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ESSAYS ON STOCK INDEX FUTURES

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INTRODUCTION

Essays on

Stock Index Futures

A BREAF HISTORY OF THE STOCK INDEX FUTURES

A stock index future is, in essence, a bet on the value of the underlying index at a specified future date. The first stock index futures contracts began on February 24, 1982, when the Kansas City Board of Trade introduced futures on the Value Line Index and, about two months later, the Chicago Mercantile Exchange introduced futures contracts on the S&P 500 index. By 1986, the S&P 500 futures contract had become the second most actively traded futures contract in the world, and it is one of the most actively traded nowadays. Since they were launched, they were an immediate success, and quickly led to a proliferation of new futures and options tied to various indexes. For instance, the Nikkei 225 futures contracts were introduced in 1988 in the Japanese market and achieved a spectacular growth. At the moment, the value of trading in the index future is of the same order of magnitude as the value of transactions in the underlying shares.

One of the reasons for this success was that index futures greatly extended the range of investment and risk management strategies available to investors by offering them, for the first time, the possibility of unbundling the market and nonmarket components of risk and return in their portfolios. They have revolutionized the art and science of equity portfolio management as practiced by mutual funds, pension plans, endowments, insurance companies and other money managers among others. Portfolio managers use the stock index futures in order to reduce the market exposure of the global position, or even take a global short position without needing to sell the individual assets. They can also sell futures to provide some cash for their clients.

Four types of trading strategies are usually recognized: speculation, spread trading, arbitrage and hedging. We are going to describe each of them briefly: (i) index futures can be used for taking large positions (long or short) in the market as a whole for the purposes of speculation on a general rise or fall in share prices. This is accomplished with low transaction costs, minimal capital and little adverse price response; (ii) spread trading involves the simultaneous purchase of one future and the sale of another - each element of a spread is called a leg. Spreads are designed to take advantage of anticipated changes in the relative price of two futures; (iii) arbitrage, on the other hand, involves exploiting pricing anomalies between the spot and futures markets to produce a riskless profit; and finally, (iv) hedging is the purchase or sale of futures contracts to offset possible changes in the values of assets or liabilities currently held, or expected to be held at some future date.

The purpose of this dissertation is to present new evidence on selected topics related to stock index futures. While many of the available studies at the moment focus mainly on the American market, it is also necessary to supply a body of empirical evidence from other countries too in order to reach general conclusions for each single issue. The broad principles governing the trading of such futures were almost standard across futures exchanges, but we will provide either new dogmas or new empirical evidence drawn from a study on this specific underlying in three different markets.

DISSERTATION EVOLUTION

This dissertation is based on the main stock index futures contracts, and more precisely on the S&P 500, DAX 30 and Nikkei 225 indexes, which are virtually the most liquid ones. The whole study investigates some of the most important topics related to futures contracts, such as the election of a rollover date to construct a long and unique future price series; calendar effects on future returns; value-at-risk techniques implying stock index futures; and finally, a new perspective of volatility taking into account contracting variables such as open and closed positions.

The dissertation thesis consists of four chapters I have worked on as part of my doctoral studies at the University of Valencia. The chapters are entitled as follows:

- Chapter 1: Rolling Over Stock Index Futures Contracts
- Chapter 2: Testing Calendar Effects on Stock Index Futures Contracts by Simulation Methods
- Chapter 3: Forecasting VaR in Spot and Futures Equity Markets
- Chapter 4: Open and Closed Positions and Stock Index Futures Volatility

During my first year as a Ph.D. student I worked on the first chapter as the last part of the Quantitative Finance Programme. The chapter was inspired by the necessity of both academics and traders to use a continuous series in the derivative contracts in order to test different academic hypotheses and trading systems. The different methodologies proposed by the literature were tested to determine the optimal rollover to create the new continuous series but the diverse criteria did not create series which were significantly different from each other, and therefore, it was inferred that the conclusions extrapolated from them would not be different. The chapter was presented in the V Workshop in Banking and Quantitative Finance in the Basque Country (Spain, 2007). The referee of the chapter, Begoña Font, made some interesting comments and suggestions that were added to the chapter. Then, the improved paper was submitted to *The Journal of Futures Markets,* accepted in July 2008, and finally it was published in 2009.

For the second chapter I spent the first six months reading about the different topics related to futures contracts, especially about returns and trading strategies. As my thesis director has three papers about calendar effects that explains some abnormal returns for the Spanish and Irish spot markets, we finally decided to come up with a paper that aggregates all the different calendar patterns that the literature has attributed to spot markets, and we aggregated another one (maturity effect), which is specific to futures markets. We tested 188 possible calendar effects by simulation methods, first using bootstrap methodology and later Monte Carlo. We reached the conclusion that the only statistically and economically significant (as well as persistent) pattern is the turn-on-the-month in the S&P 500 futures. The chapter was presented in the XII Workshop on Quantitative Finance in Padova (Italy, 2011). The referee of the chapter, Francesco Lisi, made some comments and proposals that were added to the chapter. The improved version was submitted to *The Journal of Futures Markets*, and it is under review at this moment.

During the elaboration of the dissertation, I have been attending different workshops and seminars, most of them related to the futures market. I want to emphasize the work of my director Angel Pardo, who holds the Chair in International Finance Banco Santander–Universitat de València, because it has allowed me to assist to high-level seminars in my University. Besides, I also managed to attend other seminars and workshops on the same topic in Madrid thanks to the help of the project ECO2009-14457-C04-04 whose main researcher is Francisco Climent.

Thanks to one of the events organized by the Chair, we have the opportunity to meet Svetlozar Rachev, international expert in Value at Risk methods, and we arranged a 3-month stay at the University of Karlsruhe (Germany) starting on January 15, 2010 and ending on April 30, 2010, in order to apply Value at Risk to stock index futures contracts. During this period of research I attended courses on Mathematical Finance and Financial Econometrics. While my stage at the University of Karlsruhe, I could collaborate with Young Kim, and Edward Sun from the University of Karlsruhe (Germany) and Frank Fabozzi from Yale School (USA). The two main ideas were firstly to prove the validity of the ARMA-GARCH model with tempered stable innovations to forecast one-day-ahead VaR [Kim et al. (2011)] in futures markets -very relevant due to the scarce empirical evidence about VaR for these markets; and secondly, to try to enhance the previous model using trading volume because of the extended literature linking volume and volatility, although the results only show a slight improvement in forecasting capability. The final paper was submitted to the *Review of Quantitative Finance and Accounting* and it is under review at the moment. It appears as “2010 Technical Reports” in the university webs of Stony Brook University, University of California and Santa Barbara (USA), and finally, in the University of Karlsruhe (Germany).

After testing the weak performance of the volume, we decided to research more about the different variables related to volatility. We changed the normal approach from the trading variables to the contracting variables. We found that the previous research does not take into account if the different variables are stock or flow ones, which was also unified in our study. In order to do all of this, we combined volume and open interest change in order to use open and closed positions to take a different approach at the sources of stock index futures volatility. Additionally, we included in the study the extreme contracting activity of some groups of traders, reaching some astonishing conclusions, such as the fact that days dominated by day-trading do not increase intraday volatility. The chapter was accepted to be presented in the XII Iberian-Italian Congress of Financial and Actuarial Mathematics in Lisbon (Portugal, 2011).

EMPIRICAL DATA

For this study we have chosen the daily series of future contracts of the major national stock indexes in the United States, Germany and Japan. More specifically, the contracts used have been the S&P 500, the Dax 30 and the Nikkei 225 futures. Our database has been taken from Reuters and consists of open, closed, high and low prices, trading volume and open interest data for the period comprised between December 2, 1991 and December 31, 2008. This period characterizes for different scenarios with high volatility periods and sharp upward/downward trends, high liquidity, and what is even more relevant, it is almost all the life of the DAX and Nikkei futures contracts.

The S&P 500 future contract is listed on the Chicago Mercantile Exchange as CME S&P 500 Futures. Its underlying asset is the S&P 500 index, which comprises the 500 most traded companies in the NYSE, American Stock Exchange and Nasdaq. The tick value of the futures contract is 0.1 index points or $25. The local trading hours are from 8:00 a.m. to 10:15 p.m. The last trading day is the exchange day prior to the third Friday of the delivery month (this is, a Thursday except if it is a holiday; in that case, it would be a Wednesday).

The DAX 30 future is listed on the Eurex market and its underlying asset is the Dax index. This index comprises the 30 corporations with the largest book value and the highest market capitalization on Frankfurt stock market. The tick value of the contract is 0.5 index points or €12.5. The local trading hours are from 8:00 a.m. to 10:00 p.m. and it expires the third Friday of the delivery month at 1 p.m.

The Nikkei 225 futures are listed on the Osaka Securities Exchange. Its underlying asset is the “Nikkei 225 Index”, which consists of the 225 main stocks listed in the First Section of the Tokyo Stock Exchange. The price of the futures contract fluctuates in points with a value of ¥1000 per index point. The trading hours are from 9:00 a.m. to 10:15 p.m. The last trading day is the exchange day prior to the second Friday of the delivery month (usually a Thursday except if it is a holiday; in that case, it would be a Wednesday).

All these contracts have quarterly maturities in the March-June-September-December cycle. Therefore, the sample period has 69 maturities for each one of the contracts (17 years x 4 quarters = 68, plus December 1991). The data used for each chapter comprises the periods between January 3, 2000, and December 29, 2006 (chapter 1), from December 2, 1991 to April 30, 2008 (chapter 2 and chapter

4), and from December 13, 1994 to December 31, 2008 (chapter 3); this last one includes also the spot indexes.

Taking into account the results obtained in the first chapter, we decided to construct the continuous series that we were going to use for the rest of the dissertation using the simplest criterion: making the rollover on the last day of the maturity. We can take those series from Reuters directly because they are constructed in that way. But, as the series which are linked are from different maturities, in the new long series there is a jump. In order to solve this problem, we transformed our price series in return series and, on the day of the jump, we calculated the return of the day after the rollover date as the quotient between the closing price of the following contract and the previous closing price of such a contract. In that way, all the returns are taken from the same contract, and therefore the jumps effect disappear.

SUMMARY OF THE CHAPTERS

As we have mentioned, the present dissertation thesis is focused on some of the most relevant topics related to the stock index futures contracts literature. It is composed of four chapters, and each one of them analyses, respectively: (i) the relevance of the date of the rollover in order to construct the long futures series in stock index futures; (ii) the existence of calendar anomalies in the stock index futures markets; (iii) the estimation of the one day ahead Value-at-Risk for futures contracts of stock indexes; and finally, (iv) the relationship between volatility and the number of open and closed positions in the stock index futures markets.

The Chapter 1 is entitled "**Rolling Over Stock Index Futures Contracts**". As the derivative contracts have a finite life limited by their maturity, there are not long series of futures. But the construction of continuous series, however, is crucial for academic and trading purposes. In this study, we analyse the relevance of the choice of the rollover date, defined as the point in time when we switch from the front contract series to the next one. We have used five different methodologies in order to construct five different return series of stock index futures contracts. The results show that, regardless of the criterion applied, there are not significant differences between the resultant series return. Therefore, the least complex method can be used in order to reach the same conclusions.

In the second chapter, "**Calendar Anomalies in Stock Index Futures**", we analysed the controversy about the existence or not of the calendar effects focused on stock index futures. There are a large and increasing number of papers that describe different calendar anomalies in stock markets. Although empirical evidence suggests that seasonal effects disappeared after the early 1990s, new studies and approaches assert the continuation of some anomalies in stock indexes. In this chapter, we present a comprehensive study of 188 possible cyclical anomalies in S&P 500, DAX and Nikkei stock index futures contracts from 1991 to 2008. Frictions in futures markets, unlike spot markets frictions, make it feasible to produce economically significant profits from trading rules based on calendar effects. By applying a percentile-t-bootstrap and Monte Carlo methods**,** our analysis reveals that the turn-of-the-month effect in S&P 500 futures contracts is the only calendar effect that is statistically and economically significant and persistent over time.

With Chapter 3, “**Forecasting VaR in Spot and Futures Equity Markets**”, we have three goals. First, we present evidence for the validity of the ARMA-GARCH model with tempered stable innovations to estimate one-day-ahead VaR in the cash and futures markets for three stock indexes –S&P 500, DAX 30, and Nikkei 225 – for the period December 14, 2004 to December 31, 2008. This is the first time that testing of this model has been done for equity futures. Second, based on the vast theoretical and empirical literature suggesting its strong link with volatility, we test for the first time whether adding trading volume to the classical tempered stable model improves the forecasting ability of the model. Finally, we compare the number of times that the market data drop below the corresponding one-day-ahead VaR estimations for both spot and futures equity markets in classical tempered stable models (CTS models) with and without trading volume.

Finally, in Chapter 4 -entitled “**Open and Closed Positions and Stock Index Futures Volatility**”- we analyse the relationship between volatility in index futures markets and two contracting variables: the number of open and closed positions. We observe that, in general, both positions are positively correlated with contemporaneous volatility, and the opposite effect is detected on the following day for all indexes. Additionally, we observe a stronger positive relationship on days characterized by extreme movements of these contracting movements dominating the market. Our findings suggest that days dominated by day-traders are not associated to an increment of volatility, whereas days characterized by a high number of open or closed positions, associated with hedging activity, have to do with a rise in volatility.

Chapter 1

Rolling Over Stock Index Futures Contracts

1. INTRODUCTION

Future contracts have a finite life span which is limited by their maturity. However, analysts need to create futures continuation series not only to test academic hypotheses but also to study and develop different trading systems with speculative or hedging purposes. Therefore, academics and traders have to solve a double dilemma: first, how to choose the election of a rollover date, defined as the point in time when we switch from the front contract series to the next one, and second, how to correct pricing gaps when rollover occurs.

The usual methodology was to construct a long and unique price series using only the data of the nearest future contract up to its maturity and link it with the following contract on the next day. This was the most popular method until Samuelson (1965) detected an abnormal volatility in the last weeks of life of futures contracts which did not appear in the spot series. Thus, if continuous series were constructed taking as reference the prices of the nearest future contract up to its maturity, then the (abnormal) volatility could distort the conclusions reached from the statistical inferences.

Although Samuelson (1965) focused on commodities futures, his results influenced many researchers when constructing stock index futures contract linked series. Junkus (1986), for example, constructs series without taking into account the data from the first day of the month of delivery until the day of maturity when studying weekend and day of the week effects in returns on stock index futures. Bessembinder (1992) studies the uniformity of risk pricing in futures and asset markets and compiles future return series as daily percentage changes in the settlement price of the contract with the nearest delivery date, except within the delivery month when daily percentage changes in the settlement price of the second-nearest contract are used. Östermark, Martikainen and Aaltonen (1996) and Martikainen and Puttonen (1996) analyse the Finnish stock index futures market. In both papers, a week before the maturity the analysis is shifted to the next nearest contract to avoid the so-called expiration week effects.

Finally, Geiss (1995) suggests an alternative method to construct continuous futures series producing a price index which is a weighted average of observed prices for contracts with different expiration dates.

As Ma, Mercer and Walker (1992) show, the choice of the rollover date can have unpredictable effects on the results of empirical studies. Ma, Mercer and Walker (1992) compare different methods to rollover futures and demonstrate that important biases are generated from its selection. They study several futures contracts with diverse underlying assets concluding that the differences between the return series obtained with each criterion are significant. They suggest that the election of the best methodology depends on the underlying asset. However, they indicate that, in general, the choice of rolling over at the delivery date should be avoided since it almost always generates excessive volatility.

The main purpose of this study is to provide further insights into the relevance of the choice of the rollover date when constructing a continuous returns series of future contracts. Unlike previous papers, our analysis deals with five criteria focused only on one underlying asset: the stock index. The reason for this selection is twofold. Firstly, we study only one asset because, as Ma, Mercer and Walker (1992. p. 216) indicate, the choice of the best rollover method may be contract specific, and secondly, we select stock index futures since the mechanics of such markets are more standard than other future markets such as agricultural, energy or interest rate markets. Specifically, for this study we have chosen the S&P 500, DAX, and Nikkei index futures contract. Our database has been taken from Reuters and consists of daily price, trading volume and open interest data for the period between January 3, 2000 and December 29, 2006. All these future contracts have quarterly maturities in the March-June-September-December cycle. Therefore, the sample period has 29 maturities for each one of the contracts (7 years x 4 quarters = 28, plus another maturity to complete the last one on December 29, 2006).

The plan of this paper is as follows. The different methodologies reported in the most relevant works in this line of research are analysed in Section 2. Then, taking them into account, different return series are constructed. In Section 3, some tests on the descriptive statistics of the return series and on their overall distribution are run in order to establish if there are significant differences between them depending on the criterion applied. Section 4 summarizes the article with some concluding remarks.

2. METHODOLOGY

2.1 Rollover criteria

This section discusses five criteria found in the literature in order to determine the point in time when the rollover takes place when constructing a continuous price series. The first criterion, the most used and also the most criticized, is the "Delivery Day" or "Last Day" criterion that consists of rolling on the last trading day. The main advantage of this method is its simplicity as the switch occurs when the nearest contract expires. However, if abnormal volatility occurs in the previous sessions to the contract maturity, following this method the researcher would construct a series with the maximum distortion.

The next three criteria seek the appropriate market liquidity conditions for the rollover. The rationale of these criteria is that if a trader was long or short in a future contract and wished to hold it indefinitely, he would try to find the liquidity peak to switch the contract. The criterion of "Volume" implies the switching of the contract on the day when the volume of the first maturity is always lower than the volume of the second maturity. The "Open Interest" method (*OI* in the tables) indicates the jump between series when the open interest of the second maturity is always greater than the first one. The rationale behind this criterion is found in the fact that the open interest is considered by many traders as a more reliable indicator of liquidity than volume.

The third seeking-liquidity criterion has been put forward by Lucia and Pardo (2010). In this case, the switching would occur on the day from which the number of closed positions is always larger than the number of opened positions for the nearby contract. This is, the rollover date takes place on the day when the ratio

$$R3_t = \frac{O_t - C_t}{O_t + C_t}$$

is less than zero until maturity, with O_t and C_t being, respectively, the overall number of open and closed positions in the period *t*. With this criterion, the analyst avoids taking into account information on days in which the nearby contract has lost the interest of traders.[1, 2]

The last method proposed by Geiss (1995) is based on the mathematical properties of the series. This criterion creates a “Distortion-Free or Seamless Index” (*Free* in tables). Such method relies on a rigorous theoretical justification which tries to put forward an improved method according to mathematical properties. This criterion designs an index which comprises a portfolio of future contracts with different maturities.[3] According to Geiss (1995), a distortion-free price index must possess the following properties: information, scale, level and monotonicity. It must represent the whole of the observed future contracts (or the main ones at least) and it will have to duplicate the price fluctuations in the most similar possible way.

As Geiss demonstrates, a portfolio of several contracts produces complex indexes that are indistinguishable from indexes based on simple weight functions. This is the reason why we have used the simplest variant (although equally valid), combining the two nearest to delivery contracts. Thus, the index value is a convex combination of these prices, defined by Geiss as $\Phi_t(p_t) = c_1 p_{t,i} + c_2 p_{t,i+1}$ where c_1 is the proportion of the portfolio in the nearest contract and is calculated as

[1] The majority of the papers follow seeking-liquidity criteria. This is the case of some papers from the *Applied Economics Series*. For example, the “last day criterion” is used by ap Gwylim and Buckle (2001) when examining the lead/lag relationships between the FTSE 100 stock market index and its related futures and options contracts. Ryoo and Smith (2004) also roll the contract on the last trading day when investigating the impact on the spot market of trading in KOSPI 200 futures. Lien, Tse and Tsui (2002) use the “Volume criterion” when comparing the performances of different hedge ratios. McMillan and Speight (2003) and Tsuji (2007) follow the same criterion when studying asymmetric volatility dynamics in high frequency FTSE-100 stock index futures and the dynamics of the basis of the NIKKEI 225, respectively.

[2] Attention must be paid to the fact that the choice of the rollover date matters depending on what is being tested. For example, some futures contracts, notably Eurodollar futures, have more actively traded deferred contract months than nearby contracts. In these cases, the choice among seeking-liquidity criteria would be more reliable when constructing a continuous series of liquidity-related measures such as the bid-ask spread, volume or open interest.

[3] Clark (1973), Herbst, Kare and Caples (1989) and Rougier (1996) have also proposed similar methodologies to construct continuous time series.

$c_1 = [E_i - (t + \Omega)] / [E_{i+1} - E_i]$ where E_i is the date of expiration for contract i (the nearest to maturity) plus one day; E_{i+1} is the next near maturity plus one, and Ω is the cut-off parameter and is the number of days between the expiration date and the last day that the contract prices are considered in the index. For example, if Ω is 14 it would imply that the last day the nearest contract price is included in the index is 15 days before maturity.

In our case, we have constructed series where Ω is 11, 21 and 31. Due to some lack of prices in the third maturity in the construction of the Nikkei Index future contract series, 394, 479 and 578 observations are lost when Ω is 11, 21 and 31, respectively. This problem does not appear in the other methods as they do not use the data of this maturity to construct the continuous series.

Finally, we have compared these three series in order to see if there are any significant differences between the returns of the series regarding the mean, median and variance. As Table I displays, the results show that this is not the case. As the number of lost data increases along with Ω and the "Free" series are all very alike, it has been decided to use a Distortion-Free index series with Ω equal to 11 as the one representative of this method.

2.2 Timing of rollover

Next, we consider how many days before the expiration date the rollover would be made effective by each of the proposed methods. If there is a substantial number of data which differs among the methods, then significant differences among the resultant series should be expected.

In the “Last Day” method, by definition, there are 0 days between the contract expiration and the rollover, and this would indicate that the last price with which the first maturity series contributes to the continuous series is the delivery price. Regarding the construction of the series in levels, this implies that on the delivery day we would take the close price of the first maturity and the following day, the close price of the second maturity.

Regarding the methods based on the search of liquidity, we compute the days when the second contract volume or open interest values consistently beat those of the first one up to maturity. In order to construct a price series, we take prices from the contract with more volume or open interest, respectively.

In R3 criterion, the period computed ranges from the day when the R3 variable is negative up to the maturity day. This is, when the change in the open interest is negative for the remainder of the contract life. During those days, second contract prices replace the first ones.

Finally, with respect to the “Free” series, we consider the number of days before the expiration in which the nearest contract is not weighted in the index. As Ω is 11 in this case, the number of days before maturity is 11 too (computing also the maturity day, as in the former cases).

The number of days before maturity in which the information of the nearest contract is not used any more in the construction of the series is shown in Table II. Note that the most similar methodologies in the three markets analysed are those based on the Volume and the Open Interest. The R3 series is the most variable in time, having a dispersion of 4 to 10 days for the DAX, 3 to 7 for the Nikkei and between 14 and 21 for the S&P 500. The “Free” series method is special because it is a linear combination between two contracts. However, there is a day in which the percentage of the first or the third contracts is 0% and that of the second is 100%. That is the price for the second maturity, exactly 11 days before the delivery day in our case. This is the only data which could coincide with the former series.

Table III displays the percentage differences on the number of data that vary between the different series. Here, the implications of the former table can be seen more clearly. Table III shows the “Free” series as the most different, followed by the R3. The Last day, Volume and Open Interest series only vary up to 4% for the DAX and Nikkei. In the case of the S&P, Volume and OI are practically the same, although 10% of data differ from the Last Day. Finally, remarkable differences are observed when comparing all the criteria with respect to the R3 criterion in all the indexes.

Taking into account these results, the disparity in the number of data of each of the series could make it possible to work with different samples taken from the same raw data. This is what we analyse in the next section.

3. RESULTS

Before testing the equality of the distributions, it is important to clarify how to link the series of the different maturities. Note that when we switch from using the front contract to the second nearest contract, a jump in prices takes place. This abnormal return, well-explained by the cost of carry model in stock index futures contracts, may distort the inferences obtained from the continuous series of futures.

Ma, Mercer and Walker (1992) face this problem by building two different types of series for each method. The jump remains in the first series while in the second one a price-level adjustment is made in order to avoid the gap caused by switching contracts. The correction takes place after the rollover by subtracting the difference between the prices of the new and old contracts for all the new prices, or adding up the difference to the totality of the old prices. After the subtraction, a “negative price syndrome” (noted by Ma, Mercer and Walker, 1992) may appear. This would imply negative prices in the series by the accumulation of price gaps elimination. A further criticism to this method of adjustment is that the systematic subtraction of the new prices makes the trading prices differ from those which appear in the continuous futures series. Furthermore, the historic prices actually quoted and those of the continuously readjusted series are different too, which leads to a problem when traders test the performance of the different trading systems.

In our case, to solve this problem, we have calculated the return of the day after the rollover date as the quotient between the closing price of the following contract and the previous closing price of such contract, in order to ensure that all the returns are taken from the same contract.[4] Next, considering the return series calculated by making this adjustment only on the rollover day, we have tested the equality of means, medians and variances among the futures return series constructed following the five criteria explained in Section 2.

The equality of means, medians and variances has been tested with the parametric F test, the non-parametric Kruskal-Wallis test and the Brown-Forsythe’s statistic, respectively. The results are displayed in Table IV. The p-values indicate

[4] See Holton (2003, pp. 243-245) for further details about the two approaches to remove price jumps.

that it is not possible to reject the null hypothesis of equality of means, medians and variances in any case. Furthermore, we have split all the series depending on the day of the week. We have repeated the same tests, not reported here, and we have obtained similar results. Therefore, independently of the method used to elaborate a unique and continuous future return series, we reach the same conclusions.

Taking into account the above results, it seems that the choice of the criterion to link the maturities does not matter. However, one of the criticisms made to the former tests and, by extension, to the results derived from them, is the fact that two series with the same parameters of position (media, median) and dispersion (standard deviation) could result in different distributions. This is the reason why a distribution test of the return series of each methodology studied above has been run. Specifically, we have applied the Wilcoxon/Mann-Whitney test, a non-parametric test based on ranks which determines whether or not two groups (in this case series) have the same general distribution.

The results of this test are reported in Table V. It can be shown that the null hypothesis of equality between distributions cannot be rejected in any case as all the p-values are far above 10%. Similar results have been obtained when comparing the distributions conditioned on the day of the week. [5]

In summary, our results indicate that the choice of the criterion to link the maturities does not matter. This might be considered as a surprising result. A

[5] Since there are only four contracts per year and the rollover dates are fairly close to each other, all series will have the same data for the vast majority of the days in the sample. Differences induced by choosing different rollover dates would be seen, if they exist, in short periods before the maturity dates. For this reason, all the tests have been repeated for each maturity contract choosing different short periods before the contracts expire. The results of this analysis are similar to those obtained when the entire sample period is considered. To conserve space they are not reported but are available from the authors upon request.

possible explanation for this result could be found in the papers by Daal, Farhat and Wei (2006) and Duong and Kalev (2008), in which they show that the maturity effect is absent in the majority of futures contracts, included index futures.

4. CONCLUSIONS

The purpose of this study is to investigate whether different rollover date selection criteria induce significant differences in the resulting stock index futures contract return series. We have considered five criteria that have been applied to link all the DAX, Nikkei and S&P 500 index futures contracts with maturities from January 3, 2000 to December 29, 2006.

Contrary to Ma, Mercer and Walker (1992), we find that the choice of the rollover date does not induce significant differences between return series. Consequently, the criterion used to link series is not relevant and we endorse the method of switching contracts on the last trading day of an index futures contract because of its simplicity.

REFERENCES

ap Gwylim, O., and Buckle, M. (2001). The lead-lag relationship between the FTSE100 stock index and its derivative contracts. Applied Financial Economics, 11, 385-393.

Bessembinder, H. (1992). Systematic risk, hedging pressure, and risk premiums in futures markets. The Review of Financial Studies, 5, 637-667.

Clark, P.K. (1973). A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices. Econometrica, 41, 135-155.

Daal, E., Farhat, J., and Wei, P.P. (2006). Does futures exhibit maturity effect? New evidence from an extensive set of US and foreign futures contracts. Review of Financial Economics, 15, 113-128.

Duong, H. N., and Kalev, P.S. (2008). The Samuelson hypothesis in futures markets: An analysis using intraday data. The Journal of Banking and Finance, 32, 489–500.

Geiss, G. (1995). Distortion-Free Futures Price Series. The Journal of Futures Markets, 15, 805-831.

Herbst, A.F., Kare, D.D., and Caples, S.C. (1989). Hedging effectiveness and Minimum Risk Hedge Ratios in the presence of Autocorrelation: foreign currency futures. The Journal of Futures Markets, 4, 397-407.

Holton, G.A. (2003). Value-at-risk: Theory and Practice, Academic Press, San Diego, CA.

Junkus, C. (1986). Weekend and Day of the Week Effects in Returns on Stock Index Futures. The Journal of Futures Markets, 6, 397-407.

Lien, D., Tse, Y. K., and Tsui, A. K. C. (2002). Evaluating the hedging performance of the constant-correlation GARCH model. Applied Financial Economics, 12, 791-798.

Lucia, J., and Pardo, A. (2010). On measuring hedging and speculative activities in futures markets from volume and open interest data. Applied Economics, 42, Is. 12, pp. 1549-1557.

Ma, K., Mercer, M., and Walker, M. (1992). Rolling Over Futures Contracts: A note. Journal of Futures Markets, 12, 203-217.

Martikainen, T., and Puttonen, V. (1996). Sequential information arrival in the Finnish stock index derivatives markets. The European Journal of Finance, 2, 207-217.

McMillan, D.G., and Speight, A.E.H. (2003). Asymmetric volatility dynamics in high frequency FTSE-100 stock index futures. Applied Financial Economics, 13, 599-607.

Östermark, R., Martikainen, T., and Aaltonen, J. (1995). The predictability of Finnish stock index futures and cash returns by derivatives volume. Applied Economics Letters, 2, 391-393.

Rougier, J. (1996). An optimal price index for stock index futures contracts. The Journal of Futures Markets, 16, 189-199.

Ryoo, H-J., and Smith, G. (2004). The impact of stock index futures on the Korean stock market. Applied Financial Economics, 14, 243 -251.

Samuelson, P. (1965). Proof that properly anticipated prices fluctuate randomly. Industrial Management Review, 6, 41-49.

Tsuji, C. (2007). Explaining the dynamics of the NIKKEI 225 stock and stock index futures markets by using the SETAR model. Applied Financial Economics Letters, 3, 77 – 83.

Table I. Comparison among Distortion-Free Series

Equality tests of means, medians and variances among Distortion-Free series calculated with Ω values of 11, 21 and 31. The equality of means, medians and variances has been tested with the parametric F-test, the non-parametric Kruskal-Wallis test and the Brown-Forsythe's statistic, respectively. The corresponding percentage p-values appear in all indexes at the end of the column. Sample period from January 3, 2000 to December 29, 2006.

S&P 500	**Mean**	**Median**	**Std. Deviation**
Ω = 11	-1.60E-05	5.45E-04	1.14E-02
Ω = 21	-1.61E-05	5.55E-04	1.14E-02
Ω = 31	-1.66E-05	5.35E-04	1.18E-02
p-value	100	99.31	89.07

DAX	**Mean**	**Median**	**Std. Deviation**
Ω = 11	-1.25E-05	4.80E-04	1.57E-02
Ω = 21	-1.23E-05	4.86E-04	1.57E-02
Ω = 31	-1.22E-05	4.98E-04	1.57E-02
p-value	100	100	99.99

Nikkei	**Mean**	**Median**	**Std. Deviation**
Ω = 11	-4.26E-04	-3.01E-04	1.33E-02
Ω = 21	-4.24E-04	-1.10E-04	1.32E-02
Ω = 31	-4.21E-04	1.10E-05	1.33E-02
p-value	100	99.19	99.98

Table II. Rollover Timing for each Criterion

This table indicates the mean and the standard deviation of the number of days between the rollover date and the expiration date of the contract for each of the five proposed methods. *LD, Vol, OI, R3* and Ω stand for last day, volume, open interest, R3 and Distortion-Free criterion, respectively. Sample period from January 3, 2000 to December 29, 2006.

S&P 500	**LD**	**Vol**	**OI**	**R3**	**Ω = 11**
Mean	0	5.82	5.68	17.71	11
Std. Deviation	0	0.61	0.63	3.3	0

DAX	**LD**	**Vol**	**OI**	**R3**	**Ω = 11**
Mean	0	1	2.46	7.11	11
Std. Deviation	0	0	1.45	2.57	0

Nikkei	**LD**	**Vol**	**OI**	**R3**	**Ω = 11**
Mean	0	0.78	1.15	4.81	11
Std. Deviation	0	0.42	0.46	1.84	0

Table III. Percentage Data that Differs between Continuous Futures Series.

This table indicates the difference in percentage between the number of observations that is different when constructing continuous futures series following each criterion. *LD, Vol, OI, R3* and Ω stand for last day, volume, open interest, R3 and Distortion-Free criterion, respectively. The sample period comprises all index futures contracts (S&P 500, DAX, and Nikkei) with maturities from January 3, 2000 through December 29, 2006.

S&P 500	**LD**	**Vol**	**OI**	**R3**
Vol	9.27			
OI	9.05	0.23		
R3	28.21	18.94	19.17	
Ω = 11	100	100	100	98.41

DAX	**LD**	**Vol**	**OI**	**R3**
Vol	1.57			
OI	3.88	2.3		
R3	11.19	9.61	7.31	
Ω = 11	100	100	100	100

Nikkei	**LD**	**Vol**	**OI**	**R3**
Vol	1.26			
OI	1.87	0.6		
R3	7.82	6.56	5.96	
Ω = 11	100	100	100	100

Table IV. Equality Tests

Equality tests of means, medians and variances between the continuous series constructed following the criteria explained in Section 3. *LD, Vol, OI, R3* and Ω stand for last day, volume, open interest, R3 and Distortion-Free criterion, respectively. The Distortion-Free series has been calculated with a value of Ω equal to 11. The equality of means, medians and variances has been tested with the parametric F test, the non-parametric Kruskal-Wallis test and the Brown-Forsythe's statistic, respectively. The corresponding percentage p-values appear in all panels at the end of the column. Sample period from January 3, 2000 to December 29, 2006.

S&P 500	**Mean**	**Median**	**Std. Deviation**
LD	-8.75E-05	4.55E-04	1.14E-02
Vol	-9.03E-05	4.60E-04	1.14E-02
OI	-9.11E-05	4.60E-04	1.14E-02
R3	-9.16E-05	4.60E-04	1.14E-02
Ω	-1.60E-05	5.50E-04	1.14E-02
p-value	99.96	99.9	100

DAX	**Mean**	**Median**	**Std. Deviation**
LD	-1.56E-04	4.00E-04	1.57E-02
Vol	-1.29E-04	3.85E-04	1.57E-02
OI	-1.29E-04	3.85E-04	1.57E-02
R3	-1.28E-04	3.83E-04	1.57E-02
Ω	-1.25E-05	4.76E-04	1.57E-02
p-value	99.9	99.8	100

Nikkei	**Mean**	**Median**	**Std. Deviation**
LD	4.24E-05	5.04E-04	1.38E-02
Vol	4.54E-05	5.04E-04	1.38E-02
OI	4.90E-05	5.04E-04	1.38E-02
R3	3.66E-05	0.00E+00	1.38E-02
Ω	-7.32E-06	6.21E-05	1.34E-02
p-value	100	100	96.8

Table V. Distribution Tests

This table shows the percentage p-values of the Wilcoxon/Mann-Whitney test that tests the null hypothesis that two continuous return series have the same general distribution. *LD, Vol, OI, R3* and Ω stand for last day, volume, open interest, R3 and Distortion-Free criterion, respectively. The Distortion-Free series has been calculated with a value of Ω equal to 11. Sample period from January 3, 2000 to December 29, 2006.

S&P 500	**LD**	**Vol**	**OI**	**R3**
Vol	99.97			
OI	100	99.97		
R3	99.96	99.95	99.99	
Ω	81.29	81.33	81.35	81.28

DAX	**LD**	**Vol**	**OI**	**R3**
Vol	99.3			
OI	99.06	99.77		
R3	99.49	99.81	99.59	
Ω	77.18	77.85	78.01	77.54

Nikkei	**LD**	**Vol**	**OI**	**R3**
Vol	99.75			
OI	99.35	99.1		
R3	98.22	98.52	97.62	
Ω	94.85	94.81	94.34	97.23

Chapter 2

Calendar Anomalies

in Stock Index Futures

1. INTRODUCTION

The existence of different patterns of return data has notable implications in market efficiency and in the forecast ability of profitable trading strategies. Generally speaking, a large part of the financial literature suggests that calendar effects disappeared after the early 1990s. In fact, there is some evidence that indicates the reduction of anomalies after the stock market crash in 1987 or even before. Connolly (1989) analysed the robustness of day-of-the-week effect and weekend effects to alternative estimation procedures. Although the strength of these effects appeared to depend on the testing method, both effects seemed to have disappeared by 1975. Mehdian and Perry (2002) examined the January effect in US equity markets using Dow Jones Composite, NYSE Composite and the S&P 500 and they did not find statistical support for the January effect in the US equity market after 1987. Keef and Roush (2005) studied the day-of-the-week effects in the pre-holiday returns of the S&P 500 stock index and concluded that there was a strong pre-holiday effect up to 1987, but that it had greatly diminished after that year. In the same direction are the results obtained by Hansen et al. (2005), who studied 25 stock indexes from ten countries and concluded that, beginning in the late 1980s, calendar effects have diminished except in small-cap stock indexes.

Some researchers have identified the introduction of futures markets as the key reason for the disappearance of the anomalies. Cyr and Llewellyn (1994) carried out a time series test of calendar seasonalities in the S&P 500 index since the introduction of index derivative securities and they show that calendar seasonalities documented in previous studies do not exist anymore or are mitigated to a large degree. Maberly and Pierce (2003) examined the robustness of the

Halloween strategy in Japanese equity prices and they show that the Halloween effect disappeared after the introduction of the Nikkei 225 index futures in September 1986. Finally, Szakmary and Kiefer (2004) observed that evidence of a traditional turn-of-the-year effect, in both spot and futures markets in the S&P 500 Midcap, is confined to the pre-1993 period, before the introduction of the S&P Midcap and Russell 2000 futures that year.

In spite of that, recent studies adopting new approaches assert the continuation of some anomalies in stock indexes or even the appearance of new ones. This is the case of Leontitsis and Siriopoulos (2006), who devised a method that corrects the chaotic forecasting of financial time series and shows new evidence about the existence of the day-of-the-week, the turn-of-the-month and the holiday effects in the NASDAQ Composite and TSE 300 Composite indexes from 1984 to 2003. Cooper et al. (2006) found that January returns had predictive power over the next 11 months of the year in the US markets for the period 1940-2003. McConnell and Xu (2008) observed that the turn-of-the-month effect in equity returns still exists in 31 of the 35 countries examined. Keef et al. (2009) studied the dynamics of the Monday effect in 50 international stock indexes and they assert the continuation of this anomaly. Doyle and Chen (2009) analysed 11 major stock markets during 1993-2007 and report a wandering weekday effect that changes over time. Finally, Blau et al. (2009) showed new evidence about the relationship between the weekend effect in NYSE securities and the role of short selling. Therefore, the controversy about the existence of calendar effects in spot markets continues.

Additionally, there are two important aspects we have to consider when studying calendar effects. Firstly, transactions costs in stock index futures markets are lower than those in equities markets. Secondly, stock index futures markets allow traders to take straightforwardly short positions. These two aspects, together with the high leverage in futures trading, make it easier to obtain a profit when implementing trading rules in futures markets based on seasonalities.[6]

In this paper, we present a comprehensive study of calendar effects on S&P 500, DAX and Nikkei stock index futures contracts. As far as we know, this is the first research that looks into the whole set of calendar anomalies by focusing exclusively on index futures markets. The remainder of the paper is organized as follows. Section 2 details the calendar effects considered and the financial data used in the study. In Section 3 the percentile-t-bootstrap methodology is applied to determine the statistical significance of the calendar effect in two consecutive periods. In Section 4, the Monte Carlo simulation is implemented to establish the economic significance of calendar effects that are statistical significant in both periods. Finally, Section 5 summarizes with some concluding remarks.

2. CALENDAR EFFECTS AND DATA

There exist a wide variety of possible time related patterns in index futures returns (see Sutcliffe, 2006, p. 236-244, for a comprehensive review of these

[6] It must be stressed that short-sale constraints in stock markets should not be an obstacle to taking advantage of negative anomalies. D'Avolio (2002) showed that for 90% of stocks in his sample, equity borrowing costs are less than one percent per annum, and Diether et al. (2009) found that, in terms of daily data, nearly 25% (31%) of trading volume on the NYSE (NASDAQ) is made up of short-sales. However, Fleming et al. (1996) indicate that trading index futures costs about 3% of the cost of trading an equivalent portfolio of index stocks.

patterns). In this section, we present the set of possible calendar effects we have considered in our analysis. This list is mainly based on that proposed by Hansen et al. (2005) when analysing the significance of calendar effects in 25 stock indexes. Firstly, the time related patterns studied are Day-of-the-week, Month-of-the-year, Weekday-of-the-month, Week-of-the-month, Semi-month, Turn-of-the-month, End-of-Year, Holiday-effects, Semi-month-of-the-year, and Week-of-the-month-of-the-year.[7] We also include two additional effects that have been re-analysed recently: the Friday the 13th effect and the Halloween effect.[8]

Secondly, and given that we focus on index future markets, we have also introduced another pattern called Maturity-effect, which includes the daily returns from the previous day to the day after the expiration of the futures contract in the months of March, June, September and December. The idea is to identify possible negative return patterns on the day before the maturity and on the expiration day (Pope & Yadav, 1992; Vipul, 2005; among others) followed by price reversals on the day after the expiration (Chamberlain et al., 1989).

In total, we have studied 188 possible time related anomalies. Table I gives a summary of these calendar effects and their corresponding memory aids.

The presence of these patterns has been analysed in daily returns series of future contracts of the main national stock indexes of the United States, Germany

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[7] The End-of-Year pattern consists of three calendar effects: pre-Christmas from mid-December *(pre.xmas)*, between Christmas and New Year (*inter.xm.ny)* and pre-Christmas and New Year (*pre.xm.ny).* See further details about these patterns in Hansen et al. (2005).

[8] The Friday 13th effect makes reference to the fact that returns for Fridays 13th have been significantly lower than the returns for all other Fridays. The Halloween effect considers if the returns from November to April are significantly higher than during the remainder of the year. The seminal papers where these anomalies were presented are Kolb and Rodriguez (1987) and Bouman and Jacobsen (2002), respectively.

and Japan. The reason for choosing data from future markets is twofold. Firstly, market frictions in spot markets make it difficult to produce economically significant profits from trading rules based on calendar effects. Note that most of the calendar patterns are short-term strategies; in fact, some of them involve only one day. Therefore, a market with low round-trip commissions and low bid-ask spread is needed to articulate trading rules based on seasonalities. Secondly, calendar effects in stock markets make reference to consistently higher positive returns at some times of the year. Calendar patterns in stock index futures markets, unlike what occurs in the spot market, could come from large returns, both positive and negative. Therefore, we favour a market where it is easier to take long as well as short positions.

Specifically, the future contracts studied in this paper are the S&P 500, the DAX and the Nikkei futures. Our database has been taken from Reuters and consists of daily prices for the period from December 2, 1991 to April 30, 2008. This period is characterized by different scenarios with sharp upward/downward return trends as shown in Figure 1.

All the stock index futures contracts selected have several common features: they have well-developed spot and futures markets; futures prices are quoted in index points and the contract size is the futures price times a multiplier (this is USD 250 for S&P 500, EUR 25 for the DAX, and JPY 1000 for the Nikkei); finally, all contracts have deliveries in the March quarterly cycle (March, June, September and December), and all of them are settled in cash. [9]

[9] For additional information about these futures contracts, see the following official websites: Chicago Mercantile Exchange (www.cmegroup.com) for S&P 500, EUREX (www.eurexchange.com) for the DAX and Osaka Securities Exchange (www.ose.or.jp/e/) for Nikkei 225 (last accessed on July 26, 2010).

To construct a unique continuous series, following Carchano and Pardo (2009), we have chosen the last trading day of the front contract as the rollover date. Then we have calculated the return of the day after the rollover date as the logarithm of the quotient between the closing price of the following contract and the previous closing price of such contract. By doing so, all the returns are taken from the same maturity.

Table II presents the summary statistics and the autocorrelation coefficients of the linked return series. We test the normality, the correlation in mean and the correlation in variance of the series with a Jarque-Bera test (Panel A), a correlogram of the return series (Panel B) and a correlogram of the squared returns (Panel C), respectively. The three return series are not normally distributed and present dynamics both in mean and in variance. As is well known, when return series present these features, the weakness of conventional t-tests makes them not completely reliable, making it necessary to carry out another type of test in order to determine the existence of time related patterns.

3. THE PERCENTILE-T-BOOTSTRAP METHOD

3.1 Methodology

We have applied the percentile-t-bootstrap method in order to study the statistical significance of the calendar patterns. This methodology is independent of any distributional assumption and it is appropriate for samples that present a reduced number of observations. The distribution of the original return series is non-normal and there are some calendar patterns with sparse data (i.e. pre-Christmas and New Year). Therefore, this method is appropriate for dealing with the absence of normality in the return series distribution and also solves the problem of the parametric tests when there are not enough data. However, the bootstrap methodology is not suitable for dependent data and/or heteroscedasticity, and both facts are present in our data (see Table II). To take into account both features, we estimate ARMA/GARCH models for each index return series.[10] The standardized residuals obtained from these models are not normally distributed, not correlated and homoscedastic, as we can observe in Table III, and therefore they are appropriate for the bootstrap method.

Following Wilcox (2001), we have computed a 0.95 confidence interval for the mean (μ) of the standardized residuals of calendar patterns. The steps followed have been:

1. Compute the sample mean ($\bar{X}$) and the standard deviation (*s*) for one specific calendar pattern.
2. Generate a bootstrap sample by randomly sampling with replacement *N* observations from $X_1,...,X_n$, yielding $X_1*,...,X_n*$.
3. Use the bootstrap sample to compute $T*$:

$$T* = \frac{\bar{X}* - \mu}{s*/\sqrt{N}}$$

4. Repeat steps 2 and 3 B times yielding $T_1*,...,T_B*$. In our case, B = 10,000. These B values provide an approximation of the distribution of T without assuming normality.

[10] The estimated models are an ARMA(1,1)-GARCH(1,1) in the case of the S&P 500 returns, an AR(1)-GARCH(1,1) for the DAX series, and a GARCH(1,1) without ARMA structure for the Nikkei return series.

5. Set the 2.5 (L=lower) and 97.5 (U=upper) percentiles. Therefore, the percentile-t-bootstrap interval is

$$\left(X - T^*_{(U)} \frac{s}{\sqrt{N}}, X - T^*_{(L)} \frac{s}{\sqrt{N}} \right).$$

6. Repeat the whole process for each calendar pattern. If the interval does not include the zero, we conclude that the effect is statistically significant.

Following Schwert (2003), if sufficient time elapses after the discovery of an anomaly, the analysis of subsequent data also provides a test of the anomaly. For this reason, in order to get robustness in our study, we have divided the sample into two independent subsamples with a similar number of observations. The first subsample goes from December 2, 1991 to December 1, 1999 (2021 S&P 500 returns, 1998 for DAX and 1971 for Nikkei), and the second one from December 2, 1999 to April 30, 2008 (2115 S&P 500 returns, 2138 for DAX and 2069 for Nikkei). Furthermore, by doing this, we can also ensure that the evidence supporting the existence of a significant calendar pattern is based on recent data.

3.2 Results

Firstly, the percentile-t-bootstrap method has been carried out in order to obtain the bootstrap p=0.05 confidence limits in the case of positive patterns for the pre-December 1999 and for the post-December 1999 period. The results for the first and second period are presented in Panels A and B of Table IV, respectively.

According to bootstrapping results, among the positive effects in the first sub-period (Panel A of Table IV), the pattern which is statistically significant and has the highest positive mean return in the first sub-period is detected on the "Thursdays-of-July" (*thurs.jul*) for the Nikkei future (0.72%). It is interesting to note that the "Friday the 13th" (*fri13*) is the only pattern that is statistically significant in two index futures contracts, S&P 500 and DAX.[11] Regarding the positive effects in the second sub-period (see Panel B), the highest return observed is the "pre-holiday" (*pre.hol*) in the DAX future (0.68%). A systematic pattern is observed in the three futures, the "first trading day of the month" (*tom+1*). However, the only positive calendar effect that is detected in both sub-periods is the "turn-of-the-month +1" pattern for the S&P 500.

Secondly, a similar bootstrapping exercise has been carried out for the pre-1999 and post-1999 period in the case of negative returns. The results are presented in Table V.

The lowest negative return in the first sub-period (Panel A of Table V) is the "fifth-week-of-August" (*w5.aug*) in the S&P 500 futures. "Wednesday-of- September" (*wednes.sep*) is observed statistically significant in DAX and Nikkei futures in this sub-period. The worst negative return in the second sub-period (Panel B) is "first-week-of-August" (*w1.sep*) in the DAX futures. In this period there are several calendar that appear statistically significant in both S&P 500 and DAX: "September" (*sep*), "second semi-month of February" (*sm2.feb*), "fourth-week-of-September" (*w4.sep*), and "no-Halloween" (*no.hall*). Again, a systematic pattern is observed in the three futures, the "second week" (*w2*). And finally, two negative calendar effects

[11] It is interesting to draw attention to this result because, in contrast to previous empirical evidence, our results indicate that the returns for Friday 13th days were significantly higher than the returns for all other Fridays during this period.

are detected in both sub-periods: the “fourth-week-of-September” pattern for the S&P 500 and the “fourth-week-of- July” in the Nikkei futures.

Only three effects out of the 34 that are statistically significant in the first sub-period are significant in the second sub-period: the “turn-of-the-month +1” for the S&P 500, the “fourth-week-of-September” for the S&P 500, and the “fourth-week-of-July” in the Nikkei futures. These results reveal the time-varying feature of calendar patterns suggested by Hansen et al. (2005) and Doyle and Chen (2009).

4. MONTE CARLO SIMULATION

4.1 Methodology

Following Jensen (1978), a market is efficient with respect to an information set when it is impossible to make economic profits by trading on the basis of that information. Therefore, we are going to study if the persistent anomalies we have detected in Section 3 are profitable after allowing for transactions costs. In order to determine their economic significance, we have applied the Monte Carlo simulation which is also independent of any distributional assumption. We have used a benchmark to evaluate whether the calendar pattern trading profits are significantly different from profits earned at other times by chance. Following Johnson (2001), the benchmark has been determined by using the Monte Carlo simulation. The steps followed have been:

1. Calculate the accumulated original return (subtracting costs) of the N trading days of the calendar pattern.

2. Generate a sample by randomly sampling with replacement *N* observations from the index series returns of the period of study.
3. Calculate the accumulated return of the new sample.
4. Repeat the process *B* times (i.e. 10,000).
5. Determine the corresponding percentile of the accumulated return of the calendar pattern from the simulated set.
6. Repeat the whole process for each calendar pattern. If the corresponding percentile from the calendar pattern buy-and-hold (sell-and-hold) trading strategy is higher (lower) than the 97.5 (2.5), then the strategy will be considered economically significant. [12]

4.2 Results

Table VI shows the results of the Monte Carlo simulation for the period that goes from December 2, 1999 to April 30, 2008, after taking into account an estimated round trip cost of 0.05% each time the strategy is implemented. The only effect that is economically profitable is the "turn-of-the-month +1" for the S&P 500.[13] The net accumulated profit of buying in the last trading day of the month and selling on the following trading day (that is, from close to close) yields 27.54%.

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[12] Unlike Johnson (2001) that used the 95th percentile, we are interested in both long and short positions. That is the reason why we consider the 97.5th percentile for buy-and-hold strategies and the 2.5th percentile for sell-and-hold strategies.
[13] In order to know if this strategy implies a bigger profit because it implies more risk, we have also compared the standard deviation of the "*tom+1*" data with the rest of the data of the period using Brown-Forsythe's test statistic, and we conclude that it is not possible to reject the null hypothesis of equality of variances.

4.3 Evolution of the turn-of-the-month effect

A thorough review of the turn-of-the-month literature indicates that this effect has been diminishing over the last twenty years. The effect was originally detected in the US stock markets by Ariel (1987), who coined it “monthly effect” and showed that, for the period 1963-1981, virtually all of the cumulative return of value-weighted and equally weighted daily stock index returns took place on ten consecutive trading days of the calendar month. Those days began on the last trading day of the month and extended through the first nine trading days of the following month. Lakonishok and Smidt (1988) examined returns on the Dow Jones Industrial Average index for the period 1897-1986 and found that significant average daily returns were consistent on only four consecutive trading days of the calendar month, beginning with the last trading day of the month. Although Maberly and Waggoner (2000) concluded that the effect was over in the S&P 500 futures contract and spot market, McConnell and Xu (2008) have determined its continuation. Their study examines a period which includes the last trading day of the month and the first three days of the month and they observed that the main effect is produced on both the last and the first trading days of the month. Finally, our results indicate that the “turn-of-the-month effect” is concentrated on the first trading day of the month. Therefore, after twenty years, this pattern has been reduced from ten days to only one. Additionally, it is worth noticing that we have divided the “turn-of-the-month+1” pattern on S&P 500 futures market in overnight and open-to-close returns. Only the second case is significant. Therefore, the “turn-of-the-month+1” pattern seems to be concentrated on intraday returns.

5. CONCLUSIONS

Although empirical evidence suggested that calendar effects disappeared after the early 1990s, new studies and approaches assert the continuation of some of these patterns. In this paper, we present a comprehensive study of 188 potential calendar effects in S&P 500, DAX and Nikkei stock index futures contracts from 1991 to 2008. To do this, we propose a new methodology based on simulated methods to study the statistical and economic significance of such calendar effects.

Firstly, our results indicate that there are only three effects out of 34 detected in the first period that repeat their significance in the second one: the “turn-of-the-month” effect in S&P 500 buy-and-hold strategy, the “fourth-week-of-September” for the S&P 500 sell-and-hold strategy, and the “fourth-week-of-July” in the Nikkei sell-and-hold strategy. This might be due to two reasons: the activities of practitioners who implement strategies to take advantage of anomalous behaviour can cause the anomalies to disappear (see Schwert, 2003), and/or they are time-varying, which is suggested by Hansen et al. (2005) and asserted by Doyle and Chen (2009).

Secondly, the “turn-of-the-month” effect in S&P 500 futures is the only effect that is both statistically and economically significant and persistent through time. According to the literature, the “turn-of-the-month+1” effect in S&P 500 has diminished from ten days to two in twenty years. Our results show that this pattern is currently concentrated on the trading day following the last trading day of the month. This leads us to state another plausible theory, which is that the patterns are being reduced to the minimum, as would be expected in markets as they become more efficient. In this sense, a comprehensive study of calendar effects with intraday returns might shed light on known or, perhaps, unknown patterns.

REFERENCES

Ariel, R. A., (1987). A Monthly Effect in Stock Returns. Journal of Financial Economics, 18, 161-174.

Bouman, S., & Jacobsen, B. (2002). The Halloween Indicator, “Sell in May and Go Away": Another Puzzle. American Economic Review, 92, 1618-1635.

Blau, B. M., Van Ness, B. F. & Van Ness, R. A. (2009). Short Selling and the Weekend Effect for Nyse Securities. Financial Management Autumn, 603–630.

Carchano, O. & Pardo, A. (2009). Rolling Over Stock Index Futures Contracts. The Journal of Futures Markets, 29, 684-694.

Chamberlain, T. W., Cheung S. C., & Kwan, C. C. Y. (1989). Expiration day effects of index futures and options: Some Canadian evidence. Financial Analysts Journal, 45, 67–71.

Connolly, R. A. (1989). An Examination of the Robustness of the Weekend Effect. The Journal of Financial and Quantitative Analysis, 24, No. 2., 133-169.

Cooper, M. J., McConnell J. J., & Ovtchinnikov A. V. (2006). The other January effect. Journal of Financial Economics, 82, 315–341.

Cyr, D., & Llewellyn T. (1994). A time series test of calendar seasonalities in the S&P 500 index since the introduction of index derivative securities. The Journal of Futures Markets, 14, 511-529.

D’Avolio, G. (2002). The market for borrowing stock. Journal of Financial Economics, 66, 271-306.

Diether, K., Lee, K., & Werner I. (2009). Short-Sale Strategies and Return Predictability. The Review of Financial Studies, 22, No. 2.

Doyle, J. R., & Chen, C. H. (2009). The Wandering Weekday Effect in Major Stock Markets. Journal of Banking & Finance, 33, 1388-1424.

Fleming J., Ostdiek, B., & Whaley, R. E. (1996). Trading Costs and the Relative Rates of Price Discovery in Stock, Futures, and Option Markets. Journal of Futures Markets, 16, No. 4, 353-387.

Hansen, P. R., Lunde, A., & Nason, M. (2005). Testing the Significance of Calendar Effects. Federal Reserve Bank of Atlanta Working Paper Series.

Jensen, M. C. (1978). Some anomalous evidence regarding market efficiency. Journal of Financial Economics, 6, 95-102.

Johnson, J. (2001). Using Monte Carlo simulation to assess the profitability of the pre-holiday trading strategy in the index futures market. Derivatives Use, Trading and Regulations, 6, 363-374.

Keef, S. K., & Roush., M. L. (2005). Day-of-the-week Effect in the Pre-holiday Returns of the Standard & Poor's Stock Index. Applied Financial Economics, 15, 107-119.

Keef, S. P., Khaled, M., & Zhu, H., (2009). The Dynamics of the Monday Effect in International Stock Indices. International Review of Financial Analysis, 18, 125-133.

Kolb, R. W., & Rodriguez, R. J. (1987). Friday the Thirteenth: 'Part VII'- A note. The Journal of Finance, 42, 1385-1387.

Lakonishok, J., & Smidt, S. (1988). Are Seasonal Anomalies Real? A Ninety-Year Perspective. The Review of Financial Studies, 1, 403-425.

Leontitsis, A., & Siriopoulos, C. (2006). Calendar Corrected Chaotic Forecast of Financial Time Series. International Journal of Business, 11, 367-374.

Maberly, E. D., & Waggoner, D. F. (2000). Closing the Question on the Continuation of Turn-of-the-Month Effects: Evidence from the S&P 500 Index Futures Contract. Federal Reserve Bank of Atlanta Working Paper.

Maberly, E. D., & Pierce, R. M. (2003). The Halloween Effect and Japanese Equity Prices: Myth or Exploitable Anomaly. Asia-Pacific Financial Markets 10, 319-334.

McConnell, J. J., & Xu, W. (2008). Equity Returns at the Turn of the Month. Financial Analysts Journal, 64, 49-64.

Mehdian, S., & Perry, M. J. (2002). Anomalies in US Equity Markets: a Re-examination of the January Effect. Applied Financial Economics, 12, 141-145.

Pope, P. F., & Yadav, P. K. (1992). The impact of option expiration on underlying stocks: The UK evidence. Journal of Business, Finance and Accounting, 19, 329–344.

Schwert, G. W. (2003). Anomalies and Market Efficiency. G. Constantinides, M. Harris & R. Stulz, Ed., Handbook of the Economics of Finance, North-Holland, 15, 937-972.

Sutcliffe, C. M. S. (2006). Stock Index Futures. Innovative Finance Textbooks (3rd ed.), Ashgate, Hampshire.

Szakmary, A. C., & Kiefer, D. B. (2004). The Disappearing January/Turn-of-the-year Effect: Evidence from Stock Index Futures and Cash Markets. The Journal of Futures Markets, 24, 755-784.

Vipul, (2005). Futures and options expiration-day effects: The Indian evidence. The Journal of Futures Markets, 25, 1045–1065.

Wilcox, R. R. (2001). Fundamentals of Modern Statistical Methods: Substantially Improving Power and Accuracy. New York: Springer-Verlag, ISBN 0-387-95157-1, 99-103.

Table I. Summary of Calendar Effects

This table summarizes the calendar effects investigated in the study. The first column presents the effect name, the second shows the number of individual effects and the last column gives the individual effect mnemonics employed in the text and the tables.

Effect Name	# Effect	Individual Effect Names/Apprehensions
Day-of-the-week	5	*mon, tues, wednes, thurs, fri*
Month-of-the-year	12	*jan, feb, […], nov, dec*
End-of-December	3	*pre.xmas, inter.xm.ny, pre.xm.ny*
Turn-of-the-month	8	*tom-4, […], tom-1, tom+1, […], tom+4*
Holiday-effects	2	*pre.hol, post.hol*
Semi-month	2	*sm1, sm2*
Semi-month-of-the-year	24	*sm1.jan, sm2.jan, […], sm1.dec, sm2.dec*
Week-of-the-month	5	*w1, w2, w3, w4, w5*
Week-of-the-month-of-the-year	60	*w1.jan, w2.jan, […], w5.dec, w5.dec*
Week-day-of-the-month	60	*mon.jan, tues.jan, […], thurs.dec, fri.dec*
Friday the 13th	2	*fri13,no.fri13*
Halloween	2	*hall, no.hall*
Maturity-effects	3	*mat-1,mat,mat+1*

Table II. Statistical Properties Daily Returns

This table presents some statistical properties of S&P 500, DAX and Nikkei index futures daily return series for the period from December 2, 1991 to April 30, 2008. Panel A presents the summary statistics and normality tests of Jarque-Bera statistic tests. Panel B presents the Autocorrelation (AC) and the Partial Autocorrelation (PAC) of index futures daily return series. Panel C presents the Autocorrelation (AC) and the Partial Autocorrelation (PAC) of index futures daily squared return series. The Q-statistics and the associated probability indicate the significance of the serial correlation in the series.

Panel A	**S&P 500**	**DAX**	**Nikkei**
Mean	0.032%	0.036%	-0.012%
Median	0.054%	0.077%	0.000%
Maximum	5.977%	12.344%	8.004%
Minimum	-7.758%	-9.306%	-10.192%
Std. Dev.	1.046%	1.422%	1.473%
Skewness	-0.1424	-0.0856	-0.1201
Kurtosis	7.1894	7.2320	5.1781
Jarque-Bera (p-value)	3038.555 (0)	3091.561 (0)	808.3095 (0)

Panel B	**S&P 500**				**DAX**				**Nikkei**			
Lag	**AC**	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**
1	-0.029	-0.029	3.469	0.063	-0.01	-0.013	0.725	0.394	-0.052	-0.05	10.97	0.001
5	-0.026	-0.029	15.27	0.009	-0.03	-0.025	10.28	0.068	0.005	0.003	14.81	0.011
10	0.018	0.014	22.42	0.013	0	0.002	30.03	0.001	0.036	0.034	20.84	0.022

Panel C	**S&P 500**				**DAX**				**Nikkei**			
Lag	**AC**	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**
1	0.052	0.052	11.34	0.001	0.154	0.154	97.72	0	0.144	0.144	84.37	0
5	0.027	0.02	36.75	0	0.176	0.081	806.5	0	0.128	0.084	357.1	0
10	0.027	0.017	60.4	0	0.148	0.014	1647	0	0.086	0.028	560.8	0

Table III. Statistical Properties Standardized Residuals

This table presents some statistical properties of S&P 500, DAX and Nikkei index futures daily standardized residuals series for the period from December 2, 1991 to April 30, 2008. Panel A presents the skewness and kurtosis statistics and normality tests of Jarque-Bera statistic tests. Panel B presents the Autocorrelation (AC) and the Partial Autocorrelation (PAC) of index futures daily standardized residuals series. Panel C presents the Autocorrelation (AC) and the Partial Autocorrelation (PAC) of index futures daily squared standardized residuals series. The Q-statistics and the associated probability indicate the significance of the serial correlation in the series.

Panel A	S&P 500	DAX	Nikkei
Skewness	-0.519671	-0.419957	-0.258620
Kurtosis	5.318298	5.172122	4.741109
Jarque-Bera (p-value)	1112.097 (0)	934.4358 (0)	555.3310 (0)

Panel B	S&P 500				DAX				Nikkei			
Lag	**AC**	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**
1	-0.010	-0.010	0.438	0.50	0.013	0.013	0.667	0.41	-0.025	-0.025	2.597	0.10
5	-0.002	-0.002	4.412	0.49	-0.017	-0.017	6.833	0.23	0.003	0.002	3.151	0.66
10	0.015	0.015	6.299	0.79	0.021	0.021	17.902	0.05	0.040	0.040	11.747	0.30

Panel C	S&P 500				DAX				Nikkei			
Lag	**AC**	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**
1	-0.010	-0.010	0.438	0.50	-0.021	-0.021	1.894	0.17	-0.011	-0.011	0.467	0.49
5	-0.002	-0.002	4.412	0.49	0.009	0.009	2.908	0.71	-0.003	-0.003	1.041	0.96
10	0.015	0.015	6.299	0.79	0.010	0.010	3.903	0.95	0.007	0.006	3.960	0.95

Table IV. Positive Patterns for Sub-periods

This table presents the daily average return (D.A.R.) of the statistically significant calendar effects and the bootstrapped-t p=0.05 confidence interval (upper and lower limits) for S&P 500, DAX and Nikkei index futures daily standardized residual series. Panel A shows the positive calendar effects sorted by mean of returns for the period from December 2 1991 to December 1 1999. Panel B shows the positive calendar effects sorted by mean of returns for the period from the period from December 2 1999 to April 30 2008. The name of the effect is in the heading of each column. See Table I for an explanation of the mnemonics effect.

Panel A: Positive patterns in the sub-period 1991-1999

S&P 500	**fri13**	**tom+1**	**mon.mar**	**fri.dec**	**mat**	**w2.nov**
D.A.R.	0.55%	0.35%	0.34%	0.27%	0.28%	0.18%
Upper	1.21	0.43	0.61	0.54	0.70	0.51
Lower	0.21	0.10	0.07	0.01	0.03	0.02
DAX	**w1.jul**	**fri13**	**w3.nov**	**mon.may**	**w4.dec**	**sm1.jul**
D.A.R.	0.58%	0.56%	0.56%	0.41%	0.34%	0.34%
Upper	0.96	0.92	0.61	0.77	0.86	0.49
Lower	0.20	0.12	0.14	0.04	0.02	0.07
Nikkei	**thurs.jul**	**w5.jan**	**fri.mar**			
D.A.R.	0.72%	0.70%	0.19%			
Upper	0.82	0.85	0.73			
Lower	0.14	0.10	0.09			

Panel B: Positive patterns in the sub-period 1999-2008

S&P 500	**tom+1**							
D.A.R.	0.33%							
Upper	0.51							
Lower	0.07							
DAX	**pre.hol**	**w5.jul**	**w5.dec**	**tom+1**	**tom-3**			
D.A.R.	0.68%	0.63%	0.51%	0.37%	0.31%			
Upper	0.82	0.86	0.64	0.48	0.37			
Lower	0.15	0.03	0.07	0.05	0.01			
Nikkei	**w5.nov**	**thurs.feb**	**xm.ny**	**wednes.apr**	**w1.apr**	**post.hol**	**tues.aug**	**tom+1**
D.A.R.	0.63%	0.58%	0.58%	0.57%	0.54%	0.52%	0.44%	0.38%
Upper	0.80	0.66	0.62	0.58	0.56	0.58	0.60	0.46
Lower	0.11	0.13	0.16	0.01	0.05	0.10	0.06	0.07

Table V. Negative Patterns for Sub-periods

This table presents the daily average return (D.A.R.) of the statistically significant calendar effects returns and the bootstrapped-t p=0.05 confidence interval (upper and lower limits) for S&P 500, DAX and Nikkei index futures daily standardized residual series. Panel A shows the negative calendar effects sorted by mean of returns for the period from December 2, 1991 to December 1, 1999. Panel B the positive calendar effects sorted by mean of returns for the period from the period from December 2, 1999 to April 30, 2008. The name of the effect is in the heading of each column. See Table I for an explanation of the mnemonics effect.

Panel A: Negative patterns in the sub-period 1991-1999

S&P 500	**w5.aug**	**thurs.aug**	**thurs.may**	**w4.sep**				
D.A.R.	-0.65%	-0.37%	-0.21%	-0.19%				
Upper	-0.15	-0.16	-0.04	-0.06				
Lower	-1.28	-0.83	-0.70	-0.66				
DAX	**thurs.sep**	**w5.sep**	**mon.aug**	**wednes.sep**	**sm2.sep**	**wednes.apr**	**sm2.jul**	**thurs**
D.A.R.	-0.64%	-0.42%	-0.41%	-0.41%	-0.26%	-0.18%	-0.15%	-0.10%
Upper	-0.10	-0.01	-0.16	-0.03	-0.03	-0.00	-0.01	-0.01
Lower	-0.78	-0.81	-0.79	-0.65	-0.49	-0.53	-0.55	-0.19
Nikkei	**wednes.sep**	**fri.aug**	**w4.jul**	**mon.jun**	**tom+4**	**mon**	**w4**	
D.A.R.	-0.55%	-0.53%	-0.52%	-0.47%	-0.27%	-0.17%	-0.13%	
Upper	-0.00	-0.12	-0.07	-0.02	-0.03	-0.04	-0.03	
Lower	-0.70	-0.89	-0.83	-0.76	-0.36	-0.26	-0.22	

Panel B: Negative patterns in the sub-period 1999-2008

S&P 500	**mon.mar**	**w3.jan**	**w3.jul**	**w2.apr**	**sep**	**wednes.sep**	**w4.jun**	**fri**
D.A.R.	-0.34%	-0.33%	-0.31%	-0.29%	-0.28%	-0.20%	-0.18%	-0.15%
Upper	-0.01	-0.09	-0.04	-0.03	-0.01	-0.05	-0.03	-0.03
Lower	-0.74	-0.72	-0.57	-0.83	-0.33	-0.70	-0.58	-0.22
	sm2.feb	**sm2.jan**	**sm2.jul**	**feb**	**w4.sep**	**no.fri13**	**w2**	**w4**
D.A.R.	-0.20%	-0.13%	-0.13%	-0.10%	-0.08%	-0.08%	-0.08%	-0.03%
Upper	-0.08	-0.00	-0.00	-0.03	-0.02	-0.03	-0.06	-0.02
Lower	-0.71	-0.45	-0.45	-0.36	-0.62	-0.23	-0.23	-0.19
	no.hall	**sm2**	**hall**					
D.A.R.	-0.02%	-0.01%	-0.00%					
Upper	-0.02	-0.02	-0.00					
Lower	-0.14	-0.14	-0.13					
DAX	**w1.aug**	**tues.jul**	**w2.jun**	**sep**	**tues.jan**	**w4.sep**	**thurs.aug**	**sm2.feb**
D.A.R.	-0.67%	-0.48%	-0.36%	-0.34%	-0.31%	-0.28%	-0.22%	-0.14%
Upper	-0.26	-0.02	-0.05	-0.04	-0.01	-0.04	-0.02	-0.00
Lower	-0.91	-0.74	-0.69	-0.37	-0.57	-0.75	-0.67	-0.59
	w2	**no.hall**						
D.A.R.	-0.08%	-0.05%						
Upper	-0.04	-0.01						
Lower	-0.21	-0.14						
Nikkei	**wednes.oct**	**wednes.jan**	**mat**	**w2.dec**	**w4.jul**	**tues.feb**	**tom+3**	**sm1.jul**
D.A.R.	-0.51%	-0.48%	-0.50%	-0.43%	-0.42%	-0.33%	-0.28%	-0.24%
Upper	-0.08	-0.11	-0.06	-0.08	-0.06	-0.01	-0.02	-0.01
Lower	-0.71	-0.80	-0.87	-0.60	-0.74	-0.69	-0.44	-0.43
	jul	**w2**	**wednes**	**sm1**				
D.A.R.	-0.16%	-0.16%	-0.10%	-0.06%				
Upper	-0.01	-0.04	-0.01	-0.00				
Lower	-0.30	-0.22	-0.20	-0.13				

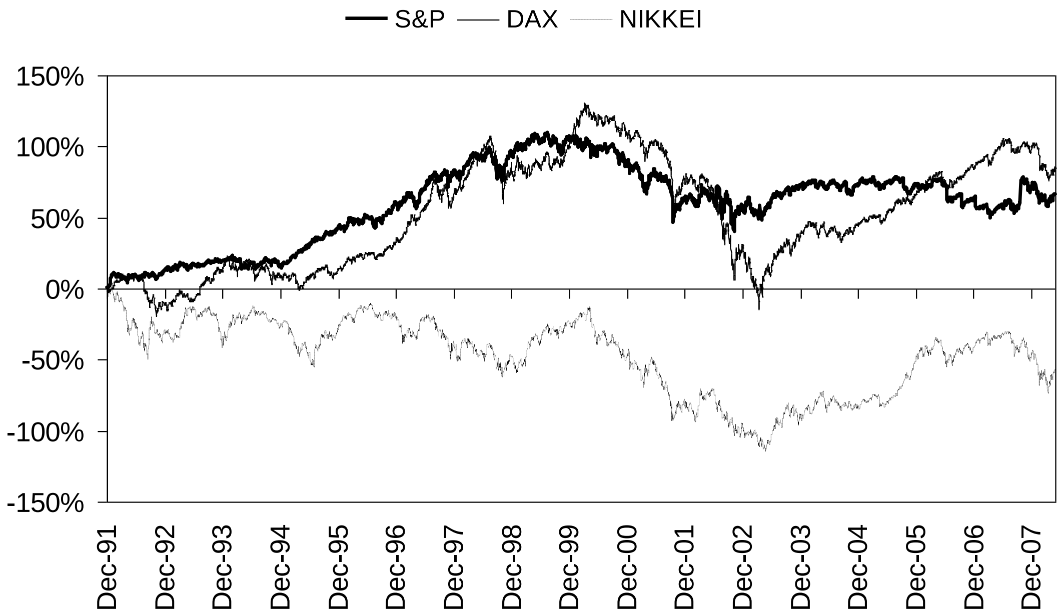
Table VI. Monte Carlo Test for Second Sub-period

This table shows the Monte Carlo test for the significant calendar effects detected with the bootstrap method that are persistent over time for the period from December 2, 1999 to April 30, 2008. N is the number of trading days for each calendar pattern; BHR indicates the buy-and-hold strategy return and P_{BHR} its corresponding percentile; SHR stands for the sell-and-hold strategy return and P_{SHR} its corresponding percentile. A buy-and-hold strategy (sell-and-hold strategy) is economically significant when its corresponding percentile is higher (lower) than 97.5 (2.5). See Table I for an explanation of the mnemonics effect.

Contract	S&P 500	S&P 500	Nikkei
Effect	tom+1	w4.sep	w4.jul
N	100	40	37
Strategy	BHR = 27.54%	SHR = 2.63%	SHR = 15.16%
Percentile	P_{BHR} = 99.38	P_{SHR} = 38.21%	P_{SHR} = 5.26

Figure 1. Accumulated Returns on Stock Index Futures Contracts

This figure represents the accumulated returns of S&P 500, DAX and Nikkei futures for the period from December 2, 1991 to April 30, 2008.



Chapter 3

Forecasting VaR

in Spot and Futures Equity Markets

1. INTRODUCTION

Predicting future financial market volatility is crucial for risk management of financial institutions. The empirical evidence suggests that a suitable market risk model must be capable of handling the idiosyncratic features of volatility, that is, daily returns time variant amplitude and volatility clustering. There is a well-developed literature in financial econometrics that demonstrates how autoregressive conditional heteroskedastic (ARCH) and generalized ARCH (GARCH) models — developed by Engle (1982) and Bollerslev (1986), respectively — can be employed to explain the clustering effect of volatility. Moreover, the selected model should consider the stylized fact that asset return distributions are not normally distributed, but instead have been shown to exhibit patterns of leptokurtosis and skewness.

Taking a different tact than the ARCH/GARCH approach for dealing with the idiosyncratic features of volatility, Kim et al. (2009) formulate an alternative model based on subclasses of the infinitely divisible (ID) distributions. More specifically, for the S&P 500 return, they empirically investigate five subclasses of the ID distribution, comparing their results to that obtained using GARCH models based on innovations that are assumed to follow a normal distribution (what we refer to as simply normal innovations). They conclude that, due to their failure to focus on the distribution in the tails, GARCH models based on the normal innovations may not be as well suited as ID models for predicting financial crashes.

Because of its popularity, most empirical studies have examined value at risk (VaR) as a risk measure. These studies have focused on stock indices. For example, Kim et al. (2011), Sun et al. (2010), and Asai and McAleer (2009) examine the S&P 500, DAX 30, and Nikkei 225 stock indices, respectively. A few researchers have studied this risk measure for stock index futures contracts: Huang and Lin (2004) (Taiwan stock index futures) and Tang and Shieh (2006) (S&P 500, Nasdaq 100, and Dow Jones stock index futures). As far as we know, there are no empirical studies comparing VaR spot and futures indices. For this reason, we compare the predictive performance of one-day-ahead VaR forecasts in these two markets.

We then introduce trading volume into the model, particularly, within the GARCH framework. There are several studies that relate trading volume and market volatility for equities and equity futures markets. Studies by Epps and Epps (1976), Smirlock and Starks (1985), and Schwert (1989) document a positive relation between volume and market volatility. Evidence that supports the same relation for futures is provided by Clark (1973), Tauchen and Pitts (1983), Garcia, Leuthold, and Zapata (1986), Ragunathan and Peker (1997), and Gwilym, MacMillan, and Speight (1999). Collectively, these studies clearly support the theoretical prediction of a positive and contemporaneous relationship between trading volume and volatility. This result is a common empirical finding for most financial assets, as Karpoff (1987) showed when he summarized the results of several studies on the positive relation between price changes and trading volume for commodity futures, currency futures, common stocks, and stock indices.

Foster (1995) concluded that not only is trading volume important in determining the rate of information (i.e. any news that affects the market), but also lagged volume plays a role. Although contemporary trading volume is positively related to volatility, lagged trading volume presents a negative relationship. Empirically, investigating daily data for several indices such as the S&P 500 futures contract, Wang and Yau (2000) observe that there is indeed a negative link between lagged trading volume and intraday price volatility. This means that an increase in trading volume today (as a measure of liquidity) will imply a reduction in price volatility tomorrow. In their study of five currency futures contracts, Fung and Patterson (1999) do in fact find a negative relationship between return volatility and past trading volume. In their view, the reversal behaviour of volatility with trading volume is generally consistent with the overreaction hypothesis (see Conrad *et al*. (1994)), and supports the sequential information hypothesis (see Copeland (1976)), which explains the relationship between return volatility and trading volume.

Despite the considerable amount of research in this area, there are no studies that use trading volume in an effort to improve the capability of models to forecast one-day-ahead VaR. Typically, in a VaR context, trading volume is only employed as a proxy for "liquidity risk" — the risk associated with trying to close out a position. In this paper, in contrast to prior studies, we analyse the impact of introducing trading volume on the ability to enhance performance in forecasting VaR one day ahead. We empirically test whether the introduction of trading volume will reduce the number of violations (i.e., the number of times when the estimated loss exceeds the observed one) in the spot and futures equity markets of the U.S., Germany, and Japan.

The remainder of this paper is organized as follows. ARMA-GARCH models with normal and tempered stable innovations are reviewed in Section 2. In Section 3, we discuss parameter estimation of the ARMA-GARCH models and forecasting daily return distributions. VaR values and backtesting of the ARMA-GARCH models are also reported in Section 2, along with a comparison of the results for (1) the spot and futures markets and (2) the normal and tempered stable innovations. Trading volume is introduced into the ARMA-GARCH model with tempered stable innovations in Section 4. VaR and backtesting of the ARMA-GARCH with different variants of trading volume are presented and compared to the results for models with and without trading volume. We summarize our principal findings in Section 5.

2. ARMA-GARCH model with normal and tempered stable innovations

In this section, we provide a review of the ARMA-GARCH models with normal and tempered stable innovations. For a more detailed discussion, see Kim et al. (2011).

Let $(S_t)_{t\geq 0}$ be the asset price process and $(y_t)_{t\geq 0}$ be the return process of $(S_t)_{t\geq 0}$ defined by $y_t = \log\frac{S_t}{S_{t-1}}$. The ARMA(1,1)-GARCH(1,1) model is:

$$\begin{cases} y_t = ay_{t-1} + b\sigma_{t-1}\varepsilon_{t-1} + \sigma_t\varepsilon_t + c_t \\ \sigma_t = \alpha_0 + \alpha_1\sigma_{t-1}^2\varepsilon_{t-1}^2 + \beta_1\varepsilon_t\sigma_{t-1}^2 \end{cases} \quad (1)$$

where $\varepsilon_0 = 0$, and a sequence $(\varepsilon_t)_{t\subset N} = 0$ of independent and identically distributed iid real random variables. The innovation ε_t is assumed to follow the standard normal distribution. This ARMA(1,1)-GARCH(1,1) model is referred to as the “normal-ARMA-GARCH model.”

If the ε_t's are assumed to be tempered stable innovations, then we obtain a new ARMA(1,1)-GARCH(1,1) model. In this paper, we will consider the standard classical tempered stable (denoted by stdCTS) distributions. This ARMA(1,1)-GARCH(1,1) model is defined as follows: CTS-ARMA-GARCH model: $\varepsilon_t \sim \mathrm{stdCTS}(\alpha, \lambda_+, \lambda_-)$. This distribution does not have a closed-form solution for its probability density function. Instead, it is defined by its characteristic function as follows: Let $a \in (0,2)\ \{1\}, C, \lambda_+, \lambda_- > 0$, and $m \in \Re$. Then a random variable X is said to follow the classical tempered stable (CTS) distribution if the characteristic function of X is given by

$$\phi_X(u) = \phi_{CTS}(u : \alpha, C, \lambda_+, \lambda_-, m) \tag{2}$$

$$= \exp\Big(ium - iuCT(1-\alpha)\big(\lambda_+^{\alpha-1} - \lambda_-^{\alpha-1}\big) +$$

$$+ C\Gamma(-\alpha)\big((\lambda_+ - iu)^{\alpha} - \lambda_+^{\alpha} + (\lambda_- - iu)^{\alpha} - \lambda_-^{\alpha}\big),$$

and we denote $X \sim \mathrm{CTS}(\alpha, \mathrm{C}, \lambda_+, \lambda_-, m)$.

The cumulants of X are defined by

$$c_n(X) = \frac{\partial^n}{\partial u^n} \log \mathrm{E}\left[e^{iuX}\right]|_{u=0}, \text{ n=1,2,3,\ldots .}$$

For the tempered stable distribution, we have $E[X] = c_1(X) = m$. The cumulants of the tempered stable distribution for *n* =2, 3, … are

$$c_n(X) = C\Gamma(n-\alpha)\big(\lambda_+^{\alpha-n} + (-1)^n \lambda_-^{\alpha-n}\big).$$

By substituting the appropriate value for the two parameters *m* and *C* into the three tempered stable distributions, we can obtain tempered stable distributions with zero mean and unit variance. That is,

$X \sim \mathrm{CTS}(\alpha, \mathrm{C}, \lambda_+, \lambda_-, 0)$ has zero mean and unit variance by substituting

$$C = \left(\Gamma(2-\alpha)\left(\lambda_+^{\alpha-2} + \lambda_-^{\alpha-2}\right)\right)^{-1}. \qquad (3)$$

The random variable *X* is referred to as the standard CTS distribution with parameters $(\alpha, \lambda_+, \lambda_-)$ and denoted by $X \sim \mathrm{stdCTS}(\alpha, \lambda_+, \lambda_-)$.

3. VaR FOR THE ARMA-GARCH MODEL

In this section, we discuss VaR for the ARMA-GARCH model with normal and tempered stable innovations.

3.1 VaR and backtesting

The definition of VaR for a significance level η is

$$VaR_\eta(X) = -\inf\{x \in \Re \mid P(X \le x) > \eta\}.$$

If we take the ARMA-GARCH model described in Section 2, we can define VaR for the information until time *t* with significance level η as

$$VaR_{\eta}\left(y_{t+1}\right) = -\inf\left\{x \in \Re \mid P_t\left(y_{t+1} \le x\right) > \eta\right\}.$$

where $P_t(A)$ is the conditional probability of a given event A for the information until time *t*.

Two models are considered: normal-ARMA(1,1)-GARCH(1,1) and stdCTS-ARMA(1,1)-GARCH(1,1). For both models, the parameters have been estimated for the time series between December 14, 2004 and December 31, 2008. For each daily estimation, we worked with 10 years of historical daily performance for the S&P 500, DAX 30, and Nikkei 225 spot and futures indices. More specifically, we used daily returns calculated based on the closing price of those indices. In the case of futures indices, we constructed a unique continuous-time series using the different maturities of each futures index following the methodology proposed by Carchano and Pardo (2009).[14] Then, we computed VaRs for both models.

The maximum likelihood estimation method (MLE) is employed to estimate parameters of the normal-ARMA(1,1)-GARCH(1,1) model. For the CTS distribution, the parameters are estimated as follows:

1. Estimate parameters $\alpha_0, \alpha_1, \beta_1, a, b, c$ with normal innovations by the MLE.
2. Extract residuals using those parameters.
3. Fit the parameters of the innovation distribution (CTS) to the extracted residuals using MLE.

[14] Thus, the last trading day of the front contract is chosen as the rollover date. Then, the return of the day after the rollover date is calculated as the quotient between the closing price of the following contract and the previous closing price of such contract. By doing so, all the returns are taken from the same maturity.

In order to determine the accuracy of VaR for the two models, backtesting using *Kupiec's proportion of failures test* (Kupiec, 1995) is applied. We first calculate the number of violations. Then, we compare the number of violations with the conventional number of exceedances at a given significance level. In Table I the number of violations and *p*-values for Kupiec's backtest for the three stock indices over the four one-year periods are reported. Finally, we sum up the number of violations and their related *p*-values for 1%-VaRs for the normal and CTS-ARMA-GARCH models.

Based on Table I, we conclude the following for the three stock indices. First, a comparison of the normal and tempered stable models indicates that there are no cases using the tempered stable model at the 5% significance level, whereas the normal model is rejected five times. This evidence is consistent with the findings of Kim et al. (2011). Second, a comparison of the spot and futures indices indicates that spot data provide less than or the same number of violations than futures data. One potential explanation is that futures markets are more volatile, particularly, when the market falls.[15] This overreaction to bad news could cause the larger number of violations.

4. INTRODUCTION OF TRADING VOLUME

In the previous section, we showed the usefulness of the tempered stable model for stock index futures. Motivated by the vast literature linking trading volume

[15] We compared the spot and futures series when the markets discount bad news (negative returns). We find that for the three stock indices, futures volatility is significantly greater than spot volatility at a 5% significance level. Moreover, for all three stock indices, the minimum return and the 1%-percentile return are also lower for futures data than spot data.

and volatility, for the first time we investigate whether the introduction of trading volume in the CTS model could improve its ability to forecast one-day-ahead VaR.

Let $(S_t)_{t\geq 0}$ be the asset price process and $(y_t)_{t\geq 0}$ be the return process of $(S_t)_{t\geq 0}$ defined by $y_t = \log \frac{S_t}{S_{t-1}}$. We propose the following ARMA(1,1)-GARCH(1,1) with trading volume model:

$$\begin{cases} y_t = a y_{t-1} + b\sigma_{t-1}\varepsilon_{t-1} + \sigma_t \varepsilon_t + c_t \\ \sigma_t = \alpha_0 + \alpha_1 \sigma_{t-1}^2 \varepsilon_{t-1}^2 + \beta_1 \varepsilon_t \sigma_{t-1}^2 + \gamma_1 Vol_{t-1}, \end{cases} \tag{4}$$

where $\varepsilon_0 = 0$, and a sequence $(\varepsilon_t)_{t\subset N} = 0$ of *iid* real random variables. The innovation ε_t is assumed to be the tempered stable innovation. We will consider the standard classical tempered stable distributions. This new ARMA(1,1)-GARCH(1,1)-V model is defined as follows:

CTS-ARMA-GARCH-V model: $\varepsilon_t \sim \mathrm{stdCTS}(\alpha, \lambda_+, \lambda_-)$.

4.1. Different variants of trading volume

For the S&P 500 cash and futures markets, we test the following versions of trading volume in order to determine which one would be the most appropriate:

- Lagged trading volume in levels: V(*t*-1)
- Logarithm of lagged trading volume: Log[V(*t*-1)]
- Relative change of lagged trading volume: Ln[V(*t*-1)/V(t-2)]

The spot series trading volume is in dollars; for the futures series, the trading value is in number of contracts. We can calculate the volume of the futures market in dollars too. The tick value of the S&P 500 futures contract is 0.1 index points or $25. Multiplying the number of contracts by the price, and finally by $250 (the contract's multiple), we obtain the trading volume series for the futures contract in dollars. Thus, for the futures contract we get three new versions of trading volume to test:

- Lagged trading volume in dollars: V$(*t*-1)
- Logarithm of trading volume in dollars: Log[V$(*t*-1)]
- Relative change of lagged trading volume in dollars: Ln[V$(*t*-1)/V$(*t*-2)]

By doing that, we can determine which series (in dollars or in contracts) seems to be more useful for the futures index.

In Table II we report the number of violations and *p*-values of Kupiec's backtest for the different versions of the CTS-ARMA-GARCH-V model for the S&P 500 spot and futures indices. We count the number of violations and the corresponding *p*-values for 1%-VaRs of both markets. From Table II, we conclude the following:

- The model with the lagged trading volume in level is rejected at the 1% significance level in all four years for the S&P 500 spot, and for the second period (2005-2006) for the S&P 500 futures.
- The logarithm of trading volume in the model is rejected at the 5% significance level for the spot market for the third period (2006-2007), but it is not rejected in any period for the futures market.

- The relative change of the lagged volume is not rejected at the 5% significance level in any period in either market. Of the three versions of trading volume tests, this version seems to be the most useful for both spot and futures markets.
- The results for trading volume in contracts and the trading volume in dollars in the futures market indicate that the former is rejected at the 1% significance level only for the lagged trading volume in level in the second period(2005- 2006). Trading volume in dollars is rejected three times, for the lagged trading volume in levels for the third period (2006-2007), and for the lagged relative trading volume change in the last two periods (2006-2007, and 2007-2008). These findings suggest that the trading volume in contracts is the preferred measure.

4.2. Lagged relative change of trading volume.

As we have just seen, the variant of trading volume that seems more useful for forecasting one-day-ahead VaR using CTS-ARMA-GARCH is the relative change of trading volume. Next, we compare the original CTS-ARMA-GARCH model with the new CTS-ARMA-GARCH-V model where V is the lagged relative change of trading volume. Table III shows the number of violations and *p*-values of Kupiec's backtest for the two models for the three stock indices and both markets. We sum up the number of violations and the corresponding *p*-values for 1%-VaRs for each case.

Our conclusions from Table III are as follows. For the spot markets, the introduction of trading volume does not mean a reduction in the number of violations in any period for any index. However, for the futures markets, the numbers of violations are the same or lower for the model with trading volume than with the original model. Thus, by introducing trading volume, we get a slightly more conservative model, increasing the VaR forecasted for futures equity markets.

4.3. Lagged trading volume or forecasting contemporaneous trading volume.

Although there is some evidence which supports the relationship between lagged trading volume and volatility, the literature is not as extensive as the studies that establish a strong link between volatility and contemporaneous trading volume. As there are countless ways to try to forecast trading volume, we begin by introducing contemporaneous trading volume relative change in the model as a benchmark to assess whether it is worthwhile to forecast trading volume.

In Table IV we show the number of violations and *p*-values of Kupiec's backtest for the CTS-ARMA-GARCH with contemporaneous and lagged relative change of trading volume for the three stock indices for both markets. We count the number of violations and the corresponding p-values for 1%-VaRs for the six indices.

Our conclusions based on the results reported in Table IV are as follows. First, with the exception of the S&P 500 futures, the introduction of the contemporaneous relative change of trading volume in the model is rejected at the 1% significance level for the last period analysed (2007-2008). In the case of the S&P 500 futures, it is rejected at the significance level of 5% for the third period (2006-2007). Second, the model with lagged relative change of trading volume is not rejected for any stock index or market. It seems to be more robust than contemporaneous trading volume (although, in general, there are fewer violations when using it).

Our results suggest that it is not worth making an effort to predict contemporaneous trading volume because the forecasts will be flawed and two variables would have to be predicted (VaR and contemporaneous trading volume). Equivalently, the lagged trading volume relative change appears to be more robust because it is not rejected in any case, although it provides a poor improvement to the model.

5. CONCLUSIONS

Based on an empirical analysis of spot and futures trading for the S&P 500, DAX, and Nikkei stock indices, in this paper we provide empirical evidence about the usefulness of using classical tempered stable distributions for predicting one-day-ahead VaR. Unlike prior studies that investigated CTS models in the cash equity markets, we analysed their suitability for both spot markets and futures markets. We find in both markets the CTS models perform better in forecasting day-ahead VaR than models that assume innovations follow the normal law.

Second, we introduced trading volume into the CTS model. Our empirical evidence suggests that lagged trading volume relative change provides a slightly more conservative model (i.e., reduces the number of violations) to predict one-day-ahead VaR for stock index futures contracts. We cannot state the same for the cash market because the results are mixed depending on the index. After that, we introduced contemporaneous trading volume in order to improve the forecasting ability of the model. We find that does not seem to be worth the effort. That is, trading volume appears not to offer enough information to improve forecasts.

Finally, we compared the number of violations of the estimated VaR in the spot and futures equity markets. For the CTS model without volume, in general, we find fewer violations in the spot indices than in the equivalent futures contracts. In contrast, our results suggest that the number of violations in futures markets is less in the case of the CTS model with trading volume in comparison to the CTS model that ignores trading volume. But if we contrast spot and futures equity markets, violations are still greater for futures than in spot markets. A possible reason is that futures markets demonstrate extra volatility or an overreaction when the market falls with respect to their corresponding spot markets.

REFERENCES

Asai, M., and McAleer, M. (2009). The structure of dynamic correlations in multivariate stochastic volatility models. Journal of Econometrics, 150, 182-192.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics, 31, 307-327.

Carchano, O., and Pardo, A. (2009). Rolling over stock index futures contracts. Journal of Futures Markets, 29, 684-694.

Clark, P. K. (1973). A subordinated stochastic process model with finite variance for speculative prices. Econometrica, 41, 135-155.

Conrad, J. S., Hameed, A., and Niden, C. (1994). Volume and autocovariances in short-horizon individual security returns. Journal of Finance, 49 (4), 1305-1329

Copeland, T. E. (1976). A model of asset trading under the assumption of sequential information arrival. Journal of Finance, 34 (1), 1149-1168.

Engle, R. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance with estimates of the variance of United Kingdom inflation. Econometrica, 50, 987-1007.

Epps, T. W., and Epps, L. M. (1976). The stochastic dependence of security price changes and transaction volumes: implications for the mixture-of-distribution hypothesis. Econometrica, 44, 305-321.

Foster, F. J. (1995). Volume-volatility relationships for crude oil futures markets. Journal of Futures Markets, 15, 929-951.

Fung, H. G., and Patterson, G. A. (1999). The dynamic relationship of volatility, volume, and market depth in currency futures markets. Journal of International Financial Markets, Institutions and Money, 17, 33-59.

Garcia, P., Leuthold, M. R., and Zapata, H. (1986). Lead-lag relationships between trading volume and price variability: New evidence. Journal of Futures Markets, 6, 1-10.

Gwilym, O., MacMillan, D., and Speight A. (1999). The intraday relationship between volume and volatility in LIFFE futures markets. Applied Financial Economics, 9, 593-604.

Huang C. Y., and Lin J. B. (2004). Value-at-risk analysis for Taiwan stock index futures: fat tails and conditional asymmetries in return innovations. Review of Quantitative Finance and Accounting, 22, 79–95.

Karpoff, J. M. (1987). The relation between price changes and trading volume: A survey. Journal of Financial and Quantitative Analysis, 22, 109-126.

Kim, Y. S., Rachev, S. T., Bianchi, M. L., and Fabozzi, F. J. (2009). Computing VaR and AVaR in infinitely divisible distributions. Technical Report, Chair of Econometrics, Statistics and Mathematical Finance School of Economics and Business Engineering University of Karlsruhe.

Kim, Y. S., Rachev, S. T., Bianchi, M. L., Mitov, I., and Fabozzi, F. J. (2011). Time series analysis for financial market meltdowns. Journal of Banking and Finance, 35 (8), 1879-1891.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models, Journal of Derivatives, 6, 6-24.

Ragunathan, V., and Peker, A. (1997). Price variability, trading volume and market depth: evidence from the Australian futures market, Applied Financial Economics, 7, 447-454.

Schwert, G. W. (1989). Why does stock market volatility change over time, Journal of Finance, 44, 1115-1153.

Smirlock, M., and Starks, L. (1985). A further examination of stock price changes and transactions volume, Journal of Financial Research, 8, 217-225.

Sun, W., Rachev, S., Chen, Y., Fabozzi, and F. J. (2009). Measuring intra-daily market risk: A neural network approach. Technical Report, Chair of Econometrics, Statistics and Mathematical Finance School of Economics and Business Engineering University of Karlsruhe.

Tang, L. T., and Shieh, J. S. (2006). Long memory in stock index futures markets: A value-at-risk approach, Physica A: Statistical Mechanics and its Applications, 366, 437-448.

Tauchen, G., and Pitts, M. (1983). The price variability-volume relationship on speculative markets, Econometrica, 51, 485-505.

Wang, G., and Yau, J. (2000). Trading volume, bid-ask spread, and price volatility in futures markets, Journal of Futures Markets, 20, 943-970.

Table I. Normal and stdCTS models Failures

Number of violations (*N*) and p values of Kupiec's proportion of failures test for the S&P 500, DAX 30 and Nikkei 225 spot and futures indices data.

Model	1 year (255 days)			
	Dec. 14, 2004 ~ Dec. 15, 2005	Dec. 16, 2005 ~ Dec. 20, 2006	Dec. 21, 2006 ~ Dec. 27, 2007	Dec. 28, 2007 ~ Dec. 31, 2008
	N(*p*-value)	*N*(*p*-value)	*N*(p-value)	*N*(*p*-value)
		S&P 500 Spot		
Normal-ARMA-GARCH	1(0.2660)	3(0.7829)	8(0.0061)	10(0.0004)
CTS-ARMA-GARCH	0	2(0.7190)	6(0.0646)	4(0.3995)
		S&P 500 Futures		
Normal-ARMA-GARCH	3(0.7829)	3(0.7829)	7(0.0211)	9(0.0016)
CTS-ARMA-GARCH	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
		DAX 30 Spot		
Normal-ARMA-GARCH	4(0.3995)	4(0.3995)	3(0.7829)	6(0.0646)
CTS-ARMA-GARCH	4(0.3995)	4(0.3995)	3(0.7829)	4(0.3995)
		DAX 30 Futures		
Normal-ARMA-GARCH	3(0.7829)	5(0.1729)	6(0.0646)	6(0.0646)
CTS-ARMA-GARCH	3(0.7829)	4(0.3995)	6(0.0646)	3(0.7829)
		Nikkei 225 Spot		
Normal-ARMA-GARCH	2(0.7190)	4(0.3995)	5(0.1729)	5(0.1729)
CTS-ARMA-GARCH	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
		Nikkei 225 Futures		
Normal-ARMA-GARCH	2(0.7190)	2(0.7190)	7(0.0211)	5(0.1729)
CTS-ARMA-GARCH	5(0.1729)	5(0.1729)	6(0.0646)	6(0.0646)

Table II. stdCTS with Volume Models Failures

Number of violations (*N*) and *p* values of Kupiec's proportion of failures test for the S&P 500 spot and futures indices with the different variants of volume into the stdCTS-ARMA(1,1)-GARCH(1,1) model. V(*t*-1), log[V(*t*-1)] and ln[V(*t*-1)/V(*t*-2)] stand for levels, logarithm and relative change of the lagged trading volume, respectively. V$(*t*-1), log[V$(*t*-1)] and ln[V$(*t*-1)/V$(*t*-2)] stand for levels, logarithm and relative change of the lagged trading volume in dollars, respectively.

Model	1 year (255 days)			
	Dec. 14, 2004 ~ Dec. 15, 2005	Dec. 16, 2005 ~ Dec. 20, 2006	Dec. 21, 2006 ~ Dec. 27, 2007	Dec. 28, 2007 ~ Dec. 31, 2008
	N(*p*-value)	*N*(*p*-value)	*N*(p-value)	*N*(*p*-value)
	S&P 500 Spot			
CTS-ARMA-GARCH-V(*t*-1)	16(0.0000)	10(0.0004)	23(0.0000)	26(0.0000)
CTS-ARMA-GARCH-log[V(*t*-1)]	1(0.2660)	4(0.3995)	10(0.0004)	6(0.0646)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	0	2(0.7190)	6(0.0646)	5(0.1729)
	S&P 500 Futures			
CTS-ARMA-GARCH-V(*t*-1)	1(0.2660)	11(0.0001)	4(0.3995)	6(0.0646)
CTS-ARMA-GARCH-log[V(*t*-1)]	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	1(0.2660)	3(0.7829)	3(0.7829)	5(0.1729)
CTS-ARMA-GARCH-V$(*t*-1)	0	1(0.2660)	15(0.0000)	3(0.7829)
CTS-ARMA-GARCH-log[V$(*t*-1)]	3(0.7829)	3(0.7829)	4(0.3995)	5(0.1729)
CTS-ARMA-GARCH-ln[V$(*t*-1)/V$(*t*-2)]	2(0.7190)	0	8(0.0061)	8(0.0061)

Table III. stdCTS and stdCTS with Volume Models Failures

Number of violations (*N*) and *p* values of Kupiec's proportion of failures test for the S&P 500, DAX 30 and Nikkei 225 spot and futures indices. stdCTS-ARMA-GARCH and stdCTS-ARMA-GARCH with lagged relative change of trading volume are compared.

Model	1 year (255 days)			
	Dec. 14, 2004 ~ Dec. 15, 2005	Dec. 16, 2005 ~ Dec. 20, 2006	Dec. 21, 2006 ~ Dec. 27, 2007	Dec. 28, 2007 ~ Dec. 31, 2008
	N(*p*-value)	*N*(*p*-value)	*N*(*p*-value)	*N*(*p*-value)
	S&P 500 Spot			
CTS-ARMA-GARCH	0	2(0.7190)	6(0.0646)	4(0.3995)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	0	2(0.7190)	6(0.0646)	5(0.1729)
	S&P 500 Futures			
CTS-ARMA-GARCH	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	1(0.2660)	3(0.7829)	3(0.7829)	5(0.1729)
	DAX 30 Spot			
CTS-ARMA-GARCH	4(0.3995)	4(0.3995)	3(0.7829)	4(0.3995)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	5(0.1729)	4(0.3995)	3(0.7829)	5(0.1729)
	DAX 30 Futures			
CTS-ARMA-GARCH	3(0.7829)	4(0.3995)	6(0.0646)	3(0.7829)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	3(0.7829)	4(0.3995)	5(0.1729)	3(0.7829)
	Nikkei 225 Spot			
CTS-ARMA-GARCH	1(0.2660)	3(0.7829)	4(0.3995)	5(0.1729)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(t-2)]	3(0.7829)	3(0.7829)	4(0.3995)	5(0.1729)
	Nikkei 225 Futures			
CTS-ARMA-GARCH	5(0.1729)	5(0.1729)	6(0.0646)	6(0.0646)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	4(0.3995)	4(0.3995)	6(0.0646)	6(0.0646)

Table IV. stdCTS with Volume and Lagged Volume Failures

Number of violations (*N*) and *p*-values of Kupiec's proportion of failures test for the S&P 500, DAX 30, and Nikkei 225 spot and futures indices. stdCTS-ARMA-GARCH with the relative change of trading volume and stdCTS-ARMA-GARCH with lagged relative change of trading volume are compared.

Model	1 year (255 days)			
	Dec. 14, 2004 ~ Dec. 15, 2005	Dec. 16, 2005 ~ Dec. 20, 2006	Dec. 21, 2006 ~ Dec. 27, 2007	Dec. 28, 2007 ~ Dec. 31, 2008
	N(*p*-value)	*N*(*p*-value)	*N*(*p*-value)	*N*(*p*-value)
	S&P 500 Spot			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	0	3(0.7829)	3(0.7829)	8(0.0061)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	0	5(0.1729)	6(0.0646)	5(0.1729)
	S&P 500 Futures			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	0	1(0.2660)	7(0.0211)	6(0.0646)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	1(0.2660)	3(0.7829)	3(0.7829)	5(0.1729)
	DAX 30 Spot			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	0	1(0.2660)	3(0.7829)	11(0.0001)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	5(0.1729)	4(0.3995)	3(0.7829)	5(0.1729)
	DAX 30 Futures			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	1(0.2660)	1(0.2660)	2(0.7190)	8(0.0061)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	3(0.7829)	4(0.3995)	5(0.1729)	3(0.7829)
	Nikkei 225 Spot			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	3(0.7829)	5(0.1729)	7(0.0211)	8(0.0061)
CTS-ARMA-GARCH-ln[V(*t*-1)/V(*t*-2)]	3(0.7829)	3(0.7829)	4(0.3995)	5(0.1729)
	Nikkei 225 Futures			
CTS-ARMA-GARCH-ln[V(*t*)/V(*t*-1)]	1(0.2660)	1(0.2660)	3(0.7829)	11(0.0001)
CTS-ARMA-GARCH-ln[V(t-1)/V(*t*-2)]	4(0.3995)	4(0.3995)	6(0.0646)	6(0.0646)

Chapter 4

Open and Closed Positions and Stock Index Futures Volatility

1. INTRODUCTION

We show that it is possible to obtain the daily total number of positions that are entered into as well as the daily total number of positions that are closed out in any futures contract, based on the daily volume of trading and open interest figures. As far as we know, this possibility is acknowledged for the first time in this paper. It allows exploring the relationships between any of the two components of the trading volume and any variable of interest, separately.

A line of research has studied the link among a given measure of price variability and some trading-related variables in futures markets. On this regard, the available literature has focused on the influence of the volume of trading and some variable related to the open interest on volatility. This previous research lacks of a homogeneous framework and has provided inconclusive results.

The objective of this paper is to study the relation between volatility and some trading-related variables under a new unifying setting, by concentrating on contracting activity (i.e., the flow of contracts entered into and closed out) instead of trading activity. To this aim, we analyse empirically the linkage between daily volatility and the number of open and closed positions for three of the most important stock index futures markets in the world.

For the DAX and Nikkei futures, we find that both contemporaneous open and closed positions have a positive relationship with volatility, while they have the opposite effect on the following day. However, for the S&P 500 futures, only the contemporaneous number of opening positions has a positive relationship with volatility, although the lagged numbers of opening and closing positions have a negative relationship again. Nonetheless, for this index futures contract, the contemporaneous number of closing positions is positively related to volatility on those days dominated by agents that are closing previously opened positions. In addition, the positive correlations between the daily volatility and the numbers of open and closed positions are usually more prominent when either the opening or the closing of positions predominates, respectively.

The remainder of the paper is organized as follows. Section 2 reviews the previous literature on the relationship between volatility, and volume and open interest. Section 3 the variables used in the empirical analysis are motivated, and it is explained how they have been computed. A daily futures volatility of the Garman-Klass type is calculated and its descriptive statistics are presented. Section 4 presents the empirical methods and results. The conclusions of the paper are summarized in Section 5.

2. REVIEW OF TRADING VOLUME

The relation between diverse measures of price variability and trading volume for index futures has been investigated extensively. Several studies show a positive and contemporaneous relationship between volume and volatility. There is evidence that documented a positive relation between volume and volatility for different stock index futures at different frequencies of data. (See, for example, Kawaller et al. (1994), and Gannon (1995) for intraday data, and Raghunathan and

Peker (1997), ap Gwilym et al. (1999), Wang and Yau (2000), Watanabe (2001), and Pati (2008) for daily data, and finally Wang (2002) for weekly data.)[16]

There are also studies that indicate that lagged volume is related to volatility as well. Nonetheless, in this case, ap Gwilym et al. (1999), and Wang and Yau (2000) found that a negative relationship between lagged trading volume and price variability, by analyzing stock index futures among other financial assets.[17]

Another variable that is thought to have a relation with volatility is the level of the open interest in the futures market. This variable is of special relevance for two reasons. Firstly, it is a trading-related measure that only appears in derivative markets. Secondly, there is a controversy about its influence on volatility. Whereas Watanabe (2001), Li (2007), and Pati (2008) observe a negative link between open interest and volatility in several stock index futures markets, Chen, Cuny, and Haugen (1995) proved a positive link for S&P 500 futures.[18] Thus, depending on the stock index futures contract, it has been showed one relationship or another.

The literature that explores the relationship between volatility, on one side, and trading volume and open interest, on the other, however, lacks of unifying explanations and interpretations. First of all, there are several competing theories on the ultimate variable of study that is getting proxied by any given observed

[16] Furthermore, there is evidence that supports the same relation in the case of equities (see Epps and Epps (1976), Smirlock and Starks (1985) and Schwert (1989)), as well as commodities (see Garcia et al. (1986)).
[17] It seems that this relationship is general for most financial assets. Foster (1995) studied crude oil futures markets, and they determined that lagged volume can partially explain current price variability. Fung and Patterson (1999) studied five currency futures markets and found the same negative relation between return volatility and past trading volume.
[18] A negative relationship is also detected in agricultural, currency, oil and metal futures contracts (see among others, Bessembinder and Seguin (1993), Fung and Patterson (1999), and Ripple and Moosa (2009)). A second group of papers finds a null or weak connection between volatility and open interest. This is the case of Martinez and Tse (2007) stock index as well as currency futures contracts. See also Yang et al. (2005), for agricultural futures markets. Figlewski (1981) proved a positive link for Government National Mortgage Association (GNMA) futures.

trading-related variable. Indeed, whereas for some researchers volume and open interest are simply two broad liquidity-related variables, for others they are related to the trading activity carried out by some specific type of traders. Thus, while volume is sometimes taken to be the simplest and more direct liquidity variable and open interest is taken to be a proxy for market depth in futures markets, other times, on the contrary, volume and open interest are assumed to be proxies, respectively, for the trading activity by informed traders, or speculators, and uninformed traders, or hedgers (see Bessembinder and Seguin (1993)).

In this vein, the multiplicity of interpretations for the trading-related variables also explains the multiple ways in which the basic trading-related variables (namely, volume and open interest) have been related to price variability. Indeed, as has been surveyed before, not only the relationship between volatility and contemporaneous volume and open interest has been studied extensively, but also the price variability has been related to lagged volume. Not surprisingly, there are also competing motivations for the inclusion of lagged volume in analysing the relationship of trading-related variables with volatility (Wang and Yau (2000), for instance, believed that a lagged trading volume increment implied a reduction in the contemporaneous volatility, taking volume as a measure of liquidity.) [19] Also several variables based on the change in open interest have been related to the variability of prices. These variables are intended to provide a good proxy for a given ultimate factor that may be related to volatility (for example, García et al. (1986), whom

[19] Foster (1995) studied crude oil futures markets, and they determined that lagged volume can partially explain current price variability. It could be due to traders conditioning their prices on previous trading volume as a measure of market sentiment. Or it could be explained also by a form of mimetic contagion where agents set their prices with reference to the trading patterns of other agents. Fung and Patterson (1999) studied five currency futures markets and found the same negative relation between return volatility and past trading volume, and they thought that it was consistent with the overreaction hypothesis what suggests a high volume of trading in the stock as well as a sharp price response –Conrad et al. (1994).

created a volume-to-open-interest ratio in order to measure the relative importance of the speculative behaviour in a given contract [20]).

We tried to shed light on this conundrum, by investigating the relationship between price variability and trading-related variables in futures markets, under a fresh perspective. Indeed, the focus on the contracting activity allows a new clear and direct explanation. Nevertheless, we also relate our results to the previous literature, which concentrates on the trading activity instead. This is due to the fact that the trading variables traditionally related to volatility can be expressed in terms of the number of open and closed positions, both contemporaneous and lagged, as we prove in the next section.

3. VARIABLES COMPUTATION AND DATA

In this paper we relate some trading-related variables to a measure of price variability in futures markets. We begin with the motivation of the trading-related variables that are used in the empirical analysis in the paper, and an explanation of the way they have been computed.

3.1. Trading related-variables

Traditionally, volume and open interest have been used as the basic trading-related variables in derivatives markets. The daily trading volume (denoted V_t) simply accounts for the amount of trading activity that has taken place in a specific

[20] See Lucia and Pardo (2010) for a critique of this line of research.

contract on a trading date *t*, whereas the daily open interest figure (OI_t) determines the number of outstanding contracts at the end of the trading date *t*.

It occurs that both daily figures are related to the total number of open positions over day *t* (denoted OP_t) as well as the total number of closed positions over day *t* (CL_t). This is because, since every trade involves two parties (long and short) and each party either opens or closes out a position, the total number of open and closed positions equals the total number of positions involved in the contracts traded (i.e. twice the number of traded contracts):

$$OP_t + CL_t = 2V_t \tag{1}$$

Furthermore, the sum of all the open positions from the first trading day of the contract up to the end of day *t* minus the sum of all the closed positions up to the same moment must be equal to the total number of outstanding (long plus short) positions at the end of day *t* (i.e. twice the open interest at the same moment), that is:

$$\sum_{s=1}^{t} \left(OP_t - CL_t \right) = 2OI_t \tag{2}$$

If we define the change in open interest as $\Delta OI_t = OI_t - OI_{t-1}$, from equation (2), it follows that:

$$OP_t - CL_t = 2\Delta OI_t \tag{3}$$

Now, from equations (1) and (3), the following accounting relationships can be easily obtained:

$$OP_t = V_t + \Delta OI_t \quad (4)$$

$$CL_t = V_t - \Delta OI_t \quad (5)$$

It is important to notice that although OI_t is a stock variable measured at the end of day *t*, the change in open interest over day *t*, ΔOI_t, is a flow variable, just as the trading volume, which depend only on the behaviour of traders on the observational day.

Finally, we can also define the relative net number of open positions on day *t*, denoted by $RNOP_t$, as opening positions minus closing positions on trading day *t* in relative terms over the total number of positions involved. Mathematically:

$$RNOP_t = \frac{OP_t - CL_t}{OP_t + CL_t} \quad (6)$$

It is easily checked that $RNOP_t$ equals $\Delta OI_t / V_t$, which shows that $RNOP_t$ coincides with the so-called $R3_t$ ratio introduced by Lucia and Pardo (2010) in the context of a critical assessment of the literature devoted to measure hedging activity from volume and open interest data.

We can also get:

$$OP_t = V_t (1 + RNOP_t) \quad (7)$$

$$CL_t = V_t (1 - RNOP_t) \quad (8)$$

As Lucia and Pardo (2010) pointed out, the *RNOP* variable (or R3, as they called it) can only take values from –1 to +1. Now, equations (6), (7), and (8) allow a convenient interpretation of the extreme values that can be taken by *RNOP* in terms of the number of open and closed positions. If *RNOP* takes the value +1 on any given day, this means that all the parties involved in every transaction occurred during the day have taken new positions in the contract. On the contrary, if *RNOP* takes the value -1, all the parties involved in every trade have liquidated old positions (i.e. they cancel their outstanding commitments). Interestingly, whenever *RNOP* equals zero, the total number of open positions equals the total number of closed positions. As Lucia and Pardo (2010) also pointed out, this may occur when every trade either is a day-trade or implies that one side involved in the trade replaces the other side in his position.

The purpose of this paper is to explore the role of open and closed positions on daily futures volatility taking into account the mechanical links reviewed in this section. Furthermore, using relative combinations of such positions, we are also able to study the effects generated by opening trades and closing trades into the price volatility of market activity when one of these groups of trades predominates in the market.

3.2. Measurement of volatility

Recall that we only deal with flow variables in this paper. Hence, to be consistent, we use a measure of daily volatility, which only depends on the behaviour of traders on the observational period that runs from time t – 1 to time t.

Taking into account open, close, high and low prices, we calculate a daily volatility measure of the Garman-Klass-type in the following manner:

$$V_{GK_t} = \left[\ln\left(\frac{O_t}{C_{t-1}}\right)\right]^2 - 0.383\left(\frac{C_t}{O_t}\right) + \frac{1.364}{4\ln 2}\left[\ln\left(\frac{H_t}{L_t}\right)\right]^2 +$$

$$+0.019\left[\ln\left(\frac{H_t}{O_t}\right)\ln\left(\frac{H_t}{C_t}\right) + \ln\left(\frac{L_t}{O_t}\right)\ln\left(\frac{L_t}{C_t}\right)\right] \quad (9)$$

where O_t, C_t, H_t and L_t are the open, close, high and low prices on day *t* and $C_{t\text{-}1}$ is the close price on day *t*-1.

Equation (9) is based on Yang and Zhang (2000). Following the assumptions of Parkinson (1980), Rogers and Satchell (1991), Rogers, Satchell and Yoon (1994) and Garman and Klass (1980), these authors make explicit the following formula for the efficient (minimum-variance) drift-independent unbiased estimator of Garman and Klass for the variance of a financial series, based on *n* observations:

$$V_{GK} = V_0^{'} - 0.383V_C^{'} + 1.364V_P + 0.019V_{RS} \quad (10)$$

where $V_0^{'}$ and $V_C^{'}$ are defined as:

$$V_0^{'} = \frac{1}{n}\sum_{i=1}^{n} o_i^2 \quad (11)$$

$$V_C^{'} = \frac{1}{n}\sum_{i=1}^{n} c_i^2 \quad (12)$$

and V_P (Parkinson (1980)) and V_{RS} (Rogers and Satchell (1991)) are defined as:

$$V_P = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{4\ln 2}\left(u_i - d_i\right)^2 \qquad (13)$$

$$V_{RS} = \frac{1}{n}\sum_{i=1}^{n}\left[u_i\left(u_i - c_i\right) + d_i\left(d_i - c_i\right)\right] \qquad (14)$$

where:

$o_i = \ln O_i - \ln C_{i-1}$, is the so-called normalized open;

$u_i = \ln H_i - \ln O_i$, is the normalized high;

$d_i = \ln L_i - \ln O_i$, is the normalized low;

and $c_i = \ln C_i - \ln O_i$, is the normalized close.

The specification in equation (9) is obtained from (10) to (14), when the computation of the estimator is based on single-period data (n = 1, in the observational period [t-1, t]).

3.3. Data series and descriptive statistics

The stock index futures contracts selected for our empirical study are S&P 500, DAX, and Nikkei. Our database has been taken from Reuters and consists of the daily open, high, low, and close prices, as well as the daily volume and open interest series for the sixteen year period that runs from December 2, 1991 to April 30, 2008.

For each variable, we construct a unique continuous-time series using the different maturities of each futures index following the methodology proposed by Carchano and Pardo (2009).[21]

Finally, we create a daily volatility series for the period from December 2, 1991 to April 30, 2008, from equation (9). Table I, Panel A, reports the descriptive statistical properties of the three volatility series.

It shows that DAX futures market has the highest daily volatility.[22] Also, all the other basic descriptive statistics take the highest values for the DAX futures contract volatility.

Before exploring the relationship between volatility and our chosen trading-related variables, we have studied the existence of seasonal volatility due to the maturity of futures contracts. Specifically, we have studied the fulfilment of the Samuelson hypothesis, which postulates that the futures price volatility increases as the futures contract approaches its expiration. If this were the case, a time-to-maturity variable would be needed to control for this effect in our empirical analysis.

In order to test the Samuelson hypothesis, we have estimated the following regression:

$$V_{GK_t} = \alpha + \beta TtM_t + \varepsilon_t \qquad (15)$$

[21] The last trading day of the front contract has been chosen as the rollover date. Following their idea, in order to avoid the rollover jump, we have calculated the normalized open of the day after the rollover date as the difference between the logarithm of the opening price of that day and the logarithm of the previous closing price of the same contract. By doing so, all the prices are taken from the same maturity.

[22] The Anova F-tests of equality of mean volatility reveal that the null is rejected at the 1% of significance level between DAX and S&P 500 and between DAX and Nikkei.

where the dependent variable V_{GK_t}, is the daily Garman and Klass volatility and TtM_t is the time to maturity measured as the number of days until expiration. For the Samuelson hypothesis to hold, the β coefficient of time to maturity should be negative and statistically significant.

Panel B of Table I shows the results. Any beta coefficient is not significant for our three stock index futures series.[23] Therefore, in our empirical study, it will not be necessary to introduce a time-to-maturity variable in order to control for the Samuelson effect.

4. EMPIRICAL METHODS AND RESULTS

4.1. Methodology

In order to investigate the relationship between the intraday volatility and the flows of entering trades and cancelling trades, we run the following regression:

$$\begin{aligned}\sigma_t^2 = \alpha_0 + \sum_{l=1}^{L} \alpha_l \, \sigma_{t-l}^2 + \beta_0 \, OP_t + \beta_1 \, OP_{t-1} + \gamma_0 \, CL_t + \gamma_1 \, CL_{t-1} \\ + \delta_1 \, RNOP(50\%)_t + \delta_2 \left[OP_t \times RNOP(95\%)_t\right] + \delta_3 \left[CL_t \times RNOP(5\%)_t\right] + \varepsilon_t\end{aligned} \qquad (16)$$

The regressors in the model can be classified into three groups. First, we include an arbitrarily long set of autoregressive coefficients to accommodate the persistence of volatility shocks in a simple way, following Schwert (1990). This is motivated by the results of Wang and Yau (2000) and Li (2007), which show the

[23] Our results are in line with those obtained recently by Duong and Kalev (2008), who analyse 20 futures contracts, including the S&P 500, and find strong support for the Samuelson hypothesis only for agricultural futures.

importance of taking into account the persistence in volatility in the analysis of the relationship between volatility and some trading-related variables, for the S&P 500 and Asian stock index futures contracts, respectively.[24] The appropriate number of lags *L* to be included is determined empirically.

Second, we include the number of open and closed positions together with their respective lagged values. We add the lagged number of open and closed positions separately in order to add flexibility to the model. We will exploit this flexibility later on.

Third, we add three variables designed to indicate a distinct relationship when either the sign of *RNOP* is undefined, or this ratio is close to one of the two possible extreme values. To this aim, we first define three dummy variables related to *RNOP*. The first one is *RNOP*(50%) and is intended to indicate those days in which almost every trader who has opened a position has closed it before the market close. This variable takes the value 1 the days that correspond to the 5% of the observations for which the RNOP value is closest to zero, and zero the remaining days. The second dummy variable is *RNOP*(95%) and selects those days with a value for RNOP which is closest to +1. It takes the value 1 the days corresponding to the 5% of the observations of *RNOP* with the highest value (the observations higher than the 95th percentile). The third is *RNOP*(5%) and selects the 5% of the days for which RNOP takes a value lower than its 5th percentile, and it takes the value 1 on those days and zero otherwise.

[24] Fleming, Kirby and Ostdiek (2006) obtain the same result for 20 stocks in the MMI (NYSE). Among the growing evidence that points out to stock volatility as a long-memory process we can find, for instance, Ding, Granger, and Engle (1993), Breidt, Crato and de Lima (1994), and Andersen et al. (2001). Nonetheless, Fujihara and Mougoué (1997) and Wang and Yau (2000) show that the introduction of the current and/or lagged volume and open interest substantially reduced the persistence of volatility for oil futures or Deutsche mark, silver, and gold futures contracts.

RNOP(50%) is directly included in the model. This is designed to capture a possible (average) additional relationship between the trading-related variables included in the model and volatility on those days in which almost every trader who has opened a position has closed it before the market close. RNOP(95%) is included in the model multiplied by OP. The multiplication is equivalent to a truncated variable that takes the value OP on those days with RNOP(95%) equal to 1, and zero otherwise. It is designed to capture a possible distinct relationship between OP and volatility precisely on those days for which most traders are opening positions. Finally, RNOP(5%) is included in the model multiplied by CL. This is equivalent to a truncated variable that takes the value of CL whenever RNOP equals 1 and zero otherwise. It will capture a possible distinct relation between CL and volatility on days for which most traders are closing positions.

We used the following estimation procedure. Firstly, the appropriate number of the lags (L) is determined with the next system: we aggregate one lag starting with lowest one (l = 1). Next, we check if the coefficient is significant or not at the 99% level of confidence. If so, we aggregate to the autoregressive model the following lag (l = 2) and we check both the significance of the two coefficients and whether the new model improves in terms of adjusted R-squared, Akaike criterion and Schwarz criterion. In that case, we introduce a new lag. If not, we get the final number of lags. All the regressions have been carried out using the Newey and West correction that accounts for heteroskedasticity and serial correlation. Secondly, we estimate the remaining coefficients and we report the adjusted R-squared, Akaike criterion and Schwarz criterion for the final model. Table II reports the estimation results.[25]

An autoregressive model of eight lags is used in the S&P 500 data, while an AR(9) is employed in DAX 30 and Nikkei 225 data.

4.2. Volatility and number of open and closed positions

Table II shows that there is a positive relationship between contemporaneous OP and volatility in the S&P 500 case, but the relationship between contemporaneous CL and volatility is rejected at the 5% of significance level. On the other hand, for DAX and Nikkei both contemporaneous OP and CL are positively related to volatility, both increasing volatility to the markets. For all three indexes, there are negative relationships between volatility and lagged OP and CL. The different sign of the contemporaneous and lagged variables coefficient can indicate that the OP and CL change play an important role with the increasing of the volatility for all indexes.[26]

[25] We checked that our data do not present multicollinearity problems between the open and closed variables. Specifically, we take the residuals of a regression of closed positions on open position, which are orthogonal to the open positions variable, and repeat the regression in equation (16) with the residuals instead of the closed positions series. We obtained similar results.

[26] Indeed, if we make the same regression with ΔOP_t and ΔCL_t instead of OP_t, OP_{t-1}, CL_t, and CL_{t-1}, in the three indexes, ΔOP_t and ΔCL_t have a positive significant relationship with volatility, even in the S&P case where CL_t is not statistically significant (the estimation results can be obtained from the authors upon request).

4.3. Other aggregated trading-related variables

It is possible to infer the empirical relationship between volatility and V and OI, from our results, given the accounting relationships that link them to the OP and CL variables (both contemporaneous and lagged). In other words, our results admit a complementary interpretation in terms of the traditional trading variables.

Developing the contracting variables of the equation (16) using the equations (4) and (5), it can be showed that the expression $\beta_0\, OP_t + \beta_1\, OP_{t-1} + \gamma_0\, CL_t + \gamma_1\, CL_{t-1}$ is equal to $\beta_0[VOL_t + \Delta OI_t] + \beta_1[VOL_{t-1} + \Delta OI_{t-1}] + \gamma_0[VOL_t - \Delta OI_t] + \gamma_1\,[VOL_{t-1} - \Delta OI_{t-1}]$, and reordering, $[\beta_0 + \gamma_0]VOL_t + [\beta_1 + \gamma_1]VOL_{t-1} + [\beta_0 - \gamma_0]\Delta OI_t + [\beta_1 - \gamma_1]\Delta OI_{t-1}$.

Now, we can infer the link between volume and open interest change with volatility testing the significance of the sum and the subtraction of the coefficients that appear in brackets in the last expression, i.e. H_0: $\beta_0 + \gamma_0 = 0$ to test the contemporaneous relationship between volume and volatility.

Test outcomes reported in Table III show that our results are in line with a positive relationship between volatility and contemporaneous volume, which is widely accepted in the literature. Secondly, our results also indicate a negative link with the volume of the previous day (which confirms previous findings by other authors in several stock indexes). Third, there should be only a (positive) relationship between volatility and the change in open interest for the S&P 500 case. Fourth and finally, that there also should be a (negative) relation between volatility and lagged change in open interest in this index. [27]

4.4. Additional effects for specific days

We now explore how the basic relationships may change for some specific days characterized by one of the three most significant values of RNOP (namely, zero, one and minus one). We turn again to the empirical results reported in Table II.

First, the dummy variable RNOP(50%), which selects those days with a RNOP value close to zero, is not significantly different from zero. This indicates that those days characterized by a number of opening positions close to the number of closing positions do not have a significantly different volatility on average.[28] Second, in general, the truncated variables, which are related to those days with an extreme value of RNOP, do have a positive relationship with volatility. In particular, the truncated variable related to RNOP(95%) shows that there is an additional positive effect of OP on those days mostly dominated by newly opened positions (a marginal effect for the S&P 500 and Nikkei). The truncated variable related to RNOP(5%) shows that there is an additional positive effect of CL on those days mostly dominated by closing positions, except for the DAX case, which shows no

[27] These results were controlled by running a regression of volatility on contemporaneous volume, lagged volume, change in open interest and lagged *OI* change (the estimation results can be obtained from the authors upon request). The coefficients and the p-values were the same as they appear in Table III. Moreover, although we change our four variables for these ones, the Adjusted R^2, Akaike info criterion, and Schwarz criterion remained the same value, and it confirms that we are working with the same amount of information -although displayed in a different way in our regression and offering new insight about it.

[28] We also checked for the significance of the truncated variable defined by: $[(OP_t + CL_t) \times RNOP(50\%)_t]$, and it was also equal to zero.

relationship. The result obtained for the truncated variable related to RNOP(5%) is particularly relevant because it shows that CL has a positive relationship with volatility, on those days mostly characterized by the closing of positions, although it may not exist a relationship on average when all the days are considered.

4.5. Alternative interpretations

Following the intuition of Bessembinder and Seguin (1993), who pointed out that many speculators (informed traders) are "day traders" who do not hold open positions overnight while hedging activity (uninformed trading) is reflected by open interest, Lucia and Pardo (2010) demonstrate that the use of open interest change, instead of the level value or the absolute value of open interest change, is more appropriate in order to reflect the hedging activity. Thus, they defined the so-called *R*3 ratio (*RNOP*) of the change in open interest to volume in order to measure speculation versus hedging activity. They also pointed out that a value of the ratio closed to zero can be associated to day-traders, whereas the two extreme values (namely, –1 and 1) can be related to uninformed traders.

Motivated by these insights, we now provide an alternative interpretation to the relevant values of *RNOP*. The results that we can extract from Table II are mainly two. Firstly, the coefficients of the *RNOP*(50%) are never significant, and therefore, we can conclude that a market session fully dominated by day-traders does not increase the volatility on that day. Secondly, we observe in the three markets that when the hedgers are opening (*RNOP*(95%)) and closing (*RNOP*(5%)) massively positions in the futures market, they add volatility to the trading day.

5. CONCLUSIONS

A number of papers have analysed the influence of both the volume and the open interest on index future volatility. However, as our results confirm, there is no general conclusions for all the stock indexes. Indeed, whereas the sign of the correlation between volatility and volume of trading (both contemporaneous and lagged) is the same in all the cases, the relationship with (the change in) open interest has not the same property.

This is an empirical puzzle that can be resolved when volatility is related to contracting activity instead of trading activity. Indeed, when open and closed positions are substituted for the traditional variables such as volume and (change in) open interest, the results are essentially the same for all the indexes, which confirms the real contribution of this change in perspective.

In summary, our results show that, for the studied stock indexes, in general both the number of contemporaneous (respectively, lagged) open and closed positions are positively (negatively) correlated with volatility. Although in the S&P 500, the contemporaneous number of closed positions and volatility is rejected at the 5% of significance level, it appears on those days mostly dominated by closing positions. The coherency of these general conclusions is achieved when volatility is analysed under a contracting perspective, which is in sharp contrast with the lack of homogeneity that is observed when volatility is related instead to traditional volume and open interest aggregated variables as it is done in the previous literature.

Our results can be reconciled with the line of reasoning that relates volatility to the activity of groups of traders, such as speculators or informed traders versus hedgers or uninformed traders. According to our findings, day-traders are not associated to an increment of the volatility, whereas uninformed traders, both opening and closing their positions, have to do with a rise in volatility.

REFERENCES

Andersen, T. G., Bollerslev, T., Diebold, F.X., and Ebens H. (2001). The distribution of stock return volatility. Journal of Financial Economics, 61, 43-76.

Ap Gwilym, O., Buckle, M., and Evans, P. (2002). The volume-maturity relationship for stock index, interest rate and bond futures contracts. EBMS Working Paper EBMS/2002/3

Ap Gwilym, O., MacMillan, D., Speight A. (1999). The intraday relationship between volume and volatility in LIFFE futures markets. Applied Financial Economics, 9, 593-604.

Bessembinder, H., and Seguin P. (1993). Price Volatility, Trading Volume, and Market Depth: Evidence from Futures Markets. Journal of Financial and Quantitative Analysis, 28, 1, pg. 21.

Breidt. F. J., de Lima, P., and Crato, N. (1994). Modeling long memory stochastic volatility. Working Papers in Economics, No 323, Johns Hopkins University.

Carchano, O., and Pardo, A. (2009). Rolling over stock index futures contracts. Journal of Futures Markets, 29, No. 7, 684-694.

Chen, N.-F., C.J. Cuny, and Haugen R.A. (1995). Stock volatility and the levels of the basis and open interest in futures contracts. Journal of Finance, 50, 281-300.

Conrad, J. S., Hameed, A., and Niden, C. (1994). Volume and autocovariances in short-horizon individual security returns. Journal of Finance, 49 (4), 1305-1329.

Ding, Z., Granger C., and Engle R. F. (1993). A long memory property of stock market returns and a new model. Journal of Empirical Finance, 1, 83-106.

Duong, H. N., and Kalev P. S. (2008). The Samuelson hypothesis in futures markets: An analysis using intraday data. Journal of Banking & Finance, 32, 489-500.

Epps, T. W., Epps, L. M. (1976). The stochastic dependence of security price changes and transaction volumes: implications for the mixture-of-distribution hypothesis. Econometrica, 44, 305-321.

Figlewski, S. (1981). Futures Trading and Volatility in the GNMA Market. Journal of Finance, 36, 445-456.

Fleming J., Kirby C., and Ostdiek B. (2006). Stochastic Volatility, Trading Volume, and the Daily Flow of Information. Journal of Business, 79, No. 3.

Foster, F. J. (1995). Volume-volatility relationships for crude oil futures markets. Journal of Futures Markets, 15, Is. 8, 929-951.

Fung, H. G., and Patterson, G. A. (1999). The dynamic relationship of volatility, volume, and market depth in currency futures markets. Journal of International Financial Markets, Institutions and Money, 17, 33-59.

Fujihara, R. A., and Mougoue M. (1997). Linear dependence, nonlinear dependence and petroleum futures market efficiency. Journal of Futures Markets, 17, Is. 1, 75-99.

Gannon, G.L. (1995). Simultaneous Volatility Effects in Index Futures. Review of Futures Market, 13, No. 4, 25-44.

Garcia, P., Leuthold, M. R., and Zapata, H. (1986). Lead-lag relationships between trading volume and price variability: New evidence. Journal of Futures Markets, 6, Is. 1, 1-10.

Garman, M. B., and Klass, M. J. (1980). On the estimation of security price volatilities from historical data. Journal of Business, 53, no 1, 67-78.

Kawaller I. G., Koch P. D., and Peterson J. E. (1994). Assessing the Intra-day Relationship between Implied and Historical Volatility. Journal of Futures Markets, 18, No 4, 399-426.

Li, J.H. (2008). The relationship between return volatility and trading activity in Asia stock index futures. Doctoral Dissertation.

Lucia, J. J., and Pardo, A. (2010). On measuring speculative and hedging activities in futures markets from volume and open interest data. Applied Economics , 42, 1549–1557.

Martinez, V., and Tse, Y. (2008). Intraday Volatility in the Bond, Foreign Exchange, and Stock Index Futures Markets. Journal of Futures Market, 28, 953-975.

Newey, W., and West, K. (1987). Hypothesis testing with efficient method of moment estimation. International Economic Review, 28, 777-787.

Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return. Journal of Business, 53, No. 1, 61-65.

Pati, P.C. (2008). The relationship between price volatility, trading volume and market depth: Evidence from an emerging Indian stock index futures market. South Asian Journal of Management, 15, No. 2, pg 25.

Ripple R.D., and Moosa I. A. (2009). The effect of maturity, trading volume, and open interest on crude oil futures price range-based volatility. Global Finance Journal, 20, 209-219.

Rogers, L. C. G., and Satchell S. E. (1991). Estimating variance from high, low and closing prices. Annals of Applied Probability, 1, No. 4, 504-512.

Rogers, L. C. G.; Satchell, S. E.; and Yoon, Y. (1994). Estimating the volatility of stock prices: A comparison of methods that use high and low prices. Applied Financial Economics, 4, Is. 3, 241-47.

Schwert, G. W. (1989). Why does stock market volatility change over time. Journal of Finance, 44, 1115-1153.

Schwert, G. W., (1990). Stock Market Volatility. Financial Analysis Journal, 46, 3, pg 23.

Smirlock, M., and Starks, L. (1985). A further examination of stock price changes and transactions volume. Journal of Financial Research, 8, 217-225.

Wang, G., and Yau, J. (2000). Trading volume, bid-ask spread, and price volatility in futures markets. Journal of Futures Markets, 20, Is. 10, 943-970.

Wang, C. (2002). The Effect of Net Positions by the Type of Trader on Volatility in Foreign Currency Futures Markets. Journal of Futures Markets, 22, No. 5, 427-450.

Watanabe, T. (2001). Price volatility, trading volume, and market depth: Evidence from the Japanese Stock Index Futures Market. Applied Financial Economics, 11, 651-658.

Yang, J., Balyeat R. B., and Leatham D. J. (2005). Futures Trading Activity and Commodity Cash Price Volatility. Journal of Business Finance & Accounting, 32, No. 1-2, 297-323.

Yang, D., Q. Zhang, (2000). Drift-Independent Volatility Estimation Based on High, Low, Open, and Close Prices. Journal of Business, 73, No. 3.

Table I. Descriptive Statistics and Samuelson Hypothesis Test

Panel A presents the maximum, mean, minimum and standard deviation statistics of the future daily return volatility of the S&P 500, DAX and Nikkei for the for the period from December 2, 1991 to April 30, 2008. Panel B presents the results for testing the Samuelson hypothesis using daily return volatility of the S&P 500, DAX and Nikkei for the for the same period. The results are based on the regression $V_{GK_t} = \alpha + \beta TtM_t + \varepsilon_t$, where the dependent variable V_{GK_t}, is the daily Garman and Klass volatility. The independent variable TtM_t is the time to maturity, measured as the number of days until expiration. The results are obtained with the Newey and West (1987) heteroskedasticity consistent covariance procedure; p-values are given in parenthesis. Finally, adjusted-R^2 is presented.

Panel A: Descriptive statistics of the futures daily return volatility

	S&P 500	DAX	Nikkei
Maximum	0.0473	0.0703	0.0388
Mean	0.0068	0.0084	0.0081
Minimum	0.0002	0.0010	0.0000
Standard Deviation	0.0043	0.0056	0.0042

Panel B: Testing for the Samuelson hypothesis using daily return volatility

	S&P 500	DAX	Nikkei
Intercept	0.0065	0.0082	0.0078
(p-value)	(0.0000)	(0.0000)	(0.0000)
TtM	9.56E-06	5.95E-06	7.15E-06
(p-value)	(0.1984)	(0.5841)	(0.3053)
Adjusted-R^2	0.0014	0.0001	0.0007

Table II. Regressions of Variance on Open and Closed Positions and *$RNOP_t$* Measure

This table presents the coefficients and their associated probability of the regressions between variance and a constant [α_0], the lags of the volatility [α_1,..., α_9], open positions on the day t and day t-1 [β_0, β_1], closed positions on day t and day t-1 [γ_0, γ_1] and *$RNOP(50\%)_t$, OP_t x $RNOP(95\%)_t$, and CL_t x $RNOP(5\%)_t$* [δ_1,δ_2,δ_3], for S&P 500, DAX and Nikkei index futures daily series for the period from December 2, 1991 to April 30, 2008.

	S&P 500		**DAX**		**Nikkei**	
	Coefficient	**Prob.**	**Coefficient**	**Prob.**	**Coefficient**	**Prob.**
α_0	-3.9900E-06	0.9798	2.8200E-04	0.0886	6.9500E-04	0.0001
α_1	2.8632E-01	0.0000	2.6533E-01	0.0000	2.7842E-01	0.0000
α_2	1.9486E-01	0.0000	1.7022E-01	0.0000	1.5404E-01	0.0000
α_3	1.0015E-01	0.0000	1.2888E-01	0.0000	1.4357E-01	0.0000
α_4	5.0161E-02	0.0064	6.4787E-02	0.0022	1.6935E-02	0.4293
α_5	5.4240E-02	0.0069	7.0105E-02	0.0133	4.5373E-02	0.0417
α_6	4.6058E-02	0.0084	4.6984E-02	0.0151	6.1982E-02	0.0034
α_7	6.5139E-02	0.0012	5.1166E-02	0.0114	8.4051E-02	0.0000
α_8	9.2391E-02	0.0000	6.4570E-02	0.0003	3.3436E-02	0.0968
α_9			4.9152E-02	0.0172	5.4615E-02	0.0062
β_0	5.3500E-08	0.0000	3.2000E-08	0.0000	5.8600E-08	0.0000
β_1	-4.3600E-08	0.0000	-2.4200E-08	0.0000	-4.0400E-08	0.0000
γ_0	6.5600E-09	0.0846	1.9100E-08	0.0034	4.3400E-08	0.0055
γ_1	-7.1400E-09	0.0448	-2.1600E-08	0.0006	-5.4100E-08	0.0000
δ_1	-1.4400E-04	0.4635	5.0300E-05	0.8674	2.8800E-05	0.8949
δ_2	1.1400E-08	0.0504	3.9200E-08	0.0016	1.4900E-08	0.0509
δ_3	6.6200E-09	0.0175	9.7400E-09	0.1038	2.6600E-08	0.0254
Adjusted R-squared	0.5705		0.6062		0.4650	
Akaike info criterion	-8.9273		-8.4614		-8.7424	
Schwarz criterion	-8.9023		-8.4349		-8.7056	

Table III. Hypothesis Testing

This table presents the coefficients of the F-statistics and their associated probability for the Wald tests for the hypothesis indicated in the main body of the text, for S&P 500, DAX and Nikkei index futures daily series for the period from December 2, 1991 to April 30, 2008.

	S&P 500		**DAX**		**Nikkei**	
H_0	**Coefficient**	**p-value**	**Coefficient**	**p-value**	**Coefficient**	**p-value**
$\beta(0) + \gamma(0) = 0$	6.01E-08	0	5.11E-08	0	1.02E-07	0
$\beta(1) + \gamma(1) = 0$	-5.07E-08	0	-4.58E-08	0	-9.46E-08	0
$\beta(0) - \gamma(0) = 0$	4.70E-08	0	1.29E-08	0.2848	1.53E-08	0.5858
$\beta(1) - \gamma(1) = 0$	-3.64E-08	0	-2.61E-09	0.8133	1.37E-08	0.3978

CONCLUSIONS

Essays on

Stock Index Futures

In this dissertation thesis, some of the most relevant topics on futures markets have been analysed. We have focused our research on stock index futures contracts of S&P 500, DAX, and Nikkei. The different issues examined have been: (i) the rollover and the construction of continuous futures series; (ii) the presence of abnormal returns based on calendar anomalies in the stock index futures; (iii) the implementation of Value-at-Risk methods in the futures markets and comparing the results with their corresponding spot markets; and finally, (iv) the connection between the contracting activity in the futures markets and their daily volatility.

The first conclusion that we have reached is that the choice of the rollover date does not imply significant differences between long return futures series. Therefore, the criterion used to link futures contract series is not relevant, because it is not expected to achieve different results. We endorse the method of switching contracts on the last trading day for stock index futures due to its simplicity. For the first time, a conclusive result about this topic for stock index futures is obtained.

Once determined how the long futures series will be constructed, we have analysed basically two topics: the returns and the volatility. In the first case, we study the possible existence of calendar anomalies in the stock index futures and we conclude that the only effect, among the 1128 individual effects studied, that is statistically and economically significant, as well as persistent through time, is the "turn-of-the-month +1" (taking long positions in futures the first trading day of the month), and only for the S&P 500 futures.

In the second case, we analyse and conclude that the CTS-ARMA-GARCH model is suitable in order to forecast the one-day-ahead Value-at-Risk at 1% level for stock index futures. We also prove that the inclusion of the relative change of the lagged volume produces a more conservative model for these markets. On the other hand, it cannot be established the same for spot markets, and it seems not to be worth the effort to forecast the contemporaneous volume to try to obtain better forecasts.

In a deeper study about the variables related to volatility, we have found that there is a strong relationship with contracting activity. Essentially, there is a positive relation with the contemporaneous number of open and closed positions during the day, but a contrary link with the lagged values. Another interesting result is when the market is dominated by traders that are opening and closing their positions during the day (associated with speculative activity), as we do not found extra volatility in the market. However, the days when the market is dominated by traders massively opening or closing positions (associated with hedging activity) seem to be related to an abnormal level of volatility.

FURTHER RESEARCH

The results obtained in this dissertation allow us to advance in some respects, but at the same time new questions have emerged, as well as new subjects of research that would be interesting to evolve in the future. Particularly, the methodologies followed in the first chapter and the results obtained make us consider carrying out similar studies for different futures underlyings, especially commodities, in order to analyze the relevance of the choice of the rollover date.

The second chapter results seem to point out the disappearance of calendar effects for daily returns, so we propose examining them for intraday returns in stock index futures. With this study we would be able to determine if the typical calendar anomalies for daily data now appear concentrated in specific moments of the session, as part of their disappearance process.

The third chapter has focused on comparing futures and spot markets, and the left tails of the returns distribution were studied. However, with the increasing number of financial instruments that allows the investors to take short positions in the markets, we consider it is of special relevance to study the right-tail risk too. Finally, in chapter four, the daily open and closed positions variables have been recovered and their role can be explored inside the set of market sentiment measures proposed by the literature.

RESUMEN

Ensayos sobre

Futuros sobre Índices Bursátiles

INTRODUCCIÓN

Dado que ninguno de los cuatro capítulos no están en alguna de las dos lenguas oficiales de la Universitat de València, y cumpliendo su normativa, a continuación se resumen los cuatro capítulos de la tesis.[29]

La presente tesis se ha centrado en el estudio de algunos de los temas más importantes relacionados con los contratos de futuros sobre índices bursátiles. Este estudio ha sido dividido en cuatro capítulos, cada uno de los cuales analiza respectivamente: (i) la relevancia de la fecha de *rollover* a la hora de construir las series continuas de futuros sobre índices bursátiles; (ii) la existencia de anomalías de calendario en los mercados de futuros sobre índices bursátiles; (iii) la estimación del *Value-at-Risk* a un día vista en los futuros sobre índices bursátiles; y finalmente, (iv) la relación entre la volatilidad y el número de posiciones abiertas y cerradas a lo largo del día en los mercados de futuros sobre índices bursátiles.

Capítulo 1: El *Rollover* en los contratos de futuros sobre índices bursátiles.

Los contratos de derivados tienen una vida limitada al vencimiento de cada uno de los contratos. Sin embargo, la construcción de las series largas de futuros es crucial para académicos y profesionales del sector de los mercados de futuros. A pesar de esto, hay muy pocos estudios sobre la metodología a seguir con el fin de construir series continuas, lo cual es muy sorprendente ya que todo investigador del mercado de futuros tiene que hacer frente a esta decisión, antes de analizar los datos. La última referencia que aparece en la literatura sobre este tema, Ma et al.

[29] El presente resumen se ha realizado en cumplimiento de la Disposición Adicional cuarta de la Normativa reguladora de los procedimientos de elaboración, autorización, nombramiento del Tribunal, defensa y evaluación de las tesis doctorales de la Universitat de València, aprobada en Consejo de Gobierno el 6 de Junio de 2006.

(1992), al estudiar futuros con distintos subyacentes, concluye que según el criterio utilizado se generan series distintas que pueden llevar a resultados diferentes.

En este estudio, analizamos la relevancia de la elección de la fecha de *rollover* –punto en el tiempo donde se cambia el contrato más próximo a vencimiento por el siguiente. Nos hemos centrado en un solo subyacente: futuros sobre índices bursátiles, específicamente, el S&P 500, DAX y Nikkei. Para ello, hemos usado cinco metodologías diferentes, dos de las cuales nunca habían sido aplicadas anteriormente, con las que hemos construido las correspondientes series largas de futuros para ser comparadas entre sí.

En concreto, los cinco criterios utilizados han sido: el último día, el volumen, el interés abierto, el R3 (Lucia y Pardo, 2010), y finalmente, el índice perfecto (Geiss, 1995). Un estudio sobre el número medio de días antes de vencimiento en que las respectivas metodologías realizan el cambio de contrato para construir las series largas de futuros nos muestra que las metodologías basadas en el "volumen negociado" y en el "interés abierto" o "volumen abierto" son muy similares en los tres mercados, e incluiríamos en esta semejanza "el último día" para los mercados alemán y el japonés. Por otro lado, los criterios R3 y el índice perfecto son muy distintos al resto.

Utilizando estas cinco metodologías, pasamos a construir las series largas correspondientes y a realizar un análisis empírico de comparación entre ellas. En primer lugar, determinamos el porcentaje de datos que difieren entre las distintas series para los diferentes mercados. Los datos del criterio del índice libre de distorsiones son prácticamente distintos en su totalidad al resto de series. El R3, por su lado, muestra entre un 5.96% y un 28.21% de datos que difieren con el resto de series construidas. Sin embargo, las series “volumen” e “interés abierto”, apenas muestran diferencias para los tres mercados, y en el caso del DAX y el Nikkei, las series también son muy similares a la serie “último día”.

A continuación, realizamos un test de igualdad de medias (el test paramétrico F-test), medianas (el test no paramétrico de *Kruskal-Wallis*), y por último, de varianzas (utilizando el estadístico de *Brown-Forsythe’s*), para cada una de las series y para cada uno de los mercados. Los resultados indican que no hay ningún *p-value* inferior al 10%, y por lo tanto, no se puede concluir que en ninguna de estas series ninguno de estos tres parámetros sea significativamente distinto a los producidos por el resto.

Para finalizar, se compara par a par la distribución de cada una de las series largas generadas por cada uno de los cinco criterios. Con ello, conseguimos descartar la posibilidad de que las series puedan tener por casualidad unos parámetros similares en media, mediana y varianza, cuando tuvieran en realidad distribuciones diferentes. Para ello, se utiliza el test de *Wilcoxon/Mann-Whitney,* comprobando que los *p-values* observados no son en ningún caso menores al 10% y que, por lo tanto, no se puede afirmar que la distribución de cada una de las series largas sea significativamente diferente al resto.

La principal contribución de este capítulo es la de realizar un análisis completo de los distintos criterios para la construcción de series largas para un subyacente específico (los futuros sobre índices bursátiles) y, por primera vez, llegar a una conclusión sobre este tema que pueda servir de utilidad a cualquier investigador que trabaje con dicho mercado.

Capítulo 2: Anomalías de calendario en los futuros sobre índices bursátiles.

Hay un gran número de estudios que describen diferentes anomalías de calendario en los mercados de contado. Aunque la evidencia empírica sugería que los efectos estacionales habían desaparecido en los inicios de los años 90, nuevos estudios y aproximaciones aseguran su continuación.

Aunque la literatura sobre las anomalías calendario se ha centrado en el mercado de contado de acciones, su investigación en los mercados de futuros sobre índices o sobre acciones individuales parece más apropiada. En primer lugar, los costes de transacción en los mercados de futuros son menores que en los de contado. En segundo lugar, los mercados de futuros permiten a los *traders* tomar posiciones cortas de forma mucho más rápida (y de nuevo, más barata). Finalmente, el alto grado de apalancamiento que permiten los contratos de futuros hace mucho más fácil obtener un beneficio cuando se implementan reglas de negociación a corto plazo basadas en estacionalidades.

En este capítulo, presentamos un estudio exhaustivo sobre 188 posibles anomalías cíclicas tanto para las estrategias de “comprar-y-mantener”, como para las de “vender-y-mantener” en tres mercados distintos (S&P 500, DAX y Nikkei). Por lo tanto, estamos considerando un total de 1128 efectos individuales. Para determinar qué efectos calendario son realmente una anomalía de mercado, hemos aplicado dos métodos basados en simulaciones: el método *bootstrap* t percentil y el Monte Carlo. Además, se ha exigido que el efecto sea persistente en el tiempo.

Primero, se determina la distribución de las series de rendimientos con el test de *Jarque-Bera*, así como si hay correlación en los rendimientos y en los rendimientos al cuadrado. Con todo ello, concluimos que los datos no siguen una distribución normal, y están correlacionados en media y presentan heterocedasticidad. Por tanto, afirmamos que no es adecuado llevar a cabo contrastes paramétricos.

De hecho, el primer test que aplicamos es el *bootstrap* t percentil. Este método es independiente a cualquier supuesto distribucional y es apropiado para un número reducido de datos. Sin embargo, esta metodología no es adecuada para datos correlacionados y/o heterocedásticos. Por ello, estimamos un modelo ARMA/GARCH para cada serie de rendimientos y trabajamos con los residuos estandarizados, los cuales, aunque no estén normalmente distribuidos, no están correlacionados y son homocedásticos Por eso, son perfectos para nuestra metodología.

Al aplicar la metodología *bootstrap* t percentil, construimos un intervalo de confianza y si este no incluye el cero, inferimos que el efecto es estadísticamente significativo. Además, dividimos la muestra en dos subperiodos y determinamos qué efectos son estadísticamente significativos en las dos submuestras distinguiendo en cada caso los efectos positivos y los efectos negativos. En el caso de los positivos, el único que es persistente fue el “cambio de mes +1”, o lo que es lo mismo, tomar posiciones largas en futuros el primer día del mes y estar fuera de mercado el resto de días. En el caso de los negativos, los dos que son persistentes son “la cuarta semana de septiembre” y “la cuarta semana de julio”.

De los 34 efectos que fueron estadísticamente significativos en el primer subperiodo, tan solo tres lo fueron también el segundo. En la literatura encontramos dos posibles explicaciones a estos resultados. La primera señala que los *traders* detectan estos efectos y los hacen desaparecer en el tiempo al aprovecharse de ellos. La segunda explicación considera que las anomalías de calendario son variantes en el tiempo.

Si no se puede obtener beneficios de las posibles anomalías detectadas, el mercado continuaría siendo eficiente. Así, una vez determinados los efectos estadísticamente significativos y persistentes en el tiempo, se ha comprobado si además son económicamente significativos. Para ello, utilizamos la simulación de Monte Carlo, con la que se establece si los efectos anteriores baten o no al mercado. Para ello se han tenido en cuenta los costes totales que implicaría la ejecución de las estrategias detectadas.

Después de realizar el constraste, se observó que el único efecto que es estadística y económicamente significativo es el “cambio de mes +1” o “primer día del mes”.

La contribución principal de este capítulo es la de aportar una panorámica completa sobre la evidencia empírica de posibles anomalías de mercado en los futuros sobre índices bursátiles. En él, se realiza una revisión exhaustiva de todos los patrones de calendario encontrados en la literatura de contado, y se añade uno específico del mercado de futuro: el efecto vencimiento.

Capítulo 3: Predicción del *Value-at-Risk* en los mercados de contado y de futuros.

La predicción del riesgo en los mercados financieros es un tema de importancia y actualidad tanto para instituciones privadas como públicas. Una buena gestión del riesgo deberá determinar fielmente el riesgo al que se está expuesto cada día para afrontarlo de forma apropiada. La evidencia empírica sugiere que un modelo de riesgo de mercado debe ser capaz de manejar las características idiosincráticas de la volatilidad, como son los *clusters* de volatilidad y una amplitud variante en el tiempo de los rendimientos diarios. En este sentido, con la aparición primero de los modelos ARCH y a continuación de los ARCH generalizados (GARCH), estas características de la volatilidad son recogidas.

Sin embargo, la crisis actual ha mostrado la ineficiencia de dichos modelos para capturar movimientos extremos en los mercados. En el estudio de Kim et al. (2011, *Time series analysis for financial market meltdowns*) se muestra que dichos modelos no contemplaban otras características de las series de rendimientos, como por ejemplo su distribución no normal y todo lo que ello puede implicar (patrones de leptocurtosis y asimetría). En este estudio, se compara el modelo de perturbaciones normales con otro modelo en que se introducen innovaciones con una distribución infinitamente divisible (ID). El estudio concluye que los modelos GARCH basados en innovaciones ID son mucho más adecuados a la hora de predecir crisis financieras.

En este capítulo, se presenta la primera evidencia de la validez del modelo ARMA-GARCH con innovaciones *tempered stable* (pertenecientes a la familia ID) para estimar el *Value-at-Risk* a un día vista en los mercados de futuros S&P 500, DAX y Nikkei. De nuevo, al revisar la literatura previa, nos encontramos con un buen número de trabajos centrados en el mercado de contado, pero la evidencia empírica previa para el mercado de futuros de nuevo es muy escasa (tanto en términos de número de estudios como de mercados analizados). Además, y también por primera vez, en este capítulo de la tesis se intenta mejorar la capacidad predictiva del modelo anterior añadiendo el volumen de negociación al mismo.

La razón que nos lleva a realizar este intento es la extensa literatura que indica la fuerte relación que existe entre la volatilidad y el volumen, tanto para contado como para futuros y, dentro de los futuros, para todo tipo de subyacentes. Incluso algunos estudios muestran que existe relación entre el volumen retardado y la volatilidad, lo cual es esencial para poder intentar predecir a un día vista.

Para ello, introducimos distintas versiones del volumen retardado para predecir el *Value-at-Risk* en los mercados de contado y futuro del S&P 500, y determinaremos qué variante es la más apropiada. En concreto, comparamos los resultados obtenidos con el volumen retardado, el logaritmo del volumen retardado, y finalmente, el cambio relativo del volumen retardado. Y, en el caso de los futuros, contrastamos además dichos volúmenes tanto en número de contratos como en valor monetario.

El análisis empírico efectuado se basa en la comparación de 4 casos: (i) del modelo con innovaciones normales con el de perturbaciones *classical tempered stable* (CTS) para los mercados *spot* y futuros; (ii) de las distintas variantes del volumen retardado para el mercado *spot* y futuros del S&P 500; (iii) de los modelos con innovaciones *tempered stable* con y sin volumen, para los mercados de contado y de futuros; y finalmente, (iv) del modelo CTS con volumen retardado respecto al volumen contemporáneo.

La forma de comparar los respectivos casos se basa en primero, desglosar los cuatro años de predicciones diarias en 4 años de 255 días, para determinar el número de veces en que la predicción del *Value-at-Risk* al 1% fue superado a la baja por el dato real en un año. Segundo, realizamos el *backtest* de Kupiec con el que determinamos si el número de errores anterior era significativamente distinto a lo normal. De este modo, se determinó que todo *p-value* asociado al número de violaciones que fuera menor al 5% implicaba que el modelo concreto cometía un número de errores anormal y por tanto era descartado. Y tercero, entre dos modelos aceptados por el test anterior, se prefirió aquel que fuera más conservador (cometiera menos errores).

Las contribuciones a destacar de este capítulo serán la aportación de evidencia empírica para los mercados de futuros de modelos válidos para la estimación del *Value-at-Risk* a un día vista, la introducción del volumen en dicho modelo para intentar mejorar su capacidad predictiva, y finalmente, la comparación de resultados para los mercados de contado y de futuros sobre índices bursátiles.

Capítulo 4: El número de posiciones abiertas y cerradas y la volatilidad en los mercados de futuros sobre índices bursátiles.

Los estudios sobre las variables relacionadas con la volatilidad muestran el fuerte vínculo existente entre esta y las variables de negociación. Esta línea de investigación en los mercados de futuros se ha centrado sobre todo en la relación entre la volatilidad y el volumen / interés abierto. Aunque toda la evidencia empírica muestra la misma relación entre el volumen y la volatilidad, no pasa lo mismo con el interés abierto. Según el subyacente utilizado, esta relación puede ser positiva, negativa o neutra.

Por otro lado, destaca que en ninguna de estas investigaciones se tienen en cuenta las distintas naturalezas de las respectivas variables. Así, aunque el volumen sea una "variable flujo", el interés abierto es una "variable stock". De la misma forma, según la medida de volatilidad utilizada, esta puede ser una variable flujo o stock. Además, también queremos recalcar que la literatura previa se ha centrado en analizar por separado volumen e interés abierto dentro de la misma regresión, cuando de hecho, existen variables clásicas que los agregan.

En este estudio, se analiza la relación entre la volatilidad y las variables de negociación desde un punto de vista unificador para los mercados de futuros del S&P 500, DAX y Nikkei. Primero, se ha tenido en cuenta que todas las variables tuvieran la misma naturaleza (en nuestro caso, todas eran variables flujo), y segundo, se ha analizado la relación de la volatilidad con variables de contratación, en concreto, el número de posiciones que se abren y se cierran en los mercados de futuros durante el día. Estas variables agregan volumen y (la variación del) interés abierto. Además, en el modelo se introducen variables ficticias que determinan momentos de contratación extrema de ciertos grupos de intermediarios, tales como especuladores y coberturistas.

Así, nuestras variables "número de posiciones abiertas" y "número de posiciones cerradas", que denotamos como OP_t y CL_t son, respectivamente, igual al volumen negociado el día *t* más la variación del interés abierto entre el día *t* y el *t-1*, para el caso de las posiciones abiertas, y menos la variación para las cerradas. Ambas variables son variables flujo. Además, en el estudio se introduce el ratio $RNOP_t$ o número relativo de posiciones abiertas que es el número neto de posiciones abiertas en el día t, respecto al número total de posiciones abiertas y cerradas ese mismo día. En concreto, nos centramos en los momentos en que dicho ratio toma valores muy cercanos a 0, 1 y -1. Al tomar valores cercanos a cero, las posiciones que se han abierto y las que se han cerrado son las mismas. Cuando es 1, tenemos que ese día el mercado ha sido dominado por posiciones que se han abierto, y para terminar, cuando es -1, el dominio lo tienen las posiciones que se cierran.

Siguiendo el criterio de escoger variables homogéneas, decidimos tomar una medida del flujo de la volatilidad diaria del tipo Garman-Klass uniperiodal, que tiene en cuenta tanto la volatilidad producida por la nueva información generada *overnight* (desde el cierre del día t-1 hasta la apertura del día t) como también la volatilidad intradiaria, y que es un estimador eficiente (en el sentido mínima-varianza), insesgado e independiente de la deriva. Para su cálculo nos basamos en el estudio de Yang and Zhang (2000, Drift-Independent Volatility Estimation Based on High, Low, Open, and Close Prices. *Journal of Business*, 73, No. 3).

Finalmente, también decidimos tener en cuenta un conjunto de retardos de la volatilidad para tener en cuenta la persistencia de la misma. Para ello, se va introduciendo un número creciente de retardos hasta que el siguiente retardo no aporte más información sobre la volatilidad contemporánea.

Así, el modelo final es:

$$\sigma_t^2 = \alpha_0 + \sum_{l=1}^{L} \alpha_l \, \sigma_{t-l}^2 + \beta_0 \, OP_t + \beta_1 \, OP_{t-1} + \gamma_0 \, CL_t + \gamma_1 \, CL_{t-1}$$
$$+ \delta_1 \, RNOP(50\%)_t + \delta_2 \left[OP_t \times RNOP(95\%)_t\right] + \delta_3 \left[CL_t \times RNOP(5\%)_t\right] + \varepsilon_t$$

el cual incluye un conjunto de retardos de volatilidad, a continuación el número de posiciones abiertas y cerradas y sus respectivos retardos, y finalmente, tres variables ficticias relacionadas con nuestro ratio $RNOP_t$. La primera intenta capturar la volatilidad extra en aquellos días en que prácticamente todos los *traders* que abren una posición la cierran antes del fin de la sesión. La segunda, multiplicada por OP, capta la relación de esta variable con la volatilidad en los días donde la mayoría de *traders* abren posiciones. Y la tercera, multiplicada por CL, determina la posible relación entre CL y la volatilidad los días donde los *traders* estén cerrando posiciones masivamente.

Este último capítulo de la tesis, aporta una visión unificada del vínculo que las variables relacionadas con la actividad negociadora tienen con la volatilidad, y más en concreto, de cómo la actividad de contratación en los mercados de futuros afecta a la misma.

CONCLUSIONES

En la presente tesis se han analizado alguno de los temas más relevantes relacionados con los mercados de futuros. En concreto nos hemos centrado en los contratos de futuros sobre índices bursátiles y en los siguientes puntos: (i) la elección de la fecha del *rollover* y la construcción de series largas de futuros; (ii) el estudio de posibles rendimientos anormales basados en anomalías de calendario en los futuros sobre índices bursátiles; (iii) la predicción del *Value-at-Risk* en los mercados de futuros y su comparación con el mercado de contado; y finalmente, (iv) la conexión entre la actividad de contratación en los mercados de futuros y la volatilidad diaria en dichos mercados.

A la primera conclusión que llegamos es que para la construcción de las series largas de futuros se puede utilizar cualquier criterio de *rollover*, ya que los distintos criterios no producen series distintas, y por lo tanto, no se espera obtener resultados distintos al trabajar con ellos. Nosotros recomendamos a los investigadores trabajar con el criterio del "último día" por ser el más sencillo. En el caso de que sean negociadores de contratos de futuros y quieran determinar el momento en el que realmente quieren cerrar una posición en el contrato más próximo a vencimiento para abrirlo en el siguiente, lo habitual es utilizar los criterios basados en la liquidez (a los que hemos añadido el R3), aunque los mercados analizados son muy líquidos en todo momento y sólo posiciones anormalmente altas podrían hacer prevalecer uno de estos criterios sobre el del "último día".

Una vez se ha determinado cómo vamos a construir nuestras series largas, pasamos a estudiar básicamente dos aspectos: los rendimientos y la volatilidad de mercado. En el primer caso llegamos a la conclusión de que prácticamente no existe ninguna anomalía de calendario hoy en día en los mercados de futuros sobre índices bursátiles. Sólo la regla del “cambio de mes +1” se puede considerar como una ineficiencia de mercado y sólo para el contrato de futuro del S&P 500.

En el segundo caso, concluimos que el modelo CTS-ARMA-GARCH es adecuado para predecir el *Value-at-Risk* al 1% a un día vista en los contratos de futuros sobre índices bursátiles, y que la inclusión del cambio relativo del volumen retardado consigue establecer un modelo ligeramente más conservador para dicho mercado. Por otro lado, ni se concluye lo mismo para el mercado de contado, ni parece adecuado intentar estimar el volumen contemporáneo para obtener mejores predicciones.

En la línea de los estudios sobre las variables relacionadas con la volatilidad, un estudio más profundo nos muestra la fuerte relación que esta tiene con la actividad de contratación. En particular, y a grandes rasgos, aparece una relación positiva con el número de posiciones que se abren y se cierran ese día, pero una relación negativa con su retardo. Por otro lado, se establece que cuando el mercado está dominado por *traders* que abren y cierran posiciones el mismo día (a los que asociamos con especuladores), no se genera una volatilidad extra, pero que si el mercado está dominado por *traders* que abren masivamente posiciones o las cierran (actividad asociada a los coberturistas) sí que se observa un incremento de la misma.

FUTURAS INVESTIGACIONES

Los resultados obtenidos en esta tesis nos han permitido avanzar en ciertos aspectos, pero a su vez han sugerido nuevas preguntas y nuevos temas de investigación que nos parece interesante desarrollar en el futuro. En concreto, las metodologías desarrolladas en el primer capítulo de la tesis y los resultados obtenidos, nos lleva a plantearnos estudios similares en contratos de futuros de distintos subyacentes, especialmente en el caso de las *commodities,* y analizar la relevancia de la elección de la fecha del *rollover* en otros subyacentes. Los resultados del segundo capítulo apuntan a una desaparición de los efectos de calendario diarios y también señalan a la búsqueda de posibles anomalías para rendimientos intradiarios de los futuros sobre índices bursátiles. Con ello se pretende dilucidar si las anomalías clásicas que habían sido detectadas para datos diarios, ahora aparecen concentradas en determinados momentos de la sesión, como parte de su proceso de desaparición.

El tercer capítulo se ha centrado en una comparativa entre contado y futuro y para ello se estudiaron las colas inferiores de las distribuciones de rendimientos. No obstante, dado el número creciente de instrumentos financieros que permiten a los inversores tomar posiciones cortas en los mercados, es de especial relevancia estudiar el riesgo localizado también en la cola derecha. Finalmente, el cuarto capítulo ha recuperado las variables “posiciones abiertas” y “posiciones cerradas” a lo largo del día y sería interesante analizar su papel dentro del abanico de medidas de *market sentiment* propuestas en la literatura.