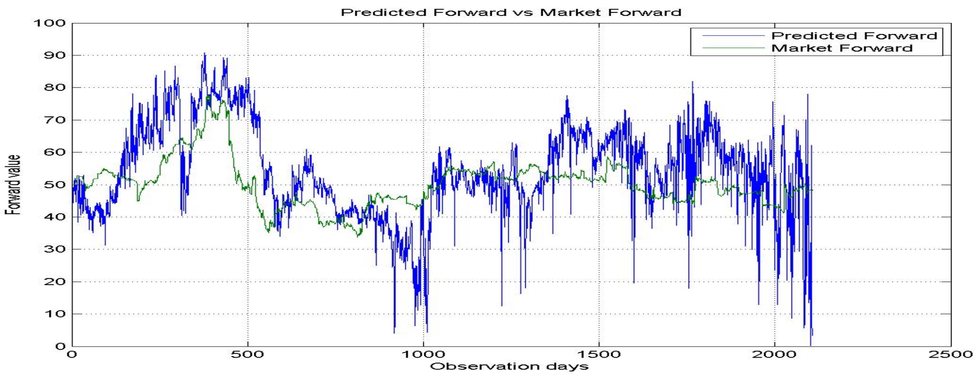


Figure 14: The simulated forward vs observed forward dynamics from July 03, 2006 until to 23,2015 -MRJD model-Q1

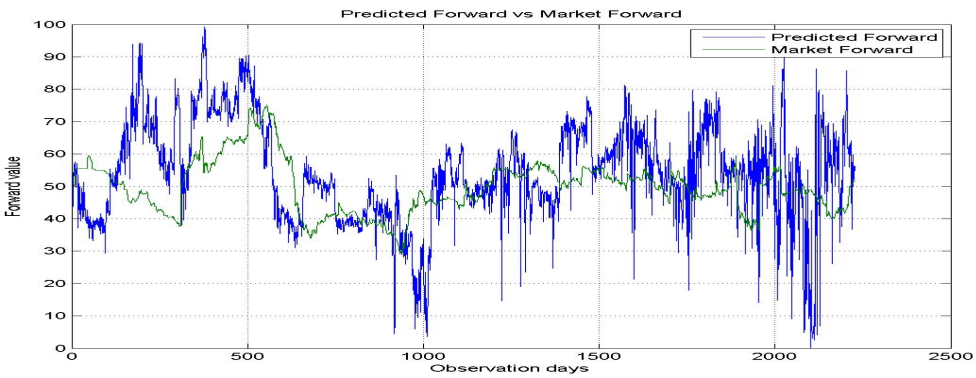


Figure 15: The simulated forward vs observed forward dynamics from January 02, 2007 to April 23,2015 -MRJD model-Q2

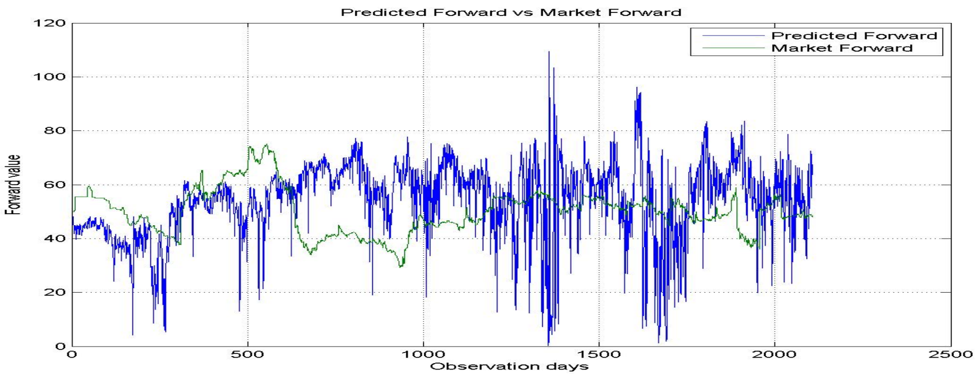
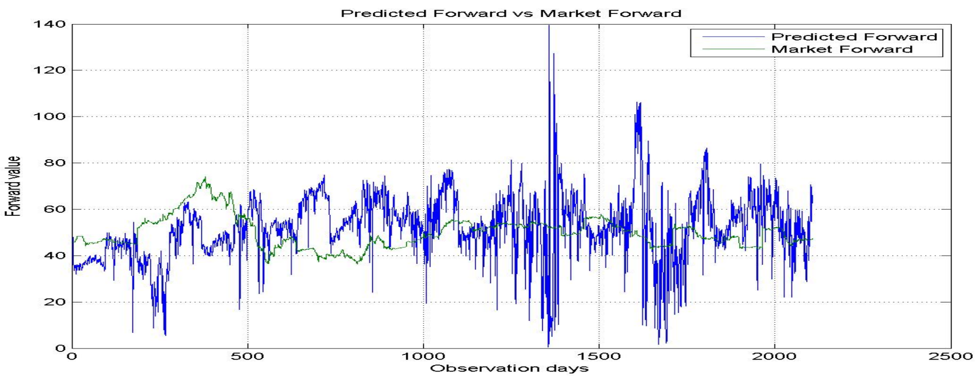


Figure 16: The simulated forward vs observed forward dynamics from January 02, 2007 to April 23,2015-MRJD model-Q3



SFP model estimations

Figure 17: The simulated forward vs observed forward dynamics from July 02, 2007 to April 23,2015 -SFP model-M+1)

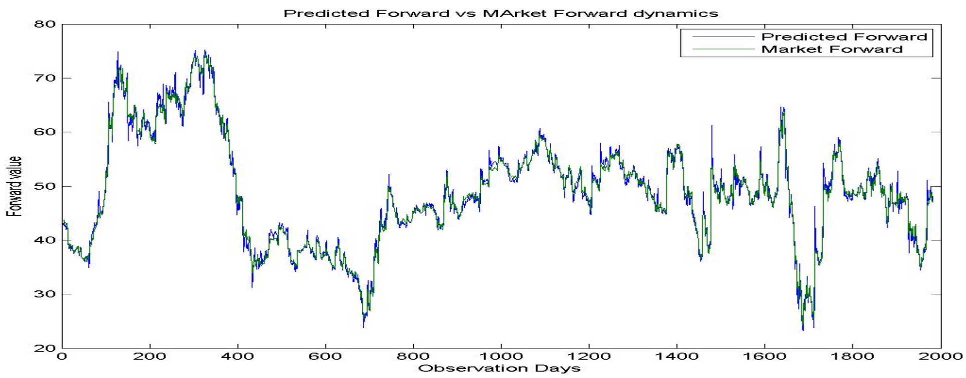


Figure 18: The simulated forward vs observed forward dynamics from July 02, 2007 to April 23,2015 -SFP model-M+2)

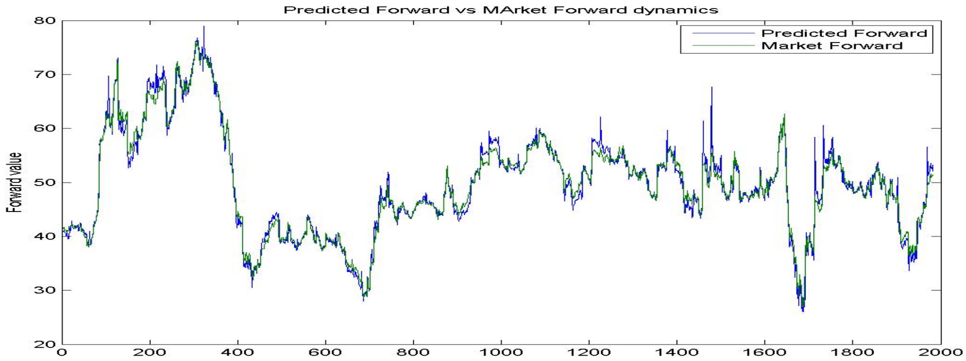


Figure 19: The simulated forward vs observed forward dynamics from January 02, 2007 to April 23,2015 -SFP model- M+3)

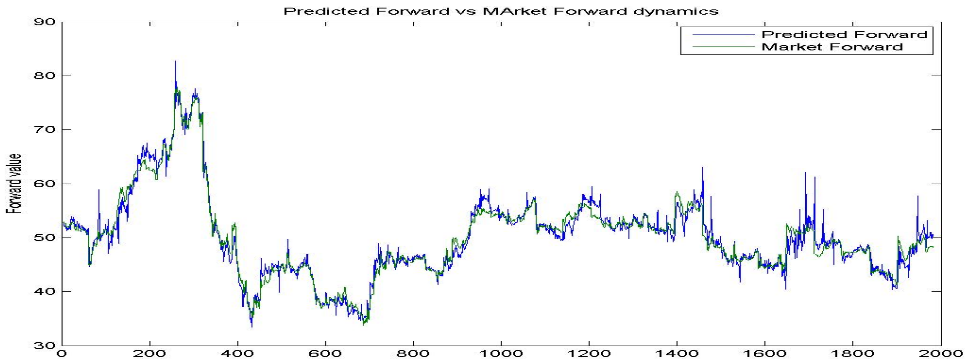


Figure 20: The simulated forward vs observed forward dynamics from July 03, 2007 to April 23,2015-SFP model-Q+1)

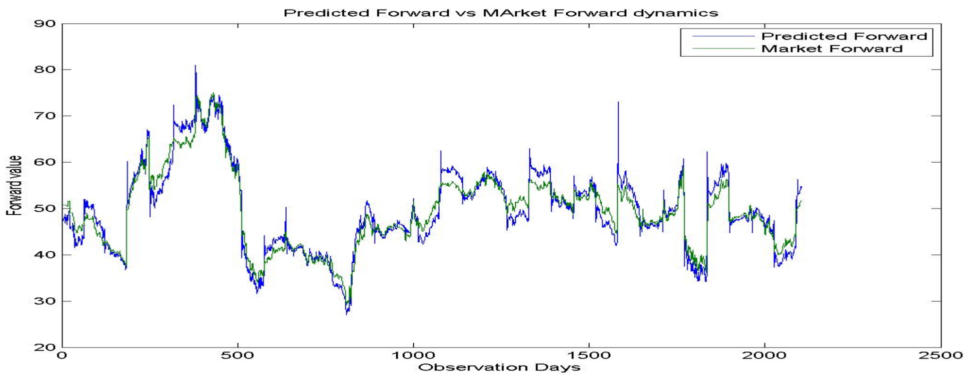


Figure 21: The simulated forward vs observed forward dynamics from January 02, 2007 to April 23,2015-SFP model-Q+2)

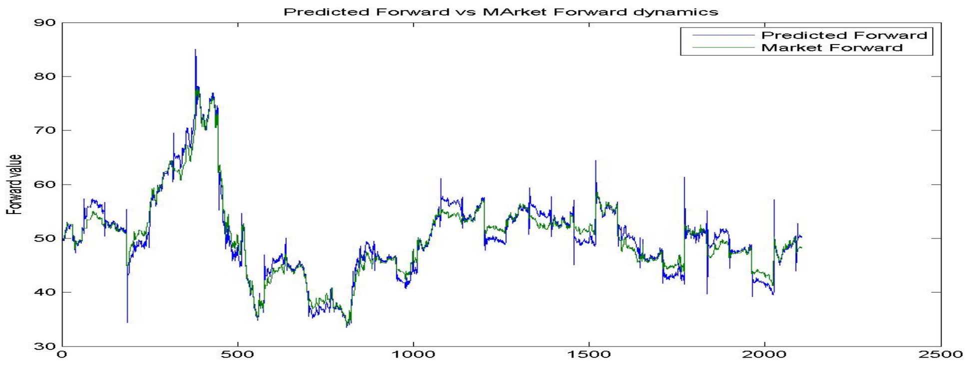
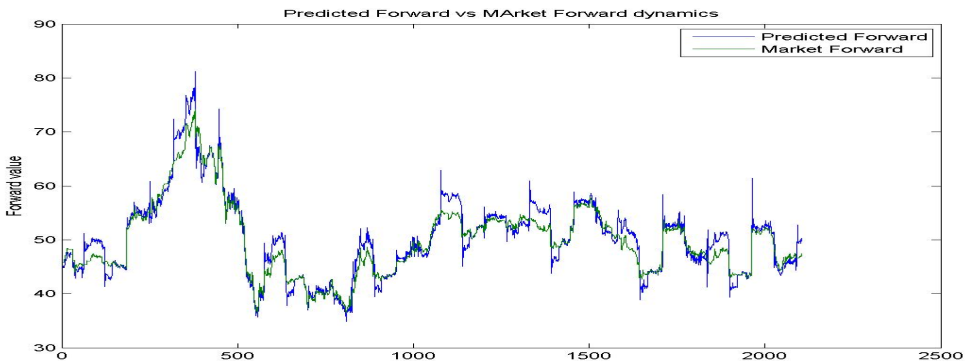


Figure 22: The simulated forward vs observed forward dynamics from January 02, 2007 to April 23,2015 -SFP model- Q+3)



5.2 Out-of-sample estimation

Generally speaking,similar conclusions obtained from the in-sample are achieved for the out-of-sample one day estimation. Thus, Table 6 evidences that the SFP model has a better 'goodness-of-fit' showing smaller values for every kind of contract than the obtained with the MRJD model. Furthermore, Table 7, confirms these results given the similarities between the historical and the simulated series obtained from the SFP model. Volatility tend to decrease with time to maturity in both models as in the historical series, however, the SFP model has the closer values to the observed series' values than the MRJD model.

.

Table 6: MSE values to evaluate the 'goodness-of-fit' of the models, November 2013. The MSE has been obtained from the returns.

	MRJD			SFP		
Parameter	M1	M2	M3	M1	M2	M3
MSE	0.22189	0.12933	0.00008	0.00038	0.000014	0.00003
Parameter	Q1	Q2	Q3	Q1	Q2	Q3
MSE	0.13093	0.13170	0.17670	0.00005	0.00003	0.00003

Table 7: Mean and Volatility of the series. The volatilities have been obtained from the returns.

	Mean			Volatility		
Model	M+1	M+2	M+3	M+1	M+2	M+3
Historical	48.15476	51.03619	44.93000	0.01649	0.00874	0.00407
MRJD model	41.7472	46.5198	59.4501	0.4862	0.3703	0.0297
SFP model	47.8368	50.70706	44.77959	0.01285	0.00806	0.00426
Parameters	Q+1	Q+2	Q+3	Q+1	Q+2	Q+3
Historical	49.21761	44.93000	51.99190	0.00471	0.00407	0.0041
MRJD model	42.8549	53.1321	66.4992	0.3730	0.3721	0.2937
SFP model	51.11986	44.90998	52.982817	0.00484	0.00394	0.00413

MRJD model estimations

Figure 23: The predicted forward vs observed forward dynamics with maturity in December 2013 (M+1)

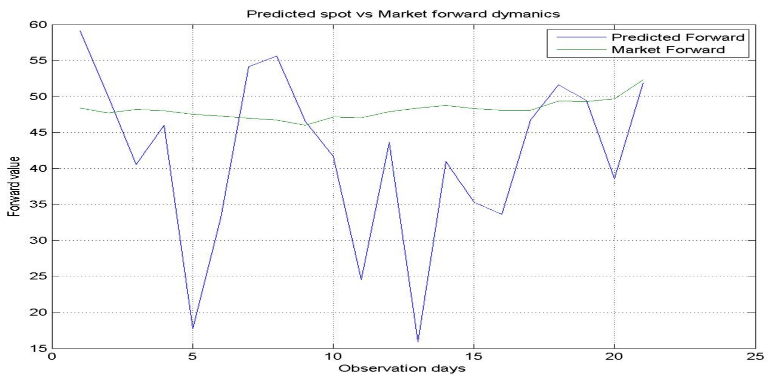


Figure 24: The predicted forward vs observed forward dynamics with maturity in January 2013 (M+2)

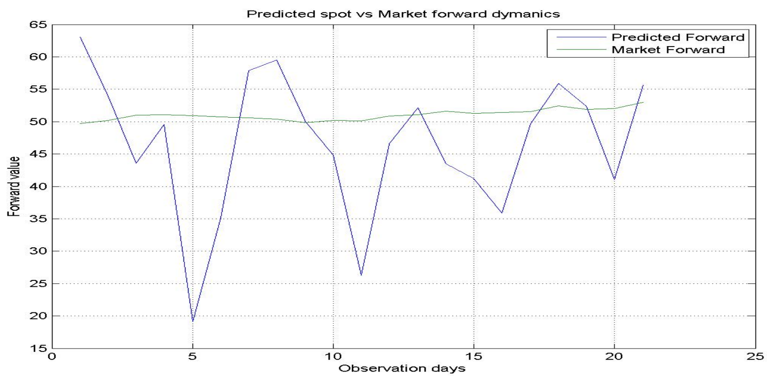


Figure 25: The predicted forward vs observed forward dynamics with maturity in February 2014 (M+2)

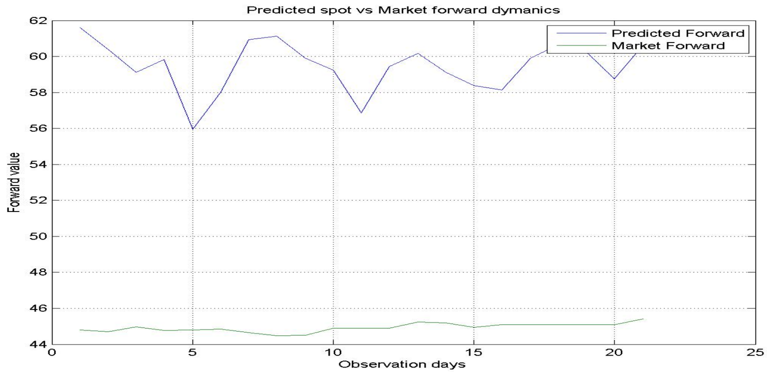


Figure 26: The predicted forward vs observed forward dynamics with maturity in the first quarter of 2014 (Q+1)

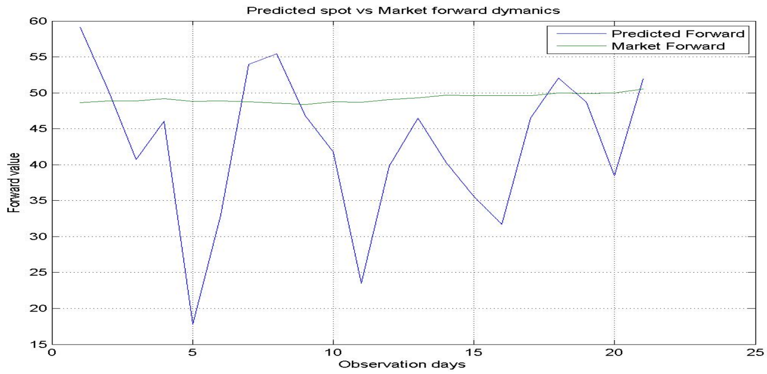


Figure 27: The predicted forward vs observed forward dynamics with maturity in the second quarter of 2014 (Q+2)

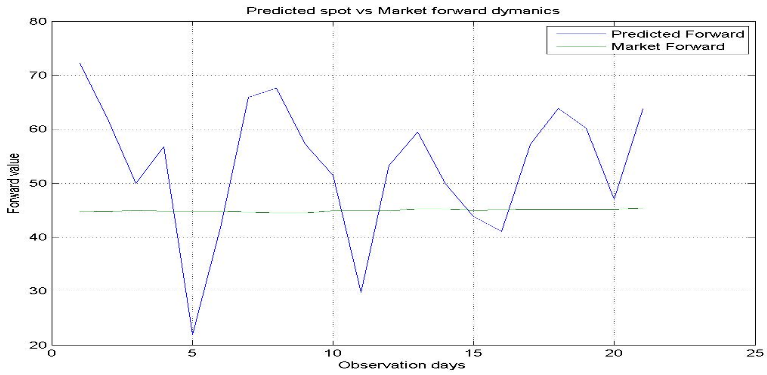
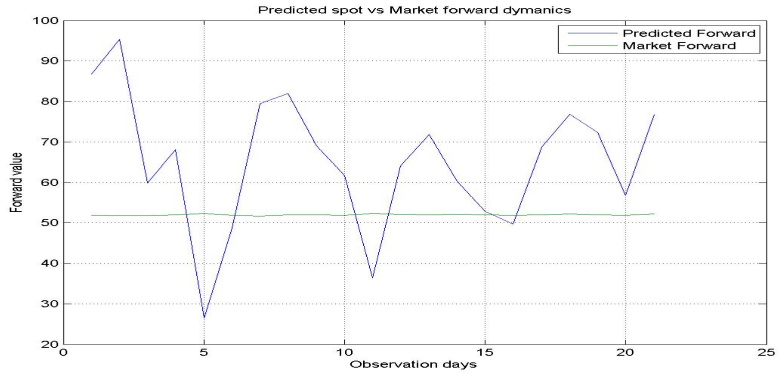


Figure 28: The predicted forward vs observed forward dynamics with maturity in the third quarter of 2014 (Q+3)



SFP model estimations

Figure 29: The predicted forward vs observed forward dynamics with maturity in December 2013 (M+1)

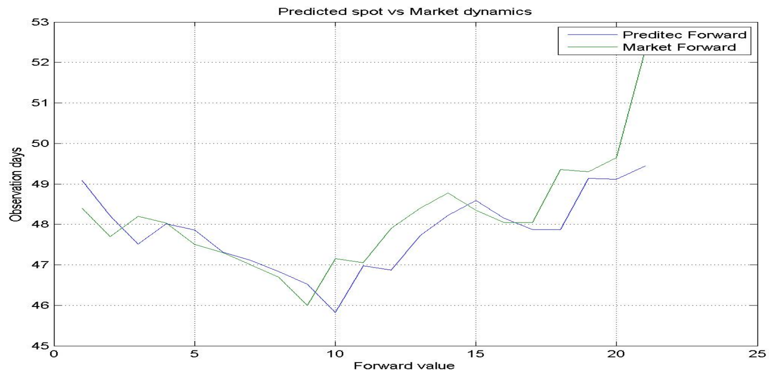


Figure 30: The predicted forward vs observed forward dynamics with maturity in January 2013 (M+2)

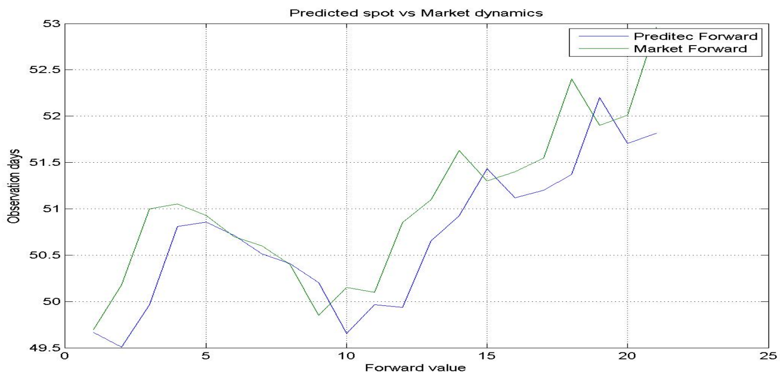


Figure 31: The predicted forward vs observed forward dynamics with maturity in February 2014 (M+3)

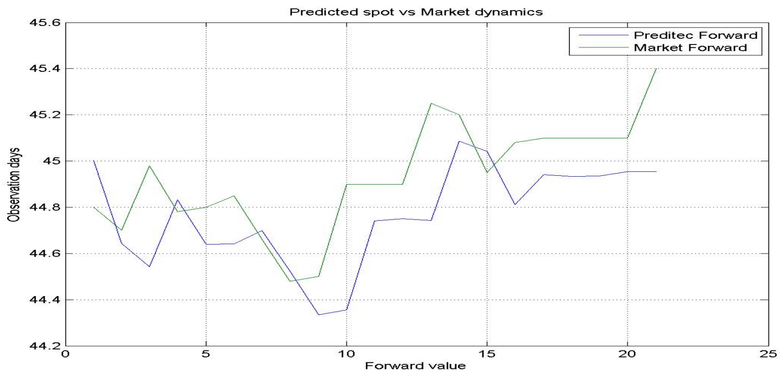


Figure 32: The predicted forward vs observed forward dynamics with maturity in the first quarter of 2014 (Q+1)

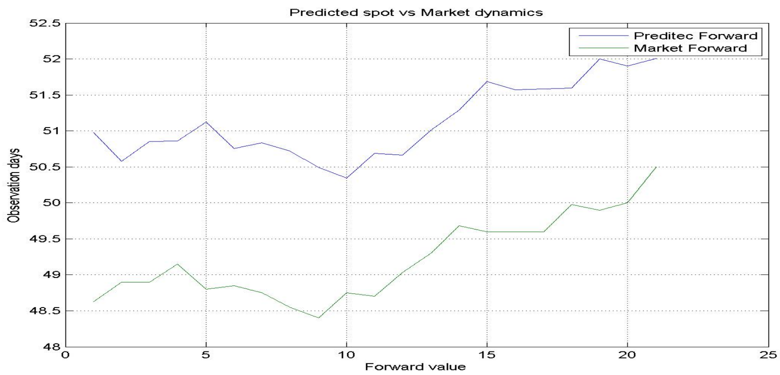


Figure 33: The predicted forward vs observed forward dynamics with maturity in the second quarter of 2014 (Q+2)

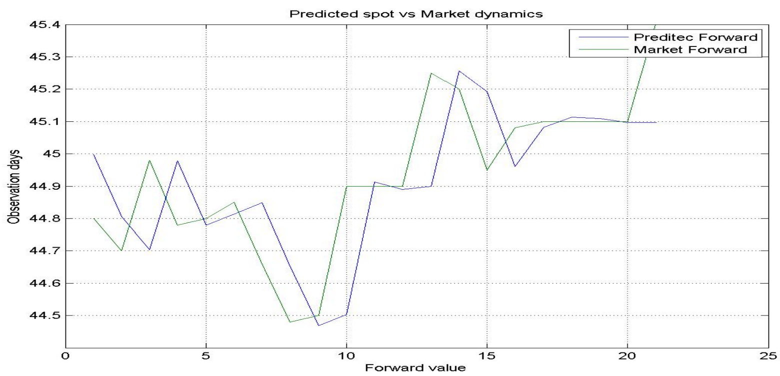
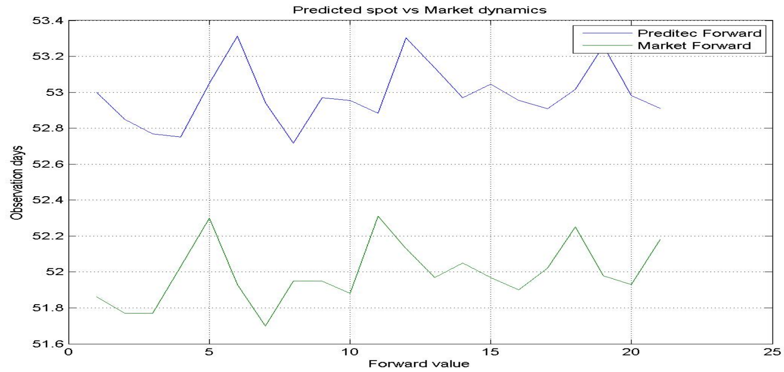


Figure 34: The predicted forward vs observed forward dynamics with maturity in the third quarter of 2014 (Q+3))



6 Conclusions

This work has carried out an empirical analysis with the aim to compare two estimation models for electricity forward prices, each of them representative of two different modelling approaches, the MRJD model, a spot-based model, and the SFP model, which uses the forward curve to price derivatives. The results of the application of these two models to the Spanish electricity market have shed light not only on the characteristics of the Spanish electricity prices, but also on the fitting level of each model to this specific market.

A preliminary visual analysis, it can be observed that the SFP model shows a better fit than that shown by the MRJD model. Then this result is confirmed by the Mean Squared Error (MSE) of the differences between the observed and estimated forward prices by each model. Within each model, in the in-sample analysis, the farthest maturities present a better adjustment. In the case of out-of-sample prediction, it is the monthly contracts the ones that show a similar behaviour to the in-sample estimation, whereas for the quarterly contracts, the opposite happens. Thus, in the MRJD model, the closer maturities show a better fit, while in the SFP model it is the farthest maturities the ones that present the best adjustment.

Although the MRJD model describes the stylized facts of electricity spot price quite well, it becomes less suitable for further analysis of derivatives pricing, being the SFP model the one that shows better abilities for modelling the Spanish forward contracts in the electricity market, and the Quarterly segment the one that can be modelled most accurately.

According to our results, the spot-based model could benefit from the introduction of a daily seasonal factor. Moreover, the analysis could be extended to yearly contracts in order to evaluate how the models work for this kind of contracts where the seasonal factor is not taken into account. This is left for further research.

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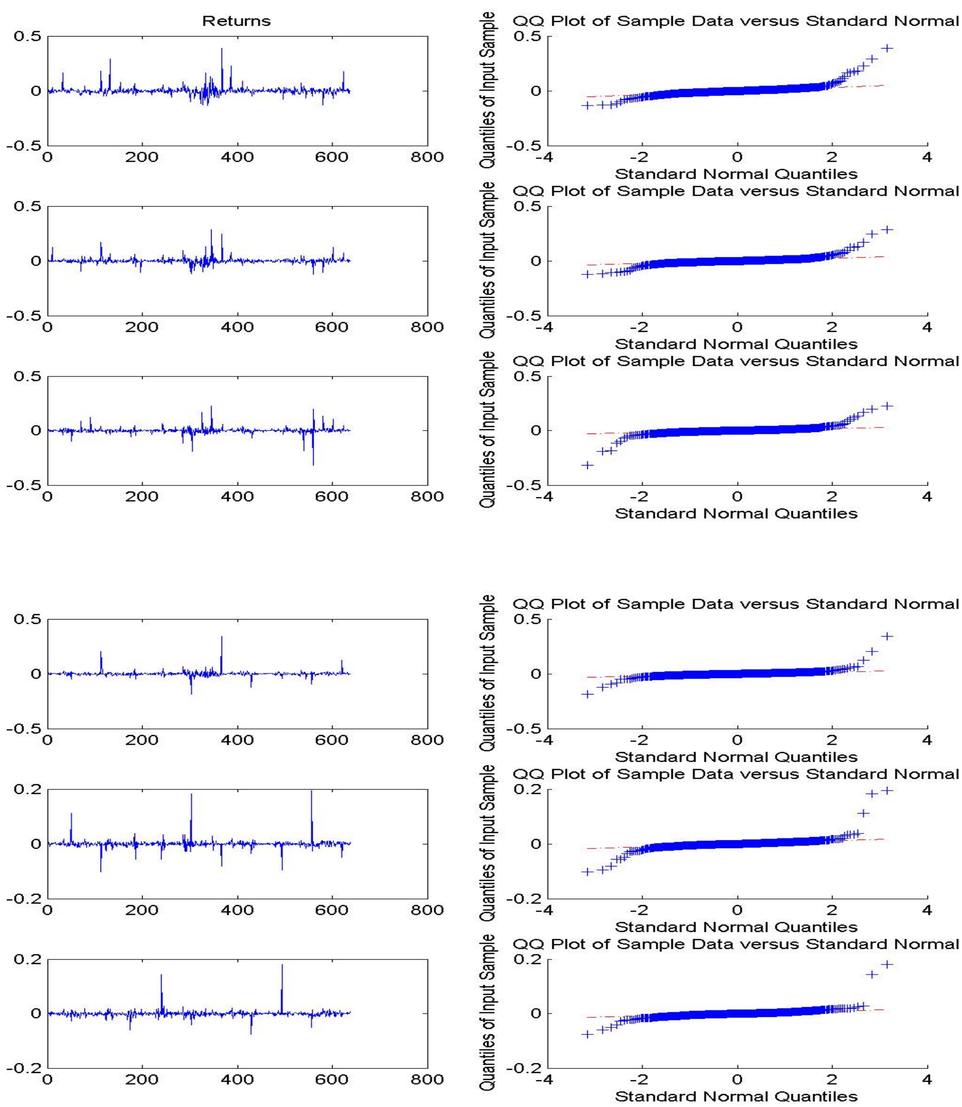
7 Appendix

7.1 Appendix A

Figure 35: Returns and their qqplots



7.2 Appendix B

Table 8: Parameters of the MRJD model for November (2013) estimation.

Parameter	November
σ_j	0.74367
l	17.69169
α	0.018835

Parameters obtained for the Fourier series of order 5 and 1 are showed in Table 9 and 10 respectively.

Table 9: Fourier series of order 5's parameters

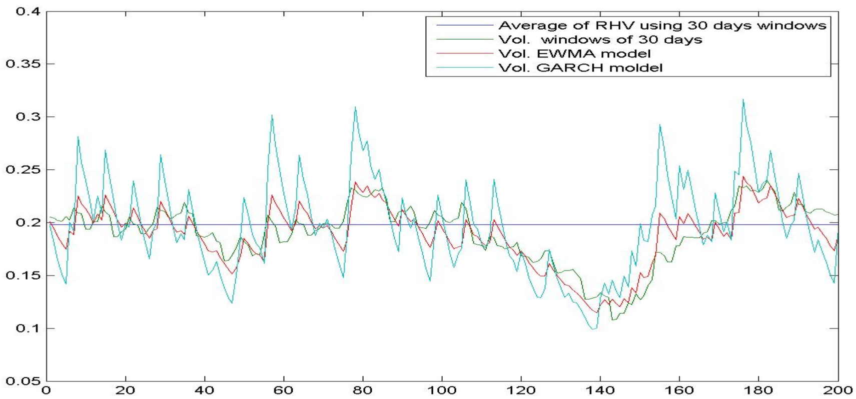
Parameter	Value	Parameter	Value
a_0	39.33	w	0.6176
a_1	2.753	b_1	-2.954
a_2	-0.1231	b_2	1.283
a_3	0.08702	b_3	-1.078
a_4	-1.14	b_4	-0.8584
a_5	-1.266	b_5	-0.6176

Table 10: Fourier series of order 1's parameters

Parameter	Value	Parameter	Value
a_0	41.37	w	1.846
a_1	3.903	b_1	-0.3111

7.3 Appendix C

Figure 36: Rolling Historical Volatility over a set of 200 observations (31/01/2000-17/08/2000



7.4 Appendix D

Figure 37: Stochastic factor evolution (γ_M) M+(1,2,3)

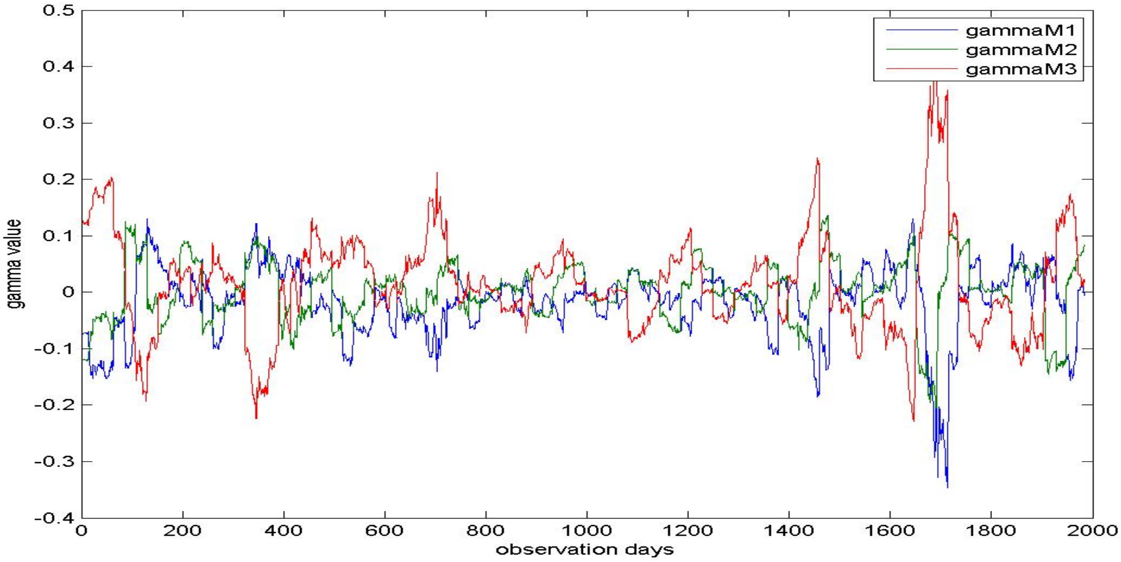
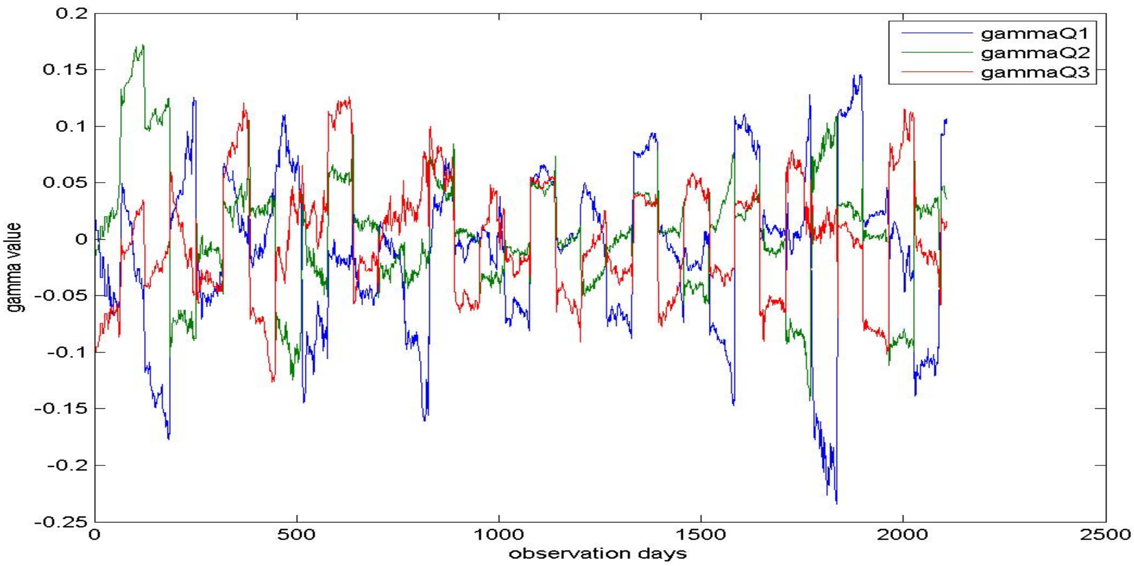


Figure 38: Stochastic factor evolution (γ_Q) for Q+(1,2,3)



Seasonal deterministic premia $s_i(K)$ estimation procedure (Borovkova and Geman, 2006):

As Borovkova and Geman (2006), we obtain the differences $s(M) - s(L)$ for all possible combinations of moths (M, L), by averaging $lnF(t, M) - lnF(t, L))$ over the entire historical dataset. Here the fact that $\gamma^{\tau}(t)$ is zero on average for all τ. In this way we obtain the matrix of the difference estimates:

$$\begin{bmatrix} \widehat{s(1)-s(2)} & & & & \\ \widehat{s(1)-s(2)} & \widehat{s(1)-s(2)} & & & \\ \widehat{s(1)-s(2)} & \widehat{s(1)-s(2)} & \widehat{s(1)-s(2)} & & \\ . & . & . & . & \\ . & . & . & . & \\ . & . & . & . & \\ \widehat{s(1)-s(2)} & \widehat{s(1)-s(2)} & ... & ... & \widehat{s(1)-s(2)} \end{bmatrix} \tag{29}$$

The individual estimates for $s(M)$ can be obtained by adding up the columns of the above matrix and using the restrictions $\sum_{M=1}^{12} s(M) = 0$. Denoting the sun of the first column by
$Sigma_1$ we have

$$\Sigma_1 = 11s(1) - \sum_{M=2}^{12} s(M) = 12s(1) \tag{30}$$

Thus, from the first column sum Σ_1 the estimation for s(1) is obtained as

$$\hat{s}(1) = \frac{\Sigma_1}{12} \tag{31}$$

The sum of the second column is denoted by

$$\Sigma_2 = 10s(1) - \sum_{M=3}^{12} s(M) = 11s(2) - s(1) \tag{32}$$

Thus, the estimation for s(2) is

$$\hat{s}(1) = \frac{\Sigma_2 + \hat{s}(1)}{11} \tag{33}$$

The procedure is similar to obtain the remaining estimations for all $s(M)$, $M = 1, 2, ..., 12$.

Once the seasonal factors have been estimated we proceed to e obtain the the estimation for the $ln\bar{F}_i$. Thus, if N denotes the number of available expiries and suppose that at date t, the first expiry month is January. Note that in this case (for a specific i)

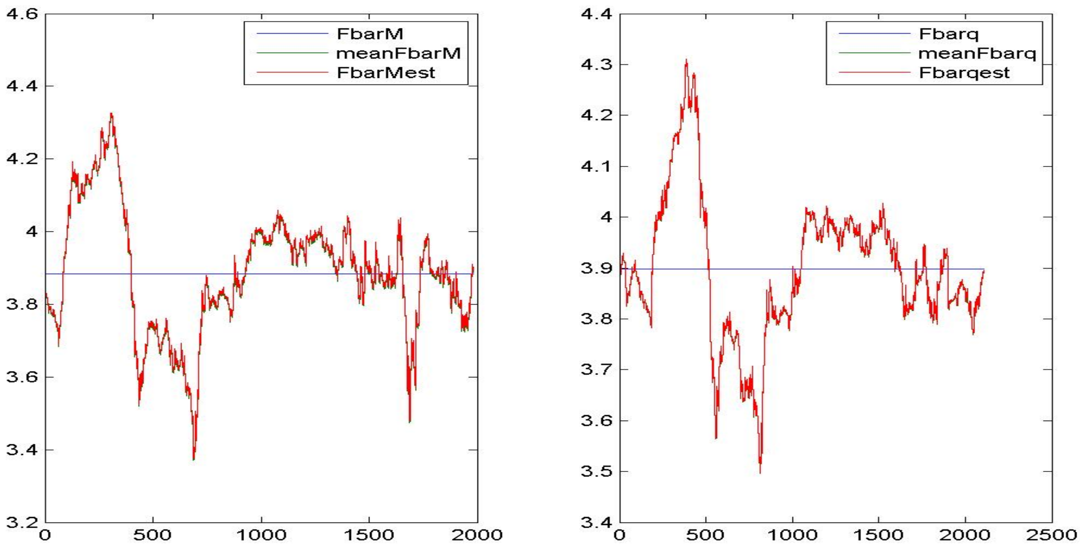
$$12ln\bar{F}_i(t) = \sum_{M=1}^{12} F_i(t, M) \approx \sum_{M=1}^{N} F_i(t, M) + (12 - N)ln\bar{F}(t) + \sum_{M=N+1}^{12} s(M) \tag{34}$$

and taking into account the fact that $\gamma^{tau}(t)$ is essentially zero. Hence, the estimate of $ln\bar{F}_i(t)$ is

$$\widehat{ln\bar{F}_i(t)} = \frac{\sum_{M=1}^{N} + \sum_{M=N}^{12} \hat{s}(M)}{N} \tag{35}$$

Figure 40 show the comparison between $ln\bar{F}$ and $ln\bar{F}$ estimated as described above.

Figure 39: Comparison between $ln\bar{F}$ and $ln\bar{F}$ estimated



7.5 Appendix E

Table 11: SURE model's parameters

Parameters	$ln\bar{F}_M$	M+1	M+2	M+3	$ln\bar{F}_Q$	Q+1	Q+2	Q+3
Coefficient	0.0216315	0.0059325	0.0224272	0.0170726	0.0033594	0.0225243	0.027109	0.027494
Constant	0.000418	-0.022999	0.000486	0.000135	-0.0130844	0.0002497	-0.000153	-0.000071

Figure 40: Monthly seasonal factor for November (2013) estimations.

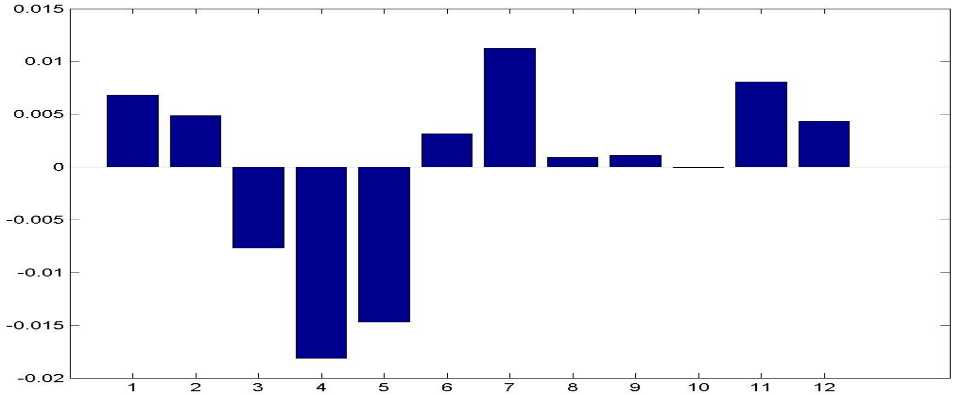
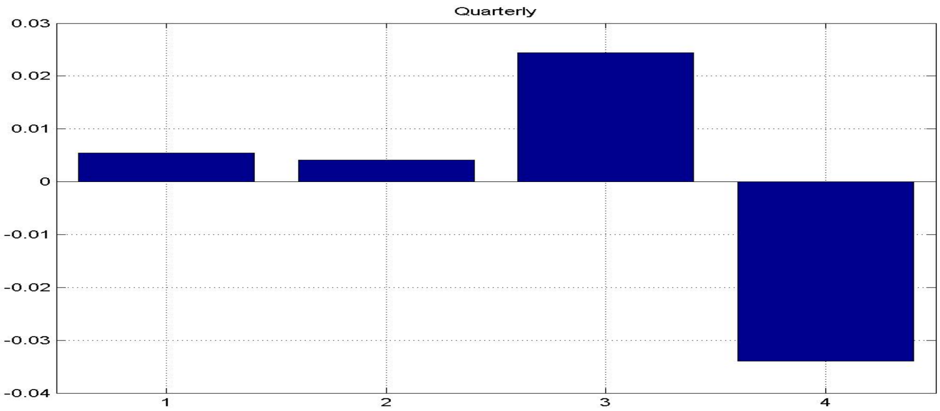


Figure 41: Quarterly seasonal factor for November (2013) estimations.



7.6 Appendix F

To estimate each path the Schoutens' (2003) indications have been followed. He states that NIG process X^{NIG} with parameters $\alpha > 0$, $|\beta| < \alpha$, $\mu \in \mathbb{R}$ and $\delta > 0$ can be obtained by time-changing a standard Brownian motion $W = (W_t, t \geq 0$ with drift by an Inverse Gaussian process $I = (I_t, t \geq 0)$ with parameters $a = 1$ and $b = \delta\sqrt{a^2 - \beta^2}$, Hence,

$$X_t^{NIG} = \mu + \beta\delta^2 I_t + \delta W_{I_t} \tag{36}$$

is a NIG process with parameters α, β and δ.

Table 12: Parameters of the MNIG distribution

MNIG	α	β	δ	μ
FBM	0.08223	-0.01734	0.20582	0.03118
M+1	0.08223	0.00143	0.20582	-0.0147
M+2	0.08223	-0.00375	0.20582	-0.02740
M+3	0.08223	-0.0123	0.20582	0.00170
FBQ	0.11140	0.00292	0.21118	-0.00975
Q+1	0.11140	0.03563	0.21118	-0.00634
Q+2	0.11140	0.01249	0.21118	0.01258
Q+3	0.11140	0.03406	0.21118	-0.02610

Glossary

EWMA Exponential Weighted Moving Average. 16

GARCH Generalized Autoregressive Conditional Heteroscedasticity. 4, 16

GLS Generalized Least Squares. 19

MGH Multivariate Generalized Hyperbolic. 7, 8

MNIG Multivariate Normal Inverse Gaussian. 6, 7, 19, 20, 40

MRJD Mean Reverting Jump Diffusion. 1, 4, 5, 6, 9, 13, 20, 21, 22, 25, 30, 34

MSE Mean Square Error. 9, 13, 21, 25, 30

NIG Normal Inverse Gaussian. 8, 20, 40

SDE Stochastic Differential Equations. 6

SFP Stochastic Forward Premium. 1, 4, 6, 7, 11, 17, 19, 20, 21, 23, 24, 25, 28, 30

SURE Seemingly Unrelated Regression Equations. 19