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REGIME-SWITCHING VOLATILITY MODELS: APPLICATION TO DYNAMIC HEDGING WITH FUTURE CONTRACTS AND THE ESTIMATION OF THE RISK PREMIUM

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INTRODUCTION

REGIME-SWITCHING VOLATILITY MODELS:

APPLICATION TO DYNAMIC HEDGING WITH FUTURE CONTRACTS AND THE ESTIMATION OF THE RISK PREMIUM

A BRIEF MOTIVATION OF THE THESIS

Since the first introduction of conditional volatility GARCH models (Engle (1982) and Bollerslev (1986)) there have been many papers which propose improvements to these models in order to consider the empirical regularities present in most financial series (see : Lien (1996), Malik (2003), Susmel (2000)). One of the last contributions in this line is based on Markov Regime Switching GARCH (MRSG) models (Hamilton, 1989, Gray, 1996, Sarno y Valente, 2000). The novelty of these models is that let us perform the volatility estimations conditioned on the existing volatility regime. They are non-linear models depending on the number of regimes considered. Generally, they consider two regimes associated with situations of low and high volatility in the stock markets. This methodology let us analyze the main conclusions of several economic theories and their empirical evidence, distinguishing if these conclusions are the same under periods of financial stability than in times of market turmoil. This type of analysis is especially relevant in the current moment when financial markets show a high degree of instability and there is an emerging stream of studies that question most of the theoretical models which the most of the modern financial economics theories are based on.

MRSG models improve standard GARCH models in three aspects (Baele, 2005): 1) reflects the fact that volatility persitence of GARCH models is lower during low volatility periods than during high volatility periods. The no consideration of this fact may cause over-estimations of the persistence (Lamoureaux y Lastrapes, 1990; Cai, 1994) which have an impact on the volatility forecast. 2) The forecast obtained using these models are more accurate than those obtained with more parsimonious models (Marcucci, 2005). 3) These models reflect an asymmetric behaviour of the correlation between two financial assets regarding the size of their returns; i.e. it tends to be higher when the returns are low and lower when the returns are high. (Ang and Bekaert 2002).

The general objective of this thesis is to analyze the differences in the empirical evidence obtained between the MRSG methodology and more "traditional" or more common methodologies used in the literature. More specifically, the two research areas are focused on:

a) The analysis of the relationship between risk premium and expected volatility, whitin the asset pricing framework drawn by Merton's (1973) ICAPM model.

b) The study of the effectiveness of dynamic hedging with futures contracts on stock indexes, comparing several methodologies in order to determine the hedge ratio and using several effectiveness metrics to evaluate the hedging performance. This study is performed for hedging practical applications both in-sample and out-sample.

Therefore, the common link in the different chapters of this thesis is the use of the Markov Regime Switching GARCH methodology.

REVIEW OF THE METHODOLOGY IN THE THESIS

The aim of this subsection is to provide the reader a wide understanding of the methodology used in this thesis. Although each chapter describes in detail the empirical model it is worthy to make a first approach to MRSG models. In this previous methodological review we start presenting traditional linear GARCH models, then Markov-Regime Switching models are introduced for the case of modelling the returns (MS in mean) and finally we present the case of state-dependent conditional volatility models where Markov-Switching and GARCH models are implemented together.

A) GARCH models

Modeling volatility has received lots of attention from academics and practitioners given its important role in several asset pricing and risk management activities. Certainly, one of the most popular approaches is that one using econometric modeling[1] via GARCH models. Since their introduction by Engle (1982) and the generalized version GARCH model by Bollerslev (1986), numerous studies have applied and extended this methodology. In ARCH models, current conditional volatility is determined by squared errors in previous *p* periods and a constant. The current conditional volatility in GARCH models is formulated as a linear function of squared errors in the previous *p* periods and conditional variances in the previous *q* periods.

We consider the simpler case. Let r_t be log-return at time t and assume conditional mean equation as:

$r_t = c + e_t$ (1) and $e_t = u_t\sqrt{h_t}$ (2) where c is a constant, h_t is the conditional variance of errors, u_t, $u_t \left| \Omega_{t-1} \right. \sim N(0,1)$ and Ω_{t-1} is the information set available to the investor up the period t-1.

The ARCH(q) specification of the conditional variance is defined as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 \qquad (3)$$

While the GARCH(p,q) approach is as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_{t-j} \sigma_{t-j}^2 \qquad (4)$$

Which in its specification *p=q=1* is quite successful in describing the patterns followed by financial series leading to the following expression GARCH(1,1):

$$\sigma_t^2 = \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2 \qquad (5)$$

Where all the parameters must be positive to ensure positive conditional variance and the restriction $\alpha + \beta < 1$ must be satisfied for guarantee the stationarity of the process. It

[1] Besides of econometric modeling (such as GARCH and stochastic volatility models), volatility can also be estimated using options prices (implied volatility) or high frequency data (realized volatility).

is worthy to mention that the use of ARCH models is not practical since those models are highly outperformed by standard GARCH models (Alexander, 2001).

This simpler specification has been improved in several ways in order to reflect properly the empirical patterns of financial data. Certainly, one limitation of this simpler GARCH models is that they are not able to reflect the asymmetric response of volatility to news of different sign (known as leverage effect). To overcome this limitation some authors develop asymmetric GARCH specifications such as the E-GARCH model (Nelson, 1991), the GJR-GARCH model (Glosten et. al, 1993) or the QARCH model developed by Sentana (1995)[2].

The estimation of the unknown parameters of this kind of models is usually done by maximizing the likelihood function assuming normal innovations.

$$f\left(r_t, \Omega_{t-1}\right) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{1}{2}\frac{\left(r_t - \mu_s\right)^2}{\sigma_t^2}\right) \qquad (6)$$

However, the description of temporal dependence in conditional second order moments is certainly more appealing in a multivariate framework since financial volatilities move together over time and across markets. These models allow the study of the relations between variances and covariances and are very useful in the volatility transmission patterns between markets[3] or the computation of time-varying hedge ratios among others. There is a vast literature proposing increasingly more efficient models for this multivariate models trying to overcome limitations in the estimation and describing statistical properties of their estimations (see Bauwens et. al (2006) for an extensively review). The multivariate models used in the development of this thesis are explained in detail in each chapter so at this point we remit to each chapter to a well-comprehension on the application of this methodology to the issues discussed in this work.

B) Markov-Switching models

Whereas the relevance of shifts in regime has increased, the literature on this topic has presented more robust methodological techniques to incorporate potential non-linear patterns on the returns evolution of financial series. The pioneer study applying this methodology to financial markets is the work of Hamilton (1989, 1990). Hamilton extended the Markov switching regression model of Goldfeld and Quandt (1973) to time series framework and analyzes the growth rate of U.S. real GNP. In this work, the returns evolution is allowed to switch stochastically among regimes, obtaining different dynamics depending on the dominant regime. Other papers such as Krolzig (1996) discuss about the statistical properties of these models and propose algorithms for more complex Markov-Switching systems in mean that are developed in a multivariate framework with cointegration relationships among series. In this previous review, we just explain the methodology used in Regime-Switching Autorregresive Systems as a first introduction to Markov-Switching models.

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[2] For an extensive survey of GARCH models see Bera and Higgins (1993)

[3] For instance, how is the volatility/shock of one market affecting the volatility of other markets or how the correlations vary among markets.

Let r_t be a financial return series, Hamilton's (1989) two state Markov regime switching AR(1)[4] model is defined as follows:

$$r_t = c_{s_t} + \alpha_{1,s_t} r_{t-1} + e_t \tag{7}$$

where we assume that the innovations e_t follow a normal distribution $N\left(0,\sigma_{s_t}^2\right)$ and s_t is an unobservable variable that determines if the process is in regime 1 at period t $\left(s_t = 1\right)$ or in regime 2 $\left(s_t = 2\right)$.

To construct the likelihood function we need a procedure of two steps. First, joint density of returns (r_t) and unobserved regime variable (s_t) can be written as:

$$f\left(r_t, s_t \left|\Omega_{t-1}\right.\right) = f\left(r_t \left|s_t, \Omega_{t-1}\right.\right) f\left(s_t \left|\Omega_{t-1}\right.\right) \tag{8}$$

Where Ω_{t-1} is the all available information up to t-1 and $f\left(r_t \left|s_t, \Omega_{t-1}\right.\right)$ is the state-dependent likelihood function defined as

$$f\left(r_t \left|s_t, \Omega_{t-1}\right.\right) = \frac{1}{\sqrt{2\pi\sigma_{s_t}^2}} \exp\left(-\frac{1}{2}\frac{\left(r_t - c_{s_t} - \alpha_{1,s_t} r_{t-1}\right)^2}{\sigma_{s_t}^2}\right) \tag{9}$$

Second, the marginal density function of r_t can be constructed as:

$$f\left(r_t \left|\Omega_{t-1}\right.\right) = \sum_{s_t=1}^{2} f\left(r_t, s_t \left|\Omega_{t-1}\right.\right) = \sum_{s_t=1}^{2} f\left(r_t \left|s_t, \Omega_{t-1}\right.\right) f\left(s_t \left|\Omega_{t-1}\right.\right) \tag{10}$$

Where $f\left(r_t \left|s_t, \Omega_{t-1}\right.\right)$ has been defined previously and $f\left(s_t \left|\Omega_{t-1}\right.\right) = \Pr\left(s_t = i \left|\Omega_{t-1}\right.\right) \quad for \ \ i = 1,2$ is the regime probability, that is, the probability that the process is in regime *i* at time t based on the all information up to time t.

So the log-likelihood function is defined as:

$$L = \sum_{t=1}^{T} \ln\left(\sum_{s_t=1}^{2} f\left(r_t \left|s_t, \Omega_{t-1}\right.\right) \Pr\left(s_t \left|\Omega_{t-1}\right.\right)\right) \tag{11}$$

To estimate this log-likelihood function the regime probabilities must be computed, but it is impossible to make inference about regime probabilities without any assumptions on the unobserved variable. So, we assume that regime switching is directed by a first order Markov Chain process with constant transition probabilities[5], where the current regime s_t only depends on the regime one period ago s_{t-1}.

$$\Pr\left(s_t \left|s_{t-1}, s_{t-2....} s_1, \Omega_{t-1}\right.\right) = \Pr\left(s_t \left|s_{t-1}\right.\right) \tag{12}$$

[4] The procedure for the AR(q) case is similar.

[5] This regime probabilities could be driven by other processes such as Independent Switching or Markov Chain process with dynamic transition probabilities but in this thesis we base our model essentially on this process for the transtion probability.

So considering two regimes with constant transition probabilities, the transition matrix which reflects the probability of switching from one regime to other regime is defined:

$$\hat{P} = \begin{pmatrix} \Pr\left(s_t = 1 \middle| s_{t-1} = 1\right) = p & \Pr\left(s_t = 1 \middle| s_{t-1} = 2\right) = (1-q) \\ \Pr\left(s_t = 2 \middle| s_{t-1} = 1\right) = (1-p) & \Pr\left(s_t = 2 \middle| s_{t-1} = 2\right) = q \end{pmatrix} \qquad (13)$$

To estimate the maximum-likelihood function we use an iterative technique designed for a general class of models where the observed time series depends on some unobservable stochastic variables. Each iteration involves a pass through the filtering and smoothing iterations, followed by an update of the first order conditions and the parameter estimates guaranteeing an increase in the value of the likelihood function. In the filtering and smoothing steps the unobserved states are estimated by their smoothed probabilities where all probabilities are computed with recursions by using the estimated parameter vector of the last maximization step. With the regime probabilities an estimation of the parameter vector is obtained as a solution and it can be used to update the filter and smooth probabilities and so on. Therefore, we have to apply the following steps:

1.- Given $\Pr\left(s_{t-1} = j \middle| \Omega_{t-1}\right)$ *for* $j = 1, 2$ at the end of period t-1, the regime probability

$\Pr\left(s_t = i \middle| \Omega_{t-1}\right)$ *for* $i = 1, 2$ is computed as: $\Pr\left(s_t = i \middle| \Omega_{t-1}\right) = \sum_{j=1}^{2} \Pr\left(s_t = i, s_{t-1} = j \middle| \Omega_{t-1}\right)$ (14)

We made the assumption that current regime s_t only depends on the regime one period ago (s_{t-1}) , therefore:

$$\Pr\left(s_t = i \middle| \Omega_{t-1}\right) = \sum_{j=1}^{2} \Pr\left(s_t = i, s_{t-1} = j \middle| \Omega_{t-1}\right) = \sum_{j=1}^{2} \Pr\left(s_t = i \middle| s_{t-1} = j\right) \Pr\left(s_{t-1} = j \middle| \Omega_{t-1}\right) \qquad (15)$$

2.- At the end of time t, using Bayesian arguments the $\Pr\left(s_{t-1} = j \middle| \Omega_{t-1}\right)$ is computed as:

$$\Pr\left(s_t = i \middle| \Omega_t\right) = \Pr\left(s_t = i \middle| r_t, \Omega_{t-1}\right) = \frac{f\left(s_t = i, r_t \middle| \Omega_{t-1}\right)}{f\left(r_t \middle| \Omega_{t-1}\right)} = \frac{f\left(r_t \middle| s_t = 1, \Omega_{t-1}\right) \Pr\left(s_t = i \middle| \Omega_{t-1}\right)}{\sum_{i=1}^{2} f\left(r_t \middle| s_t = 1, \Omega_{t-1}\right) \Pr\left(s_t = i \middle| \Omega_{t-1}\right)} \qquad (16)$$

Then, the regime probabilities for all periods can be computed by iterating these two steps that are determined by the likelihood function itself.

3.- An estimation of the parameter vector is obtained as a solution of the first order conditions of the likelihood function when the regime probabilities used are those obtained in the previous two steps. Equipped with the new parameter vector the filtered and smoothed probabilities are updated and the algorithm starts again since the optimum is achieved.

C) Regime-Switching GARCH models

Some literature combines the two types of processes described above (Markov-Switching and GARCH) ant it has focused on developing state-dependent time-varying volatility models. Hamilton and Susmel (1994) and Cai (1994) proposed Markov Regime Switching ARCH (SWARCH) model independently by combining Markov Regime Switching model with ARCH models. In this model, each regime is characterized by a different ARCH (q) process and parameters of conditional variance take different values for each regime. Gray (1996) extends this approach and proposes the Markov-Switching GARCH model.

$$r_{s_t} = \mu_{s_t} + e_{t,s_t} \qquad e_{t,s_t} \sim N\left(0, \sigma^2_{t,s_t}\right) \tag{17}$$

$$\sigma^2_{t,s_t} = \omega_{s_t} + \alpha_{s_t} e_{t-1} + \beta_{s_t} \sigma^2_{t-1} \tag{18}$$

for $s_t = 1,2$. μ_{s_t} and σ^2_{t,s_t} are the state-dependent mean and state-dependent conditional variances respectively. Both are allowed to switch between two regimes. To ensure positivity of conditional variance in each regime, necessary conditions are similar to the necessary conditions in uni-regime GARCH (1,1) model[6]. The unobserved regime variable s_t is governed by a first order Markov chain with transition probability matrix similar than (13).

Then, conditional distribution of return series r_t becomes a mixture-of-distribution model in which the weight variable is given by the *ex ante probability* $\left(\Pr\left(s_t = i \middle| \Omega_{t-1}\right)\right)$ *for* $i = 1,2$ for each of the two-state dependent marginal densities:

$r_t \left| \Omega_{t-1} = f\left(r_t \middle| s_t = 1, \Omega_{t-1}\right)\right.$ with probability $\Pr\left(s_t = 1 \middle| \Omega_{t-1}\right)$

$r_t \left| \Omega_{t-1} = f\left(r_t \middle| s_t = 2, \Omega_{t-1}\right)\right.$ with probability $\Pr\left(s_t = 2 \middle| \Omega_{t-1}\right)$

Where $f\left(r_t \middle| s_t = i, \Omega_{t-1}\right)$ *for* $i = 1,2$ represents the distributions assumed for the innovations. So, the log-likelihood function can be written as:

$$L = \sum_{t=1}^{T} \ln\left[f\left(r_t \middle| s_t = 1, \Omega_{t-1}\right) \Pr\left(s_t = 1 \middle| \Omega_t\right) + f\left(r_t \middle| s_t = 2, \Omega_{t-1}\right) \Pr\left(s_t = 2 \middle| \Omega_t\right) \right] \tag{19}$$

which can be estimated similarly than the process for Markov-Switching models in mean.

Hamilton and Susmel (1994) and Cai (1994) limited their estimation to the Markov Regime Switching ARCH model because there is an infinite path dependence problem inherent in SW-GARCH models. In SWARCH models, the conditional variance at time t depends on past q squared residuals and past q regime variables (s_t , … , s_{t-q}).

[6] However, the conditions are not the exactly the same. For instance, non-stationarity in one state does not imply non-stationarity in the whole process. See Abramson and Cohen (2007) for details.

However, in SW-GARCH model, the conditional variance at time t depends on the conditional variance at time t-1 and regime variable at time t (s_t) while the conditional variance at time t-1 depends on the conditional variance at time t-2 and regime variable at time t-1 (s_{t-1}), and so on. Therefore, the conditional variance at time t depends on the entire history of regimes up to time t. Both Hamilton and Susmel (1994) and Cai (1994) stated that path dependence nature of SW- GARCH model makes estimation infeasible and impossible for large sample size.

For example, in a SW-GARCH with M-regimes model, the number of paths enlarges by a factor of M in each period and integrating all possible paths is required to construct the likelihood function. For the t^{th} observation, there are M^t components of likelihood function and this makes estimation intractable for large sample sizes.

In order to solve this problem of path dependency in SW-GARCH models, Gray (1996)[7] proposed to use conditional expectation of the lagged conditional variance $E_{t-2}\left(\sigma_{t-1}^2\right)$ instead of lagged conditional variance σ_{t-1}^2. This approach preserves the natural essential of the GARCH process and allows tractable estimation of model. Gray's approach recombines $\sigma_{t-1,s_t=1}^2$ and $\sigma_{t-1,s_t=2}^2$ into σ_{t-1}^2, and recombines $e_{t-1,s_{t-1}=1}$ and $e_{t-1,s_{t-1}=2}$ into e_{t-1} by taking conditional expectations of σ_{t-1}^2 and e_{t-1} based on the ex ante probabilities. That is,

$$e_{t-1} = r_{t-1} - E\left(r_{t-1} \middle| \Omega_{t-2}\right) = r_{t-1} - \left(\Pr\left(s_t = 1 \middle| \Omega_t\right)\mu_{s_{t-1}=1} + 1 - \Pr\left(s_t = 1 \middle| \Omega_t\right)\mu_{s_{t-1}=1}\right) \tag{20}$$

$$\begin{aligned}\sigma_{t-1}^2 &= E_{t-2}\left(\sigma_{t-1}^2\right) = E\left[r_{t-1}^2 \middle| \Omega_{t-1}\right] - E\left[r_{t-1} \middle| \Omega_{t-1}\right]^2 = \\ &= \left[\Pr\left(s_t = 1 \middle| \Omega_t\right)\left[\mu_{s_{t-1}=1}^2 + \sigma_{t-1,s_{t-1}=1}^2\right] + \left(1 - \Pr\left(s_t = 1 \middle| \Omega_t\right)\right)\right] - \left[\Pr\left(s_t = 1 \middle| \Omega_t\right)\mu_{s_{t-1}=1} + \left(1 - \Pr\left(s_t = 1 \middle| \Omega_t\right)\right)\mu_{s_{t-1}=1}\right]^2\end{aligned} \tag{21}$$

The use of conditional expectation of the lagged conditional variance $E_{t-2}\left(\sigma_{t-1}^2\right)$ instead of lagged conditional variance σ_{t-1}^2 makes conditional variance at time *t* depends on only current regime s_t and inference about s_{t-1}. Therefore, the Gray's *collapsing procedure* simplifies and makes tractable the estimation of SW-GARCH models.

Given the initial values for conditional mean and conditional variance in each regime, the parameters of SW-GARCH model can be obtained by maximizing numerically the log-likelihood function in equation (19). The state-dependent log-likelihood functions are constructed recursively similar to that in a uni-regime GARCH models:

$$f\left(r_t \middle| s_t = i, \Omega_{t-1}\right) = \frac{1}{\sqrt{2\pi\sigma_{t,s_t=i}^2}} \exp\left(-\frac{1}{2}\frac{\left(r_t - \mu_{s_{t=i}}\right)^2}{\sigma_{t,s_{t=i}}^2}\right) \quad for \quad i = 1,2 \tag{22}$$

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[7] Posterior studies to Gray also develop other recombining methods to collapse the state-dependent variances and errors and solve the path-dependency problem. Two of them are the procedure used by Dueker (1997) and the one proposed by Klaasen (2002).

THESIS STRUCTURE

This thesis is structured in 4 chapters, besides the introduction and the last chapter dedicated to the general conclusions.

A) ICAPM

Chapters 2, 3 and 4, besides the application of MRSG methology, share a common objective; validate empirically the Capital Asset Pricing Model (CAPM) considering the intertemporal approximation proposed by Merton (1973). According to this paper, the trade-off between return and conditional volatility does not exclusively depend on the market risk factor. It must consider additional risk factors reflecting the investment opportunity set available to the investors. However, this model does not specify which these additional risk factors are. There are many works (Capiello et. al. (2008); Capiello and Guéné (2005), Bali and Engle (2010)) analyzing which factors could be reflecting this "intertemporal" risk premium. Anyway, from the conclusion derived in this theoretical model one expect a positive and significant relationship between the expected return and conditional volatility.

However, the empirical evidence does not show conclusive evidence according either the sign or significane of this trade-off. In this sense, the results have been different regarding the sample period analyzed, the frequency of the returns and the methodology employed. We are not intended to be exhaustive in this introduction since they will be presented in more detail in the following chapters, but the main methodologies used in the empirical studies of the risk-return trade-off are: GARCH (French et al. (1987), Campbell (1987), Glosten et al. (1993), Scruggs (1998), Engle and Lee (1999), Scruggs and Glabadanidis (2003), Matallín et. al. (2004)); MRSG (Chauvet and Potter (2001), Whitelaw (2000), Mayfield (2004)); MIDAS (Ghysels, et. al., 2005; León, et. al.(2007)) or models using variables reflecting the businness-cycles to perform the forecasts of returns and conditional volatility (Fama and French (1988,1989)). A further extension of the last models considers additional risk factors obtained through economical/financial series (Ludvigson and Ng (2007).

So, chapter 2 is entitled: **Re-examining the risk-return relationship: The influence of financial crisis (2007-2009)**. This paper analyzes the risk–return trade-off in Europe using several empirical methodologies (GARCH, MIDAS, and RS-GARCH) and considering the impact of the recent financial crisis between 2007 and 2009. It is shown that when non-linear patterns in the risk–return trade-off are considered, a significant positive risk–return relationship can be obtained. This result is robust among countries despite the short span used in the empirical analysis and the lack of consideration of an alternative investment set suggesting that the lack of significance in previous studies may be because of the strong linear assumptions in the modeling of the risk–return trade-off. The risk premiums obtained are higher than those found in previous studies, mainly because of the impact of the financial crisis. Although risk prices in different countries exhibit different patterns during the crisis, the extreme increase in non-diversifiable risk during this period explains the higher risk premiums observed

The title of chapter 3 is: **The risk–return tradeoff in Emerging Markets.** This paper studies the risk-return tradeoff in some of the main emerging stock markets in the world. Although previous studies on emerging markets were not able to show a positive and significant tradeoff, favorable evidence can be obtained if a non-linear framework between return and risk is considered. Using 15 years of weekly data observations on 25 MSCI stock index: 5 latin American, 9 asian, 5 eastern european, 3 africans and 3 aggregate index for Asia, Eastern Europe and Latin America, in a Regime Switching-GARCH framework, favorable evidence is obtained in most of the emerging markets during low volatility periods, but not for periods of financial turmoil or using the traditional linear GARCH-M approach.

Chapter 4 has the following title: **Non-linear trade off between risk and return: A regime-switching multi-factor framework.** This paper examines the risk-return trade-off in Spain during the last 15 years. The study is developed in a multi-factor framework where not only the market risk is considered but also potential changes in the investment opportunity set. Although previous studies find no clear evidence about a positive and significant relation between return and risk, favorable evidence can be obtained if a non-linear relation between return and risk is established. Despite the importance of the intertemporal hedging component in the risk premium demanded by investors, the evidence obtained is independent of the choice of the proxy used. Different patterns for the risk premium dynamics in low and high volatility periods are obtained, both in risk prices and risk (conditional second moments) patterns.

B) Hedging with future contracts

In chapter 5 the main objective of the research changes, although the methodology used still being the same applied in previous chapters (MRSG). The research topic in this chapter is hedging with future contracts on stock indexes. The development of derivative markets caused the appareance of many literature focused on the study of hedging techniques with futures contracts which get a reduction in the investment risk. Most of this literature is focused on the determination of the optimal hedge ratio (Myers and Thompson (1989), Cheung et. al (1990), Chen et al. (2003); Aragó (2009)). Among the different approaches, the most used is that one minimazing the variance of returns in a hedged portfolio which contains spot and futures positions (Johnson, 1960). The pioneer work in constant hedge ratios is made by Ederington (1979). According to this approach, the optimal hedge ratio is obtained as the quotient between the covariance of spot and futures returns, and the variance of the futures returns. The estimation of this ratio is made through the slope of the OLS regression between the spot and future returns. However, this approach assume constant conditional second moments, and therefore, static hedging strategies. To solve this problem, in the vast literature on this research area, it has been proposed the use of bivariate GARCH models (Myers (1991), Kroner and Sultan (1993), Park and Switzer (1995), Brooks et. al (2002)), which let us estimate conditional second moments conditioned to the information set available to the investor, and therefore, to the available information set.

However, these models exhibit several problems. One of them[8] is the no consideration of structural changes in the unconditional volatility (Lamoreux and Lastrapes (1990), Wilson et. al (1996)) or even the possibility of shifts in the parameters of the model. These aspects can be considered with Markov Regime Switching GARCH models (Hamilton and Susmel (1994), Susmel (2000), Sarno and Valente (2000), Alizadeh and Nomikos (2004), Alizadeh et. al (2008)). The evidence of the studies that include regime-switches (MRSG) conclude that more robust estimations can be obtianed if we let the volatility to follow several processes according to the market states, causing a higher hedging effectiveness of the strategies which consider them.

The title of chapter 5 is: **Measuring hedging effectiveness of index futures contracts. Do dynamic models outperform static models? A regime-switching approach**. This paper estimates linear and non-linear GARCH models to obtain optimal hedge ratios with futures contracts for some of the main European stock indexes. Introducing non-linearities through a regime-switching we can obtain more efficient hedge ratios and superior hedging performance both in and out sample analysis compared to other methods usually performed in the literature (constant hedge ratios and linear GARCH). Moreover, the non-linear models also reflect different patterns followed by the dynamic relationship between spot and futures returns during low and high volatility periods

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CHAPTER 1

RE-EXAMINING THE RISK–RETURN RELATIONSHIP: THE INFLUENCE OF THE FINANCIAL CRISIS, 2007–2009

Abstract

This paper analyzes the risk–return trade-off in Europe using several empirical methodologies: GARCH, MIDAS and Regime-Switching GARCH considering the impact of the last financial crisis. It is shown that when non-linear patterns in the risk-return trade-off are considered, a significant positive risk–return relationship can be obtained. This result is robust among countries despite the short span used in the empirical analysis (Lundblad, 2007) and the no consideration of an alternative investment set (Scruggs, 1998) suggesting that the lack of significance in previous studies may be due to strong linear assumptions in the modelling of the risk-return trade-off. The risk premiums obtained are higher than in previous studies, due mainly to the impact of the financial crisis. Although risk prices in different countries exhibit different patterns during the crisis, the extremely increase of non-diversifiable risk during this period explains the higher risk premiums observed.

1. Introduction

One of the most discussed topics in financial economics is that tries to establish a relationship between return and risk. There are several attempts to explain and understand which are the dynamics and interactions followed between these two fundamental variables. From a theoretical framework, one of the most cited works analyzing this risk-return trade-off is the Merton's (1973) ICAPM model. Merton demonstrates that there is a linear relationship between conditional excess market return and its conditional variance, and with its covariance with investment opportunities:

$$\mu_M - r_f = A\sigma_M^2 + BX_{M,S} \qquad (1)$$

where $\mu_M - r_f$ is the excess return of the portfolio over the risk-free asset; σ_M^2 is the conditional variance of excess market returns (known as idiosyncratic portfolio risk), $X_{M,S}$ is the conditional covariance between excess market returns and the state variable that represents the investment opportunities (known as hedge component), and A, B are the prices of these sources of risk.

Despite the important role of this trade-off in the financial literature, there is no clear consensus about its empirical evidence. In the theoretical framework, all the parameters (the risk prices A,B) and the variables (the sources of risk) are allowed to be time-varying. However, to make this model empirically tractable one must make several assumptions; the most common is considering constant risk prices (Goyal and Santa-Clara 2003, Bali et. al 2005). Another common assumption is considering a set of investment opportunities constant over time, remaining the market risk as the only source of risk (Glosten et al. 1993, Shin (2005), Lundblad (2007)). It is also necessary to assume specific dynamics for the conditional second moments. The most common are the GARCH models (Bollerslev 1986). Finally, the empirical model is established in a discrete time economy instead of the continuous time economy used in the equilibrium model of the theoretical approach.

Given the assumptions mentioned above, there are many papers explaining alternatives empirical models in order to obtain favorable evidence as suggest the theoretical intuition. The methodology most commonly used in the empirical analysis of the risk-return trade-off is the GARCH-M approach (Engle et. al (1989)). This framework is simple to implement but the results obtained are often poor at best. In a recent paper, Lundblad (2007) shows that the typically insignificant relationship between the market risk premium and its expected volatility may be due to a statistical artifact[9] of the GARCH-M framework. A large data span is required in this approach to find successfully a positive risk–return trade-off, showing in the Monte-Carlo simulation that even 100 years of data constitute a small sample from which one is forced to make inferences, obtaining sometimes no favorable evidence. That paper reveals that for the analysis of the risk-return trade-off using a shorter 'span' data, the GARCH-M approach

[9] Small sample inference is plagued by the fact that conditional volatility has almost no explanatory power for realized return.

usually obtain disappointing results and that may be one of the main causes of the controversial existing in the literature.

Therefore, we need alternative approaches to the usual GARCH-M methodology in order to analyze the risk-return trade-off in a shorter ‘span’. The main important frameworks developed in the financial literature are the followings. Ghysels et al. (2005) propose an alternative empirical methodology to counteract the disadvantages of the GARCH-M estimations, using different data frequencies to estimate the mean (with lower data frequency) and the variance (with higher data frequency) equations. Ludvigson and Ng (2007) use a factor approach to summarize a large amount of economic information in their risk–return trade-off analysis. Bali (2008) proposes an alternative approach considering not only the time series dimension of the portfolio market but also the cross-sectional dimension that allows the consideration of the whole market. Whitelaw (1994) uses an instrumental variables specification for the conditional second moments. Harrinson and Zhang (1999) use nonparametric techniques in their study instead of the parametric approaches used above. Whitelaw (2000) and Mayfield (2004) employ methodologies whereby states of the world are essentially defined by volatility regimes.

Among these approaches, some of them use information not only about the market portfolio but also about additional risk factors such as other asset portfolios, economic indicators, etc. extending their empirical model to a multi-dimensional framework. However, there are several alternatives that try to obtain favourable evidence using only the information in the market portfolio. These approaches modify the empirical methodology to overcome the limitations of the traditional GARCH-M methodology. These main alternative frameworks[10] are the inclusion of Regime-Switching in the empirical model, and the use of the MIDAS regression. The first one proposes a non-linear relationship between return and risk which is based on an equilibrium framework developed in the paper of Whitelaw (2000). This theoretical framework is slightly different from Merton’s approach because a complex, non-linear, and time-varying relationship between expected return and volatility is obtained. The second one presents an alternative specification, the MIDAS regression, for modelling conditional second moments against GARCH models.

This paper analyzes the risk–return trade-off in Europe and tries to shed light to the dynamics between these two variables in a shorter ‘span’ analysis. The main result obtained is that a non-linear specification is necessary to reflect a positive and significant trade-off between return and risk. When several volatility states are considered, the risk–return relationship becomes significant, even ignoring possible changes in the set of investment opportunities. When linear patterns in the risk specification (GARCH and MIDAS) are considered, no significant relationship in any market could be obtained. More specifically, when non-linear patterns are considered (RS–GARCH models), a positive and significant trade-off between return and risk for

[10] Non-parametric GARCH could be viewed as an alternative from the traditional parametric GARCH-M estimation but exhibits similar problems. Therefore, it is not included in the study.

the state that governs the variance process is obtained. However, for the secondary state (the state that does not govern the volatility process), this relationship becomes insignificant. These results are robust for all the stock indexes analyzed. Furthermore, we also find a significant trade-off between return and risk in secondary volatility states in markets such as Spain and the United Kingdom after controlling for the global financial crisis from 2007–2009. This result shows that the lack of empirical evidence in previous studies may be due to a strong assumption of a linear risk-return relation rather than non-linear and reveals the perils of using linear frameworks in order to analyze empirically this trade-off.

The principal contributions of our paper are as follows. First, we show that a positive and significant risk–return trade-off is obtained after considering non-linearities in the conditional variance process even ignoring the hedge component. Secondly, we show the evolution of the risk premium in Europe in recent years, including the period of the global financial crisis, and, finally, we analyze whether the risk premium and its components (risk-price and non-diversifiable risk) present different patterns during this period.

The remainder of this paper is structured as follows. Section 2 describes the data used in the study and develops the methodology. Section 3 reports and analyzes the main results obtained. Finally, section 4 summarizes.

2. Data and methodology

We use 1130 weekly[11] excess returns for the period between January 1988 and August 2009 for the GARCH and RS–GARCH specifications. Additionally, we use daily[12] data for the same sample period for the MIDAS model. Excess returns are computed using the log-returns of the main European stock index, DJEurostoxx (Europe hereafter), subtracting the proxy for risk-free[13] investment. The index data is obtained from Datastream, and the risk-free rates from International Financial Statistics[14]. For robustness, we also use data from some European countries[15] such as France (CAC-40), the United Kingdom (FTSE-100), and Spain (IBEX-35). Sample periods and databases are the same as for the European case.

In next subsections we develop the methodology proposed in each one of the empirical models that we use to analyze the risk-return trade-off.

[11] Following papers such as Capiello and Fearnley (2000) or Ghysels et al. (2007), we analyze this relationship using weekly data rather the monthly data used in other studies Even though there are slight differences in the parameter estimations using different data frequency, there is no particular reason that the conclusions in this study should be affected by the selection of data frequency. Some authors remark on this point in their studies (Lundblad 2007).

[12] MIDAS approach proposes the estimation of the mean equation using low frequency data (in our case weekly data) and high frequency data for the variance equation (daily data in our case).

[13] Following Leon et al. (2007), we use an equal-weighted average of the suitably compounded monthly market money rate of Germany, France, the United Kingdom and Spain as a proxy for risk-free investment in Europe.

[14] For brevity, the descriptive statistics are not presented, but they are available from the authors upon request.

[15] For individual markets, a compounded monthly market money rate for each country is used as the proxy for the risk-free investment.

2.1. Standard GARCH

The first approach is the traditional GARCH–M model of Engle et al. (1987). This framework is the most used in the financial literature to study the risk-return trade-off despite the problems explained above.

The mean equation is defined as follows:

$$r_t = c + \lambda h_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, h_t) \qquad (2)$$

where r_t is the excess market return, h_t is the conditional variance and ε_t represents the innovations, which are assumed to follow a normal distribution.

In this approach, the conditional volatility is obtained as in Bollerslev (1986):

$$\varepsilon_t = h_t z_t \qquad z_t \sim N(0,1) \qquad (3)$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \qquad (4)$$

where $\hat{\alpha} + \hat{\beta} < 1$ guarantees the stationarity of the process.

We estimate this first model using the Quasi Maximum-Likelihood (QML) function of Bollerslev–Wooldrige (1992) that allows us to obtain robust estimates of standard errors.

$$L(\theta) = \sum_{t=1}^{T} \ln\left[f\left(r_t, \Omega_t; \theta\right)\right] \qquad where \qquad f\left(r_t, \Omega_t; \theta\right) = \left(2\pi h_t\right)^{-\frac{1}{2}} e^{-\frac{(\varepsilon_t)^2}{2h_t}} \qquad (5)$$

However, this approach has not presented favourable evidence on the significance of the risk aversion parameter in many previous studies, such as Baillie and De Gennaro (1990), Glosten et al. (1993), Shin (2005), Leon et al. (2007)).

2.2. Regime-Switching (RS) GARCH

An explanation for these results may lie in a wrong specification for the relationship between risk and return that follows non-linear rather than linear patterns. Therefore, an immediate extension is to consider non-linearities in this trade-off against the linear framework usually implemented. We use a Regime Switching (RS)–GARCH specification, based on a model originally proposed by Hamilton (1989) that allows us to distinguish between different volatility states governed by a hidden state variable that follows a Markov process.

In this model, the mean equation is not exactly as shown in Equation 2 because it is state-dependent:

$$r_{t,s_t} = c_{s_t} + \lambda_{s_t} h_{t,s_t} + \varepsilon_{t,s_t} \qquad \varepsilon_{t,s_t} \sim N(0, h_{t,s_t}) \qquad (2')$$

where r_{t,s_t}, h_{t,s_t} and ε_{t,s_t} are the state-dependent returns, variances and innovations, and s_t = 1 (state 1), or 2 (state 2).

The state-dependent innovations follow a normal distribution, with two possible variances depending on the state of the process. The state-dependent variances are modelled as in Equation 4 allowing different parameters, depending on the state[16].

$$\varepsilon_{t,s_t} = h_{t,s_t} z_t \qquad z_t \sim N(0,1) \qquad (3')$$

$$h_{t,s_t} = \omega + \alpha_{s_t} \varepsilon_{t-1}^2 + \beta_{s_t} h_{t-1} \qquad (4')$$

The shifts from one state to another are governed by a hidden state variable following a Markov process with transition matrix

$$\hat{P} = \begin{pmatrix} \Pr\left(s_t = 1 \middle| s_{t-1} = 1\right) = p & \Pr\left(s_t = 1 \middle| s_{t-1} = 2\right) = (1-q) \\ \Pr\left(s_t = 2 \middle| s_{t-1} = 1\right) = (1-p) & \Pr\left(s_t = 2 \middle| s_{t-1} = 2\right) = q \end{pmatrix} \qquad (6)$$

Because of this state-dependence, the model is econometrically intractable[17]. We must therefore obtain state-independent estimates of variances and innovations. We averaged out according to the ex-ante probability[18] of being in each state (Dueker (1997)).

$$h_t = P\left(s_t = 1 \middle| \Omega_{t-1}; \theta\right) h_{t,s_t=1} + P\left(s_t = 2 \middle| \Omega_{t-1}; \theta\right) h_{t,s_t=2} \qquad (7)$$

$$\varepsilon_t = P\left(s_t = 1 \middle| \Omega_t; \theta\right) \varepsilon_{t,s_t=1} + P\left(s_t = 2 \middle| \Omega_t; \theta\right) \varepsilon_{t,s_t=2} \qquad (8)$$

where h_t and ε_t are the state-independent variances and disturbances and

$$P\left(s_t = 1 \middle| \Omega_{t-1}; \theta\right) = p * P\left(s_{t-1} = 1 \middle| \Omega_{t-1}; \theta\right) + (1-q) P\left(s_{t-1} = 2 \middle| \Omega_{t-1}; \theta\right) \qquad (9)$$

and

$$P\left(s_t = 2 \middle| \Omega_{t-1}; \theta\right) = 1 - P\left(s_t = 1 \middle| \Omega_{t-1}; \theta\right) \qquad (10)$$

are the ex-ante probabilities, where

$$P\left(s_t = k \middle| \Omega_t; \theta\right) = \frac{P\left(s_t = k \middle| \Omega_{t-1}; \theta\right) f\left(r_t \middle| s_t = k, \Omega_t; \theta\right)}{\sum_{k=1}^{2} P\left(s_t = k \middle| \Omega_{t-1}; \theta\right) f\left(r_t \middle| s_t = k, \Omega_t; \theta\right)} \qquad (11)$$

for k=1, 2 are the filtered probabilities.

We estimate this model, maximizing the QML function of Bollerslev-Wooldrige (1992), weighted by the filtered probability of being in each state.

$$L(\theta) = \sum_{t=1}^{T} \ln\left[\sum_{k=1}^{2} P\left(s_t = k \middle| \Omega_t; \theta\right) f\left(r_t, \Omega_t; \theta\right)\right] \quad \textit{where} \quad f\left(r_t \middle| s_t, \Omega_t; \theta\right) = \left(2\pi h_{t,s_t}\right)^{-\frac{1}{2}} e^{-\frac{\left(\varepsilon_{t,s_t}\right)^2}{2h_{t,s_t}}} \qquad (5')$$

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[16] Following Capiello and Fearnley (2000), to facilitate convergence, the constant variance term is not allowed to switch between regimes.

[17] See e.g. Gray (1996) or Dueker (1997)

[18] Following Hamilton (1994), the ex-ante probability is defined as $P\left(s_t = k \middle| \Omega_{t-1}; \theta\right)$ for k=1,2 i.e. the probability of being in the k[th] state, given the information up to t-1.

2.3. MIDAS regression

In recent years, a new methodology has been developed to capture a significant relationship between return and risk using data from different frequencies to obtain expected returns and variances, namely, MIDAS (Mixed Data Sampling) regression, Ghysels et. al (2005) (hereafter GSV). These authors found evidence of a significant positive trade-off between return and risk and argue that MIDAS allows the use of monthly returns in the mean equation and daily returns in the variance equation. We use this specification with weekly returns (r_t) combined with D daily[19] lag squared returns (R_t^2) to obtain the weekly variance; i.e., the mean equation of this model is similar to Equation 2.

$$r_t = c + \lambda VAR(r_t) + \varepsilon_t \qquad \varepsilon_t \sim N(0, VAR(r_t))$$

However, the MIDAS estimator of weekly conditional variance is a function of D lag squared daily returns (R_t^2):

$$VAR(r_t) = \sum_{d=0}^{D} \omega\left(k_1, k_2, d\right) R_{t-d}^2 \qquad (12)$$

where $$\omega\left(k_1, k_2, d\right) = \frac{\exp\left(k_1 d + k_2 d^2\right)}{\sum_{i=0}^{D} \exp\left(k_1 i + k_2 i^2\right)} \qquad (13)$$

is the weight function[20].

Assuming normality in returns $r_t \sim N\left(c + \lambda VAR(r_t), h_t\right)$, we estimate this model by maximizing the Bollerslev-Wooldrige QML function, as in Equation 4[21].

2.4. Asymmetric case

The symmetric models presented above can easily be extended to the asymmetric case in which the variance responses more after negative returns than after positive returns (leverage effect). For GARCH and RS–GARCH models we add a new variable $\eta_t = \min\left(\varepsilon_t, 0\right)$ in the variance process using the asymmetric GJR model (Glosten et al. (1993)).

[19] In the original specification, these summations are infinite. We truncate them at 250 daily lag squared returns to estimate the weekly variance.

[20] GSV develop several weight functions for the MIDAS estimator, but due to its tractability, the Almon Lag specification is the most frequently used in the literature.

[21] Although some authors estimate this specification with Non-linear Least Squares, GSV used the QML estimate in their original paper.

These models are estimated in a similar way to that presented above, substituting Equations 4 and 4´ for 14 and 14´ respectively.

$$h_t = \omega + \alpha\varepsilon_t^2 + \beta h_{t-1} + \delta\eta_{t-1}^{\ 2} \qquad (14)$$

$$h_{t,s_t} = \omega + \alpha_{s_t}\varepsilon_t^2 + \beta_{s_t} h_{t-1} + \delta_{s_t}\eta_{t-1}^{\ 2} \qquad (14´)$$

We estimate the MIDAS model for the asymmetric case substituting Equation 12 for Equation 15:

$$Var(r_t) = \theta\sum_{d=0}^{D}\omega\left(k_1^-, k_2^-, d\right) r_{t-d}^2 \cdot 1_{t-d}^- + (2-\theta)\sum_{d=0}^{D}\omega\left(k_1^+, k_2^+, d\right) r_{t-d}^2 \cdot 1_{t-d}^+ \qquad (15)$$

where $\theta, k_1^-, k_2^-, k_1^+, k_2^+$ are the parameters to be estimated, and $1_{t-d}^-, 1_{t-d}^+$ are the indicator functions for $\{r_{t-d} < 0\}$ and $\{r_{t-d} \geq 0\}$, respectively. We use Equations 5 and 5´ again to estimate these models.

3. Empirical results

In this section we present the main empirical findings of our study. First, we show the main results on the risk–return trade-off for the European case and then we check whether these findings are supported in some individual European markets. In the remainder of the section, we analyze the risk premium evolution and the influence of the financial crisis (2007–2009).

3.1 Estimations for Europe

The estimated parameters for the models proposed for the European case are shown in Table 1.

[INSERT TABLE 1]

The results for the GARCH model are similar to those presented in the literature, cf. Glosten et al. (1993), Shin (2005), Leon (2007). The results indicate a positive but non-significant relationship between return and risk. The estimated risk aversion coefficient is similar to other studies that obtain estimates of this parameter between 1 and 4 for US data[22], Bali (2008). Furthermore, the variance parameters present the typical patterns reported in the literature with a high persistence of the GARCH term. This fact has led some authors ((Lameroux & Lastrapes (1990), Marcucci (2005)) to consider different regimes for the variance process. They suggest that if these regime shifts are ignored, GARCH models tend to overestimate persistence in periods of financial instability and underestimate it in calm periods.

The RS–GARCH estimations show some interesting findings. In this case we can associate state 1 with low volatility periods and state 2 with high volatility periods because the median of the estimated volatility in each state is 2,584 and 3,687

[22] The risk aversion coefficient is divided by 100 because we are using excess returns multiplied by 100.

respectively. For $s_t = 1$, corresponding to the low volatility state, there is a significant positive relationship between return and risk. The risk aversion parameter for this case ($\lambda_{s_t=1}$) has a value of approximately 3.5. Another important finding is related to the variance parameter estimates in this state. The persistence of the GARCH term is even greater than for the non-switching case. This fact confirms the evidence from the literature (Marcucci (2005)). This author concludes that in low volatility periods there is a greater persistence in variance and it is underestimated if this regime switching is ignored.

For the state $s_t = 2$ we obtain a positive but non-significant relationship between return and risk. Moreover, the risk aversion coefficient ($\lambda_{s_t=2}$) is lower than for the low volatility regime. This finding is not consistent with the spirit of the theoretical models that suggest that higher volatility should be compensated with higher returns. However, some papers such as Mayfield (2004), Lettau and Ludvigson (2003), and Lundblad (2007) found the same evidence. This fact indicates that in high volatility periods the investor's risk aversion is lower. This may be due to the existence of a different risk price depending on the volatility regime. An investment considered too risky in calm periods (low volatility) is less risky when there is a period of market instability with more uncertainty and any investment involving risk. This finding also could be explained by investors' characteristics in high volatility states. In these periods, the more risk-averse investors leave the market, letting the less risk-averse investors adjust the price of risk according to their less demanding preferences. However, the specification presented here may be confounding expected returns with realized returns, particularly in the less common high volatility states (corresponding generally with recession periods) often associated with low or even negative markets returns (Lundblad, 2007).The estimated parameters for the variance equation show a lower persistence of the GARCH term and a higher presence of shocks in the volatility process. The reason for this finding is that in high volatility periods there are a high number of shocks. The persistence is overestimated in high volatility periods if regime switching is ignored (Marcucci (2005)).

Also note that the expected duration[23] for the low volatility state is approximately 12 weeks, about 4 times higher than the high volatility state. Figure 1 shows the smoothed probabilities[24] of being in state 1 for the sample period.

[INSERT FIGURE 1]

The bottom of Table 1 shows the results for MIDAS methodology. The main difference between this and previous models is the different data frequencies used to obtain

[23] We obtain the expected duration of being in each state $S_t = 1,2$ as $\frac{1}{1-p}$ *and* $\frac{1}{1-q}$ respectively

[24] The smoothed probability is defined as the probability of being in each state considering the entire information set $P(s_t = 1|\Omega_T;\theta) = P(s_t = 1|\Omega_t;\theta)\left[p\frac{P(s_{t+1} = 1|\Omega_T;\theta)}{P(s_{t+1} = 1|\Omega_t;\theta)}\right] + \left[(1-p)\frac{P(s_{t+1} = 2|\Omega_T;\theta)}{P(s_{t+1} = 2|\Omega_t;\theta)}\right]$

expected returns (weekly data) and variances (daily data). The risk aversion coefficient is similar to that obtained in other models. The results indicate a positive but not significant relationship between return and risk. Our results are different from previous studies that obtain favourable evidence using this methodology[25]. The variance estimates also indicate a high degree of persistence, because a great number of daily lags are needed to accurately estimate the variance. Specifically, 25.64% of the total weekly variance corresponds to the first 10 daily-lag returns, 44.51% to the 10-30 daily-lag returns, and 34.11% to higher lags.

Table 2 shows the estimates for the mean equation in the asymmetric case. Basically, the estimates for the risk aversion coefficient are similar than those obtained for the symmetric case in all specifications and support the above findings showing the consideration of leverage effect has no impact on the significance of the risk-return trade-off.

[INSERT TABLE 2]

3.2 Estimates for European countries

For robustness, Table 3 shows[26] the estimates for the mean equation in the symmetric case for three European countries: France, the United Kingdom and Spain. Basically, the estimates for the risk aversion coefficient are similar to those obtained for the European case in all specifications and support our findings.

[INSERT TABLE 3]

GARCH and MIDAS specifications still provide non-significant estimates of the risk–return trade-off in all cases presented. The RS–GARCH model shows significant estimates in the low volatility states in France and Spain, but in the United Kingdom we obtain the significant trade-off in high volatility states. Introducing non-linearities in the variance process we obtain a significant relationship between return and risk in the state that dominates the variance process. This fact could be observed most clearly by the smoothed probabilities of being in each state as shown in Figure 2.

[INSERT FIGURE 2]

The figure represents the probability of being in a low volatility state in the three countries considered. In France and Spain the process follows the low volatility state in most of the sample. However, the high volatility state dominates[27] the variance process

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[25] These differences may be due to our use of mixed daily and weekly data, while most studies use mixed daily (variance) and monthly (returns) data, as in Ghysels et. al (2005) and Leon et. al (2007). Some studies analyzing risk premium with MIDAS and weekly data in returns with statistical significance of the risk aversion parameter use intraday data (Ghysels et al. (2007)) in the variance equation. However, the consideration of the financial crisis period in the empirical analysis could blur the evidence in a linear framework.

[26] For brevity, we only show the estimates for the symmetric case. The results for the asymmetric case are essentially the same and support our findings. These estimates are available from the authors upon request.

[27] The number of periods in which variance process is in a low volatility state (low volatility state probability lower than 0.5) in Spain, Europe and France is slightly greater than for high volatility states

in the United Kingdom. Therefore, significant estimates of the risk-averse coefficient in the state that dominates the variance process are obtained. Following our findings, only when the process is in the main state which governs the conditional second moments dynamics do we observe a clear relationship between expected return and risk; when the market is out of this dominant state, the risk–return relationship becomes insignificant. This result may suggest that only a significant risk-return trade-off is observed under market 'normal' conditions. When the market is in secondary states this relationship between return and risk becomes non significant. Strong assumptions of a linear relation between return and risk could lead to model misspecification and an inability of the empirical model to capture a significant risk-return relationship since the existence of periods where a risk-return trade-off is not observed could lead to non-significant estimation of this relation for the entire sample.

3.3. Risk premium evolution in Europe

Figure 3 shows the risk premium evolution in Europe[28] during the sample period. The risk premium is given by λh_t where λ is the risk aversion (or also the risk price in our case) parameter and h_t represents the non-diversifiable risk obtained for each methodology. For the RS–GARCH specification we obtain the two variables described above with a weighted average using the filter probabilities (similar to method use to obtain independent variances and disturbances).

[INSERT FIGURE 3]

The figures show similar patterns for the risk premium evolution. The premiums only differ by the scale of the risk price, because risk exposure is similar for all methodologies. The high increase in the risk premium in recent years coincides with a period of high financial instability. The median[29] of the weekly risk premiums series shows that over the past 20 years the risk premium in Europe has remained at approximately 4% to 7% per annum[30]. We present in Table 4 an average risk premium (using the median of risk premium series) for the four stock market indexes considered.

[INSERT TABLE 4]

The risk premiums obtained present a higher or lower value depending the market considered. Spain and France are markets with a higher risk premium demanded by investors whereas Europe and the United Kingdom show lower risk premiums. In all markets observe that RS-GARCH risk premiums are the highest. All these premiums represent higher values than the 3% to 5% obtained in other studies for US data (Bali

(589, 583 and 591 periods) but in the United Kingdom is the opposite (only 396 periods in a low volatility state).

[28] We only show the risk premium evolution for Europe, but the risk premium evolutions for France, the United Kingdom and Spain are available from the authors upon request.

[29] We use the median rather the mean of the conditional second moments as a proxy for the average non-diversifiable risk in each period because it is less affected by outliers.

[30] For the sake of brevity the descriptive statistics for the risk premiums are not shown, but they are available from the authors on request.

(2008)) without considering the global financial crisis. One of the reasons of the higher risk premium obtained in this paper may be due to the recent years of financial crisis.

3.4. Influence of financial crisis on the risk premium

To check the influence of the more recent financial crisis, we analyze both the risk aversion coefficient and the non-diversifiable risk distinguishing the period from August 2007 to 2009. We analyze this possibility because a structural break is detected around this period. We use an ICSS algorithm (Sansó et al.(2004)) to detect potential structural breaks that may affect the trade-off between return and risk and do we obtain a common sudden change around the observation 1020 (20 July 2007) for all series considered[31].

We introduce a dummy variable (D_t) into the mean equation that takes a value of 0 for periods prior to August 2007 and a value of 1 for the periods corresponding to August 2007–2009, following (15):

$$r_t = c + \lambda_1 h_t + \lambda_2 h_t D_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, h_t) \qquad (15)$$

The conditional second moments are obtained using the three different specifications presented in this paper.

Table 5 shows the risk aversion parameters and the non-diversifiable risk distinguishing the two periods presented above (1988–2009 and August 2007–2009). Panel A shows the estimated risk-aversion parameters for the mean equation using the different symmetric approaches distinguishing the period of the global financial crisis 2007–2009 for the different markets considered. The robust *t*-stats are presented in parenthesis. Panel B shows the median of the estimated variance series using each methodology (in the RS–GARCH model we present the estimated variance for the two states) for the two periods considered as a proxy for the non-diversifiable risk in each market.

[INSERT TABLE 5]

Panel A shows non-significant estimates for the risk–return trade-off in GARCH and MIDAS specifications in all cases considered. Even after controlling for the crisis period, these methodologies cannot reflect a significant relationship in the risk aversion parameter. However, with the RS–GARCH framework we can obtain a significant positive relationship in the state that governs the process in all markets. This significant trade-off is also observed in the secondary volatility states in markets such as Spain and the United Kingdom when we control for the financial crisis period.

Moreover, another interesting result is obtained in the RS–GARCH case. In markets such as Spain and the United Kingdom, the parameter λ_2 is negative and significant for the high volatility regimes (which dominate[32] the period October 2007–2009). This indicates a reduction of investor risk aversion in this period. The results show two

[31] The exact period for the structural change vary slightly among countries, but is close to the date mentioned above.

[32] See Figures 1 and 2. In this period the probability of being in a high volatility state is higher than that of being in a low volatility state.

patterns in the risk price evolution during the financial crisis corresponding to France and Europe in one side and Spain and United Kingdom in the other side. These differences among countries coincides to the results obtained in Laporta et al. (1999) and Schmeling (2009) that found significant differences in the returns patterns of these countries (mainly France and Europe versus Spain and the United Kingdom) due to idiosyncratic cultural parameters (such as the investor sentiment) or due to different ownership structure of the firms in each country. The intuition of the result obtained in Spain and United Kingdom about a decrease of the risk price in the high volatility state is not easy to see and could be due to a wide range of factors which are difficult to validate empirically (risk aversion level, investor behaviour). A possible explanation of this result may be the more averse investors tend to leave the market letting the riskier investors to establish a lower price per unit of risk in Spain and the UK. In contrast, in market such as France and Europe λ_2 is positive and significant for low volatility states during the crisis period. In these markets, the more risk adverse investors do not leave the market and continue to work in a context of market jitters, demanding a higher risk price in accordance with their conservative preferences.

Panel B shows the median of estimations for the non-diversifiable risk in each model. All models capture a high degree of risk during the 2007-2009 period caused by the financial crisis occurred. Among the linear models (GARCH and MIDAS), the MIDAS approach often leads to higher estimation of risk during this period. The results for the RS-GARCH also reflect an increase in estimated volatility for both states (low and high volatility) during this period. Therefore, despite the differences in the risk price observed in each market the common increase of non-diversifiable risk leads to the higher risk premium obtained in figure 3.

4. Conclusion

This paper analyzes the risk–return relationship for different European stock indexes considering the influence of the most recent financial crisis (2007–2009). We demonstrate that, even ignoring the hedge component, a significant risk–return trade-off is obtained when non-linear dynamics for conditional volatility (Regime Switching–GARCH) is considered. Only when we consider this particular dynamic for the risk-return trade-off do we obtain the results suggested in the theoretical model for the state that governs the volatility process. Linear specifications (GARCH, MIDAS) lead to non-significant estimations for the risk–return trade-off in all markets considered. The omission of the hedge component does not bias the significance of the relationship if the second moments are estimated adequately. These results (robust across all markets) support the evidence that the lack of significance in previous studies may be due to strong linear assumptions in the modelling of the risk-return trade-off. Furthermore, after controlling for the period of financial crisis, we increase the favourable evidence for the RS–GARCH specification in some markets, obtaining a significant trade-off not only in the dominant state of volatility process but also in the secondary state.

The risk premium estimates for Europe are generally higher than that obtained in previous studies for US data, due mainly to the period of financial instability generated by the global crisis of 2007–2009. We obtain an average risk premium between 4% and 8%, depending on the market and the methodology used. Although the risk prices show different patterns depending on the market considered, there is a common and extremely high non-diversifiable risk observed in all European markets during the recent financial crisis period. This is the main cause for the rise of the market risk premium demanded by investors during the financial crisis period.

The differences between risk prices across countries show different patterns in investor behaviour in pricing the risk during the financial crisis (2007–2009). In Spain and the United Kingdom, the results present a lower risk price for high volatility states, whereas in France and Europe (DJEurostoxx), a rise in these risk prices for low volatility states is observed. The profile of the investors who still trade in the market in times of market jitters and the idiosyncratic cultural parameters in each country considered may be the reasons of this differential behaviour in the risk price among countries.

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TABLE 1. Estimated parameters for symmetric models in Europe

Panel A. GARCH estimates						
	c	λ	ω	α	β	LL
Coefficient *(t-stat)*	0.0624 (0.0830)	0.0160 (0.0127)	0.1022 (2.1290)	0.1335 (4.4322)	0.8644 (41.1728)	-2583.56

Panel B. RS-GARCH estimates							
	$c_{s_t=k}$	$\lambda_{s_t=k}$	$\omega_{s_t=k}$	$\alpha_{s_t=k}$	$\beta_{s_t=k}$	p	q
$s_t = 1$ Coefficient *(t-stat)*	0.1320 (1.6460)	0.0348 (2.4151)	0.1341 (3.3743)	0.0567 (1.3806)	0.8822 (25.6158)	0.9752 (115.75)	0.5838 (6.0001)
$s_t = 2$ Coefficient *(t-stat)*	-4.7834 (-4.8147)	0.0243 (0.5992)	0.1341 (3.3743)	0.3041 (3.4892)	0.6842 (7.6367)	LL: -2542.32	

Panel C. MIDAS estimates							
	c	λ	k_1	***% weights days 1-5***	***% weights days 10-30***	***% weights days >30***	LL
Coefficient *(t-stat)*	0.0007 (1.0042)	-0.0357 (-0.2688)	0.0016 (5.3599)	25.64%	44.51%	34.11%	-3805.79

This table shows the estimated parameters for the different models presented in the paper (t-stats in parenthesis).

TABLE 2. Estimated parameters for asymmetric models

Panel A. GARCH		***Panel B. RS-GARCH***			***Panel C. MIDAS***	
c	λ_1	State	c	λ_1	c	λ_1
0.0695 (0.8781)	-0.0062 -(0.5149)	$s_t = 1$	-0.7088 (-0.9468)	0.3760 (1.9765)	0.0907 (2.8113)	-0.0280 -(0.9462)
		$s_t = 2$	-0.1973 (-1.4721)	0.0139 (0.6412)		

This table shows the estimated parameters for the mean equation using the different asymmetric approaches presented above (robust t-stats in parenthesis).

TABLE 3. Estimated parameters for symmetric models in different European countries

<table>
<tr><th colspan="7">Panel 1. FRANCE</th></tr>
<tr><th colspan="2">Panel 1.A. GARCH</th><th colspan="3">Panel 1.B. RS-GARCH</th><th colspan="2">Panel 1. C. MIDAS</th></tr>
<tr><td>c</td><td>λ_1</td><td>State</td><td>c</td><td>λ_1</td><td>c</td><td>λ_1</td></tr>
<tr><td rowspan="2">-0.0123
(-0.0986)</td><td rowspan="2">0.0209
(1.2882)</td><td>$s_t = 1$</td><td>-1.0316
(-1.8894)</td><td>0.3496
(2.3583)</td><td rowspan="2">0.0004
(0.4619)</td><td rowspan="2">0.0606
(0.0041)</td></tr>
<tr><td>$s_t = 2$</td><td>-0.6973
(-1.9420)</td><td>0.0331
(1.1219)</td></tr>
<tr><th colspan="7">Panel 2. UNITED KINGDOM</th></tr>
<tr><th colspan="2">Panel 2.A. GARCH</th><th colspan="3">Panel 2.B. RS-GARCH</th><th colspan="2">Panel 2. C. MIDAS</th></tr>
<tr><td>c</td><td>λ_1</td><td>State</td><td>c</td><td>λ_1</td><td>c</td><td>λ_1</td></tr>
<tr><td rowspan="2">-0.0019
(-0.0207)</td><td rowspan="2">0.0181
(0.9949)</td><td>$s_t = 1$</td><td>-0.4911
(-1.0693)</td><td>0.3274
(1.435)</td><td rowspan="2">0.0006
(0.8492)</td><td rowspan="2">-0.0964
(-0.5675)</td></tr>
<tr><td>$s_t = 2$</td><td>-0.8196
(-3.1993)</td><td>0.06544
(2.2581)</td></tr>
<tr><th colspan="7">Panel 3. SPAIN</th></tr>
<tr><th colspan="2">Panel 3.A. GARCH</th><th colspan="3">Panel 3.B. RS-GARCH</th><th colspan="2">Panel 3. C. MIDAS</th></tr>
<tr><td>c</td><td>λ_1</td><td>State</td><td>c</td><td>λ_1</td><td>c</td><td>λ_1</td></tr>
<tr><td rowspan="2">0.0055
(0.0508)</td><td rowspan="2">0.0178
(1.1498)</td><td>$s_t = 1$</td><td>0.2993
(1.4880)</td><td>0.0352
(2.0720)</td><td rowspan="2">-0.0019
(-0.0526)</td><td rowspan="2">0.0250
(0.8026)</td></tr>
<tr><td>$s_t = 2$</td><td>-1.8992
(-4.3101)</td><td>0.0149
(0.2471)</td></tr>
</table>

This table shows the estimated parameters for the mean equation using the different symmetric approaches for France, United Kingdom and Spain main stock indexes

TABLE 4. Average risk premium for Europe

Panel A.- Risk premium with GARCH models			
Europe	***France***	***United Kingdom***	***Spain***
4.3931 %	7.5033 %	4.0352 %	5.9531 %
Panel B.- Risk premium with RS-GARCH models			
Europe	***France***	***United Kingdom***	***Spain***
7.3451 %	9.4471 %	6.6492 %	8.5273 %
Panel C.- Risk premium with MIDAS models			
Europe	***France***	***United Kingdom***	***Spain***
6.4314 %	8.8970%	5.3786 %	7.9675 %

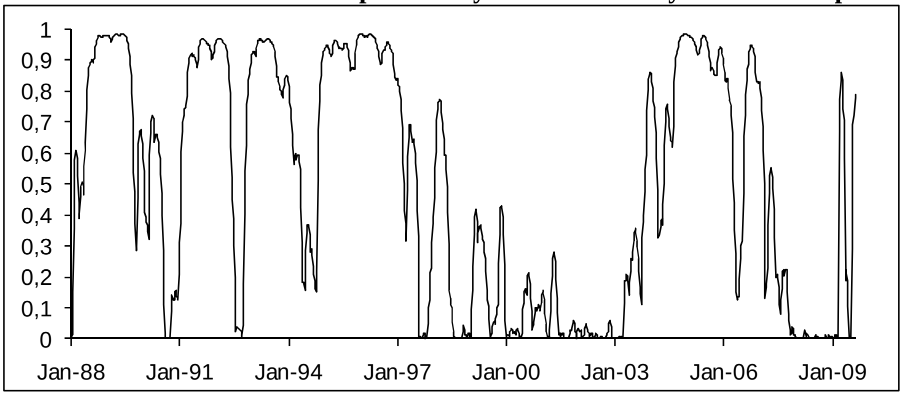
This table shows the average risk premium estimated for 4 stock indexes during the sample period.

Table 5. Influence of the global financial crisis on the risk premiums

Panel A.- Differences in risk aversion parameters: Full sample (1988-2009) vs crisis period (October 2007-2009)																
Risk price	*1.- EUROPE*				*2.- FRANCE*				*3.- UNITED KINGDOM*				*4.- SPAIN*			
	GARCH	**RS-GARCH**		**MIDAS**	**GARCH**	**RS-GARCH**		**MIDAS**	**GARCH**	**RS-GARCH**		**MIDAS**	**GARCH**	**RS-GARCH**		**MIDAS**
λ_1	0.0177 (0.9304)	*k=1*	0.5143 (1.9881)	0.0370 (0.3469)	0.0313 (1.4047)	*k=1*	0.2748 (2.1804)	0.0703 (0.5895)	0.0255 (0.8488)	*k=1*	0.3920 (2.3189)	0.0140 (0.2841)	0.0225 (1.0967)	*k=1*	0.3316 (2.6917)	0.0652 (1.7752)
		k=2	0.0386 (1.1661)			*k=2*	0.0436 (1.1369)			*k=2*	0.1465 (3.1886)			*k=2*	0.1214 (2.6443)	
λ_2	-0.0037 (-0.1233)	*k=1*	0.4437 (3.1267)	-0.1029 (-1.6911)	-0.0208 (-0.8396)	*k=1*	0.2599 (2.8980)	-0.0961 (-1.326)	-0.0106 (-0.316)	*k=1*	0.5517 (1.9910)	-0.0799 (-1.611)	-0.0101 -(0.376)	*k=1*	0.1352 (1.7898)	-0.1144 -(1.067)
		k=2	-0.0476 (-1.4278)			*k=2*	-0.0503 (-1.467)			*k=2*	-0.0883 (-2.2745)			*k=2*	-0.0656 -(2.284)	
Panel B.- Non-diversifiable risk: Full sample (1988-2009) vs crisis period (2007-2009)																
Non-diversif. risk	**GARCH**	**RS-GARCH**		**MIDAS**	**GARCH**	**RS-GARCH**		**MIDAS**	**GARCH**	**RS-GARCH**		**MIDAS**	**GARCH**	**RS-GARCH**		**MIDAS**
h_t [88-09]	5.9300	*k=1*	2.5836	3.7822	6.4040	*k=1*	3.9635	5.9506	3.9503	*k=1*	2.3380	3.2719	5.9300	*k=1*	3.9052	5.5248
		k=2	3.6872			*k=2*	7.8078			*k=2*	4.5438			*k=2*	4.0076	
h_t [07-09]	8.2813	*k=1*	6.6148	9.1857	8.6360	*k=1*	4.9657	16.1326	6.1365	*k=1*	3.1705	12.7150	8.2813	*k=1*	6.6647	10.3975
		k=2	8.6583			*k=2*	10.6774			*k=2*	7.1754			*k=2*	8.9858	

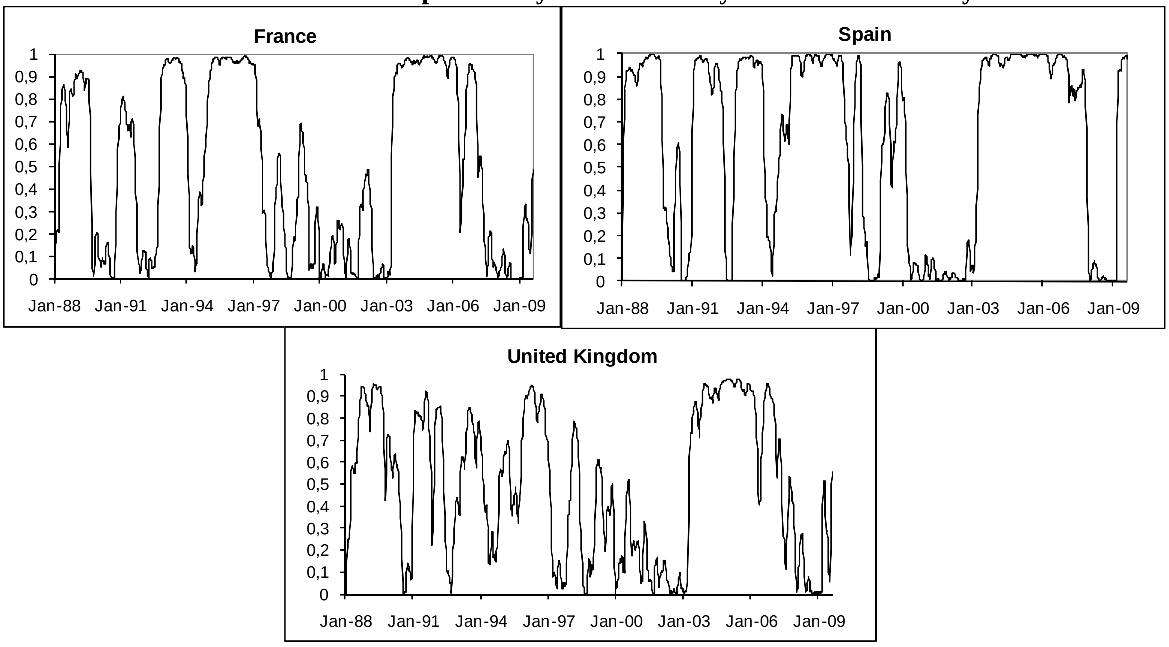
Panel A shows the risk price for the full period considered and for the financial crisis period. Panel B shows the average non-diversifiable risk for these two periods.

FIGURE 1.- Smooth probability for low volatility state in Europe



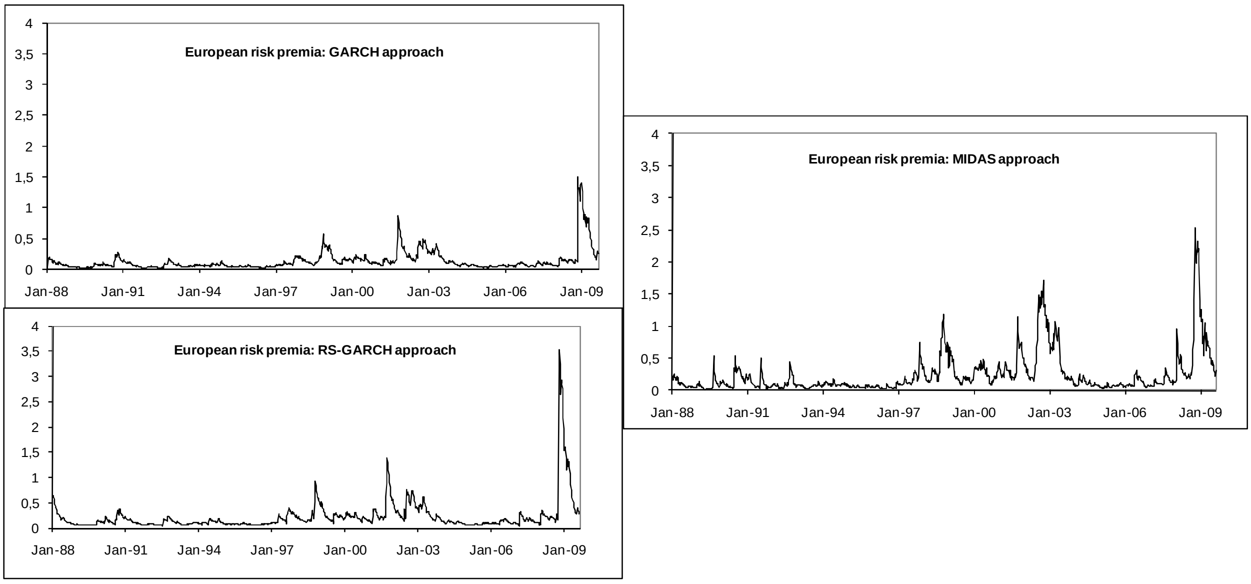
This figure represents the probability of being in a low volatility state in Europe

FIGURE 2.- Smoothed probability of low volatility state in each country



These figures represent the probability of being in a low variance state in France, UK and Spain.

FIGURE 3.- Risk premium evolution in Europe



These figures show the risk premium Evolution in Europe for GARCH, RS-GARCH and MIDAS models

CHAPTER 2:

THE RISK–RETURN TRADE-OFF IN EMERGING MARKETS

Abstract

This paper studies the risk-return tradeoff in some of the main emerging stock markets in the world. Although previous studies on emerging markets were not able to show a positive and significant tradeoff, favorable evidence can be obtained if a non-linear framework between return and risk is considered. Using 15 years of weekly data observations for 25 Emerging Markets MSCI index (5 Latin American, 9 Asian, 5 Eastern European, 3 Africans and 3 aggregate index for Asia, Eastern Europe and Latin America) in a Regime Switching-GARCH framework, favorable evidence is obtained for most of the emerging markets during low volatility periods, but not for periods of financial turmoil or using the traditional linear GARCH-M approach.

1.- Introduction

The relationship between return and risk has motivated lots of research in both the theoretical and the empirical field for many years. Many of the asset pricing models are based on this fundamental financial relationship and a good comprehension of the dynamics of return and risk is essential to understand these models. One of the most cited theoretical works in the financial literature analyzing the relationship between return and risk is Merton's (1973) intertemporal capital asset pricing model (ICAPM). Merton shows a linear relationship between the expected return on a wealth portfolio and its conditional variance and its conditional covariance with the investment opportunity set:

$$E_t\left(R_{W,t+1}\right)=\left[\frac{-J_{WW}W}{J_W}\right]\sigma^2_{M,t}+\left[\frac{-J_{WB}}{J_W}\right]\sigma_{MB,t} \qquad (1)$$

where $J(\cdot)$ is the utility function (subscripts represent partial derivatives), $W(\cdot)$ is the wealth function, $B(\cdot)$ is a variable that describes the state of investment opportunities in the economy, $E_t\left(R_{W,t+1}\right)$ is the expected excess return on aggregate wealth, σ^2_M and σ_{MB} are the conditional variance and the conditional covariance with the investment opportunity set and $\left[\frac{J_{WB}}{J_W}\right]$, $\left[\frac{J_{WW}W}{J_W}\right]$ could be viewed as the risk prices of the sources of risk.

Despite the important role of this trade-off in the financial literature, there is no clear consensus about its empirical evidence. In a theoretical framework, all the parameters (the risk prices in brackets) and the variables (the sources of risk) are allowed to be time varying (Campbell and Cochrane (1999), Whitelaw (2000)). However, to make this model empirically tractable one must make several assumptions; the most common is that of constant risk prices (Goyal and Santa-Clara (2003), Bali et al. (2005)). Another common assumption made in the empirical analysis of the risk–return tradeoff is that of a set of investment opportunities constant over time, leaving the market risk as the only source of risk (Baillie and De Gennaro (1990), Glosten et al. (1993)). Finally, the empirical model is established in a discrete time economy instead of the continuous time economy used in the equilibrium model of the theoretical approach. Many empirical papers studying the risk-return use one or more of the assumptions explained above.

In the studies focused in the emerging markets, the most common empirical framework is the GARCH-M approach developed by Engle et al. (1987). De Santis and Imrohoroglu (1997) find some weak evidence[33] for a positive risk–return trade-off in Latin American stock markets, but no evidence in those of Asia using weekly series from December 1988 to May 1996 in a GARCH(1,1)-M framework. Karmakar (2007) estimates an EGARCH model for Indian stock market data between July 1990 and December 2004, finding no relationship between return and risk. Chiang and Doong (2001) estimate a TAR-GARCH(1,1)-M model using data from Hong Kong, South Korea, Malaysia, the Philippines, Singapore, Taiwan, and Thailand. They find a significant positive relationship in daily data, but the impact of volatility (or risk) on market returns is weak in weekly data and insignificant in monthly data. Shin (2005) estimates both parametric and semiparametric GARCH-M models using weekly data from January 1989 to May 2003 to investigate the risk–return trade-off in emerging Latin American, Asian, and European stock markets. The results show a positive but insignificant tradeoff in most cases.

However, there are several important alternatives to the usual GARCH-M methodology in the financial literature. Ghysels et al. (2005) propose an alternative empirical methodology to counteract the disadvantages of the GARCH-M estimations, using different data frequencies to estimate the mean (with lower data frequency) and the variance (with higher data frequency) equations. Ludvigson and Ng (2007) use a factor approach to summarize a large amount of economic information in their risk–return tradeoff analysis. Bali (2008) proposes an alternative approach considering not only the time series dimension of the portfolio market but also the cross-sectional dimension that allows the consideration of the whole market. Whitelaw (1994) uses an instrumental variables specification for the conditional second moments. Harrinson and Zhang (1999) use nonparametric techniques in their study instead of the parametric approaches used above. Whitelaw (2000) and Mayfield (2004) employ methodologies whereby states of the world are essentially defined by volatility regimes.

Among the alternative methodologies to the GARCH-M framework existing in the literature, in this paper is considered the RS-GARCH[34] approach following the papers of Whitelaw (2000) and Mayfield (2004). This methodology is based on an equilibrium framework developed in the paper of Whitelaw (2000). This theoretical framework is slightly different from Merton's approach because a complex, non-linear, and time-varying relationship between expected return and volatility is obtained.

As remarked above, the evidence of a risk–return tradeoff in emerging markets using the GARCH-M approach is poor. In a recent paper, Lundblad (2007) shows that the typically insignificant relationship between the market risk premium and its expected

[33] These authors find essentially no evidence of a relationship between expected return and country-specific volatility, which is our main point in this paper; but when they generalize the model assuming regional or global international integration, they find support for a reward–risk relationship in Latin American countries.

[34] The main reason for this choice is that this framework introduces non-linearities in the analysis of the risk–return trade-off against the linear relationship of the GARCH-M framework.

volatility may be because of a statistical artifact[35] of the GARCH-M framework. A large data span is required in this approach to find successfully a positive risk-return tradeoff, showing in the Monte-Carlo simulation that even 100 years of data constitute a small sample from which one is forced to make inferences, obtaining sometimes disappointing results. To avoid this limitation of analyzing the risk–return tradeoff in a shorter span, it is proposed an alternative methodology which shows favorable evidence in most emerging markets. It is showed that for shorter span empirical analysis, the relationship between expected return and volatility follows non-linear rather than linear patterns as suggested the GARCH-M framework. The RS-GARCH approach proposed in this study let obtain favorable evidence for a positive and significant risk–return tradeoff.

This study examines the relationship between risk and expected return in several emerging markets, using Latin American, Asian, Eastern European and African countries. Despite the multitude of literature focused on developed markets, there has been insufficient attention on emerging markets. The main contributions of this paper are the following. Firstly, an alternative empirical methodology through a Regime Switching (RS) model is considered against most of the previous studies that use a GARCH-M framework. The weak evidence for a risk–return tradeoff in emerging markets in previous studies could be due to a potential misspecification of the empirical model. The main results show that a specification of a non-linear relationship between return and risk in the short-term is more appealing than the common assumption of a linear risk–return trade-off. Non-linear specifications also allow distinguishing between the patterns followed by this relationship between low and high volatility states. This point is especially interesting in the current period, when the global financial crisis that started in October 2007 still questioning most of the classic theoretical models. Furthermore, differences in risk aversion levels and significance during high and low volatility periods are also detected in these emerging markets. Using this methodology, a positive and significant risk–return tradeoff for the most recent data in most of the emerging markets is obtained. Secondly, the study also shows that for shorter time span strong linear assumption in the risk-return relationship may lead to misleading results. Thirdly, the risk-free rate for each country is considered in contrast to previous studies (De Santis and Imrohoroglu (1997), Shin (2005)). Finally, it is showed that the risk-return trade-off is essentially observed in low volatility periods where stock markets behave according the economic intuition; however, in high volatility periods this basic relationship between return and risk is not observed.

The paper is organized as follows. Section 2 provides the data. Section 3 develops the empirical framework used in the paper. Section 4 shows the empirical results. Section 5 provides a battery of robustness tests and section 6 concludes.

[35] Small sample inference is plagued by the fact that conditional volatility has almost no explanatory power for realized return.

2.- Data description

This empirical study uses weekly observations for five of the main stock markets in Latin America: Argentina, Brazil, Chile, Mexico, and Peru, nine Asian markets such as China, Indonesia, Malaysia, Thailand, India, Korea, Philippines and Taiwan, five Easter European Countries as Czech Republic, Hungary, Poland, Russian and Turkey and finally three African emerging markets: Morocco, Egypt and South Africa . Ii is also used an aggregate index for Asia, Eastern Europe and Latin America emerging markets[36]. The proxy used for the market portfolio is the Emerging Markets (EM) Morgan Stanly Capital International (MSCI) index computed in US dollars for each country considered. This market portfolio presents two main advantages: first, this data is more reliable than those of local markets given the well-documented exchange rate and inflation problems in these countries; second, allows the comparison between countries because all markets are considered in the same currency.

For each country, we considered weekly data from January 1995 to December 2010 for a total of 835 observations. The frequency and length of the time series allow the comparison of the conclusions with previous studies analyzing the risk–return trade-off in emerging markets such as De Santis and Imrohoroglu (1997) and Shin (2005). Against the works cited above, the risk-free rate is also considered to compute the excess market returns. The monthly money market rate in each country suitably compounded at a weekly frequency[37] is used as a proxy for the risk-free rate. Thomson Datastream is used to obtain the data about the MSCI indexes and International Financial Statistics for the data corresponding to the risk-free rate. After having computed logarithmic returns[38] for the market portfolio and having obtained the risk-free rate proxies, the excess market return in each market is obtained as the difference between the two of them.

[INSERT TABLE 1]

Table 1 contains summary statistics for the excess market returns in each country. All excess market return series exhibit non-normal distributions with strong evidence for skewness and kurtosis. This result suggests fat tails in the unconditional distributions. Moreover, the series also show conditional heteroskedasticity problems (autocorrelation in squared market excess returns). GARCH models fit properly to the data with these patterns (fat tails and conditional heteroskedasticity). There is also a common high value of the skewness statistic for all markets.

[36] The EM MSCI aggregate index for African countries only contains data since 2003, so I decided not to include it to avoid misleading results due to the difference in the length of the sample.

[37] This approach is used in Leon et al. (2007) to avoid the limitations in the availability of the risk-free rate at higher frequencies than monthly.

[38] To facilitate the convergence of the models I consider the logarithmic returns multiplied by 100.

3.- Empirical specifications

This section presents and discusses the empirical models proposed in this study to analyze the risk–return trade-off. Assuming GARCH dynamics for the conditional second moments, I built two models considering linear and non-linear relationships between expected return and conditional variance.

3.1.- GARCH-M framework

The empirical analysis relating to expected return and conditional volatility is traditionally validated using a GARCH-M methodology. Considering the theoretical framework shown above and the assumptions usually established in the previous literature[39] lead to the following model:

$$r_t = c + \lambda h_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, h_t) \qquad (2)$$

$$\varepsilon_t = h_t z_t \qquad z_t \sim N(0,1) \qquad (3)$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \qquad (4)$$ where $\hat{\alpha} + \hat{\beta} < 1$ guarantees the stationarity of the process.

In this model, r_t is the excess market return, h_t is the conditional variance, and ε_t represents the innovations, which are assumed to follow a normal distribution. This first model is estimated using the Quasi Maximum-Likelihood (QML) function of Bollerslev–Wooldridge (1992), which allows us to obtain robust estimates of standard errors.

$$L(\theta) = \sum_{t=1}^{T} \ln\left[f\left(r_t, \Omega_t; \theta\right)\right] \qquad where \quad f\left(r_t, \Omega_t; \theta\right) = \left(2\pi h_t\right)^{-\frac{1}{2}} e^{-\frac{(\varepsilon_t)^2}{2h_t}} \qquad (5)$$

In this model, the variance appears in the mean equation as a regressor and its parameter can be viewed as the market risk price or the risk aversion coefficient of a representative investor. Therefore, this parameter reflects the presence or lack of a risk–return trade-off and the sign of this relationship.

In this empirical model, the relationship between market risk premium and conditional variance is linear as suggested by Merton's model. However, several previous studies using this methodology fail to obtain favorable empirical evidence (French et. al (1987), Baillie and De Gennaro (1990)). It is showed in the next subsection an alternative empirical specification to avoid some of the limitations of the GARCH-M methodology.

3.2.- RS-GARCH framework

The model explained above proposes a linear relationship between return and risk. In this section, I show an empirical model that allows us to introduce non-linearities into this relationship. This specification could be viewed as the empirical validation of the theoretical equilibrium developed in Whitelaw (2000). Whitelaw (2000) concludes that empirical models imposing a strong, often linear relationship between expected returns

[39] These assumptions often include (De Santis and Imrohoroglu (1997), Shin (2005), Karkamar (2007)) constant risk prices, time-varying risk and a constant set of investment opportunities.

and volatility (such as GARCH-M models) need to be employed with caution. Given the importance of regime shifts to the results, an RS-GARCH specification is proposed, based on the model originally proposed by Hamilton (1989) and Hamilton and Susmel (1994) that allows us to distinguish between different volatility states governed by a hidden state variable that follows a Markov process.

In this model, the mean equation is not exactly as shown in Equation 2 because it is state-dependent:

$$r_{t,s_t} = c_{s_t} + \lambda_{s_t} h_{t,s_t} + \varepsilon_{t,s_t} \qquad \varepsilon_{t,s_t} \sim N(0, h_{t,s_t}) \qquad (6)$$

where r_{t,s_t}, h_{t,s_t}, and ε_{t,s_t} are the state-dependent returns, variances, and innovations, and s_t = 1 (low volatility state) or 2 (high volatility state).

The state-dependent innovations follow a normal distribution, with two possible variances depending on the state of the process. The state-dependent variances are modeled as in Equation 4 allowing different parameters, depending on the state[40]:

$$\varepsilon_{t,s_t} = h_{t,s_t} z_t \qquad z_t \sim N(0,1) \qquad (7)$$

$$h_{t,s_t} = \omega + \alpha_{s_t} \varepsilon_{t-1}^2 + \beta_{s_t} h_{t-1} \qquad (8)$$

The shifts from one state to another are governed by a hidden state variable following a Markov process with a probability transition matrix:

$$\hat{P} = \begin{pmatrix} \Pr\left(s_t = 1 \middle| s_{t-1} = 1\right) = p & \Pr\left(s_t = 1 \middle| s_{t-1} = 2\right) = (1-q) \\ \Pr\left(s_t = 2 \middle| s_{t-1} = 1\right) = (1-p) & \Pr\left(s_t = 2 \middle| s_{t-1} = 2\right) = q \end{pmatrix} \qquad (9)$$

Because of this state dependence, the model is econometrically intractable[41]. Therefore, we must obtain state-independent estimates of variances and innovations; one simple method consists on averaging out each state-dependent error/variance according to the *ex ante* probability[42] of being in each state (Dueker (1997)):

$$h_t = P\left(s_t = 1 \middle| \Omega_{t-1}; \theta\right) h_{t,s_t=1} + P\left(s_t = 2 \middle| \Omega_{t-1}; \theta\right) h_{t,s_t=2} \qquad (10)$$

$$\varepsilon_t = P\left(s_t = 1 \middle| \Omega_t; \theta\right) \varepsilon_{t,s_t=1} + P\left(s_t = 2 \middle| \Omega_t; \theta\right) \varepsilon_{t,s_t=2} \qquad (11)$$

where h_t and ε_t are the state-independent variances and disturbances and

$$P\left(s_t = 1 \middle| \Omega_{t-1}; \theta\right) = p * P\left(s_{t-1} = 1 \middle| \Omega_{t-1}; \theta\right) + (1-q) P\left(s_{t-1} = 2 \middle| \Omega_{t-1}; \theta\right) \qquad (12)$$

[40] Following Capiello and Fearnley (2000), to facilitate convergence, the constant variance term is not allowed to switch between regimes.
[41] See e.g. Gray (1996) or Dueker (1997).
[42] Following Hamilton (1989), the *ex ante* probability is defined as $P\left(s_t = k \middle| \Omega_{t-1}; \theta\right)$ for k=1,2 i.e. the probability of being in the k[th] state, given the information up to t-1.

and

$$P\left(s_t = 2 \middle| \Omega_{t-1};\theta\right) = 1 - P\left(s_t = 1 \middle| \Omega_{t-1};\theta\right) \qquad (13)$$

are the *ex ante* probabilities, where

$$P\left(s_t = k \middle| \Omega_t;\theta\right) = \frac{P\left(s_t = k \middle| \Omega_{t-1};\theta\right) f\left(r_t \middle| s_t = k, \Omega_t;\theta\right)}{\sum_{k=1}^{2} P\left(s_t = k \middle| \Omega_{t-1};\theta\right) f\left(r_t \middle| s_t = k, \Omega_t;\theta\right)} \qquad (14)$$

where k=1, 2 are the filtered probabilities.

This model is estimated maximizing the QML function of Bollerslev–Wooldridge (1992), weighted by the filtered probability of being in each state:

$$L(\theta) = \sum_{t=1}^{T} \ln\left[\sum_{k=1}^{2} P\left(s_t = k \middle| \Omega_t;\theta\right) f\left(r_t, \Omega_t;\theta\right)\right] \quad \text{where} \quad f\left(r_t \middle| s_t, \Omega_t;\theta\right) = \left(2\pi h_{t,s_t}\right)^{-\frac{1}{2}} e^{-\frac{\left(\varepsilon_{t,s_t}\right)^2}{2h_{t,s_t}}} \qquad (15)$$

<u>3.3.- Asymmetric specifications</u>

To robustness purposes it is also considered the well-known fact that a negative shock has a greater impact in volatility than a positive shock. In all the series analyzed there is a common high value of the skewness statistic. For this reason, it is worthy proposing the consideration of the 'leverage effect' in the empirical model because let us treat in a different way the impact of positive and negative shocks. To reflect this, the GJR specification of Glosten et. al (1993) is used in the variance equation in both linear and non-linear specifications. I just estimate the same models presented above but instead of using equation (4) and (8) I replace them by the following equations:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} + \delta\eta_{t-1}^2 \qquad (4')$$

$$h_{t,s_t} = \omega + \alpha_{s_t}\varepsilon_{t-1}^2 + \beta_{s_t} h_{t-1} + \delta_{s_t}\eta_{t-1}^2 \qquad (8')$$

Where δ is a new parameter to be estimated reflecting the impact of negative shocks and $\eta_t = \min\left(e_t, 0\right)$. The rest of parameters are the same defined above and I estimate the unknown parameters again maximizing the QML functions in (5) and (15).

4.- Empirical results

This section shows the main empirical results of the risk–return analysis in the emerging markets. I focus my attention on the relationship between expected market returns and conditional volatility rather than the well-known patterns and dynamics followed by volatility in these markets[43]. It is worthwhile noting the results of this relationship because it is the inconclusive point of the previous literature; the expected returns and volatility dynamics are similar in previous studies of emerging markets (Choudry (1996), De Santis and Imrohoroglu (1997), Shin (2005)). This study is directly comparable with previous studies because the choice of data (in terms of frequency[44] and sample size) is similar. Furthermore, the data selection also includes the recent period of the global financial crisis (from October 2007), which is not treated in any previous study for emerging markets.

The left side of Table 2 shows the estimated parameters for the mean equation[45] using the GARCH-M framework for the emerging markets considered. The parameter c represents the constant term (the intercept) and the parameter λ represents the risk aversion parameter; that is the risk–return relationship.

[INSERT TABLE 2]

The main conclusion of these results is that the GARCH-M framework fails to show favorable evidence of the risk–return trade-off in emerging markets. There is no clear evidence about either the sign or significance of the relationship using this approach. Brazil is the only country where a significant trade-off is obtained but is negative. Therefore, the influence of volatility on stock markets is not enough to be significant in the linear framework drawn here. This result is inconsistent with the theoretical model that it is based on. Following Merton's ICAPM, we expect a positive and significant risk–return tradeoff. However, some previous studies also obtained similar results using this framework for both developed and emerging markets (Baillie and Di Gennaro (1990), Glosten et al. (1993), Shin (2005)).

A potential reason for these results may be that in shorter periods the risk–return trade-off follows a non-linear relationship. The limitations imposing a linear relationship between return and risk are clearly observable in inconclusive previous studies. Whitelaw (2000) states the concerns about the importance of non-linear risk and develops a theoretical framework analyzing the relationship between return and risk in a two-regime economy, remarking the perils of linear models such as GARCH-M.

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[43] Previous papers (De Santis and Imrohoroglu (1997) and Shin (2005)) analyzing emerging (and Latin American markets) reach similar conclusions about the volatility dynamics. For almost all these countries, there is evidence of time-varying volatility, which exhibits clustering and predictability.

[44] The selection of the data frequency may be a concern. Most previous studies use weekly data in emerging markets. Even though there are slightly differences in the parameter estimations using different data frequencies, there is no particular reason that the conclusions in this study should be affected by the selection of data frequency. Some authors note this point in their studies (De Santis and Imrohoroglu (1997), Shin (2005), Lundblad (2007)).

[45] Estimations for variance equation are not presented to save space. Moreover, the results for the variance equation do not provide any relevant contribution about the risk–return trade-off. They only suggest the volatility dynamics (which is not the objective of the paper).

Right side of table 2 shows the estimations for the RS-GARCH model proposed. In this approach, there are two intercepts and two risk prices (aversion coefficients) corresponding to low and high volatility states. The introduction of regime switching in the empirical analysis lets us establish a non-linear relationship between expected return and conditional volatility as an alternative to the disappointing results obtained when we assume a linear relationship.

The main results for the RS-GARCH estimations show positive and significant estimations for the risk–return relationship in low volatility periods but the results turn non-significant in the high volatility state. With the sample used in this study, it is found favorable evidence for a positive and significant risk–return trade-off in most of the emerging markets. In some countries such as Peru, Philippines and Russia this evidence is very strong with significance even at 1% confidence level. In several countries as Argentina, Brazil, Mexico, Chile, Thailand, Egypt, Morocco, Poland, Turkey and the aggregate Asian index the trade-off is significant at 5% level. In some countries, the evidence is weaker just at 10% confidence level as China, Indonesia, India, Korea, South Africa, Hungary and the aggregated index for Latin America. Finally in some emerging markets I cannot find evidence of a risk-return trade-off even in the low volatility periods as in the cases of Malaysia and the aggregated European index. This positive evidence is essentially observed in low volatility states where the financial markets are stable. However, the results for the high volatility state reveal a lack of a trade-off in periods of market jitters. None of the parameters in this state is significant at any confidence level (except for Turkey which is significant negative at 5%). Therefore, what this evidence suggests is that a positive and significant risk-return trade-off is only observed during periods of financial stability but this fact is not observed in times of financial turmoil in the emerging stock markets.

Moreover, some interesting results deserve some attention as well. First, the risk aversion coefficients in state 1 (corresponding to low volatility states) are higher than those corresponding to state 2 (high volatility states). This result suggests that there is a lower risk aversion level in high volatility states. This finding is not consistent with the spirit of the theoretical models that suggest that higher volatility should be compensated with higher returns. However, some papers such as Mayfield (2004), Lettau and Ludvigson (2003), and Lundblad (2007) found the same evidence; in high volatility states, there is a decreasing level of risk aversion. One possible explanation could be the different risk aversion profiles for the investors in each state. During calm (low volatility) periods, more risk-averse investors are trading in the markets, but in high volatility periods only the less risk-averse investors remain in the market because they are the only investors interested in assuming such risk levels, decreasing the risk premium demanded during these periods. However, the specification presented here may be confounding expected returns with realized returns, particularly in the less common high volatility states (corresponding generally with recession periods) often associated with low or even negative markets returns (Lundblad, 2007).

The evidence obtained in this paper about a significant trade-off in calm periods but non-significant during high volatility situations may also be related to the findings in papers as Nyberg (2011) and Kim and Lee (2008). These authors find similar evidence in developed markets but establishing the state-dependence of the risk aversion on the business cycles instead of volatility regimes. In a certain way, they are different forms of introducing the non-linear relationship between return and risk, but very similar in the sense that many periods corresponding to recessions are associated with high volatility situation states and boom cycles often coincide with low volatility periods in stock markets. In our case, we also support the procyclical risk aversion observed in the paper of Kim and Lee (2008) since in low volatility states (boom periods) the investors show are stronger risk-aversion than during high volatility (recession) periods.

Another interesting result is related with the significance of the constant term. In many countries this parameter presents a significant value. Some authors (Leon et. al(2007)) relate this significance with structural market imperfections. This interpretation is totally plausible in the markets analyzed here which are in developing process and may present some of these imperfections. Moreover, due to the significance of this parameter, its omission could lead to misleading results because the model would be misspecified. However, I explain this issue in more detail in the next section.

Finally, note that the volatility persistence estimated with linear models is usually very high (around 0.9). However, considering two regimes we get a reduction of this persistence overall in the high volatility state where there is a greater impact of the shocks and the impact of these decay more quickly (Marcucci, 2005). Considering just one volatility process could be another of the reasons of the inconclusive results obtained with linear models.

[INSERT FIGURE 1]

Figure 1 presents the smooth probability of being in a low volatility state in each of the emerging markets analyzed. It is not possible to extract a common pattern among all these countries because each country follows its own idiosyncratic volatility process. However, it is worthy to note that in most cases low volatility states governs the volatility process and high volatility states are just present during the crisis periods in each specific country.

4.1.- Diagnosis tests

In this subsection, I perform some specification tests on the standardized residuals from the estimations. The objective is to detect potential misspecifications in our empirical model that could lead to wrong or spurious results. Table 3 shows the diagnosis tests using the standardized residuals for the aggregated Asian, European and Latin American countries case as a representation of all emerging markets[46].

[INSERT TABLE 3]

[46] The choice for these markets is purely arbitrary and is done in order to save space. The results for other markets are similar and are available upon request.

The first rows in Table 3 show summary statistics for the standardized residuals ($\in_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}}$), in levels and squares for both GARCH-M and RS-GARCH. The mean values for residuals in levels are around 0 and variance values are around 1. The degree of skewness and kurtosis is also reduced compared with the original series. This reduction is even higher in the RS-GARCH approach, suggesting a better fit for the fat tails in the unconditional distribution. Table 6 also shows the Ljung-Box autocorrelation test; the results show that there is no evidence for autocorrelation in standardized residuals for levels or squares. Finally, at the bottom of the table, there are two order moment tests (developed by Bollerslev and Wooldridge (1992)) to validate the consistency of the QML estimations for deviations from normality. These authors demonstrate that the estimations obtained for the QML estimations are consistent even in the case of deviations from normality if: $E_{t-1}\left(\hat{\in}_{i,t}\right) = 0$, $E_{t-1}\left(\hat{\in}^2_{i,t}\right) = 1$. The results support consistency in our estimation results despite the non-normality patterns of the original series. All the analysis performed for the standardized residuals show that the models proposed reflect the dynamics of both the market risk premium and the conditional second moments. Any sign or evidence of a potential model misspecification is found at a significant level.

5.- Robustness test

The results in the previous section show a significant relationship between expected returns and risk in almost all the emerging markets analyzed. In this section the empirical analysis both from a linear and non-linear point of view is repeated using different specifications proposed in the literature to model the mean and the variance equations[47]. More specifically, in the variance equation I consider the asymmetric response of volatility against shock of different sign (the 'leverage effect') and I propose a model omitting the constant term in the mean equation (Lanne and Saikkonen (2006), Guo and Neely (2008)).

[INSERT TABLE 4]

Table 4 shows the estimations for the original model with an asymmetric GJR specification in the variance equation[48]. In this case it is observed a significant risk-return trade-off of at least at 90% confidence level in 19 of our 24 index analyzed during the low volatility periods. The results for high volatility periods and for the GARCH-M framework are similar than the symmetric case. If anything, these results support the findings obtained above.

[INSERT TABLES 5 AND 6]

[47] All the estimations have been replicated assuming a t-student distribution for the innovation term and the results are very similar to those reported in the paper.

[48] The results are very similar to the symmetric case. For the sake of brevity I just describe bravely the main implications on the risk-return trade-off observed.

Table 5 and Table 6 represent the risk aversion coefficient in the case the constant term is omitted in the mean equation for the symmetric and asymmetric variance specification respectively. Lanne and Saikkonen (2006) have pointed out that in many empirical studies analyzing the risk-return trade-off the intercept is included in the model for the conditional mean in the GARCH-M model although, based on the ICAPM, it is not theoretically justified. They failed to find a positive risk-return tradeoff in the U.S stock returns when the intercept is included in the model. However, a positive and statistically significant GARCH-M estimate (using the notation employed in this paper) is obtained when the intercept is excluded. The results of Tables 5 and 6 do not support this evidence for emerging markets. Among the 24 indexes markets analyzed, using the linear framework without constant in only 5 (4 in the asymmetric case) of them it is found a positive and significant tradeoff between return and risk and in some cases this relationship is negative. The results for the non-linear cases show that a significant tradeoff is obtained in 21 (only 13 in the asymmetric case) for low volatility periods and essentially a negative and non-significant relationship is obtained during high volatility periods. But the evidence omitting the constant term in the mean equation are generally weaker than including it. So, in a linear framework one is more likely by imposing the restriction of no constant term in the return equation to find a positive risk-return relation but in the non-linear framework this fact is not observed and the omission of the constant could lead to weaker results. Anyway, as the true data generating process is not known, with the restricted models one could be estimating misspecified models and, therefore, is preferably including the constant term (Guo and Neely (2008)).

Suming up, the main result here is that favorable evidence of a positive and significant risk–return tradeoff with a time ‘span’ of approximately 15 years can be obtained for almost all the emerging countries considered, as it is suggested by the theoretical intuition. However, only in the case of (i) a proper relationship between return and risk (that is, non-linear rather than linear); and (ii) periods identified as low volatility states, the empirical evidence supports the theoretical models. The results shown in this study demonstrate the importance of non-linear risk and RS in the patterns followed by the dynamics and the trade-off between return and risk in emerging markets. Strong linear assumptions about the risk–return tradeoff in shorter ‘spans’ could be the reason for the weak evidence documented in the previous literature.

6.- Conclusions

This study provides a risk–return analysis for almost all the main stock markets known as Emerging Markets. It analyzes different countries in several world regions as Asia, Latin America, Eastern Europe and Africa. Using the standard GARCH-M framework (similar to previous studies in emerging markets), I cannot show favorable evidence about a significant risk–return trade-off. However, using a RS-GARCH approach to explore this trade-off I obtain a significant estimation for the risk aversion parameter with a relatively short time span (15 years of data). The results suggest that the RS-GARCH framework can identify a non-linear relationship between expected return and risk for 'shorter' time spans in contrast to the disappointing results of the GARCH-M framework. So, strong linear assumptions analyzing the risk–return relationship in emerging markets must be taken with caution.

The results also provide a relationship between volatility regimes and risk aversion level. The risk aversion level in emerging markets is higher in low volatility states and lower in high volatility states. The investor profile in each context may also have an influence on this lower risk aversion coefficient during high volatility periods. Generally, high volatility regimes correspond to periods of recession or low expansion in the country's economy, whereas low volatility regimes correspond with periods of economic expansion and let us link the findings in this paper with other works focused in developed markets that obtain similar results (Kim and Lee, 2008). This study also support the procyclical risk aversion of investors documented for developed markets.

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Table 1.- Summary statistics for weekly excess market returns

	Summary statistics for weekly excess market returns						
	Mean	*Variance*	*Skewness*	*Kurtosis*	*JB test*	*LB(6)*	*LB(6) squares*
Argentina	-0.0540	28.634	-0.7764	8.9830	1493.26***	28.755*	351.818***
Brazil	-0.6793	37.848	-0.8169	6.1116	482.73***	125.61***	529.395***
Peru	0.1962	18.566	-0.1714	7.4347	773.23***	16.309	430.190***
Mexico	-0.0125	20.939	-0.8160	9.7620	1891.19***	41.583***	219.274***
Chile	0.0370	11.634	-1.3262	16.4009	7293.65***	34.061**	186.583***
China	-0.0791	23.981	-0.2772	5.5502	236.97***	18.039	173.181***
Indonesia	-0.1887	49.361	-0.9430	18.2803	8247.16***	80.460***	448.258***
Malaysia	-0.0414	19.159	-1.0231	25.2975	17443.3***	97.788***	333.309***
Thailand	-0.1412	29.283	-0.0942	6.7539	491.50***	51.946***	582.739***
India	0.1616	16.743	-0.4341	5.3038	210.88***	40.135***	191.021***
Korea	-0.0223	33.480	-0.9590	14.4785	4711.96***	49.281***	224.261***
Philippines	-0.0664	18.211	-0.6377	7.7093	828.18***	28.445*	167.802***
Taiwan	-0.0053	15.345	-0.0327	4.8785	122.91***	19.068	86.061***
Egypt	0.1047	15.759	-0.5329	6.7209	521.20***	74.570***	266.139***
Morocco	0.1831	6.206	-0.4811	5.9916	343.59***	34.912**	235.534***
South Africa	-0.0667	16.434	-0.2080	7.6092	745.18***	22.392	519.180***
Hungary	0.0299	28.698	-1.1089	11.4632	2663.13***	46.434***	123.216***
Poland	-0.0882	24.959	-0.4728	5.9675	337.48***	27.847	216.849***
Turkey	-0.5001	55.551	-1.1520	16.6894	6704.63***	28.463*	92.242***
Czech Republic	0.1323	16.434	-0.6497	8.8403	1245.44***	31.395*	329.386***
Russia	0.0161	57.533	-0.1659	7.5360	719.68***	36.294**	455.892***
MSCI Asia	-0.0674	12.245	-0.5714	5.8076	319.70***	35.425***	309.663***
MSCI Europe	-0.0941	20.590	-0.4939	11.3763	2475.02***	52.336***	561.192***
MSCI Latin America	-0.0023	19.154	-0.8682	9.6571	1646.79***	47.227***	513.575***

This table shows the statistics for the sample used in the study (excess market returns (multiplied by 100) in each market). J-B test is the Jarque-Bera test for normality. LB (6) is the Ljung-Box autocorrelation test including six lags for the series in levels. L-B (6) squares is the Ljung-Box autocorrelation test including six lags for the series in squares.***, **, and * represent significance at 1%, 5%, and 10% levels.

Table. 2- Estimations for the MSCI index using the GARCH-M and RS-GARCH-methodology.

This table shows the estimations for the intercept, the risk aversion parameter and the shock persistence in the emerging markets considered using the symmetric variance specification. T-stats in parenthesis. ***,

	GARCH-M			***RS-GARCH-M***					
				State k=1			***State k=2***		
Paramete r *(t-stat)*	c	λ_1	Persist.	c	λ_1	Persist.	c	λ_1	Persist.
Argentina	**0.0033*** *(1.6648)*	**-0.0077** *(-1.4123)*	**0.9984**	**-0.6094** *(-0.5204)*	**0.0939**** *(2.0765)*	**0.9674**	**-0.8429** *(-0.4925)*	**-0.0058** *(-0.1990)*	**0.1631**
Brazil	**-0.0003** *(-0.1192)*	**-0.0160**** *(-2.2417)*	**0.9986**	**-3.8899*** *(-1.8796)*	**0.3283**** *(2.1170)*	**0.9718**	**-0.3735** *(-0.4452)*	**-0.0356** *(-1.1974)*	**0.2460**
Peru	**0.0034** *(1.0102)*	**-0.0134** *(-0.4942)*	**0.9394**	**-2.082***** *-(3.0990)*	**0.0930***** *(2.6489)*	**0.9920**	**1.2366***** *(4.2526)*	**-0.0287** *(-1.4156)*	**0.8906**
Mexico	**0.0048**** *(2.0823)*	**-0.0092** *(-0.6837)*	**0.9539**	**-1.332***** *(-2.9639)*	**0.0278**** *(2.0743)*	**0.9581**	**0.7536***** *(3.2841)*	**-0.0055** *(-0.2179)*	**0.8653**
Chile	**-0.0025** *(-1.1054)*	**0.0278** *(1.1145)*	**0.9582**	**-1.9582** *(-1.6070)*	**0.3749**** *(2.0506)*	**0.9574**	**-1.6822***** *(-2.9749)*	**0.0245** *(1.6416)*	**0.1275**
China	**0.0017** *(0.7048)*	**-0.3891** *(-0.3468)*	**0.9879**	**-1.1206** *(1.5205)*	**0.7943*** *(1.6949)*	**0.9834**	**-1.0071*** *(-1.8155)*	**0.0061** *(0.7329)*	**0.0595**
Indonesia	**0.0019** *(1.0531)*	**0.2078** *(0.3460)*	**0.9885**	**-1.6542** *(-1.5241)*	**0.9417*** *(1.6534)*	**0.9858**	**-1.0883*** *(-1.8517)*	**0.0069** *(0.8024)*	**0.0509**
Malaysia	**0.0013** *(1.2275*	**0.4308** *(0.4836)*	**0.9988**	**-0.2799** *(-1.3606)*	**0.0069** *(0.2958)*	**0.9806**	**0.6476** *(2.5412)*	**0.0043** *(0.4530)*	**0.9725**
Thailand	**0.0020** *(0.8707)*	**-0.3323** *(-0.3157)*	**0.9982**	**-2.2689**** *(-2.0499)*	**0.1119**** *(2.2787)*	**0.9865**	**-0.4497** *(-0.7871)*	**0.0535** *(1.4824)*	**0.8403**
India	**0.0022** *(0.9382)*	**0.3082** *(0.1999)*	**0.9737**	**0.2172** *(0.1700)*	**0.1188*** *(1.8061)*	**0.9281**	**-1.8606**** *(-2.4293)*	**0.0586** *(0.6636)*	**0.1971**
Korea	**0.0006** *(0.3642)*	**0.6988** *(0.9322)*	**0.9837**	**-0.8231**** *(-2.1287)*	**0.0244*** *(1.9266)*	**0.9789**	**0.7654***** *(2.9612)*	**0.0012** *(0.1305)*	**0.9492**
Philippines	**0.0004** *(0.1378)*	**0.6996** *(0.4227)*	**0.9665**	**-0.4497** *-(0.7871)*	**0.1119***** *(2.2787)*	**0.9723**	**-2.2689**** *(-2.0499)*	**0.0535** *(1.4824)*	**0.7873**
Taiwan	**0.0015** *(0.7407)*	**0.2053** *(0.1443)*	**0.9709**	**-1.5389**** *(-2.0643)*	**0.0508*** *(1.6628)*	**0.9629**	**0.5468** *(1.5661)*	**0.0370** *(0.6903)*	**0.3070**
Egypt	**-0.0013** *(-1.2003)*	**1.0308** *(1.2702)*	**0.9868**	**1.7453***** *(3.3960)*	**0.0519**** *(2.0974)*	**0.9759**	**-0.7881***** *(-4.6347)*	**0.0112** *(0.9432)*	**0.7383**
Morocco	**0.0010** *(0.7648)*	**1.3871** *(0.5496)*	**0.9370**	**-0.826***** *(-3.8264)*	**0.0998**** *(2.0660)*	**0.9385**	**0.8595***** *(2.9842)*	**-0.0470** *(-0.8947)*	**0.9206**
South Africa	**-0.0012** *(-0.5483)*	**1.5616** *(0.9941)*	**0.9630**	**-1.4903*** *(-1.9311)*	**0.0672*** *(1.8277)*	**0.9649**	**0.4619** *(1.6020)*	**0.0165** *(0.6672)*	**0.6798**
Hungary	**0.0022** *(0.7426)*	**0.1386** *(0.1255)*	**0.9481**	**-0.7771** *(-0.4731)*	**0.1231*** *(1.7172)*	**0.9818**	**-2.3796***** *-(2.7573)*	*0.0306* *(0.8941)*	**0.0360**
Poland	**0.0004** *(0.1365)*	**0.1476** *(0.1576)*	**0.9601**	**-2.850***** *(-2.6421)*	**0.0952**** *(2.0089)*	**0.9962**	**0.9420**** *(2.3069)*	**-0.0244** *(-1.1923)*	**0.8815**
Turkey	**0.0041** *(0.9713)*	**-1.3477** *(-1.5430)*	**0.9750**	**-12.154**** *(-2.4855)*	**0.5203**** *(2.3014)*	**0.9184**	**1.1348** *(1.4090)*	**-0.0267**** *(-2.4106)*	**0.2182**
Czech Republic	**0.0039*** *(1.7196*	**-0.7682** *(-0.4747)*	**0.9306**	**0.1893** *(0.3387)*	**0.0795*** *(1.7160)*	**0.9540**	**-1.4449*** *(-1.6520)*	**0.0245** *(0.9446)*	**0.1839**
Russia	**0.0045*** *(1.6775)*	**-0.3516** *(-0.5465)*	**0.9898**	**-8.714***** *(-11.014)*	**0.1307***** *(6.7283)*	**0.9585**	**0.7273**** *(2.0076)*	**0.0109** *(1.3975)*	**0.9023**
MSCI Asia	**0.0009** *(0.7896)*	**0.6225** *(0.5238)*	**0.9874**	**-1.3243** *(-1.6044)*	**0.5028**** *(2.0055)*	**0.9877**	**-0.1258** *(-0.1643)*	**-0.0195** *(-0.4128)*	**0.1757**
MSCI Europe	**0.0001** *(-0.0188)*	**-0.1216** *(-0.1098)*	**0.9728**	**-1.1167** *(-1.9689)*	**0.0428** *(1.2895)*	**0.9713**	**0.6415** *(2.3894)*	**-0.0198** *(-1.3140)*	**0.9326**
MSCI Latin America	**-0.0002** *(-0.0713)*	**0.5455** *(0.3832)*	**0.9480**	**-6.9891** *(-1.2647)*	**0.7638*** *(1.7149)*	**0.9274**	**-1.5876**** *(-2.0133)*	**0.0189** *(0.7707)*	**0.0626**

**, and * represent significance at 1%, 5%, and 10% levels. Persist. means the persistence of an unexpected shock in the market volatility and is computed as the sum of the parameters (α+β) in the variance equation.

Table 3.- Summary statistics for standardized residuals

Index	Model	Stand. resid	Mean	Variance	J-B test	L-B (6)	t-stat for H0:	t-stat for H1:
MSCI ASIA	GARCH	$\hat{\epsilon}_{m,t}$	-0.0052	0.9987	104.6***	53.17***	0.1072	-
		$\hat{\epsilon}^2_{m,t}$	0.9991	3.2501	63235***	17.991	-	0.9899
	RS-GARCH	$\hat{\epsilon}_{m,t}$	-0.0055	1.0328	76.487***	23.164	0.8704	-
		$\hat{\epsilon}^2_{m,t}$	1.0325	2.3578	14148***	6.5294	-	0.8031
MSCI EUROPE	GARCH	$\hat{\epsilon}_{m,t}$	-0.0021	1.0003	208.9***	37.35**	0.9483	-
		$\hat{\epsilon}^2_{m,t}$	0.9996	3.9287	85573***	13.8586	-	0.9959
	RS-GARCH	$\hat{\epsilon}_{m,t}$	-0.0086	1.0548	31.189***	23.911	0.6701	-
		$\hat{\epsilon}^2_{m,t}$	1.0566	3.1097	30146***	0.1861	-	0.5839
MSCI LATIN AMERICA	GARCH	$\hat{\epsilon}_{m,t}$	-0.0418	0.99375	231.4***	30.683*	0.2001	-
		$\hat{\epsilon}^2_{m,t}$	0.9994	3.9964	112873***	12.9796	-	0.9931
	RS-GARCH	$\hat{\epsilon}_{m,t}$	-0.0098	1.0895	35.26***	22.056	0.7829	-
		$\hat{\epsilon}^2_{m,t}$	1.0899	3.5056	8635.2***	2.7955	-	0.4322

This table shows the statistics for the standardized residuals ($\epsilon_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}}$) for both models used: GARCH-M and RS-GARCH. J-B test is the Jarque-Bera test for normality. L-B (6) is the Ljung-Box autocorrelation test including six lags. This also displays the first two order condition test of Bollerslev-Wooldridge (1992) of the standardized residuals to validate consistent estimations of the QML procedure from deviations to normality. .***, **, and * represent significance at 1%, 5%, and 10% levels.

Table 4.- Estimations for the MSCI index using the GJR- GARCH-M and GJR- RS-GARCH-methodology.

	GARCH-M			***RS-GARCH-M***					
				State k=1			***State k=2***		
Parameter *(t-stat)*	c	λ_1	Persist.	c	λ_1	Persistence	c	λ_1	Persist.
Argentina	**0.2465** *(1.1983)*	**-0.0104**** *(-1.9959)*	**0.9927**	**-0.5065** *(-0.4848)*	**0.0906**** *(2.0931)*	**0.9746**	**0.6784** *(-0.9280)*	**-0.0078** *(-0.4925)*	**0.1855**
Brazil	**1.2171***** *(2.9806)*	**-0.0621***** *(-4.7613)*	**0.9813**	**-3.8809*** *(-1.7449)*	**0.3277**** *(1.9818)*	**0.9719**	**-0.3739** *(-0.4412)*	**-0.0356** *(-1.1505)*	**0.2463**
Peru	**-0.2428** *(-1.0764)*	**0.0267** *(1.0729)*	**0.9583**	**-2.0797***** *(-3.7862)*	**0.0929***** *(2.8484)*	**0.9920**	**1.2368***** *(3.8078*	**-0.0287** *(1.2978)*	**0.8909**
Mexico	**0.5719** *(0.2266)*	**-0.0235** *(0.0145)*	**0.9231**	**-2.5091***** *(-3.6486)*	**0.0981***** *(2.8791)*	**0.8766**	**0.7778***** *(3.8334)*	**-0.0109** *(-0.8750)*	**0.6242**
Chile	**-0.2428** *(-1.0764)*	**0.0267** *(1.0729)*	**0.9586**	**-1.0830*** *(-1.9072)*	**0.0830**** *(1.9850)*	**0.9231**	**0.6728***** *(3.3803)*	**-0.0215** *(-1.3274)*	**0.9075**
China	**0.3981** *(0.5491)*	**-0.0142** *(-0.7097)*	**0.9746**	**-1.2315** *(-1.3072)*	**0.1910*** *(1.7866)*	**0.9857**	**-0.5112** *(-0.7200)*	**-0.0026** *(-0.1183)*	**0.2833**
Indonesia	**0.2201** *(1.1952)*	**-0.0050** *(-0.7771)*	**0.9777**	**-8.4867** *(-1.0467)*	**1.0324***** *(3.0577)*	**0.9874**	**-1.0759** *(-0.3190)*	**0.0072** *(0.0436)*	**0.0484**
Malasia	**0.1032** *(1.0026)*	**0.0028** *(0.3204)*	**0.9935**	**0.6455***** *(3.6438)*	**0.0032** *(0.2612)*	**0.9807**	**-0.2349** *(-1.2471)*	**0.0054** *(0.3606)*	**0.9721**
Thailand	**0.1284** *(0.5397)*	**-0.0051** *(-0.4463)*	**0.9871**	**-0.2359** *(-0.4102)*	**0.0674***** *(2.1222)*	**0.9832**	**-0.3847** *(-0.6931)*	**-0.0137** *(-0.7104)*	**0.8897**
India	**0.2772** *(1.2178)*	**-0.0025** *(-0.1666)*	**0.9687**	**-11.225***** *(-20.200)*	**0.1619***** *(3.7981)*	**0.9999**	**0.1439** *(0.6080)*	**0.0234** *(1.3371)*	**0.7360**
Korea	**0.0009** *(0.0054)*	**0.0021** *(0.2946)*	**0.9747**	**-0.8469**** *(-2.3028)*	**0.0242*** *(1.6435)*	**0.9793**	**0.7807**** *(1.9605)*	**0.0013** *(0.0619)*	**0.9943**
Philippines	**0.0798** *(0.3046)*	**-0.0013** *(-0.0769)*	**0.9587**	**-0.3445** *(-0.4684)*	**0.1107*** *(1.6971)*	**0.9739**	**-1.9929***** *(-3.5068)*	**0.0456** *(1.6290)*	**0.8012**
Taiwan	**0.2141** *(0.9112)*	**-0.0068** *(-0.4135)*	**0.9708**	**-1.7062**** *(-2.2161)*	**0.0601*** *(1.6903)*	**0.9247**	**0.5258** *(1.1218)*	**0.0387** *(0.5164)*	**0.2266**
Egypt	**0.0139** *(1.5272)*	**-0.1422** *(-1.3135)*	**0.9971**	**-0.8151***** *(-4.2283)*	**0.0267** *(1.5545)*	**0.9999**	**1.3689***** *(3.7862)*	**-0.0238** *(-0.9963)*	**0.9242**
Morocco	**0.1287** *(0.0243)*	**0.0065** *(0.0243)*	**0.9382**	**-0.7708***** *(-3.4016)*	**0.0830*** *(1.7333)*	**0.9349**	**0.8995***** *(2.8602)*	**-0.0463** *(-0.8154)*	**0.9186**
South Africa	**0.0001** *(0.0006)*	**0.0025** *(0.1469)*	**0.9231**	**-1.5907***** *(-2.5682)*	**0.0663*** *(1.9340)*	**0.9772**	**0.4485** *(1.9100)*	**0.0236** *(1.1818)*	**0.5980**
Hungary	**0.2227** *(0.8450)*	**-0.0068** *(-0.6371)*	**0.9433**	**-1.5550** *(-1.2636)*	**0.1707*** *(1.9258)*	**0.9684**	**-1.4575**** *(-2.2946)*	**0.0309** *(1.6154)*	**0.3432**
Poland	**0.0369** *(0.1440)*	**0.0007** *(0.0738)*	**0.9591**	**-1.9868**** *(-2.1793)*	**0.0397*** *(1.6503)*	**0.8209**	**0.0898** *(0.0181)*	**0.0277** *(0.0689)*	**0.0397**
Turkey	**0.4060** *(1.0651)*	**-0.0143**** *(-2.144)*	**0.9703**	**-2.1126** *(-1.3860)*	**0.0762*** *(1.6858)*	**0.9881**	**-0.2158** *(-0.2913)*	**-0.0191*** *(-1.9397)*	**0.7428**
Czech Republic	**0.3981*** *(1.6908)*	**-0.0142** *(-0.8269)*	**0.9082**	**0.2329** *(0.2211)*	**0.0719** *(0.4884)*	**0.9305**	**-1.1934**** *(-2.1098)*	**0.0215** *(1.0363)*	**0.1772**
Russia	**0.4428** *(1.6442)*	**-0.0037** *(-0.5830)*	**0.9893**	**-0.1313** *(-0.2300)*	**-0.0002** *(-0.0203)*	**0.9944**	**1.1275*** *(1.8828)*	**-0.0043** *(-0.2979)*	**0.9330**
MSCI Asia	**0.1637** *(1.3389)*	**-0.0101** *(-0.8105)*	**0.9716**	**-1.2134** *(-1.2538)*	**0.4734*** *(1.7354)*	**0.9871**	**0.1510** *(0.4284)*	**-0.0307** *(-1.1566)*	**0.1339**
MSCI Europe	**0.0125** *(0.0698)*	**-0.0038** *(-0.3297)*	**0.9660**	**-1.3635**** *(-2.3256)*	**0.0711**** *(2.0025)*	**0.9649**	**0.5780**** *(2.0865)*	**-0.0221** *(-1.5195)*	**0.9450**
MSCI Latin America	**-0.0082** *(-0.5151)*	**0.0805** *(0.3399)*	**0.9163**	**-1.7886***** *(-2.9394)*	**0.1509** *(0.3485)*	**0.9547**	**-10.4221** *(-0.3296)*	**0.0245** *(1.3025)*	**0.0402**

This table shows the estimations for the intercept, the risk aversion parameter and the shock persistence in the emerging markets considered using the asymmetric variance specification. T-stats in parenthesis. ***, **, and * represent significance at 1%, 5%, and 10% levels. Persist. means the persistence of an unexpected shock in the market volatility and is computed as the sum of the parameters ($\alpha+\beta$) in the variance equation.

Table 5.- Estimations for the MSCI index using the GARCH-M and RS-GARCH-methodology without including constant.

LATINOAMERICA									
Parameter *(std. error)*	***GARCH-M***			***RS-GARCH-M***					
				State k=1			***State k=2***		
Country	*BRA*	*ARG*	*PER*	*BRA*	*ARG*	*PER*	*BRA*	*ARG*	*PER*
λ_1	**-0.0034** *(-0.9746)*	**-0.016*** ** *(-4.5717)*	**0.0100** *(1.0480)*	**0.0299*** ** *(2.5970)*	**0.0227** ** *(2.5276)*	**0.0306** ** *(2.3202)*	**-0.0046** *(-1.5287)*	**-0.0103** *(-1.2474)*	**0.0018** *(0.1471)*
Country	*CHI*	*MEX*	*MSCI LATIN*	*CHI*	*MEX*	*MSCI LATIN*	*CHI*	*MEX*	*MSCI LATIN*
	0.0029 *(0.2870)*	**0.0142*** *(1.9421)*	**0.0046** *(0.5831)*	**0.0416** ** *(2.2275)*	**0.0509** ** *(2.3994)*	**0.0502** ** *(2.0400)*	**-0.0091** *(-0.7313)*	**-0.0081** *(-0.6510)*	**-0.0247** *(-1.5677)*
ASIA									
Parameter *(std. error)*	***GARCH-M***			***RS-GARCH-M***					
				State k=1			***State k=2***		
Country	*CHI*	*INDON*	*MAL*	*CHI*	*INDON*	*MAL*	*CHI*	*INDON*	*MAL*
λ_1	**0.0033** *(0.4884)*	**0.0059** *(1.2332)*	**0.0104** *(1.5173)*	**0.0275** ** *(2.0109)*	**0.0592*** ** *(4.0240)*	**0.0820*** ** *(4.4463)*	**-0.0162** ** *(-2.0097)*	**-0.0085** *(-1.4869)*	**-0.0087** *-(1.0901)*
Country	*THAI*	*INDIA*	*KOR*	*THAI*	*INDIA*	*KOR*	*THAI*	*INDIA*	*KOR*
λ_1	**0.0040** *(0.6330)*	**0.0155*** *(1.8953)*	**0.0089*** *(1.6602)*	**0.0245*** *(1.9556)*	**0.0650** ** *(2.2727)*	**0.0337*** ** *(2.8609)*	**-0.0110** *(-1.4344)*	**-0.0243** *(-0.8771)*	**-0.0030** *(-0.3862)*
Country	*PHIL*	*TAIW*	*MSCI ASIA*	*PHIL*	*TAIW*	*MSCI ASIA*	*PHIL*	*TAIW*	*MSCI ASIA*
λ_1	**0.0090** *(1.1150)*	**0.0109** *(1.3831)*	**0.0132** *(1.3269)*	**0.0397*** ** *(3.1041)*	**0.0457** ** *(2.4795)*	**0.0853*** ** *(3.9435)*	**-0.0173*** *(-1.8609)*	**-0.0138** *(-0.9572)*	**-0.0227** ** *(-2.0111)*
EUROPE									
Parameter *(std. error)*	***GARCH-M***			***RS-GARCH-M***					
				State k=1			***State k=2***		
Country	*CZECH*	*HUNG*	*POL*	*CZECH*	*HUNG*	*POL*	*CZECH*	*HUNG*	*POL*
λ_1	**0.0156*** *(1.8596)*	**0.0086** *(1.4797)*	**0.0025** *(0.4875)*	**0.0427*** ** *(2.7466)*	**0.0281** ** *(2.0847)*	**1.1154*** ** *(3.6167)*	**-0.0150** *(-1.0595)*	**-0.0193** ** *(-2.1362)*	**-0.0055** *(-1.0552)*
Country	*RUSS*	*TURK*	*MSCI EURO*	*RUSS*	*TURK*	*MSCI EURO*	*RUSS*	*TURK*	*MSCI EURO*
λ_1	**0.0033** *(0.8056)*	**-0.0072*** *(-1.8645)*	**-0.0014** *(-0.199)*	**0.0184*** *(1.7901)*	**-0.0055** *(-0.7175)*	**0.0205** *(1.3067)*	**-0.0014** *(-0.2689)*	**-0.0098*** *(-1.6814)*	**-0.0084** *(-0.9834)*
AFRICA									
Parameter *(std. error)*	***GARCH-M***			***RS-GARCH-M***					
				State k=1			***State k=2***		
Country	*MOR*	*EGYP*	*SOU AF*	*MOR*	*EGYP*	*SOU AF*	*MOR*	*EGYP*	*SOU AF*
λ_1	**0.0104** *(1.5173)*	**0.0040** *(0.6218)*	**0.0089** *(1.1349)*	**0.0539*** *(1.6552)*	**0.0130** *(1.2054)*	**0.0616*** ** (3.5583)	**0.0223** *(1.4414)*	**-0.0096** *(-0.4153)*	**-0.0132** *(-1.0773)*

This table shows the estimations for the risk aversion parameter in the emerging markets considered in the symmetric case omitting the constant term in the mean equation. T-stats in parenthesis. ***, **, and * represent significance at 1%, 5%, and 10% levels.

Table 6.- Estimations for the MSCI index using the GJR-GARCH-M and GJR- RS-GARCH-methodology without including constant.

LATINOAMERICA									
Parameter *(std. error)*	**GARCH-M**			**RS-GARCH-M**					
				State k=1			**State k=2**		
Country	*BRA*	*ARG*	*PER*	*BRA*	*ARG*	*PER*	*BRA*	*ARG*	*PER*
λ_1	**-0.0336**** *(-7.0351)*	**-0.0073*** *(-1.7552)*	**0.0064** *(0.6470)*	**0.0214** *(1.3816)*	**0.0070** *(0.5363)*	**0.0326**** *(2.3744)*	**-0.0413** *(-0.5913)*	**-0.0093** *(-0.7125)*	**-0.0160** *(-1.376)*
Country	*CHI*	*MEX*	*MSCI LATIN*	*CHI*	*MEX*	*MSCI LATIN*	*CHI*	*MEX*	*MSCI LATIN*
	0.2910 *(0.2741)*	**0.0056** *(0.7745)*	**-0.0038** *(-0.431)*	**0.0312**** *(1.7331)*	**0.0323**** *(1.7605)*	**0.0255** *(1.3195)*	**-0.0260** *(-1.5689)*	**-0.0113** *(-1.2577)*	**-0.0159** *(-1.578)*
ASIA									
Parameter *(std. error)*	**GARCH-M**			**RS-GARCH-M**					
				State k=1			**State k=2**		
Country	*CHI*	*INDON*	*MAL*	*CHI*	*INDIA*	*MAL*	*CHI*	*INDIA*	*MAL*
λ_1	**-0.0027** *(-0.4240)*	**-0.0004** *(-0.0884)*	**0.0076** *(1.0269)*	**0.0428**** *(2.9401)*	**0.0217*** *(1.7528)*	**0.0278** *(1.2406)*	**-0.0439***** *(3.2648)*	**-0.0164** *(-1.186)*	**-0.0033** *(-0.2599)*
Country	*THAI*	*INDIA*	*KOR*	*THAI*	*INDIA*	*KOR*	*THAI*	*INDIA*	*KOR*
λ_1	**-0.0004** *(-0.0611)*	**0.0137** *(1.6408)*	**0.0021** *(0.3429)*	**0.0199** *(1.4150)*	**0.0561**** *(2.2478)*	**0.0209** *(1.2333)*	**-0.0098** *(-1.2041)*	**-0.0064** *(-0.5194)*	**-0.0102** *(-0.9600)*
Country	*PHIL*	*TAIW*	*MSCI ASIA*	*PHIL*	*TAIW*	*MSCI ASIA*	*PHIL*	*TAIW*	*MSCI ASIA*
λ_1	0.0033 *(0.4033)*	0.0062 *(0.7689)*	-0.0032 *(-0.431)*	**0.0297**** *(2.1527)*	**0.0434**** *(2.2992)*	**0.0463**** *(2.0185)*	**-0.0131** *(-1.1998)*	**-0.0129** *(-1.218)*	**-0.0112** *(-0.9485)*
EUROPE									
Parameter *(std. error)*	**GARCH-M**			**RS-GARCH-M**					
				State k=1			**State k=2**		
Country	*CZECH*	*HUNG*	*POL*	*CZECH*	*HUNG*	*POL*	*CZECH*	*HUNG*	*POL*
λ_1	**0.0098** *(1.0895)*	**0.0003** *(0.0517)*	**0.0018** *(0.2997)*	**0.0450***** *(2.8740)*	**0.0200** *(-0.688)*	**0.0054** *(0.3914)*	**-0.0217** *(-1.5636)*	**-0.0062** *(1.2479)*	**0.0038** *(0.4984)*
Country	*RUSS*	*TURK*	*MSCI EURO*	*RUSS*	*TURK*	*MSCI EURO*	*RUSS*	*TURK*	*MSCI EURO*
λ_1	**0.0067** *(0.8658)*	**-0.0081**** *(-2.0835)*	**-0.0032** *(-0.431)*	**0.0192**** *(1.9619)*	**0.0020** *(0.3037)*	**0.0084** *(0.5263)*	**-0.0190** *(-1.4980)*	**-0.0159** *(-1.419)*	**-0.0076** *(-0.9216)*
AFRICA									
Parameter *(std. error)*	**GARCH-M**			**RS-GARCH-M**					
				State k=1			**State k=2**		
Country	*MOR*	*EGYP*	*SOU AF*	*MOR*	*EGYP*	*SOU AF*	*MOR*	*EGYP*	*SOU AF*
λ_1	**0.0263*** *(1.9221)*	**0.0067** *(0.8658)*	**0.0025** *(0.3150)*	**0.0424** *(0.9808)*	**0.0541***** *(3.8911)*	**0.0491***** *(2.1914)*	**0.0257** *(1.2030)*	**-0.046***** *(-2.869)*	**-0.0065** *(-0.5017)*

This table shows the estimations for the risk aversion parameter in the emerging markets considered in the asymmetric case omitting the constant term in the mean equation. T-stats in parenthesis. ***, **, and * represent significance at 1%, 5%, and 10% levels.

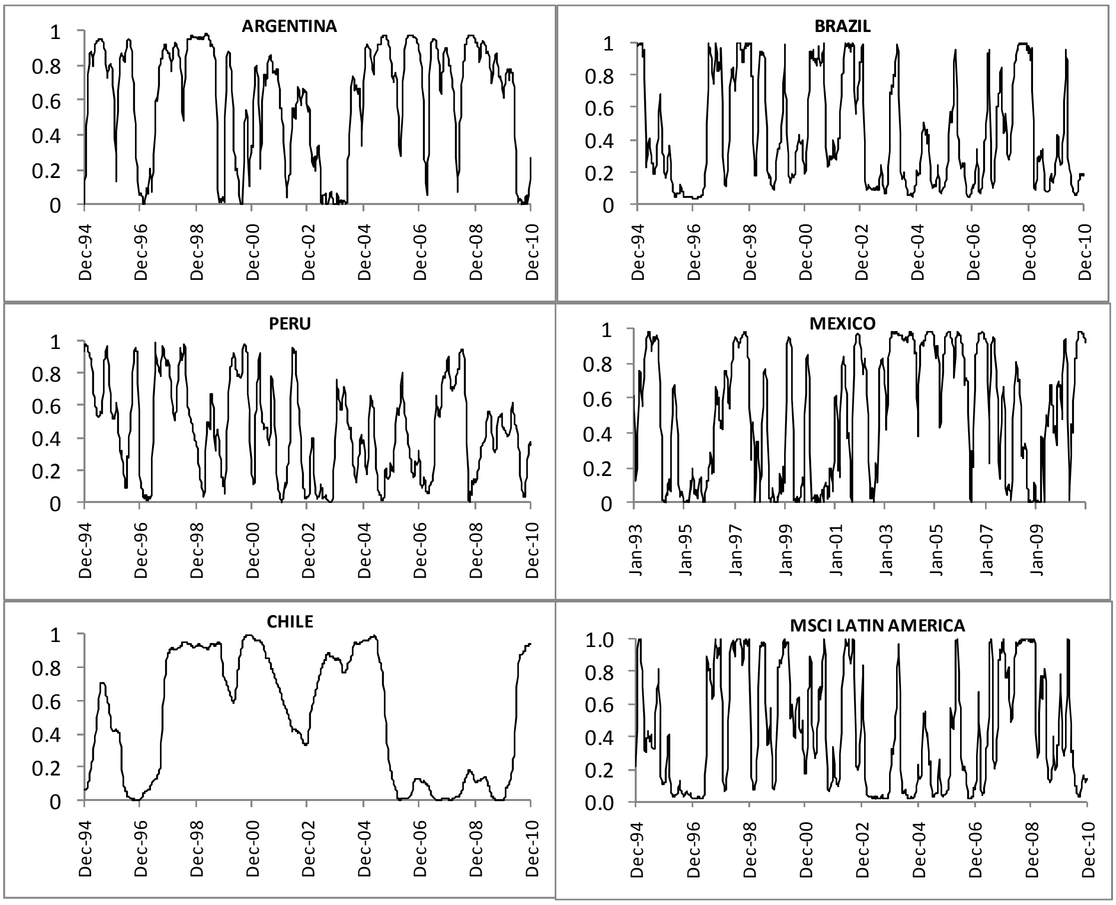

Figure 1.A.- Charts showing the smooth probability of being in a low volatility state in each country during the period 1995–2010 in Latin American Emerging Markets.

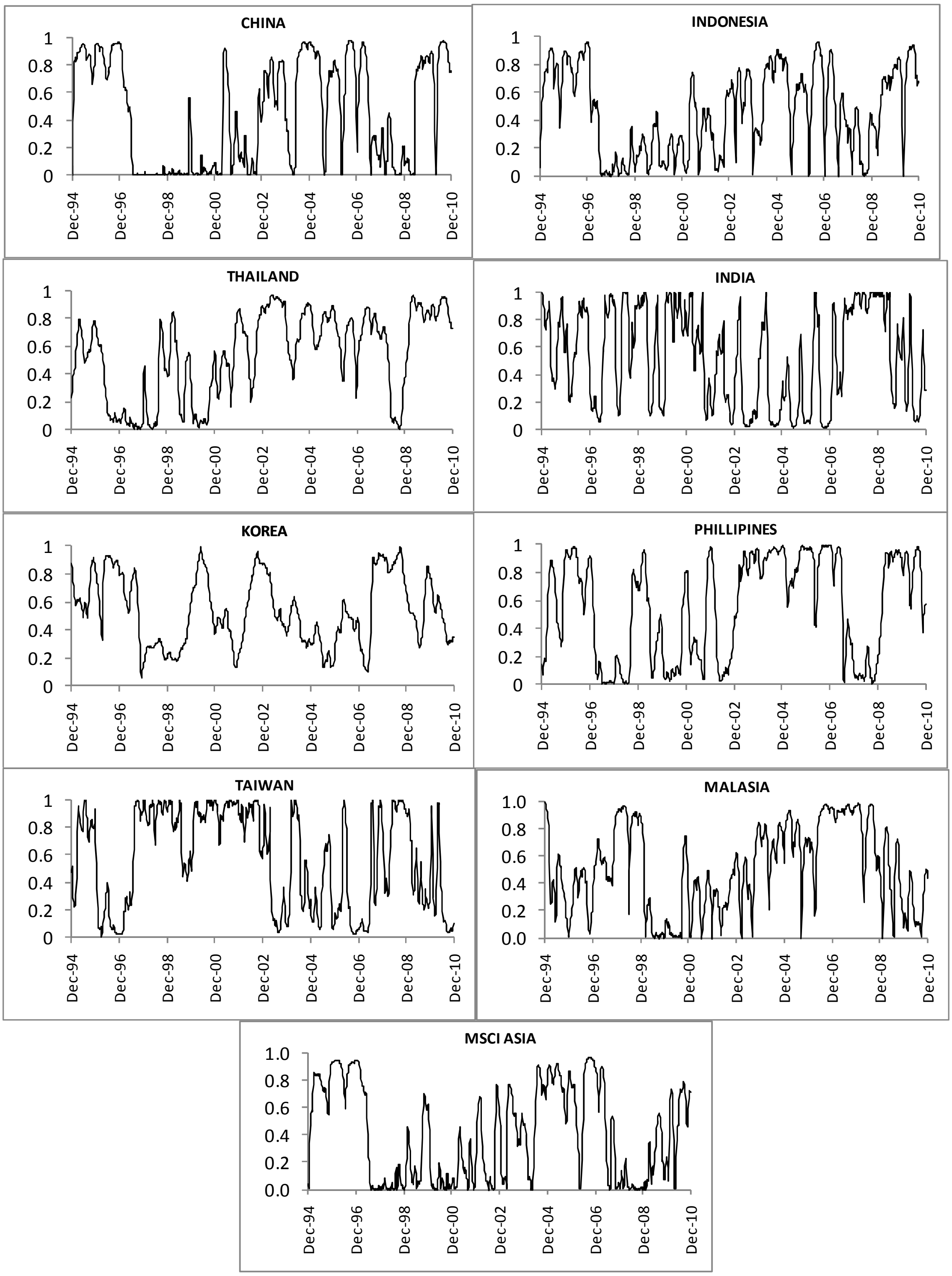

Figure 1.B- Charts showing the smooth probability of being in a low volatility state in each country during the period 1995–2010 in Asian Emerging Markets.

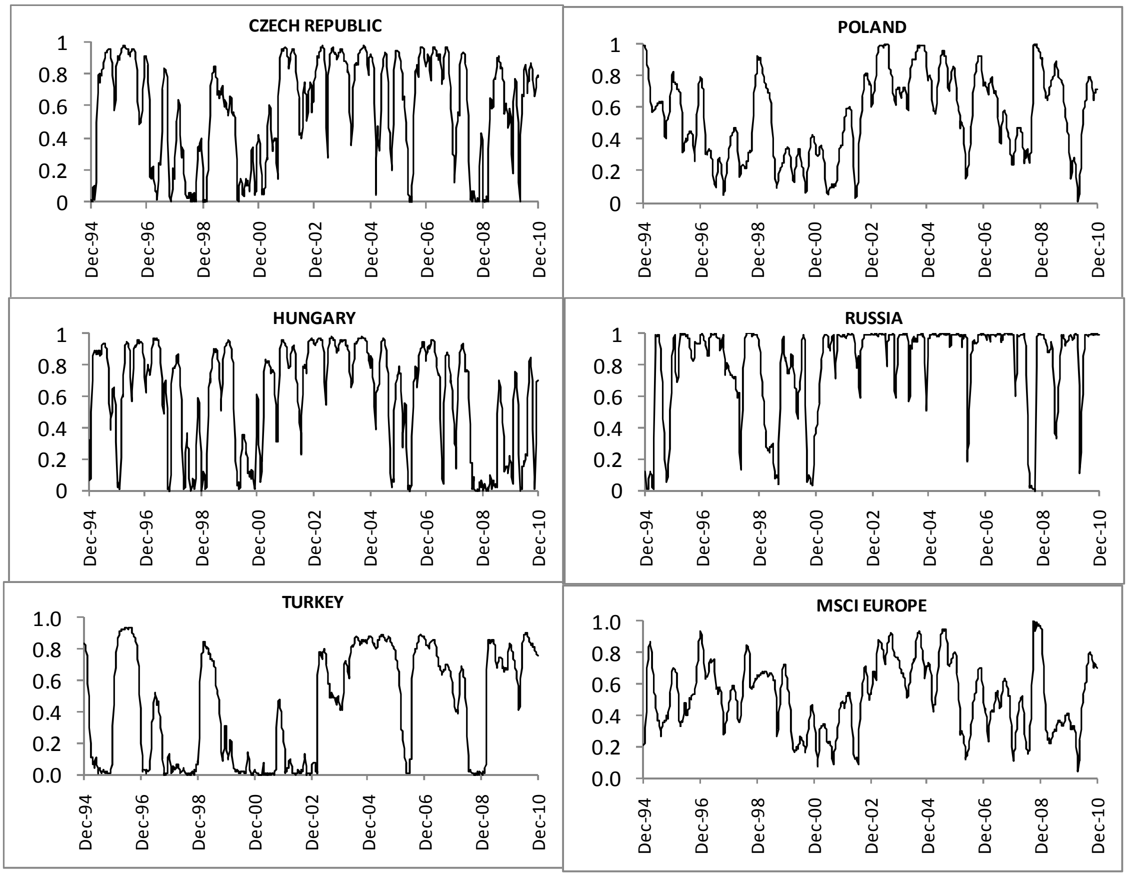

Figure 1.C- Charts showing the smooth probability of being in a low volatility state in each country during the period 1995–2010 in European Emerging Markets.

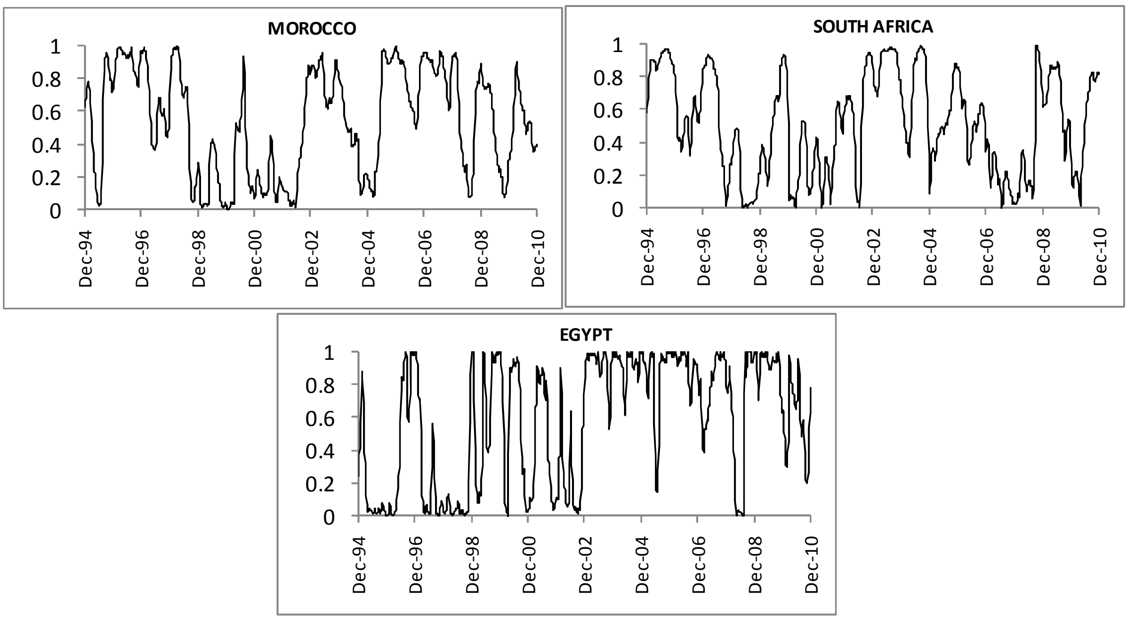

Figure 1.D- Charts showing the smooth probability of being in a low volatility state in each country during the period 1995–2010 in African Emerging Markets.

CHAPTER 3:

NON-LINEAR TRADEOFF BETWEEN RISK AND RETURN: A REGIME-SWITCHING MULTI-FACTOR FRAMEWORK

Abstract

This study develops a multi-factor framework where not only the market risk is considered but also potential changes in the investment opportunity set. Although previous studies find no clear evidence about a positive and significant relation between return and risk, favourable evidence can be obtained if a non-linear relation is established. The positive and significant tradeoff between return and risk is essentially observed during low volatility periods suggesting a procyclical risk aversion of investors. Different patterns for the risk premium dynamics in low and high volatility periods are obtained, both in risk prices and risk (conditional second moments) patterns.

1. Introduction

The relation between expected return and risk has motivated many studies in the financial literature. Most of the recent asset pricing models are based in this fundamental trade-off, so understanding the dynamics of this relation is a key issue in finance. One of the first studies establishing a theoretical relation between expected return and risk is the Sharpe (1964) and Lintner (1965) CAPM model. These authors proposed a positive linear relationship between the expected return of any asset and its covariance with the market portfolio; in other words, the expected return of the market portfolio is proportional to its conditional variance. This static model has been analyzed empirically in several studies obtaining no clear evidence about the sign and significance of this relationship (Campbell (1987), Harvey (1989), Glosten et. al 1993). Merton (1973) proposed an extension of this model adding a second risk factor in the relationship that may improve the static CAPM model. The market risk premium in the Merton's model is proportional to its conditional variance and the conditional covariance with the investment opportunity set (hedging component). This framework established in a time-continuous economy is an extension of the static CAPM model assuming a stochastic set of investment opportunities. The expected market risk premium in equilibrium is:

$$E_t\left(R_{W,t}\right) = \left[\frac{-J_{WW}W}{J_W}\right]\sigma^2_{W,t} + \left[\frac{-J_{WB}}{J_W}\right]\sigma_{WB,t} \qquad (1)$$

Where J(W(t), B(t), t) is the utility function (subscripts representing partial derivatives), W(t) is the wealth level, B(t) is a variable that describes the state of investment opportunities in the economy, $E_t\left(R_{W,t}\right)$ is the expected excess return on aggregate wealth, $\sigma^2_{W,t}$ and $\sigma_{WB,t}$ are, respectively, the conditional variance and the conditional covariance of the excess returns with the investment opportunity set, and $\left[\frac{J_{WW}W}{J_W}\right]$, $\left[\frac{J_{WB}}{J_W}\right]$ could be viewed as the risk prices of the sources of risk.

Assuming risk-averse investors $J_W > 0$ and $J_{WW} < 0$, the model establishes a positive relation between risk premium and market volatility. However, the relation between the risk premium with the second risk factor (σ_{WB}) depends on the sign of J_{WB} and σ_{WB}. If J_{WB} and σ_{WB} share the same sign the investors demand a lower risk premium, but if the sign is different a higher risk premium is demanded. Assuming that Equation 1 is the proper model for the empirical study of the risk-return trade-off, the omission of this risk factor could lead to misspecifications of the empirical models and misleading evidence about the risk-return relationship.

Despite the important role of this trade-off in the financial literature, there is no clear consensus about its empirical evidence. In the theoretical framework, the parameters (the risk prices in brackets) are considered constant over time[49] and the variables (the sources of risk) are allowed to be time-varying. However, to make this model empirically tractable one must make several assumptions; the most common is considering constant risk prices (Goyal and Santa-Clara 2003, Bali et. al 2005). Another common assumption made in the empirical analysis of the risk-return trade-off is considering a set of investment opportunities constant over time, remaining the market risk as the only source of risk (Baillie and Di Gennaro 1990, Glosten et al. 1993). This assumption leads to the validation of the static CAPM model. It is also necessary to assume specific dynamics for the conditional second moments. The most common are the GARCH models (Bollerslev, 1986)[50]. Finally, the empirical model is established in a discrete time economy instead of the continuous time economy used in the equilibrium model of the theoretical approach. Many of the empirical papers studying the risk-return use one or more of the assumptions explained above.

The great controversy in the empirical validation of the risk-return trade-off is motivated by the disappointing results obtained about the sign and significance of this relation. There is no consensus about whether these results are due to: (1) wrong specifications of conditional second moments, Guo and Neely (2008), Leon et al. (2007); (2) misspecifications of the empirical models caused by the omission of the hedge component, Scruggs (1998); (3) both causes.

However, another potential problem related with the empirical validation of the risk-return trade-off is the assumption of a linear relationship between return and risk. Some authors (Whitelaw 2000, Mayfield 2004) are concerned with this point and develop alternative theoretical models for the risk-return trade-off where non-linear patterns are included through regime-switching models. The equilibrium model in Whitelaw (2000) is slightly different from Merton's approach. A more complex, non-linear and time-varying relation between expected return and volatility is obtained. Whitelaw also remarks the importance of the hedge component in the determination of the risk-return trade-off in his non-linear framework.

This study tries to shed light on the empirical validation of the risk-return trade-off. Although there is a large literature focused on this empirical validation, there are only few studies using multi-factor models that consider the hedge component[51]. The main

[49] There are other general equilibrium models where time-varying risk aversion coefficients are obtained in models with habit persistence such as Campbell and Cochrane (1999) or other theoretical frameworks where a non-linear and time-varying relation between risk and return is considered (Whitelaw (2000)).

[50] Ghysels et al. (2005) proposes an alternative specification, the MIDAS regression, for modelling conditional second moments against GARCH models.

[51] One of the most common assumptions in the literature is the consideration of a constant set of investment opportunities, or, alternatively, independent and identically distributed rates of return. This assumption implies that the market risk premium only depends on its conditional variance and could be validated using univariate rather than multi-factor models.

empirical approach used in the literature is the GARCH-M framework, which assumes a linear relation between return and risk. However, there are other empirical approaches to analyze empirically the risk-return trade-off. Most of them use different econometric techniques to validate a linear relationship between return and risk based on the Merton's ICAPM model (i.e. Ghysels et al. (2005) using the MIDAS regression, Ludvigson and Ng (2007) using a factor analysis with macroeconomic variables or Bali and Engle (2010) using a temporal and cross-sectional analysis of a wide range of portfolios comprising the whole market). However, in this paper we use another econometric approach based on the equilibrium model of Whitelaw (2000) in which we do not consider a linear relationship between return and risk but non-linear. It is showed in this paper that for shorter span empirical analysis, the relationship between expected return and volatility follows non-linear rather than linear patterns as suggested the ICAPM model. The RS-GARCH approach proposed in this study let obtain favorable evidence for a positive and significant risk–return tradeoff.

The main contributions of this paper are the followings: Firstly, this study analyzes the risk premium evolution in Spain during the last few years. Secondly, according to the papers of Mayfield (2004) and Whitelaw (2000) it proposes a multi-factor model (considering a stochastic set of investment opportunities) where both the risk prices and sources of risk are state-dependent, allowing us to consider non-linear relationships between return and risk. Thirdly, it shows differences in the patterns followed by risk prices and conditional volatilities in different states (defined as low and high volatilities), being the risk price values lower during high volatility states and the conditional volatility more persistent during low volatility states. Fourth, it shows that a significant risk-return tradeoff can be obtained when it is assumed a non-linear relationship between return and risk. This evidence is essentially observed during low volatility states but not during high volatility states or when a linear relationship between return and risk is analyzed suggesting a procyclical risk-aversion of investors. Fifth, it seems that the relevant aspect for this evidence is the assumption of a non-linear relation between return and risk although the hedge component is important overall in the non-linear framework.

This paper is organized as follows. Section 2 provides a description of the data. Section 3 develops the empirical framework used in the paper. Section 4 gives the main empirical results and Section 5 concludes.

2. Data Description

This study uses 720 weekly (Capiello and Fearnley (2000), Shin (2005)) excess market returns from the Spanish market, including observations from 1 January 1996 to 15 October 2009. Even though there are slight differences in the parameter estimations using different data frequency, there is no particular reason that the conclusions in this study should be affected by the selection of data frequency. Some authors remark this point in their studies (De Santis and Imhoroglu 1997, Shin 2005, Lundblad 2007).

The excess market returns are computed using the quotations of the IBEX-35 index, first obtaining logarithmic returns[52] and then subtracting from these returns the risk-free rate. Following Leon et al. (2007) the market money rate suitably compounded at weekly frequency is used as the proxy for the risk-free rate. The choice for the proxy used as the hedging component against changes in the investment opportunity set are the followings rates for the Spanish market (Bali and Engle (2009, 2010) use similar proxies for the American case): 1-year Treasury bill, 3-year Treasury bond, 5-year Treasury bond, 10-year Treasury bond, an equally averaged portfolio with the previous 3 bonds and the difference between the yields on the 10-year and the 3-year Treasury bond. Thomson Datastream is used to obtain the data about the stock index, International Financial Statistics for the data corresponding to the risk-free rate and the AFI (*Analistas Financieros Internacionales*) database[53] for the data about the proxies used as the intertemporal hedging component. Table 1 shows the main summary statistics for excess market returns and the intertemporal hedging alternatives rates.

[INSERT TABLE 1]

All series included in this study present non-normal unconditional distributions with strong evidence for skewness and kurtosis. This result suggests fat tails in the unconditional distributions. Furthermore, all series exhibit conditional heteroskedasticity features (serial autocorrelation in square returns). With these serial patterns, the use of GARCH models to represent the dynamics of conditional second moments, which has a large support in the previous literature, is totally understandable. It is also observed that the temporal series in levels do not exhibit in general serial autocorrelation so the inclusion of any structure[54] in the mean equation is not necessary. Finally, the correlation matrix for the different proxies shows a low correlation between the excess returns of the market portfolio and the potential alternative investments. This result shows that the last series could be considered as proxies reflecting the alternative investment set available to the investors. Due to the lack of consensus in the literature about the best proxy representing the alternative investment set (Scruggs and Glabadanidis 2003, Guo and Whitelaw (2006), Bali 2008), this study uses the different assets shown above which present different characteristics (in their terms and maturity) and add robustness to the study.

[52] We use logarithmic returns multiplied by 100 to facilitate the convergence of the empirical models.

[53] AFI is a Spanish private consulting company.

[54] The 1-year T-Bill and the excess market return series exhibit these problems, but after modelling the variance as a GARCH specification the serial autocorrelation disappears without including any lag in the mean equation.

3. Empirical Methodology

This section presents the empirical models used in the study. The main contribution of this paper is the assumption of a state-dependent risk price and state-dependent conditional volatilities, which implies a non-linear relationship between return and risk, following the equilibrium model in Whitelaw (2000). So, assuming bivariate GARCH dynamics for conditional volatilities, (more specifically, the BEKK model of Baba et. al (1990)), state-independent multi-factor models that establish a linear relation between return and risk are presented in Section 3.1, followed by state-dependent multi-factor models that establish a non-linear risk-return trade-off through regime-switching both in the risk premium and conditional volatilities, in Section 3.2.[55]

3.1. State-independent multi-factor model

This section presents a multi-factor model derived from the Merton's (1973) ICAPM model. The 'general' model allows time-varying conditional second moments, but the risk aversion (risk price) coefficients for market risk $\left[\frac{J_{WW}W}{J_W}\right]$ and intertemporal component risk $\left[\frac{J_{WB}}{J_W}\right]$ are constant over time (Scruggs and Glabadanidis 2003).

$$\begin{aligned} r_{m,t} &= \lambda_{10} + \lambda_{11}\sigma_{m,t}^2 + \lambda_{12}\sigma_{mb,t} + \varepsilon_{m,t} \\ r_{b,t} &= \lambda_{20} + \lambda_{21}\sigma_{bm,t} + \lambda_{22}\sigma_{b,t}^2 + \varepsilon_{b,t} \end{aligned} \qquad (2)$$

where λ_{ij} for i=1,2 and j=0,1,2 are the parameters to estimate and represent the different risk prices and $\sigma_{m,t}^2$, $\sigma_{b,t}^2, \sigma_{mb,t}$ represent the conditional second moments (market variance, intertemporal hedging component variance and covariance between market portfolio and hedging component). A restricted model is also estimated, where the alternative investment set is time invariant ($\lambda_{21} = \lambda_{22} = 0$) (Scruggs 1998).

As we explained above, it is necessary make an assumption about the dynamics of the volatilities in order to empirically validate the theoretical ICAPM model. To analyze bivariate relationships, one of the most used models in the literature is the BEKK model of Baba et al. (1990). This model sets the following variance equation:

[55] The asymmetric response of volatility to news of different signs (leverage effect) is not considered for severak reasons: (1) there is no improvement about the significance of the risk-return trade-off in previous studies (Aragó and Salvador 2010); (2) the convergence of the proposed models is harder to achieve due to the inclusion of the new parameters. These reasons lead to the consideration of a more parsimonious model. Moreover, Lundblad (2007) states that the choice of volatility specification in the GARCH-M context is of second-order importance providing different specifications similar results.

$$H_t = \begin{pmatrix} \sigma^2_{m,t} & \sigma_{mb,t} \\ \sigma_{mb,t} & \sigma^2_{b,t} \end{pmatrix} = CC' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BH_{t-1}B' \qquad (3)$$

where C is a lower triangular 2x2 matrix of constants, A and B are 2x2 matrices of parameters, ε_{t-1} is a Tx2 vector of innovations and H_{t-1} is the lagged covariance matrix.

The model is estimated by the maximization of the Quasi-Maximum Likelihood function of Bollerslev-Wooldrige (1992), assuming that the innovations follow a normal bivariate distribution $\varepsilon_t \sim N(0, H_t)$, which allows us to obtain robust estimates of standard errors.

$$L(\theta) = \sum_{t=1}^{T} \ln\left[f(r_t, \Omega_t; \theta) \right] \quad where \quad f(r_t, \Omega_t; \theta) = (2\pi)^{-1} |H_t|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \varepsilon'_t H_t^{-1} \varepsilon_t \right) \qquad (4)$$

where | H_t | represents the determinant of the covariance matrix and the remaining terms have been defined above.

3.2. Regime-switching multi-factor model

This section introduces a new multi-factor model where both the risk prices and the conditional second moments are dependent of the state in the economy. In this case, we propose two states[56]. The consideration of regime-switching in the empirical relation allows us to obtain state-dependent estimations for the risk prices and conditional second moments. This implies a non-linear and state-dependent relation between expected return and risk following the general equilibrium model developed in Whitelaw (2000).

The mean equation specification in this model is

$$\begin{aligned} r_{m,t,s_t} &= \lambda_{10,s_t} + \lambda_{11,s_t}\sigma^2_{m,t,s_t} + \lambda_{12,s_t}\sigma_{mb,t,s_t} + \varepsilon_{m,t,s_t} \\ r_{b,t,s_t} &= \lambda_{20,s_t} + \lambda_{21,s_t}\sigma_{bm,t,s_t} + \lambda_{22,s_t}\sigma^2_{b,t,s_t} + \varepsilon_{b,t,s_t} \end{aligned} \qquad (5)$$

for $s_t = 1,2$ where λ_{ij,s_t} for i=1,2 and j=0,1,2 are state-dependent parameters, r_{m,t,s_t} and r_{b,t,s_t} are the state-dependent excess market and hedging component returns, σ^2_{m,t,s_t}, σ^2_{b,t,s_t} and σ_{bm,t,s_t} are the state-dependent conditional second moments, and ε_{m,t,s_t} and ε_{b,t,s_t} are the state-dependent innovations[57].

[56] Previous studies considering three states (e.g., Sarno and Valente 2000) show that the third state only reflects odd jumps in the return series.The explanatory power of the third state is low and it is worthless in light of the difficulties of the estimation process that it produces.

[57] We also estimate a restricted model where $\lambda_{21} = \lambda_{22} = 0$.

It is assumed that the state-dependent conditional second moments follow a GARCH bivariate dynamics (more specifically, a BEKK model). That is, there are as many covariance matrices as states. The state-dependent covariance matrices are

$$H_{t,s_t} = \begin{pmatrix} \sigma^2_{m,t,s_t} & \sigma_{mb,t,s_t} \\ \sigma_{mb,t,s_t} & \sigma^2_{b,t,s_t} \end{pmatrix} = C_{s_t} C_{s_t}{}' + A_{s_t} \varepsilon_{t-1} \varepsilon'_{t-1} A_{s_t}{}' + B_{s_t} H_{t-1} B_{s_t}{}' \qquad (6)$$

The consideration of several states leads to a noteworthy rise in the number of parameters to estimate. In order to reduce this over-parameterization we only let parameters accompanying lagged innovations and lagged variances to be regime-switching[58][10]. Furthermore, the difference among states is defined by two new parameters *sa* and *sb* that properly weight the estimations obtained in one state for the other state. Therefore, the state-dependent covariance matrices in our model are:

$$H_{t,s_t=1} = \begin{pmatrix} \sigma^2_{m,t,1} & \sigma_{mb,t,1} \\ \sigma_{mb,t,1} & \sigma^2_{b,t,1} \end{pmatrix} = CC' + A_1 \varepsilon_{t-1} \varepsilon'_{t-1} A_1{}' + B_1 H_{t-1} B_1{}' \qquad (6.1)$$

$$H_{t,s_t=2} = \begin{pmatrix} \sigma^2_{m,t,2} & \sigma_{mb,t,2} \\ \sigma_{mb,t,2} & \sigma^2_{b,t,2} \end{pmatrix} = CC' + A_2 \varepsilon_{t-1} \varepsilon'_{t-1} A_2{}' + B_2 H_{t-1} B_2{}' \qquad (6.2)$$

where $A_2 = sa \cdot A_1$ and $B_2 = sa \cdot B_1$, A_1 and B_1 are 2x2 matrices of parameters, and C is a 2x2 lower triangular matrix of constants (the same for the 2 states).

The shifts from one regime to another are governed by a hidden variable following a first-order Markov process with transition matrix[59]

$$\hat{P} = \begin{pmatrix} \Pr(s_t = 1 | s_{t-1} = 1) = p & \Pr(s_t = 1 | s_{t-1} = 2) = (1-q) \\ \Pr(s_t = 2 | s_{t-1} = 1) = (1-p) & \Pr(s_t = 2 | s_{t-1} = 2) = q \end{pmatrix} \qquad (7)$$

where p and q are the probability of being in state 1 and 2 if in the previous period the process was in state 1 and 2 respectively.

Due to this state-dependence and the recursive nature of GARCH models, the construction and estimation of the maximum likelihood function would be intractable unless independent estimates for innovations and covariances were obtained. In order to solve this problem, we use a recombinative method similar to that used in Dueker (1997) that let us obtain state-independent estimations for the covariance matrix and the innovations weighting the state-dependent covariance matrix and innovations by the ex-ante probability of being in each state.

$$H_t = P(s_t = 1 | \Omega_{t-1}; \theta) H_{t,s_t=1} + P(s_t = 2 | \Omega_{t-1}; \theta) H_{t,s_t=2} \qquad (8)$$

$$\varepsilon_t = P(s_t = 1 | \Omega_t; \theta) \varepsilon_{t,s_t=1} + P(s_t = 2 | \Omega_t; \theta) \varepsilon_{t,s_t=2} \qquad (9)$$

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[58] Capiello and Fearnley (2000) make a similar assumption to avoid potential convergence problems.
[59] Hamilton (1989, 1994) was the first to use this kind of inference in non-linear models

where H_t and ε_t are the state-independent estimations for the covariance matrix and the innovations

The ex-ante probabilities (the probabilities of being in each state in the period t using the information set at t-1) are (10.1) and (10.2):

$$P\left(s_t = 1 \middle| \Omega_{t-1};\theta\right) = p * P\left(s_{t-1} = 1 \middle| \Omega_{t-1};\theta\right) + (1-q)\, P\left(s_{t-1} = 2 \middle| \Omega_{t-1};\theta\right) \qquad (10.1)$$

$$P\left(s_t = 2 \middle| \Omega_{t-1};\theta\right) = 1 - P\left(s_t = 1 \middle| \Omega_{t-1};\theta\right) , \qquad (10.2)$$

where

$$P\left(s_t = k \middle| \Omega_t;\theta\right) = \frac{P\left(s_t = k \middle| \Omega_{t-1};\theta\right) f\left(r_t \middle| s_t = k, \Omega_t;\theta\right)}{\sum_{k=1}^{2} P\left(s_t = k \middle| \Omega_{t-1};\theta\right) f\left(r_t \middle| s_t = k, \Omega_t;\theta\right)} \qquad (11)$$

for k=1, 2 are the filtered probabilities (the probabilities of being in each state in the period t with the information set up to t).

Assuming state-dependent innovations following a normal bivariate distribution $\varepsilon_{t,s_t} \sim N\left(0, H_{t,s_t}\right)$, the vector of unknown parameters θ is estimated by maximizing the following maximum-likelihood function:

$$L(\theta) = \sum_{t=1}^{T} \ln\left[\sum_{k=1}^{2} P\left(s_t = k \middle| \Omega_t;\theta\right) f\left(r_t, \Omega_t;\theta\right)\right] \quad where \quad f\left(r_t, \Omega_t;\theta\right) = (2\pi)^{-1} \left|H_t\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\varepsilon_t' H_t^{-1} \varepsilon_t\right) \qquad (12)$$

where the state-dependent likelihood function is weighted by the ex-ante probability of being in each state.

4.- Empirical Results

This section presents the empirical results for the models proposed. We estimate the models explained in the previous section for the different proxies used as the intertemporal hedging component; models using the 1-year T-bill, the 3-year T-bond, the 5-year T-bond, the 10-year T-bond, the equally-weighted bond portfolio and the term spread are named .a, .b, .c, .d, .e, .f for brevity. Section 4.1 shows the results for the linear models (without regime-switching) in the two cases mentioned: general and restricted version. Section 4.2 explains the results for the non-linear multi-factor models (general and restricted), including regime switching. Section 4.3 describes the risk premium evolution in Spain during the last years according to each model and analyzes the reason for the differences between them. Finally, Section 4.4 performs some specification tests over the estimation residuals in order to detect any problems related with a potential misspecification of the empirical model.

4.1.- Multi-factor models estimations

The estimated models in this section are those introduced in section 3.1. The case without restrictions is named general model and the restricted version are the models where we assume constant risk premiums for the hedge component $\lambda_{21} = \lambda_{22} = 0$. The estimated parameters for the mean equation are presented in Table 2.

[INSERT TABLE 2]

It is clear that most of the parameters in this multi-factor model are non-significant for the mean equation. The coefficients that reflect the market risk price (λ_{11}) are positive but non-significant in all cases considered. Similar results are obtained for the hedging component risk factor (λ_{12}).

Table 3 shows the parameter estimates for the variance equation. These parameters define the dynamics and patterns followed by the conditional second moments.

[INSERT TABLE 3]

The results reflect that the bivariate GARCH specification fit and properly capture the conditional second moments dynamics. Significance in the parameters representing shocks in volatility (a_{11}, a_{22}) and persistence of past variance (b_{11}, b_{22}) is observed for both risk factors (market risk and investment opportunity set component). However, the impact of one risk factor in the composition of the other factor's volatility is not significant, neither the impact of shocks (a_{12}, a_{21}) nor persistence (b_{12}, b_{21}). There is another remarkable result about volatility dynamics; the persistence level in the two sources of risk—market risk (b_{11}) and hedging component (b_{22})—are relatively high using multi-factor models, with values close to 1. This high persistence level suggests the presence of several regimes in the volatility process (Lameroux and Lastrapes 1990). Ignoring these regime shifts could lead to inefficient volatility estimations. Regime-Switching (RS)-GARCH models let us consider different states in the volatility, process as we explain in the next sub-section, and overcome this limitation.

4.2- Regime-Switching multi-factor models estimation

This section shows the estimations for the state-dependent models presented in Section 3.2. These models exhibit state-dependent risk prices and conditional moments. Table 4 describes the estimation for the state-dependent mean equation in all cases considered. As we explain below in Figure 2, we can associate states 1 and 2 with low and high volatility periods respectively.

[INSERT TABLE 4]

Positive and significant estimations for the market risk price in low volatility states ($\lambda_{11,s=1}$) are obtained in all cases considered (for all proxies used as the intertemporal hedging component in the general and restricted version of the model)[60]. A positive and significant influence over the market risk premium of the risk price is also observed, representing the covariance between risk premium and hedging component ($\lambda_{12,s=1}$) in

[60] The results for the intercept are also significant. Some authors (Ghysels et. al. 2005, Leon et al. 2007) interpret this fact as market imperfections.

low volatility states. Generally, this covariance exhibits a negative influence in the total risk premium demanded (see Figure 1). So, the product of the risk price times the covariance between excess market return and hedging component $\left(\lambda_{12}\sigma_{mb,t,s_t=1}\right)$ shows that the total risk premium required by the investor $\left(\lambda_{11}\sigma^2_{m,t,s_t=1} + \lambda_{12}\sigma_{mb,t,s_t=1}\right)$ is slightly lower than the market risk premium. Only when the covariance is positive does the premium associated with the hedging component lead to higher values of the total risk premium regarding the market risk premium.

[INSERT FIGURE 1]

Panel B of Table 4 shows the results obtained for state 2. Generally, a significant relation is not observed between expected return and risk in high volatility states. A positive but no significant estimation is obtained for the risk price (market risk ($\lambda_{11,s=2}$), and covariance between market risk and hedge component ($\lambda_{12,s=2}$)). Moreover, the risk aversion coefficients in state 1 (corresponding to low volatility states) are higher than those corresponding to state 2 (high volatility states). This result suggests that there is less risk aversion in high volatility states. This finding is not consistent with the spirit of the theoretical models that suggests that higher volatility should be compensated with higher returns. However, Mayfield (2004), Lettau and Ludvigson (2003), and Lundblad (2007) found the same evidence: during high volatility states there is a decreasing level of risk aversion. One possible explanation could be the different risk aversion profile for investors in each state (Schmeling, 2009). During calm (low volatility) periods more risk-averse investors are trading in markets, but in high volatility periods only the less risk-averse investors remain in the market because they are the only investors interested in assuming such risk levels, decreasing the risk premium demanded during these periods. Moreover, recent papers such as Kim and Lee (2008) have reported similar evidence obtaining a significant risk-return trade-off during boom periods. In this study we do not define the states of the market depending on the business cycle (boom/crisis) but we use regime volatilities. However, the evolution for regime volatilities is very close to those of business cycles and very often low volatility states corresponds with calm periods while the less common high volatility states are associated with crisis periods (Lundblad, 2007). The procyclical risk-aversion (investors show more risk-aversion during boom periods than during crises periods) documented in the paper of Kim and Lee (2008) is also supported in this approach using volatility regimes where investors show more risk-aversion during low volatility periods than during high volatility periods .

Table 5 shows the estimations for the state-dependent variance equations. Again, significant estimates are obtained for the parameters accompanying the shock impact (a_{11}, a_{22}) and the persistence (b_{11}, b_{22}) in the volatility formation in both risk factors. Most of the cross-relationships between factors (a_{12}, a_{21}, b_{12}, b_{21}) in the volatility construction are non-significant, that is, shocks or volatility persistence in one factor has no effect in the other volatility factor.

[INSERT TABLE 5]

Furthermore, the volatility formation depends on the regime considered in this framework. For low volatility regimes there is observed a higher influence of the lagged variance (matrix B) even than the non-switching case (with values higher than unity in some cases)[61]. Moreover, in these states, there is also a lower impact of shocks (matrix A) in volatility formation. This result means that the volatility observed in a period t in a low volatility state is determined overall by the variance observed in the previous period and less by the shock occurring in period t. However, there is an increase of the shock influence in the volatility formation in high volatility regimes (determined by the product sa·A). There is also a decrease of the volatility persistence in these high volatility states (sb·B). In this case, the volatility observed in a period t in a high volatility state is less determined by the variance observed in the previous period and more by the shock occurring in this period t. These results suggest that linear GARCH models could lead to sub-estimation of volatility persistence in high volatility periods and over-estimation of volatility persistence in high volatility periods, where there is a higher presence of shocks in volatility formation (Marcucci 2005).

In addition, the non-linear multi-factor model lets us associate the different states that follow the volatility process with low (state 1) and high volatility (state 2) market periods. The median of the estimated volatility for state 1 are $\hat{\sigma}^2_{M,s_t=1}$ =6.8718 , $\hat{\sigma}^2_{B,s_t=1}$ = 0.3740 and $\hat{\sigma}_{MB,s_t=1}$ = -0.0982 while the median of estimated volatility series in state 2 are $\hat{\sigma}^2_{M,s_t=2}$ = 8.5479 , $\hat{\sigma}^2_{B,s_t=2}$ = 0.4496 and $\hat{\sigma}_{MB,s_t=2}$ = -0.1215. These results (jointly with Figure 2) let us associate the states defined in the non-linear model with low (state 1) and high volatility states (state 2).

Figure 2 show the smooth probabilities[62] of being in state 1 during the sample period for the 10-year T-bond[63] as hedging component case.

[INSERT FIGURE 2]

There are four patterns in the volatility process. The first part of the sample (until 2000 approximately) shows market uncertainty about the main regime in the market with sudden regime shifts (as the 1997 crisis). After that, high volatility periods seem to govern the process during the 2000-latest 2002 period, coinciding with the dot-com bubble. After this turbulent period, low volatility regimes govern again the Spanish market during the 2003-latest 2007 period, coinciding with a great expansion period of the Spanish economy. Then, coinciding with the global financial crisis of late 2007, high volatility regimes govern again the volatility process.

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[61] See Abramson and Cohen (2007) for necessary and sufficient conditions in MRS-GARCH processes

[62] The smooth probability is defined as the probability of being in each state considering the entire information set. $P(s_t=1|\Omega_T;\theta)=P(s_t=1|\Omega_t;\theta)\left[p\frac{P(s_{t+1}=1|\Omega_T;\theta)}{P(s_{t+1}=1|\Omega_t;\theta)}\right]+\left[(1-p)\frac{P(s_{t+1}=2|\Omega_T;\theta)}{P(s_{t+1}=2|\Omega_t;\theta)}\right]$

[63] For brevity, only the figure for the 10-year T-bond as alternative investment in the general model is considered; the dynamics of the probability in the rest of the cases are very similar. Results are available from the authors upon request

Despite these continuous changes in regime, low volatility regimes show a higher presence during the sample period governing the volatility process. The number of periods where the volatility process is in a low volatility state (probability of being in a low volatility states is higher than 0.5) are 496 periods, corresponding to 69% of the total sample

The results obtained about the significance of the risk-return trade-off in both multi-factor models suggest that the lack of empirical evidence in previous studies could be due to the strong assumption of a linear risk-return trade-off. Non-linear assumptions lead us to favorable evidence of the risk-return trade-off in low volatility states but we cannot obtain favorable evidence when a linear trade-off is assumed. We also obtain a significant impact of the intertemporal component in the risk-return relation similar to Whitelaw (2000).

Summing up, we can only obtain favorable evidence for a positive and significant risk-return trade-off for low volatility regimes (state 1). As the differences in the risk price show, there is a real risk-return trade-off in this state, but such a relation is not observed in high volatility states. The lack of evidence in the linear case could be due to the existence of several periods in the sample where there is not a risk-return trade-off (corresponding to high volatility states), causing a non-significant risk-return trade-off for the whole sample. However, if we distinguish among states we can identify low and high volatility states and identify a significant trade-off essentially in the low volatility state.

4.3.- Risk premium evolution

This section describes the risk premium evolution demanded by the investors in Spain, distinguishing between what proportions of the risk premium correspond to each risk factor: the market risk and the hedging component. We compute the premium associated with the market risk by the product of the risk price with idiosyncratic risk $\lambda_{11}\sigma^2_{m,t}$ for linear multi-factor models (and similarly for the hedging component premium). For the non-linear case, this risk premium is obtained using the state-dependent market risk premium weighted by the smooth probability of being in each state $P\left(s_t = 1 \middle| \Omega_T;\theta\right)\lambda_{11,s_t=1}\sigma^2_{m,t,s_t=1} + P\left(s_t = 2 \middle| \Omega_T;\theta\right)\lambda_{11,s_t=2}\sigma^2_{m,t,s_t=2}$ (and similarly for the hedging component premium). The total risk premiums are computed by the sum of the two factor premiums.

For brevity, we only show the results corresponding to the 10-year T-bond as alternative investment case.[64] Figure 3 describes the risk premium for the linear and non-linear cases.

[INSERT FIGURE 3]

[64] The dynamics of the risk premium evolution in the rest of the cases are very similar. Results are available from the authors upon request.

Both figures share similar patterns and only differ because of the scale of the risk price. The dynamics for the source of risk are very similar. There is a common rise of the market risk premium coinciding with high volatility periods (dot-com bubble period (2000-2002) and the last financial crisis (2007-2009)). The median[65] of the weekly risk premiums series shows that over the past 15 years the risk premium in Spain has remained at approximately 4% to 7% per annum[66] depending on the model used. Furthermore, the total risk premium is essentially defined for the risk associated with the market. The percentage of the total risk premium corresponding to the hedging component is relatively small for the linear model. More specifically, over the total risk premium estimated, only 95.5% and 74% of the premium are due to the market factor in the linear and non-linear multi-factor models respectively.

In order to detect the differences in the risk premium between the models proposed, Figure 4 presents the evolution of the differences between the total risk premium obtained in each model[67].

[INSERT FIGURE 4]

A similar evolution of the total risk premium is observed in both models during low volatility states (2002-2007). However, non-linear models exhibit higher estimations of the risk premium during high volatility periods (such as 2000-2002 and 2008 periods). According to this evidence, the assumption of linear patterns in the risk-return trade-off could lead to underestimations of the risk premium in high volatility periods.

4.4.- Specification test

This section performs several specification tests in order to check the adequacy of the QML estimations of the multi-factor models. For this reason, we analyze the properties of the standardized residuals ($\in_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}}$) and the product of the standardized residuals for the models proposed. Only the results for the 10-year T-bond case[68] are shown for brevity for the linear and non-linear models.

[INSERT TABLE 6]

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[65] We use the median rather the mean of the conditional second moments as a proxy for the average non-diversifiable risk in each period because it is less affected by outliers.

[66] The descriptive statistics for the risk premiums are not shown but they are available from the authors upon request.

[67] For brevity, only the figure for the 10-year T-bond as alternative investment in the general model are shown; the dynamics of the differences in the risk premium evolution in the rest of the cases are very similar. Results are available from the authors upon request.

[68] Results for all models are available from the authors upon request

The first part of the table shows summary statistics for the standardized residuals of the estimated multi-factor models. The mean value is around 0 in both cases with a standard deviation nearly to 1. The two cases (linear and non-linear) exhibit good properties. A reduction in the skewness and kurtosis of the residuals is observed compared to the original series. A reduction even higher is observed in the skewness and kurtosis in the non-linear case, suggesting a more accurate description and fit of the conditional second moment dynamics. The Ljung-Box test performed over the standardized residuals reveal a lack serial autocorrelation neither in levels nor in their cross-products. It is also removed the original heterokedasticty problem present in the original series.

The bottom of the table presents two moment tests to analyze the consistence of the QML estimations performed (Bollerslev and Wooldrige (1992)). These authors explain that, even in deviations from normality, consistent estimations are obtained if

$E_{t-1}\left(\hat{\in}_{i,t}\right)=0$, $E_{t-1}\left(\hat{\in}_{i,t}^{2}\right)=1$ and $E_{t-1}\left(\hat{\in}_{i,t}\hat{\in}_{j,t}\right)=0$ for i,j = m,b where $\hat{\in}_{i,t}$ are the standardized residuals.

The results obtained do not reject the null hypothesis assumed about the considered value of the two first order moments. These results confirm the consistency of the estimations of our models even for deviations from normality.

5.- Conclusion

This paper analyzes empirically the risk-return trade-off for the Spanish market using several proxies for the alternative investment set. We propose two multi-factor models considering conditional second moments according a bivariate GARCH specification based on theoretical frameworks which develop linear and non-linear relationships between return and risk. The results show that only a positive and significant risk-return trade-off is obtained in the non-linear case and only in the states governed by low volatility process (State 1). However, it is found no favorable evidence either in the linear framework or in high volatility states. These results support the findings of previous papers which present a procyclical risk aversion behaviour of investors. During low volatility states (associated with boom periods) investors are more risk averse than during high volatility periods (associated with crises). The investor profile in each context may also have influence in this lower risk aversion coefficient. The weight of the hedging component in the risk premium is less important than the market risk factor although the former has also a significant impact in low volatility periods. Strong assumptions of a linear relation between return and risk could lead to model misspecification and an inability of the empirical model to capture a significant risk-return relationship since the existence of periods where a risk-return trade-off is not observed could lead to non-significant estimation of this relation for the entire sample.

The risk premium evolution in Spain is close to the market volatility. The risk premium demanded for the investors presents a higher value than other sample periods during 2000-2003 and 2007-2009 (coinciding with crisis periods). Despite the decrease in the risk price during these periods, there is an extremely rise in the market risk that lead to higher risk premiums during the high volatility periods. The two multi-factor models also estimate noteworthy different risk premium during these periods. Non-linear models estimate higher risk premium during these periods, although for the rest of the sample the estimations are quite similar. Furthermore, the linear framework presents higher persistence of volatility shocks in the volatility formation during low volatility periods (and vice-versa). This fact is corrected with the introduction of the regime-switching, obtaining lower persistence volatility estimation in high volatility periods and higher persistence volatility estimation in low volatility periods.

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TABLE 1.- Summary statistics for excess market returns and intertemporal hedging proxies

Panel A.- Summary statistics							
	Excess market return	1-year T-bill	3-year T-bond	5-year T-bond	10-year T-bond	Averaged portfolio	Term Spread
Minimum	-23.032	-0.7516	-0.9480	-1.319	-3.220	-1.893	-1.298
Maximum	13.784	0.6022	1.1246	1.854	2.363	1.662	1.698
Median	0.1514	0.0116	0.0398	0.0606	0.0815	-0.0335	0.0489
Std. deviation	3.105	0.1039	0.2698	0.4447	0.6705	0.4706	0.4163
Skewness	-0.7825	0.4097	-0.1785	-0.1001	-0.3863	-0.3149	-0.0790
Kurtosis *(standarized)*	8.808	13.386	4.1338	3.7837	4.271	3.781	3.775
J-B	1085.68**	3256.52**	42.392**	19.631**	66.450**	30.198**	18.798**
L-B (6)	42.186**	61.842**	30.622	21.217	18.997	15.924	20.596
L-B^2 (6)	224.899**	251.798**	132.371**	151.362**	68.018**	67.018**	152.579**

Panel B.- Correlation matrix							
	Excess market return	1-year T-bill	3-year T-bond	5-year T-bond	10-year T-bond	Averaged portfolio	Term Spread
IBEX-35	1	-0.0105	-0.0830	-0.0508	-0.0317	-0.0523	-0.0516
1-year T-bill	·	1	0.4319	0.3813	0.3059	0.3603	0.1576
3-year T-bond	·	·	1	0.9525	0.8420	0.9265	0.9096
5-year T-bond	·	·	·	1	0.9313	0.9815	0.9729
10-year T-bond	·	·	·	·	1	0.9773	0.9184
Averaged portfolio	·	·	·	·	·	1	0.9585
Term Spread	·	·	·	·	·	·	1

*Panel A shows summary statistics for excess markets returns and alternative hedging proxies. JB is the Jarque-Bera test for normality distribution. LB(6) and LB^2(6) are the Ljung-Box test for serial autocorrelation in levels and squares respectively. (**denotes significance at 5% level). Panel B presents the correlation matrix for all the series included in this study.*

Table 2. Mean equation estimations for multi-factor models

		$r_{m,t} = \lambda_{10} + \lambda_{11}\sigma^2_{m,t} + \lambda_{12}\sigma_{mb,t} + \varepsilon_{m,t}$ $r_{b,t} = \lambda_{20} + \lambda_{21}\sigma_{bm,t} + \lambda_{22}\sigma^2_{b,t} + \varepsilon_{b,t}$					
		Model 2.a	Model 2.b	Model 2.c	Model 2.d	Model 2.e	Model 2.f
λ_{10}	R	0.1639	0.1192	0.1388	0.1851	0.1487	0.1297
	G	0.1722	0.1858	0.1706	0.1992	0.1782	0.1626
λ_{11}	R	0.0125	0.0221	0.0151	0.0126	0.0190	0.0221
	G	0.0107	0.0162	0.0163	0.0139	0.0176	0.0205
λ_{12}	R	-0.2247	0.4998	0.2283	-0.0171	0.2779	0.4398
	G	-0.0433	0.7222	0.3634	0.0127	0.4165	0.5212
λ_{20}	R	0.0008	0.0243**	0.0387**	0.0631**	-0.0505***	0.0173**
	G	-0.0033	-0.0103	-0.0187	-0.0040	-0.1169*	-0.0091
λ_{21}	G	0.2417	0.0701	0.0864	0.0402	0.0688	0.0495
λ_{22}	G	0.2597	0.6618	0.4032*	0.1720	0.3700	0.6155

*Estimated parameters for the mean equation in multifactor models. ***, **and * represents significance at 1%, 5% and 10% levels.*

Table 3. Variance equation estimations for multi-factor models

		$H_t = \begin{pmatrix} \sigma^2_{m,t} & \sigma_{mb,t} \\ \sigma_{mb,t} & \sigma^2_{b,t} \end{pmatrix} = CC' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BH_{t-1}B'$					
		Model 2.a	Model 2.b	Model 2.c	Model 2.d	Model 2.e	Model 2.f
c_{11}	R	0.3068***	0.3403**	0.3180***	0.2912***	0.2971**	0.3298**
	G	0.3017***	0.3276**	0.3139***	0.2886**	0.2972**	0.3276*
c_{12}	R	-0.0178**	-0.0256	-0.0350	-0.0427	-0.0622	-0.0183
	G	-0.0172	-0.0178	-0.0283*	-0.0407	-0.0475**	-0.0168
c_{22}	R	0.0241***	-0.0102	-2.80E-05	0.1192***	9.00E-06	0.01669
	G	0.0242**	8.76E-04	-2.50E-07	0.1162***	1.60E-08	-0.0102
a_{11}	R	0.2815***	0.2932***	0.2912***	0.2801***	0.3005***	0.2933***
	G	0.2777***	0.2890***	0.2903***	0.2804***	0.2965***	0.2948***
a_{12}	R	-0.0024	0.0052	0.0092	-0.0166	0.0034	0.0043
	G	-0.0019	0.0072*	0.0111**	-0.0141	0.0078	0.0052
a_{21}	R	-1.4984*	-0.7233	-0.4042	0.0651	-0.3442	-0.6339
	G	-1.5390	-0.4929	-0.3152	0.0618	-0.2850	-0.4930
a_{22}	R	0.5630***	0.2190***	0.1846***	0.2421***	0.1807***	0.2264***
	G	0.5659***	0.1968***	0.1733***	0.2397***	0.1603***	0.2146***
b_{11}	R	0.9556***	0.9509***	0.9527***	0.9586***	0.9514***	0.9526***
	G	0.9566***	0.9537***	0.9538***	0.9587***	0.9530***	0.9529***
b_{12}	R	0.0012*	-0.0022	-0.0040**	0.0047	-0.0019	-0.0020*
	G	0.0010	-0.0026	-0.0044***	0.0038	-0.0033	-0.0022
b_{21}	R	0.8897**	0.3620	0.1950	0.0038	0.1849	0.3643
	G	0.8853	0.2680**	0.1627	0.0084	0.1566	0.3219
b_{22}	R	0.8118***	0.9675***	0.9761***	0.9491***	0.9734***	0.9664***
	G	0.8121***	0.9740***	0.9786***	0.9515***	0.9795***	0.9706***

*Estimated parameters for the variance equation in the multi-factor models. ***, ** and * represents significance at 1%, 5% and 10% levels.*

Table 4.- Mean equation estimations for non-linear multi-factor models

$$r_{m,t,s_t} = \lambda_{10,s_t} + \lambda_{11,s_t}\sigma^2_{m,t,s_t} + \lambda_{12,s_t}\sigma_{mb,t,s_t} + \varepsilon_{m,t,s_t}$$

$$r_{b,t,s_t} = \lambda_{20,s_t} + \lambda_{21,s_t}\sigma_{bm,t,s_t} + \lambda_{22,s_t}\sigma^2_{b,t,s_t} + \varepsilon_{b,t,s_t}$$

		Model 2.a	Model 2.b	Model 2.c	Model 2.d	Model 2.e	Model 2.f
		Panel A. Low volatility state (s_t =1)					
$\lambda_{10,s_t=1}$	R	-1.1540**	-0.8954	-2.3156***	-2.3689***	-2.7943***	-2.4819**
	G	-0.8077	-1.5745***	-2.6614***	-2.5942***	-2.2375***	-4.0322***
$\lambda_{11,s_t=1}$	R	0.4044**	0.1169**	0.1311**	0.1867***	0.2980**	0.2270**
	G	0.3415**	0.1682**	0.1144***	0.1982***	0.0758**	0.3011**
$\lambda_{12,s_t=1}$	R	2.7521	4.6265	3.7124**	2.0452***	1.6691**	1.3646***
	G	7.6601***	1.8731**	-0.3294	2.5449***	0.0169**	1.6573***
$\lambda_{20,s_t=1}$	R	0.0099	0.0406**	0.0742**	0.0785	-0.0745**	0.0524**
	G	-0.0009	0.0096	0.4503	-0.0172	-0.2907**	0.1715***
$\lambda_{21,s_t=1}$	G	-0.1518	0.1997***	0.4438**	-0.0479	-0.8829***	-0.4219**
$\lambda_{22,s_t=1}$	G	1.8914	0.4396	0.9609	0.3025	0.2076	-1.3879**
Panel B. High volatility state (s_t =2)							
$\lambda_{10,s_t=2}$	R	-1.4062**	0.3597***	0.3436*	0.3502**	0.2323***	0.2662*
	G	-1.2897**	0.1993**	0.1891	0.3652**	0.3583***	0.2908**
$\lambda_{11,s_t=2}$	R	0.0733	0.0198	0.0137	0.0043	0.0191	0.0153
	G	0.0662	0.0302***	0.0337	0.0111	2.8310	0.0169
$\lambda_{12,s_t=2}$	R	-1.2216	0.3280	-0.3172	-0.3739	-0.0507	-0.7152
	G	-1.4985	0.3132	0.7456*	-0.4473	-0.0323	-0.2767
$\lambda_{20,s_t=2}$	R	-0.0027	0.0210	0.0354	0.0605	-0.0404**	0.0183**
	G	-0.0117***	-0.0355	-0.1469*	0.03480	-0.0768	-0.0184
$\lambda_{21,s_t=2}$	G	-0.1059	-0.1202*	0.1151	0.0637	0.0540	-0.0215
$\lambda_{22,s_t=2}$	G	0.9297	1.0195	1.1358**	0.0635	0.1869	0.7740*

*This table shows the estimated parameters for the mean equation in the non-linear multi-factor model. ***, ** and * represents significance at 1%, 5% and 10% levels.*

Table 5. Variance equation estimations for non-linear multi-factor models

$$H_{t,s_t} = \begin{pmatrix} \sigma^2_{m,t,s_t} & \sigma_{mb,t,s_t} \\ \sigma_{mb,t,s_t} & \sigma^2_{b,t,s_t} \end{pmatrix} = C_{s_t} C_{s_t}' + A_{s_t} \varepsilon_{t-1} \varepsilon_{t-1}' A_{s_t}' + B_{s_t} H_{t-1} B_{s_t}'$$

$$A_2 = sa \cdot A_1 ; \; B_2 = sa \cdot B_1$$

		Modelo 2.a	Modelo 2.b	Modelo 2.c	Modelo 2.d	Modelo 2.e	Modelo 2.f
c_{11}	R	1.3614***	0.6107***	0.4977***	0.5162***	0.3707***	0.4648***
	G	1.2338***	0.3035***	0.6063***	0.5194***	0.4119***	0.3766***
c_{12}	R	-0.0026	-0.0299*	-0.0254*	-0.0155	-0.0486***	-0.0072
	G	-0.0004	-0.0095	-0.0508***	-0.0073	-0.0170	0.0038
c_{22}	R	0.0346***	0.0689***	0.0798***	0.1836***	0.0590***	0.0577***
	G	0.0315***	-0.0273***	0.1324***	0.1988***	0.1173***	-0.0513***
a_{11}	R	0.1161**	0.0933	0.0114	0.2236***	0.3318***	0.2073***
	G	0.1011	0.4130***	0.2184***	0.2401***	0.2074***	0.1215***
a_{12}	R	0.0002	0.0053	-0.0001	-0.0165**	-0.0029	-0.0024
	G	0.0095	0.0014	0.0203*	-0.0230**	-0.0086*	0.0001
a_{21}	R	0.2580	0.1096	0.0040	-0.0143	-0.4763**	0.0340
	G	-0.0404	-0.1642	0.0716*	0.0687	0.2727*	0.1531
a_{22}	R	0.2748***	0.1227	0.0207	0.2369***	0.2318***	0.2699***
	G	0.2654***	0.2776***	0.3075***	0.2473***	0.2163***	0.1572***
b_{11}	R	0.9778***	1.0321***	1.0486***	1.0049***	0.9764***	1.0562***
	G	0.9851***	0.9545***	1.0183***	1.0021***	1.0405***	1.0502***
b_{12}	R	-7.50E-04	-0.0020*	-0.0008	0.0032	-0.0006	0.0007
	G	-0.0014*	-0.0011	-0.0064*	0.0059**	0.0017	-0.0009
b_{21}	R	0.9677	0.1022	0.0740	0.0211	0.1758***	-0.0523
	G	1.2557	0.1564	0.1670*	-0.0343	-0.0615	-0.0162
b_{22}	R	0.9111***	0.9857***	1.0132***	0.9742***	0.9856***	1.0213***
	G	0.9187***	0.9677***	0.9477***	0.9671***	1.0128***	1.0167***
sa	R	2.7440**	3.2140***	17.1565***	1.1736***	1.1979***	1.1136***
	G	2.8049***	1.0111***	1.1758***	1.0667***	1.0861***	1.8867***
sb	R	0.6393***	0.8370**	0.8918***	0.9111***	0.3590***	0.8743***
	G	0.6524***	0.2353**	0.8845***	0.9035***	0.8939***	0.8979***
p	R	0.98	0.97	0.98	0.98	0.98	0.97
	G	0.97	0.97	0.98	0.96	0.96	0.97
q	R	0.97	0.98	0.98	0.98	0.98	0.97
	G	0.96	0.96	0.97	0.98	0.97	0.96

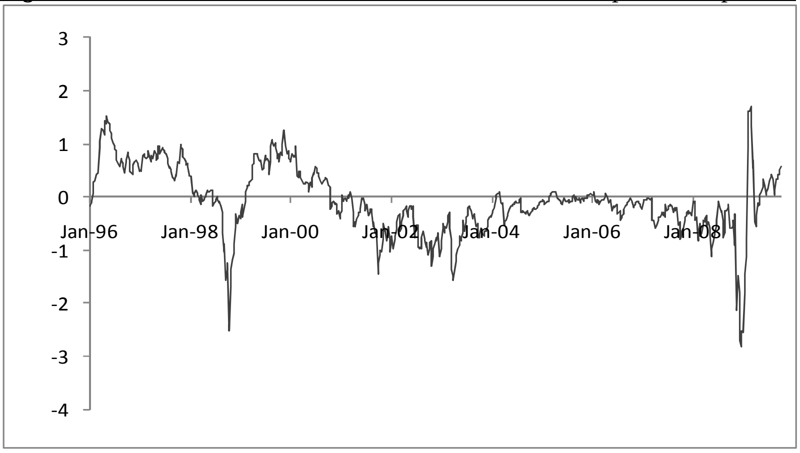
*Estimated parameters for the variance equation in the non-linear multi-factor models. ***, ** and * represents significance at 1%, 5% and 10% levels.*

Table 6.- Specification test for the standardized residuals

Panel A.- Linear Model	$\hat{\epsilon}_{m,t}$	$\hat{\epsilon}_{b,t}$	$\hat{\epsilon}^2_{m,t}$	$\hat{\epsilon}_{m,t}\hat{\epsilon}_{b,t}$	$\hat{\epsilon}^2_{b,t}$
Mean	-0.0643	0.0078	0.9850	0.0162	1.013
Std. Dev	0.9910	1.007	2.9696	1.377	1.760
Skewness	-1.072	-0.4084	18.9559	7.897	5.684
Kurtosis	9.857	4.0290	443.3336	138.792	61.417
J-B test	1546.78**	51.714**	5 851 793.34**	559 890.43**	106 109.44**
L-B (6)	24.507	16.609	6.927	20.2143	15.106
t-stat for H0:	-1.740	0.2096			
t-stat for H1:			-0.1354	0.3156	0.1993
Panel B.- Non linear-Model	$\hat{\epsilon}_{m,t}$	$\hat{\epsilon}_{b,t}$	$\hat{\epsilon}^2_{m,t}$	$\hat{\epsilon}_{m,t}\hat{\epsilon}_{b,t}$	$\hat{\epsilon}^2_{b,t}$
Mean	0.0271	-0.0037	1.075	0.0374	0.9877
Std. Dev	1.0375	0.9945	2.250	1.236	1.5652
Skewness	-0.4701	-0.3271	10.508	2.550	3.5096
Kurtosis	5.42261	3.50276	176.96414	32.28573	22.115248
J-B test	202.31153**	20.40111**	919 878.21**	26 473.38**	12 422.63**
L-B (6)	28.57888	17.44660	17.37963	10.78957	19.30694
t-stat for H0:	0.70157	-0.10188			
t-stat for H1:			0.90324	0.81193	-0.21049

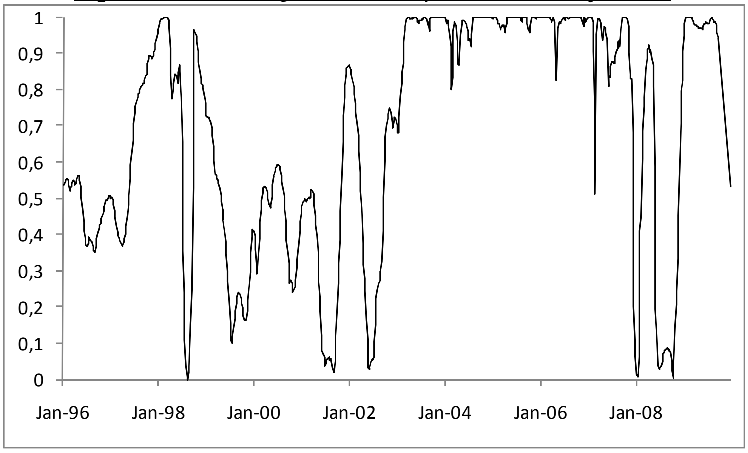
This table shows the statistics for the standardized residuals for both models used: GARCH-M and RS-GARCH framework. J-B test is the Jarque-Bera test for normality. L-B (6) is the Ljung-Box autocorrelation test including 6 lags. It also presents tests about the first two moments of the standardized residuals to validate consistent estimations of the QML procedure from deviations to normality. .***, **, * represent significance at 1%, 5% and 10% levels. H0 and H1 represent the t-statistic for the two moment order test developed in Bollerslev-Wooldrige (1992).

<u>Figure 1- Covariance excess market returns and intertemporal component</u>



Covariance between excess market returns and 10-year T-bond used as intertemporal hedging component.

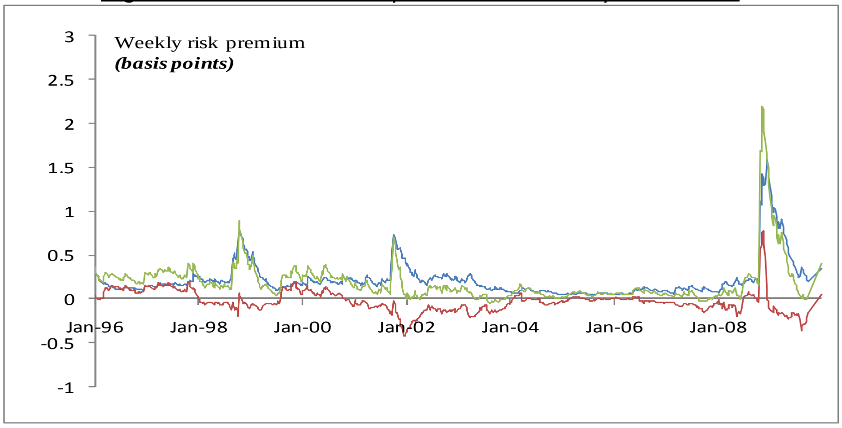
Figure 2.- Smooth probabilities for low volatility states



Probability of being in a low probability state for the case where the 10-yearT-bond is the alternative investment.

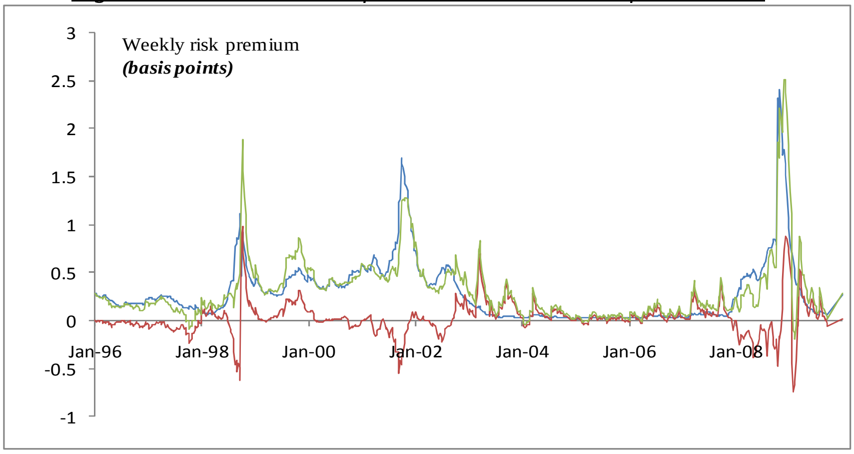
Figure 3 Risk Premium evolution in Spain

Figure 3.a.- Risk Premium for the linear multi-factor model



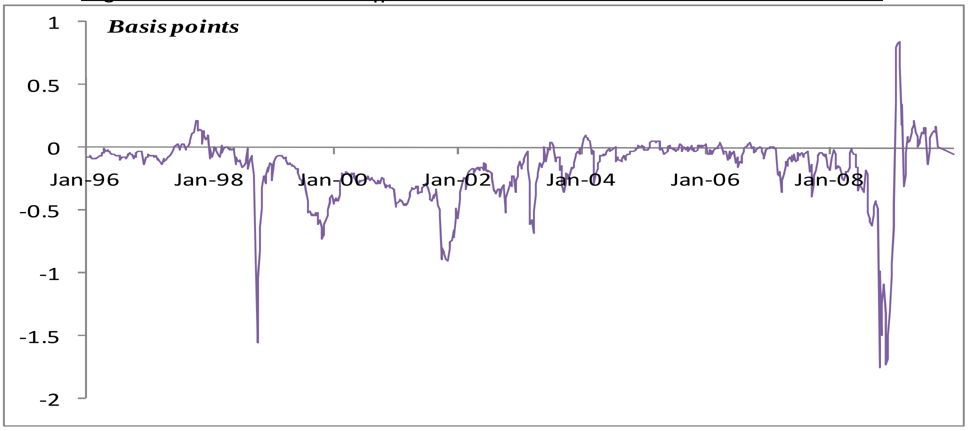
Estimated risk premium for the Spanish market using the linear multi-factor model. The greenline is the market risk, the red line is the premium associated with the hedging component and blue line represents the total risk premium.

Figure 3.b.- Risk Premium for the non- linear multi-factor model



Estimated risk premium for the Spanish market using the non- linear multi-factor model. The greenline is the market risk, the red line is the premium associated with the hedging component and blue line represents the total risk premium.

Figure 4.- Risk Premium differences between linear and non-linear models



Differences in the total risk premium estimated using linear and non-linear multi-factor model.

CHAPTER 4:

MEASURING THE HEDGING EFFECTIVENESS OF INDEX FUTURES CONTRACTS: DO DYNAMIC MODELS OUTPERFORM STATIC MODELS? A REGIME-SWITCHING APPROACH

Abstract

This paper estimates linear and non-linear GARCH models to find optimal hedge ratios with futures contracts for some of the main European stock indexes. By introducing non-linearities through a regime-switching model, we can obtain more efficient hedge ratios and superior hedging performance in both in- and out-sample analysis compared with other methods (constant hedge ratios and linear GARCH). Moreover, the non-linear models also reflect different patterns followed by the dynamic relationship between the volatility of spot and futures returns during low and high volatility periods.

1. Introduction

Over the past two decades with the development of derivatives markets, plenty of literature has focused on techniques to reduce investment risk. One simple technique for this purpose is hedging with futures contracts, which despite its simplicity has received extensive research attention. The literature on this subject is extensive and much of it focuses on determining the optimal hedge ratio (Myers and Thompson, 1989; Cheung et al., 1990; Chen et al., 2003). The most common approach is one that minimizes the variance of returns in a portfolio of spot and futures positions (Johnson, 1960).

The pioneering work using constant hedge ratios was performed by Ederington (1979). In this approach, the hedge ratio is $\left(HR = \sigma_{sf} / \sigma_f^2 \right)$. This hedge ratio is estimated through the slope of the ordinary least squares (OLS) regression between the spot and futures returns.

However, this approach exhibits several problems. One of them is that it does not account for the long-run disequilibrium between spot and futures markets (Ghosh, 1993; Lien, 1996). Another problem is that it assumes constant conditional second-order moments and, therefore, static hedging being not conditional on the arrival of information into the market. There are essentially two approaches to obtain dynamic hedge ratios. The first one consists of allowing hedge ratios to be time-varying coefficients and estimating these coefficients directly (Alizadeh and Nomikos, 2004; Lee et al., 2006). The second approach (Kroner and Sultan, 1991; Brooks et al., 2002) uses conditional second-order moments of the spot and futures returns from multivariate GARCH models, which allow for the estimation of hedge ratios at time *t* adjusted to the information set available to the investor at t–1: $\left(RC_t = \frac{\sigma_{s,f}}{\sigma_f^2} \middle| \Omega_{t-1} \right)$

Most of the literature has focused on this second approach, proposing increasingly complete models that more accurately capture the characteristics of the financial data and thereby overcome the limitations of the simpler GARCH models. One of the limitations of GARCH models is that they are incapable of reliably capturing the patterns of financial data series, specifically the asymmetric impact of news (Glosten et al., 1993; Engle and Ng, 1993; Kroner and Ng, 1998). Negative shocks are widely known to have a greater impact on financial series than do positive shocks. This fact should be taken into account when estimating hedge ratios. Brooks et al. (2002) conclude that hedging effectiveness is greater when this asymmetric behavior is considered. A further limitation of GARCH models is that they consider high volatility persistence. This high persistence level suggests the presence of several regimes in the volatility process (Marcucci, 2005). Ignoring these regime shifts could lead to inefficient volatility estimations. Therefore, the consideration of several regimes in the volatility process could lead to more accurate estimations of volatility and thus a better performance of hedging strategies. This approach is described in Hamilton and Susmel (1994), who use a switching ARCH (SWARCH) model to introduce regime switches.

Susmel (2000) analyzes the possibility of regime switches, but uses an E-SWARCH specification that also considers asymmetry, and concludes that both ARCH and asymmetric effects are reduced when regime switches are introduced.

In recent years, regime-switching models have taken on a new dimension with the development of Markov regime switching (MRS) models. Sarno and Valente (2000) propose a multivariate version of Hamilton's (1989) MRS model. Alizadeh and Nomikos (2004) were the first to use this methodology to estimate time-varying hedge ratios. Lee and Yoder (2007a) develop a new MRS-BEKK model in which they extend the work of Gray (1996) to the bivariate case. These studies propose a recombining method for conditional covariance matrices that allow the models to be tractable. They focus on modeling the variance and disregard the behavior of the mean. Alizadeh et al. (2008) incorporate an error correction term (ECT) that allows series characteristics to be related in the short- and long-run. The evidence from studies including regime switches shows more robust estimates are generated if volatility is allowed to follow different regimes depending on the market conditions, with the result that the hedge effectiveness will be greater (Alizadeh et al., 2008).

The main objective of this paper is to analyze the influence of non-linear patterns and regime switching on the effectiveness of dynamic hedging strategies and assess whether these models show an improvement over the simpler models usually performed in the literature. We compare the results for the estimated hedge ratios and the effectiveness found assuming linear and non-linear dynamics between the patterns followed by spot and futures returns. The study is performed for several European markets using the main stock index in each case (namely FTSE for the UK, DAX for Germany and Eurostoxx50 for Europe) and their future contracts considering an ex post and ex ante analysis, with the last approach closer to the decision process followed by an investor/hedger. The out-sample analysis also includes the last financial crisis to show the best hedging models in periods of market jitters.

In our empirical study, we use multivariate GARCH models. More specifically, we use the traditional BEKK model (Baba, Engle, Kraft and Kroner, 1990; Engle and Kroner, 1995) and estimate asymmetric BEKK models (Brooks et al., 2002) to include the well-known 'leveraged effect'[69] of volatility. Moreover, the existence of cointegration relationships between spot and futures markets leads us to the incorporation of an ECT in the mean equation (Ghosh, 1993; Lien, 1996). Finally, we also propose more complex models that consider non-linear relationships by using a regime-switching specification (Alizadeh et al., 2008), thereby allowing hedge ratios to be dependent on the state of the market and analyze whether the use of these more complex models leads to a significant hedging improvement. This approach let us compare the effectiveness of linear GARCH models with that of non-linear GARCH models.

[69] The 'leverage effect' is the different response of volatility to shocks of different sign (Nelson, 1991; Glosten et al., 1993).

The effectiveness of the hedging strategy is measured through several approaches. Firstly, we compute the variance reductions of the different hedging strategies over the unhedged portfolio (Ederington, 1979). Secondly, we analyze the economic significance of the risk reduction in terms of investor utility (Kroner and Sultan, 1993). Variance reduction is a good risk measure of a hedge strategy if the returns follow a normal distribution but this assumption is not always satisfied (Jei and Park, 2010). To avoid this problem, we also estimate alternative effectiveness measures based on loss distribution tails such as Value at Risk (VaR) (Jorion, 2000) and Expected Shortfall (ES) (Artzner et al., 1999).

Several authors (Myers, 1991; Kroner and Sultan, 1993; Park and Switzer, 1995) show that dynamic hedge ratios outperform constant hedge ratios in terms of reducing portfolio risk. However, there are some papers where the main conclusion is just the opposite, even considering several effectiveness measures (Lien and Tse, 2002; Cotter and Hanly, 2006; Jei and Park, 2010). One of the motivations behind this paper is to provide empirical evidence on these contradictory results and analyze whether more complex models better fit financial series patterns. Nevertheless, there is no strong evidence, as pointed out, on the ability of these models to improve the effectiveness found with simpler models, even the static OLS model[70].

The main contributions of the paper are the following. This empirical study is the first to apply such a database, both considering the time horizon analyzed and the different stock indexes used. It also introduces a model that includes different volatility processes with a MRS-GARCH approach that also considers the asymmetric response of volatility to shocks of different signs and the cointegration (long-run equilibrium) price followed by futures and spot markets[71] to analyze the effectiveness of the hedging strategy. The findings show that considering non-linearities in the volatility specification leads to differences in the estimations and forecasts of volatility. These differences have an impact on the hedge ratios obtained and the effectiveness reached, causing non-linear models to achieve better effectiveness. The last result coincides with Lien (2009), who points out that the existence of structural breaks in financial series may improve the performances of dynamic models or at least that the consideration of these in estimated models improves effectiveness. Finally, this result is robust across countries and independent of the effectiveness measure considered.

The outline of the paper is as follows. Section 2 presents the database used in the study. Section 3 introduces the empirical methodology. Section 4 shows the main empirical results of the study analyzing the optimal hedge ratio estimations and the effectiveness measures proposed. Finally, we present the main conclusions of the study.

[70] Lien (2009) analyzes and demonstrates why static models (OLS) may outperform more complex models.

[71] In their respective studies, Alizadeh et al. (2008) use an ECT-MRS-diagonal-BEKK specification and Jei and Park (2010) use several linear bivariate GARCH models; however, to the best of our knowledge, no paper considers cointegration, regime switching and volatility asymmetries in the same model.

2. Description of the data and preliminary analyses

The data used in this study include weekly closing prices[72] (Alizadeh and Nomikos, 2004; Alizadeh et al., 2008; Chen and Tsay, 2011) for some of the main European stock indexes and their futures contracts. Specifically, we use the information on the UK (FTSE100), Germany (DAX30), and Europe (Eurostoxx50). The time horizon includes observations from 1 July 1998 to 30 September 2010. We divide this data into two sub-samples: observations from 1 July 1998 to 31 December 2008 (548 observations) are used for the in-sample analysis and observations from 1 January 2009 to 30 September 2010 (92 observations) are used for the out-sample study. We obtained the indexes data from Thomson DataStream and the futures information from the Institute of Financial Markets Data Center.

We construct the continuous futures series using the contract closest to maturity[73]. Weekly returns series are computed as the logarithmic differences multiplied by 100.

$$r_{i,t} = 100\left(\log\frac{P_{i,t}}{P_{i,t-1}}\right) \qquad para \quad i=\{s, f\} \tag{1}$$

Descriptive statistics are presented in Table 1. Panel A shows the main summary statistics for the spot and futures indices. Certain results are noteworthy. For the returns, negative values are present in the third-order moments. These statistics further justify including the asymmetric term when finding hedge ratios. There is also excess kurtosis in the returns (fat tails); this finding suggests that the variances of the series may be time varying. Finally, note that the Jarque–Bera normality test (1980) is rejected because of the asymmetric and leptokurtic characteristics of the series. Results for the out-sample period differ only slightly from those of the in-sample period[74]. Panel B displays the serial autocorrelation tests for the series in levels and squares. The Ljung–Box statistics for the squared series suggest evidence of conditional heteroscedasticity for both series. There is also evidence of serial correlation for returns in levels so it is necessary include structure (lags) in the mean equation. Panel C reflects the stationarity tests performed over the price series and reveals that the price series are I(1), so we have to work with the returns series for stationarity reasons. Finally, panel D presents the results of the cointegration tests for the series studied. The results also show that both series are cointegrated. Therefore, these relationships will be introduced in the specification of the model used to calculate the hedge ratios, since otherwise we would obtain inefficient hedges (Lien, 1996).

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[72] Wednesday closing prices are used as weekly observations. If a Wednesday is not available in a week is replaced by the Tuesday in that week.

[73] Carchano and Pardo (2008) show that rolling over the futures series has no significant impact on the resultant series. Therefore, the least complex method can be used for series construction to reach the same conclusions.

[74] The out-sample data run from 1 January 2009 to 30 September 2010 (observations). The descriptive statistics, not presented in the paper, are available from the authors upon request.

3. Methodology

This section explains and develops the empirical models used to estimate time varying volatilities and hedge ratios. We start with the symmetric and asymmetric linear specifications (BEKK and GJR-BEKK) to model the dynamic relationship between spot and futures returns. After that, we assume non-linear dynamics through a regime-switching process, thereby allowing hedge ratios to be dependent on the state of the market.

3.1. Linear bivariate GARCH models

Linear bivariate GARCH models have been widely used in the analysis of dynamic hedge ratios (Baillie and Myers, 1991; Park and Switzer, 1995). One of the most frequently used is the BEKK model (Baba et al., 1990) since it incorporates certain characteristics[75] that make it particularly attractive for this type of study. In this specific case, we incorporate an ECT in the mean equation because both series are cointegrated. Let $r_{s,t}$ and $r_{f,t}$ be the spot and futures returns at period t respectively; thus, we define the mean equation as:

$$r_{s,t} = a_0 + a_1 r_{s,t-1} + a_2 r_{f,t-1} + a_3 ECT_{t-1} + \varepsilon_{s,t} \tag{2}$$

$$r_{f,t} = b_0 + b_1 r_{f,t-1} + b_2 r_{s,t-1} + b_3 ECT_{t-1} + \varepsilon_{f,t} \tag{3}$$

$$\varepsilon_t \mid \Omega_{t-1} = \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix} \mid \Omega_{t-1} \sim BN(0, H_t) \tag{4}$$

where a_i, b_i for $i = \{0,1,2,3\}$ are the parameters to be estimated. The sub-indices *s* and *f* indicate spot or futures respectively, $\varepsilon_{s,t}$ and $\varepsilon_{f,t}$ indicate innovations, Ω_{t-1} denotes the information set available up to t–1, BN refers to the bivariate normal distribution and H_t is a positive definite time-varying 2×2 matrix defined as follows:

$$H_t = \begin{pmatrix} h^2_{s,t} & h_{fs,t} \\ h_{sf,t} & h^2_{f,t} \end{pmatrix} = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \tag{5}$$

where C is a lower triangular matrix of constants and A and B are 2×2 square matrices of coefficients to be estimated.

[75] The main advantage of this model is that it guarantees that the covariance matrix will be a positive definite by construction (quadratic form).

Assuming that the innovations follow a bivariate normal distribution, the unknown parameters $\theta = (a_i, b_i, C^l_{2x2}, A_{2x2}, B_{2x2})$ for $i = \{0,1,2,3\}$ are estimated by maximizing the following likelihood function with respect to θ:

$$f(r_t;\theta) = (2\pi)^{-1} \left|H_t(\theta)\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\varepsilon_t(\theta)' H_t^{-1} \varepsilon_t(\theta)\right) \tag{6}$$

$$L(\theta) = \sum_{i=1}^{T} \log f(r_t;\theta) \tag{7}$$

where T is the number of observations.

GARCH models allow us to obtain an estimation of the variance–covariance matrix for each period. We obtain the dynamic hedge ratio (HR_t) estimations, according to the expression (8):

$$HR_t = \frac{\hat{h}_{sf,t}}{\hat{h}^2_{f,t}} \tag{8}$$

This simplest variance specification (shown in 5) can be used to incorporate other financial series characteristics such as asymmetries in volatility. One of the most popular approaches in the literature is the GJR model of Glosten et al. (1993), which uses specific variables to incorporate this asymmetric behavior.

$$H_t = \begin{pmatrix} h^2_{s,t} & h_{fs,t} \\ h_{sf,t} & h^2_{f,t} \end{pmatrix} = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1} A + B' H_{t-1} B + D'\eta_{t-1}\eta'_{t-1} D \tag{9}$$

where D is a diagonal 2×2 matrix of parameters to be estimated and $\eta_t = \min\ (\varepsilon_t, 0)$. The remaining parameters and variables are the same as those in equations 2–4 and the estimation procedure is similar to that above.

3.2. Non-linear bivariate GARCH models

In contrast to previous models, in which the dynamic relationship between spot and futures returns is characterized by linear patterns, the model presented by Lee and Yoder (2007a) allows regime shifts, which suggests that one can obtain more efficient hedge ratios and superior hedging performance compared with other methods. These types of non-linear models open up a new line for dynamic hedging in which the returns process is state-dependent.

Let $r_{s,t,st}$ and $r_{f,t,st}$ be the state-dependent spot and futures returns at t respectively; we define the state-dependent mean equations as:

$$r_{s,t,st} = a_0 + a_1 r_{s,t-1} + a_2 r_{f,t-1} + a_{3,st} ECT_{t-1} + \varepsilon_{s,t,st} \tag{11}$$

$$r_{f,t,st} = b_0 + b_1 r_{f,t-1} + b_2 r_{s,t-1} + b_{3,st} ECT_{t-1} + \varepsilon_{f,t,st} \tag{12}$$

$$\varepsilon_{t,st} \mid \Omega_{t-1} = \begin{pmatrix} \varepsilon_{s,t,st} \\ \varepsilon_{f,t,st} \end{pmatrix} \mid \Omega_{t-1} \sim BN(0, H_{t,st}) \tag{13}$$

where a_i, b_i for $i = \{0,1,2,3\}$ are the parameters to be estimated. For computational tractability, they are not considered to be state-dependent. However, following Alizadeh et al. (2008), the parameters accompanying the ECT depend on regime $s_t = \{1,2\}$.

The state-dependent innovations $\varepsilon_{t,st}$ follow a bivariate normal distribution that depends on state $s_t = \{1,2\}$. This state variable follows a two-state first-order Markov process with transition probabilities:

$$\hat{P} = \begin{pmatrix} \Pr\left(s_t = 1 \middle| s_{t-1} = 1\right) = p & \Pr\left(s_t = 1 \middle| s_{t-1} = 2\right) = (1-q) \\ \Pr\left(s_t = 2 \middle| s_{t-1} = 1\right) = (1-p) & \Pr\left(s_t = 2 \middle| s_{t-1} = 2\right) = q \end{pmatrix} \tag{14}$$

where p represents the probability of continuing in state 1 if it was previously in state 1 and q represents the probability of continuing in state 2 if it was previously in state 2.

The state-dependent conditional second-order moments $H_{t,st}$ follow an asymmetric BEKK[76] specification model that takes different values depending on the value of $s_t = \{1,2\}$. Because of this state dependence, the model will become intractable as the number of observations increases. In order to resolve this problem we apply the recombining method used in Gray (1996) where the path dependency problem is solved for univariate models. Lee and Yoder (2007a) extend this recombining method for the bivariate case. Thus, the variance specification in each state is defined as follows:

$$H_{t,st} = \begin{pmatrix} h^2_{s,t,st} & h_{fs,t,st} \\ h_{sf,t,st} & h^2_{f,t,st} \end{pmatrix} = C'_{st} C_{st} + A'_{st} \varepsilon_{t-1} \varepsilon'_{t-1} A_{st} + B'_{st} H_{t-1} B_{st} + D'_{st} \eta_{t-1} \eta'_{t-1} D_{st} \tag{15}$$

where $h^2_{c,t,st}$ and $h^2_{f,t,st}$ are the conditional variances of the spot and futures in period t for each state s_t and $h_{cf,t,st}$ is the conditional covariance in t for each s_t. C_{st}, A_{st}, B_{st} and D_{st} are the matrices of parameters to be estimated as in previous models.

[76] We also present the results for the symmetric MRS-BEKK model. This model is similar to that presented in the paper except for the variance equation where the last summation $D'_{st}\eta_{t-1}\eta_{t-1}D_{st}$ is not considered.

The consideration of several states leads to a noteworthy rise in the number of parameters to estimate. In order to reduce this over-parameterization the difference among states is defined by four new parameters *sa*, *sb*, *sc* and *sd* that properly weight the estimations obtained in one state for the other state[77]. Therefore, the state-dependent covariance matrices in our model are:

$$H_{t,s_t=1} = \begin{pmatrix} h^2_{s,t,1} & h_{fs,t,1} \\ h_{sf,t,1} & h^2_{f,t,1} \end{pmatrix} = C_1C_1' + A_1\varepsilon_{t-1}\varepsilon'_{t-1}A_1' + B_1H_{t-1}B_1' + D_1'\eta_{t-1}\eta'_{t-1}D_1 \tag{6.1}$$

$$H_{t,s_t=2} = \begin{pmatrix} h^2_{s,t,2} & h_{fs,t,2} \\ h_{sf,t,2} & h^2_{f,t,2} \end{pmatrix} = C_2C_2' + A_2\varepsilon_{t-1}\varepsilon'_{t-1}A_2' + B_2H_{t-1}B_2' + D_2'\eta_{t-1}\eta'_{t-1}D_2 \tag{6.2}$$

where $C_2 = sc{\cdot}C_1$, $A_2 = sa{\cdot}A_1$, $B_2 = sb{\cdot}B_1$, $D_2 = sd{\cdot}D_1$, A_1 and B_1 are 2×2 matrices of parameters, C_1 is a 2×2 lower triangular matrix of constants and D_1 is a diagonal 2×2 matrix of parameters.

The basic equations of the recombining method[78] used to collapse the variances and covariances of the spot and futures errors and to ensure the model is tractable are described below:

$$h^2_{i,t} = \pi_{1,t}\left(r^2_{i,t,1} + h^2_{i,1,t}\right) + \left(1-\pi_{1,t}\right)\left(r^2_{i,t,2} + h^2_{i,2,t}\right) - \left(\pi_{1,t}r_{i,t,1} + \left(1-\pi_{1,t}\right)r_{i,t,2}\right)^2 \quad \text{for i}=\{s, f\} \tag{16}$$

$$\varepsilon_{i,t} = \Delta S_t - \left(\pi_{1,t}r_{i,t,1} + \left(1-\pi_{1,t}\right)r_{i,t,2}\right) \tag{17}$$

$$\begin{aligned} h_{cf,t} &= \pi_{1,t}\left(r_{s,t,1}r_{f,t,1} + h_{sf,1,t}\right) + \left(1-\pi_{1,t}\right)\left(r_{s,t,2}r_{f,t,2} + h_{sf,2,t}\right) \\ &-\left(\pi_{1,t}r_{s,t,1} + \left(1-\pi_{1,t}\right)r_{s,t,2}\right)\ -\left(\pi_{1,t}r_{f,t,1} + \left(1-\pi_{1,t}\right)r_{f,t,2}\right) \end{aligned} \tag{18}$$

where $h^2_{i,t}$, $h_{sf,t}$ are the state-independent variances and covariances aggregated by the recombining method and $h^2_{i,st,t}$, $h_{sf,st,t}$ are the state-dependent variances and covariances for $s_t = \{1,2\}$.

The terms $r_{i,t,st}$ represent the state-dependent mean equations and $\pi_{1,t}$ is the probability of being in state 1 at time t obtained by the expression:

$$\pi_{1,t} = p\left(\frac{g_{1,t-1}\pi_{1,t-1}}{g_{1,t-1}\pi_{1,t-1} + g_{2,t-1}\left(1-\pi_{1,t-1}\right)}\right) + \left(1-q\right)\left(\frac{g_{2,t-1}\left(1-\pi_{1,t-1}\right)}{g_{1,t-1}\pi_{1,t-1} + g_{2,t-1}\left(1-\pi_{1,t-1}\right)}\right) \tag{19}$$

[77] The economic interpretation of the parameters sc, sa, sb and sd is how much the constant term, the weight of the shocks, the weight of the past variance and the impact of negative shocks on the volatility formation differ between each state respectively.
[78] For further details on the recombining method, see Gray (1996) and Lee and Yoder (2007a).

where

$$g_{i,t} = f\left(r_t \middle| s_t = i,\ \Omega_{t-1}\right) = (2\pi)^{-1} \left|H_{t,i}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\varepsilon_{t,i}^{'} H_{t,i}^{-1} \varepsilon_{t,i}\right\} \quad \text{for } i = \{1,2\} \tag{20}$$

and p and q are as described in equation 14.

Thus, the parameters of the model can be estimated with the following maximum likelihood function:

$$\begin{aligned} f\left(r_t \middle| \theta\right) = \pi_{1,t} &\left[(2\pi)^{-1} \left|H_{t,1}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\varepsilon_{t,1}^{'} H_{t,1}^{-1} \varepsilon_{t,1}\right\}\right] + \\ \pi_{2,t} &\left[(2\pi)^{-1} \left|H_{t,2}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\varepsilon_{t,2}^{'} H_{t,2}^{-1} \varepsilon_{t,2}\right\}\right] \end{aligned} \tag{21}$$

$$L(\theta) = \sum_{i=1}^{T} \log\ f(r_t;\theta) \tag{22}$$

Based on the estimations obtained, we calculate the optimal hedge ratio from the results of the state-independent covariance matrix given by the recombining method, substituting the resulting second-order moments in expression (8).

4. Empirical results

This section presents the main empirical results of the study. Section 4.1 shows the parameter estimation results for all the models proposed. Section 4.2 describes the volatility evolution and the hedge ratios estimated using each model. Section 4.3 proposes several effectiveness measures to analyze the performances of the different hedging policies. Finally, section 4.4 performs specification tests over the estimation residuals to detect any problems related with a potential misspecification of the empirical model.

4.1 Model estimation

In this section, we show the evolution of the patterns followed by the volatility[79] in the linear and non-linear frameworks proposed in the study. The estimations of the models are presented in Table 2 for all the European markets considered. A two-state specification is used for the MRS models. This specification allows the states to be associated with high and low volatility regimes[80].

For each market in Table 2, the first two columns show the parameter estimations for the linear models (BEKK and ASYM-BEKK). We can observe that the linear models reflect in most cases a weak significance of the parameters representing the persistence of the impact of shocks in volatility (a11, a22). Furthermore, the impact of one market's shocks on the other markets’ volatility is generally not significant (a12, a21).

[79] We focus mainly on the interpretation of the variance equation parameters since this determines the estimated covariance matrix and, therefore, the optimal hedge ratio.
[80] Sarno and Valente (2000) use a three-state process, but the third state seems to capture spurious state changes that are not related to market regime switches.

The evidence for a significant influence of past volatility on volatility formation is more evident both for spot (b11) and futures markets (b22) but this is not observed for the cross parameters (b12, b21). Generally, there is also an asymmetric response of volatility against negative shocks, although in markets such as Europe and the UK this evidence is only observed in the futures markets (d22).

Finally, there is another remarkable result about volatility dynamics; the persistence level in linear models is relatively high. This result suggests the presence of several regimes in the volatility process and, therefore, potential non-linearities and the adequacy of using MRS-GARCH models.

The last two columns of Table 4 reflect the estimations for non-linear models (MRS-BEKK and MRS-ASYM-BEKK). In our model, the dynamic relationship between spot and futures returns is dependent on two states of the market. The states can be associated with low and high volatility periods using the median of the estimated state-dependent volatilities for the stock indexes[81], which present a value of 6,766 *(6,510)* for state 1 and 8,188 *(7,021)* for state 2 in Europe, 7,192 *(2,426)* for state 1 and 2,588 *(5,693)* for state 2 in the UK and 8,882 *(7,660)* for state 1 and 9,359 *(8,153)* for state 2 in Germany[82]. Therefore, the state with the highest value of estimated conditional variance in each model corresponds to the high volatility state.

Moreover, Figure 1 shows the smooth probability of being in the low volatility state in each data series used[83]. The figure corresponding to Europe is governed essentially by this state, which corresponds with a calm period in financial markets (2003–2007). When the state governing the process is state 2, this corresponds to periods of market jitters such as the dot-com bubble (2002–2003) and the last financial crisis (2008). The probabilities for the rest of markets share these periods of high volatility states and, moreover, present other high volatility periods probably related with their own country-idiosyncratic market evolution.

It is also interesting to analyze the differences in the volatility parameters between states of the market in non-linear models. For example, the constant term is usually lower in low volatility states than it is in high volatility states[84]. That is understandable because the constant term in our model reflects the unconditional volatility, and this is supposed to be higher in high volatility states. Second, the presence of shocks on volatility formation is higher in high volatility states than it is in low volatility states.[85] However, the impact of past variance on the formation of volatility is lower in high volatility

[81] The estimated volatility for the futures indexes follow the same order and they are not displayed to save space. The results are available from the authors upon request.
[82] Values in parentheses refer to medians in the asymmetric models.
[83] The estimation process itself determines whether state 1 corresponds to high or low volatility states. Depending on the country, state 1 could refer to a high volatility state in one market and to a low volatility state in another market. The figure represents the probability of low volatility states.
[84] In the models where state 1 corresponds to low volatility periods this is observed because the scale sc for state 2 (high volatility) is higher than 1; in the cases where state 1 corresponds to high volatility periods, the scalar sc is lower than 1.
[85] Similar to footnote 13, namely using the scalar sa instead of sc.

states than it is in low volatility states[86]. There seems to be a trade-off between the impact of shocks and past variance on the formation of volatility between states. In low volatility states, there is a greater past variance persistence and a lower presence of shocks in volatility. In high volatility states, there is a higher presence of shocks but a lower impact of past variance. These results are similar to those of Marcucci (2005), who explain these differences in volatility dynamics between low and high volatility periods by arguing that there is a greater amount of news during high volatility periods. Therefore, the continuous arrival of new information into the market causes volatility formation to occur largely because of the impact of these shocks rather than the past variance observed in the market, as occurs in low volatility periods when less news affects the markets. Finally, we find that the asymmetric response of volatility is significant in spot and futures markets in the non-linear specification. We also find that there is a different asymmetric response of volatility in low and high volatility periods. However, there is no common result on how the asymmetric response changes with volatility regime. In Europe and Germany, this asymmetric response is higher in low volatility periods, while it is less acute in high volatility periods but in the UK, the opposite occurs.

We also considered it interesting to determine the average durations of the different states in the economy. This duration value can be obtained according to the transition probability estimates p and q in equation 14. For example, Europe presents a value of $p=0.966$ and $q=0.962$; this means that once in state 1, the probability of remaining in that state is 96.6%, while the probability of remaining in state 2 is 96.2%. Therefore, the average duration of being in state 1 when the volatility process is governed by this state will be approximately 29 weeks (1/(1–0.966)). A similar duration can be calculated in the high volatility regime state (1/(1–0.962)). This indicates that the regime switches present a smooth evolution, keeping the process in each state during relatively long periods. For the remainder of markets these values are very similar.

4.2.- Volatility and hedge ratios

At this point, it is interesting to analyze the evolution and differences in the estimated variances obtained in each model, which will then lead us to the differences in the estimated hedge ratios. The estimated covariance matrix for the linear models is obtained using equation 5 for the symmetric and equation 9 for the asymmetric cases. For a proper comparison between models, we use the estimation for the independent covariance matrix (equations 15, 16 and 18) for the non-linear models. Figure 2 shows the estimated variance for the spot market[87] for all markets considered.

[86] In this case, when state 1 corresponds to low volatility periods the scale sb for state 2 (high volatility) is lower than unity; in the cases where state 1 corresponds to high volatility periods, the scalar sb is higher than 1.

[87] For brevity, only the spot market volatility is shown. The estimated volatilities for futures markets and the covariance between spot and futures markets are similar. The results are available from the authors on request.

All figures seem to exhibit similar patterns, although there are obvious differences between them. Common to all the estimations, there are two periods corresponding to 2001–2003 and 2008 that present higher estimations of volatility. These periods of high volatility coincide with the dot-com bubble and the last financial crisis, which are periods of market jitters. Figure 1 shows that the mentioned periods correspond with periods governed by high volatility states and the rest of the sample is often governed by low volatility states. The volatility estimations in high volatility periods using non-linear models are higher than are those obtained with linear models, but in the rest of the sample coinciding with calm periods the volatility estimations using linear models are higher than are those obtained with non-linear models[88]. If we do not distinguish between states, one state would define the volatility process, and this may not properly reflect the patterns during turbulent periods, which exhibit different dynamics than do those present during calm periods. Therefore, the volatility estimations tend to be underestimated using linear GARCH models in the periods corresponding to high volatility states and overestimated in low volatility periods, and this may influence the effectiveness of the hedge policy.

Finding the optimal hedge ratios for the in-sample analysis is simple. For linear GARCH models, we use equation 8 and the covariance matrix estimates at each moment t (Kroner and Sultan, 1993). For non-linear models, we also use equation 8 and the state-independent estimations of the covariance matrix.

Finding hedge ratios for the out-sample period is more complex and differs by model. Common to all models is the construction of a rolling window in which the model is re-estimated for each window period, removing the first observations and adding new ones as the window advances. The parameter values are found for each estimation period, which allows us to make one period ahead forecasts of the covariance matrix. Note that this procedure is performed for the linear BEKK models both with and without asymmetries.

The process of forecasting the covariance matrix for the non-linear BEKK models with (and without) asymmetries is more complex because of the existence of two possible states. This forecast is performed in a three-stage process (Alizadeh et al., 2008). In the first stage, we use the estimations of the transition matrix in t (equation 14) and the smoothed probabilities in t to obtain the prediction of the probability of being in each one of the two states $s_t = 1, 2$ in the period t+1.

$$E\begin{bmatrix} \pi_{1,t+1} \\ \pi_{2,t+1} \end{bmatrix}' = \begin{pmatrix} \hat{\pi}_{1,t} \\ \hat{\pi}_{2,t} \end{pmatrix} \begin{pmatrix} \hat{p} & (1-\hat{q}) \\ (1-\hat{p}) & \hat{q} \end{pmatrix} \tag{23}$$

[88] Using the filtered probability in each market, we find that the average for high volatility states using symmetric linear models in Europe, the UK and Germany are 8.83, 21.14 and 18.51 respectively, while for the non-linear case they are 9.10, 22.33 and 22.93 respectively. For low volatility states, the average estimated volatility is 4.51, 9.12 and 11.92 for linear models against 3.14, 8.55 and 10.98 for non-linear models.

In the second stage, we make a prediction one period ahead of the state-dependent mean and variance equations (equations 11–13 and 15) using the parameters estimated. In the third stage, the recombining method is used as in equations 16–18 to obtain the predictions of the state-independent covariance matrix. Once we have the one period ahead prediction of the covariance matrix for each model, we obtain the predicted hedge ratio using the equation 8 for t+1.

Figure 3 presents the hedge ratios obtained for both the in-sample and out-sample period, together with their evolution[89]. The top figures show the evolution for the in-sample analysis and the bottom graphs reflect the forecasts performed for each model. We compare symmetric (MRS-BEKK) against asymmetric (MRS-ASYM-BEKK) non-linear models on the left-hand figures and linear (BEKK) against non-linear (MRS-BEKK) on the right-hand side, with the continuous line, the MRS-BEKK model and the alternatives in each case plotted with dashed lines.

The differences among models are evident both between linear (dashed line) and non-linear (continuous line) specifications and between symmetric (continuous line) and asymmetric (dashed line) specifications (Table 3). There exist differences in the averages and in the variability of the estimated and forecasted ratios. Therefore, it seems as though the omission or inclusion of one of these characteristics could lead to significant differences in the estimated hedge ratios and, therefore, in the effectiveness reached. Therefore, concerning the evident differences between the estimated and forecast hedge ratios obtained in each strategy, we try to explain in the next section which hedge strategy allows us to achieve a more effective hedge policy. The study in the next section is especially appealing because the out-sample analysis is performed over the period of the recent financial crises and could thus prove which models work better in periods of market uncertainty.

4.3.- Hedging effectiveness

To analyze hedging effectiveness we consider four different measures. The first two measures are based on the variance of the loss distribution of the hedge portfolio. The first approach is the variance of the hedged portfolio (Ederington, 1979) for each model compared with an unhedged portfolio, that is $RC_t = 0$ for all t. The variance of the hedged portfolio is:

$$Var\left(x_t \left|\Omega_{t-1}\right.\right) = Var\left(\left(\Delta S_t - RC_t * \Delta F_t\right)\left|\Omega_{t-1}\right.\right) \qquad (24)$$

Another commonly used approach is to analyze the economic benefits of the hedging (Kroner and Sultan, 1993) by constructing the investor’s utility function based on the return and risk of the hedge portfolio.

[89] The estimated hedge ratios of the remaining models are not presented here for brevity, but are available from the authors upon request.

This measure is motivated by the fact that dynamic strategies are most costly to implement since they require a frequent updating of the hedge portfolio. In line with studies such as Park and Switzer (1995) and Meneu and Torró (2003), the utility function is constructed in a mean/variance context:

$$E\left[U(x_t\left|\Omega_{t-1}\right.)\right]=E\left[x_t\left|\Omega_{t-1}\right.\right]-\lambda Var\left[x_t\left|\Omega_{t-1}\right.\right] \qquad (25)$$

where λ is the investor's level of risk aversion (normally $\lambda = 4$) and the hedged portfolio returns are also assumed to present an expected value equal to 0 (Alizadeh et al., 2008).

The third metric proposed is based on the VaR measure (Jorion, 2000). The VaR of the hedged portfolio at the confidence level q is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1-q)$. In our case, this is calculated by the sample quantiles using the empirical distribution of the hedge portfolio returns.

$$VaR_q=\inf\left\{l\in\Re : P\left(L>l\right)\leq 1-q\right\} \qquad (26)$$

The last effective measure is based on the ES of the hedged portfolio (Artzner et al., 1999). ES is an alternative to VaR in that it is more sensitive to the shape of the loss distribution in the tail of the distribution. The ES at the $q\%$ level is the expected return on the portfolio in the worst $q\%$ of the cases.

$$ES_q=E\left(x\left|x<\mu\right.\right) \qquad (27)$$

where μ [90] is determined by $\Pr\left(x<\mu\right)=q$ and q is the given threshold, while x is a random variable that represents profit during a specified period.

Table 4 summarizes the hedging strategy effectiveness for all the series used in the study. It shows the four effectiveness measures both for in-sample and out-sample analysis and for all linear and non-linear models proposed as well as the effectiveness achieved by using a constant OLS strategy and by the unhedged portfolio.

Panel A presents the effectiveness analysis for the in-sample period in all countries considered. The highest effectiveness considering the reduction of variance of the hedge portfolio is observed in the MRS-ASYM-BEKK in the UK, Germany and Europe. That is, non-linear models outperform the effectiveness of the rest of the models in terms of variance reduction. Another interesting result arises here. The effectiveness of the OLS strategy outperforms in all cases (except the UK) the linear GARCH hedging strategies[91]. This result is the same as those found in studies such as Lien (2009), Lien

[90] Note that μ is the value at risk.

[91] Lien (2009) shows that variance-based metrics reflect the reduction of the unconditional volatility of the hedge portfolio. Therefore, OLS strategies reach the greatest variance reduction by definition, whereas the linear GARCH strategies achieve a reduction on the conditional variance.

and Tse (2002), Cotter and Hanly (2006) and Jei and Park (2010). These authors find that constant strategies present better effectiveness than do dynamic strategies. However, when we consider non-linear strategies, these more complex models outperform the rest of the policies. Generally, the introduction of non-linearities in the models lets us achieve a greater fit to the data because of the identification of different regimes in the volatility process and the more accurate estimation. Therefore, this non-linear specification outperforms both the linear models and constant strategies. The utility analysis reaches a similar conclusion because these first two measures are both based on the variance of the hedge portfolio loss distribution. However, as Jei and Park (2010) remark, this measure could present problems when the return distribution deviates from normality.

If we consider tail-based measures, we obtain most of the greater risk reduction in the non-linear models but using this metric the evidence is less clear than it is with the variance reduction. For VaR metrics, we find that MRS-BEKK performs best for the UK at 1% and 10% significance levels, Germany at 1% and Europe at all levels. The asymmetric non-linear model (MRS-ASYM-BEKK) performs best for the UK at 5% and Germany at 10% . However, for Germany at 5% significance the asymmetric linear GARCH achieves the best hedging performance. The result for the ES, which reflects the expected loss when we consider only the worst scenarios, again non-linear models performs better than linear models in most cases. However, there are some cases where linear models outperform non-linear ones, such as Germany at 1% significance. Using these last two metrics, the dominance of non-linear models is again evident outperforming in almost all cases linear and constant models[92].

Panel B presents the effectiveness analysis for the out-sample analysis. The highest effectiveness considering the reduction of variance of the hedge portfolio is observed in the MRS-ASYM-BEKK in the UK and the MRS-BEKK in Europe and Germany. Non-linear models outperform the effectiveness of the rest of the models in terms of variance reduction in the out-sample analysis. The utility results are similar. With this evidence, it seems clear that more complex non-linear models lead to better forecasts of the hedge ratio and a greater risk reduction using variance-based metrics. However, if we compare linear GARCH models to constant strategies we find a greater variance reduction for constant strategies. This result reveals an issue widely discussed in the empirical literature. Most of the literature comparing dynamic (i.e. linear GARCH models) with constant strategies obtain a better performance from the latter (Lien, 2009; Lien and Tse, 2002; Cotter and Hanly, 2006; Jei and Park, 2010). However, when non-linear dynamics models through regime switching are introduced, a better performance compared with constant and linear GARCH models is achieved.

[92] Cotter and Hanly (2006) find that some performance metrics (especially VaR) yield different results in terms of the best hedging model compared with the traditional variance reduction criterion.

The tail loss distribution measures also reflect the higher performances of non-linear models in most cases. VaR measures show that MRS-BEKK presents the highest effectiveness in Europe and Germany at 1%, while the MRS-ASYM-BEKK is the best strategy in the UK, Germany and Europe at 5% and 10% levels. For the UK at 1%, the linear BEKK model is most effective. The ES results show similar conclusions to those of the VaR results in the out-sample analysis. This metric also shows the greater effectiveness of non-linear models (the symmetric case for Europe at all levels, Germany at 1% and 10%, and the asymmetric model for the UK at all levels and Germany at 5%)[93].

This implies that non-linear models exhibit a higher hedging effectiveness than do constant and dynamic linear models using variance-based metrics. The evidence with tail loss metrics also supports the more complex models in most cases, although in a few scenarios linear models beat them. This greater out-sample effectiveness of non-linear models may be because they offer more accurate forecasting than do more parsimonious models (Marcucci, 2005). When the dynamic relationship between spot and futures returns is characterized by regime shifts, allowing the hedge ratio to be dependent upon the state of the market, one can obtain more efficient hedge ratios and hence, superior hedging performance compared with other methods in the literature.

4.4. Specification test

To test robustness, this section performs several specification tests to check the adequacy of the QML estimations of the multivariate models. For this reason, we analyze the properties of the standardized residuals ($\in_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}}$) *for i=s,f* and the product of the standardized residuals for the models proposed.

Table 5 displays the main results of these specification tests. The first part of the table shows summary statistics for the standardized residuals of the estimated models. The mean value is around zero in all cases with a standard deviation close to one. A reduction in the skewness and kurtosis of the residuals is observed compared with the original series. The Ljung–Box test performed over the standardized residuals reveals a lack of serial autocorrelation in both levels and their cross products. This also removed the heteroscedasticity problem present in the original series.

The bottom of the table presents two moment tests to analyze the consistency of the QML estimations performed (Bollerslev and Wooldridge, 1992). These authors explain that even in the case of deviations from normality, consistent estimations are found if:

$E_{t-1}\left(\hat{\in}_{i,t}\right)=0$, $E_{t-1}\left(\hat{\in}_{i,t}^{2}\right)=1$ and $E_{t-1}\left(\hat{\in}_{i,t}\hat{\in}_{j,t}\right)=0$ for i,j = m,b where $\hat{\in}_{i,t}$ are the standardized residuals.

[93] Alizadeh and Nomikos (2004) and Alizadeh et al. (2008) also find a general outperforming of regime-switching models regarding other strategies in their studies but in a few scenarios, the more complex models they propose are beaten.

The findings do not reject the null hypothesis assumed about the considered values of the two first-order moments. These results confirm the consistency of the estimations of our models even for deviations from normality.

5. Conclusions

This paper analyzes hedging effectiveness using complex non-linear GARCH models in some of the main European stock indexes. It presents MRS-BEKK specifications that assume non-linear dynamics between spot and futures returns to overcome the traditional linear GARCH limitations and reflect properly the characteristics of the financial data.

The estimation of the models reveals that significant differences exist in the variance equation parameters between states. This may reflect the fact that the volatility process is not defined by a unique process as proposed by linear GARCH models but by two different volatility processes observed during high and low volatility periods. The consideration of one instead of two volatility processes leads to poor estimations of volatility and this may influence the estimated hedge ratios. Differences in volatility between low and high volatility states are observed in terms of the (asymmetric) impact of shocks and past variance on the volatility formation in each state. Another interesting result is related to the state governing the process in each period. Usually, high volatility states are present in contexts of market uncertainty such as the dot-com bubble or the last financial crisis.

The volatility estimations and forecasts are also different between linear and non-linear models. These differences affect the effectiveness reached by each strategy as our empirical results demonstrated. Non-linear models generally outperform the rest of the models in both in-sample and out-sample analysis. The presented results are robust across countries and for most of the effectiveness measures proposed. Because the out-sample analysis was performed during the last financial crisis it seems that non-linear models improve the rest of the models during these periods of market jitters. This may be because the consideration of different volatility processes (distinguishing between calm and uncertain periods) lets these models achieve a better performance than can those models that cannot make this distinction.

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TABLE 1.- Summary statistics for prices and returns of spot and futures on the selected European indexes

Panel A.- Summary statistics						
In-sample[a]	*United Kingdom*		*Europe*		*Germany*	
	Spot	*Futures*	*Spot*	*Futures*	*Spot*	*Futures*
Mean	*-0,0792*	*-0,0671*	*-0,0710*	*-0,0726*	*-0,0462*	*-0,0470*
Standard deviation	*7,0904*	*6,5581*	*10,7594*	*11,1216*	*13,0599*	*12,7500*
Skewness	*-1,2664*	*-0,4259*	*-0,8156*	*-0,4338*	*-0,6041*	*-0,3824*
Kurtosis (excess)	*16,1331*	*9,1974*	*10,2306*	*6,9790*	*8,0840*	*6,2083*
JB test	*4077,29******	*891,91******	*1252,21******	*378,00******	*622,37******	*247,93******
Panel B.- Autocorrelation test						
	United Kingdom		*Europe*		*Germany*	
	Spot	*Futures*	*Spot*	*Futures*	*Spot*	*Futures*
LB-Q (7)	*37,256***	*23,541*	*35,065***	*35,457***	*33,335***	*34,082***
LB-Q^2 (7)	*107,73****	*124,58****	*87,11****	*110,64****	*137,39***	*159,92***
Panel C- Stationarity test						
	United Kingdom		*Europe*		*Germany*	
	Spot	*Futures*	*Spot*	*Futures*	*Spot*	*Futures*
Dickey-Fuller	*-25,170**** **(-0,4435)**	*-23,954**** **(-0,4782)**	*-25,071**** **(-0,6358)**	*-24,760**** **(-0,6514)**	*-24,190**** **(-0,3491)**	*-23,515**** **(-0,3565)**
Phillips-Perron	*-25,171**** **(-0,4435)**	*-23,955**** **(-0,4782)**	*-25,071**** **(-0,6358)**	*-24,760**** **(-0,6514)**	*-24,191**** **(-0,3491)**	*-23,516**** **(-0,3565)**

Panel D.- Cointegration test				
In-sample	***Lags***	***H_0***	***Trace Statistic***	***Eigen Statistic***
Europe	1	r <= 0	*63,999****	*62,708****
		r <= 1	*1,2935*	*1,2935*
United Kingdom	1	r <= 0	*89,394****	*87,298****
		r <= 1	*2,0961*	*2,0961*
Germany	1	r <= 0	*199,648****	*197,485****
		r <= 1	*2,1585*	*2,1585*

This table presents the descriptive statistics for the in-sample series (spot and futures) for Europe, the UK and Germany. The JB test is the Jarque–Bera (1980) test for normality. LB-Q (7) and LB-Q2 (7) are the Ljung–Box (1978) test for series autocorrelation for the series in levels and squares.

Table 2.- Estimations of the linear and non-linear GARCH models

$r_{s,t} = a_0 + a_1 r_{s,t-1} + a_2 r_{f,t-1} + a_3 ECT_{t-1} + e_{c,t}$ $\quad r_{f,t} = b_0 + b_1 r_{f,t-1} + b_2 r_{s,t-1} + b_3 ECT_{t-1} + e_{f,t}$

$$H_{t,s_t} = \begin{pmatrix} \sigma^2_{m,t,s_t} & \sigma_{mb,t,s_t} \\ \sigma_{mb,t,s_t} & \sigma^2_{b,t,s_t} \end{pmatrix} = C_{s_t} C_{s_t}' + A_{s_t}\varepsilon_{t-1}\varepsilon_{t-1}' A_{s_t}' + B_{s_t} H_{t-1} B_{s_t}' + D_{st}' \eta_{t-1}\eta_{t-1} D_{st} \quad where \quad A_2 = sa\cdot A_1 ;\ B_2 = sb\cdot B_1\ C_2 = sc\cdot C_1\ D_2 = sd\cdot D_1$$

	Europe				UK				Germany			
	Linear		Non-linear		Linear		Non-linear		Linear		Non-linear	
	Sym	Asym	Sym	Asym	Sym	Asym	Sym	Asym	Sym	Asym	Sym	Asym
c_{11}	0,1505 (0,1094)	0,2783*** (0,0552)	0,4862*** (0,0764)	0,2738* (0,1601)	0,2137** (0,0952)	0,9132*** (0,2160)	2,1490*** (0,2536)	0,9566*** (0,1565)	0,4509*** (0,0881)	0,9530*** (0,1533)	0,4609* (0,2459)	0,5139*** (0,1111)
c_{12}	0,2944*** (0,1037)	0,2518*** (0,0588)	0,4555*** (0,0922)	0,3136** (0,1468)	0,1457 (0,1017)	0,9060*** (0,2211)	2,1572*** (0,2602)	1,0047*** (0,1603)	0,6210*** (0,0922)	1,0090*** (0,1488)	0,4977* (0,2765)	0,5508*** (0,1140)
c_{22}	0,0029 (0,0347)	0,1020 (0,0883)	0,2700*** (0,0312)	0,3105*** (0,0409)	0,0001 (0,0467)	0,00001 (0,0072)	-0,241*** (0,0361)	-0,061** (0,0294)	0,0012 (0,0651)	0,2841** (0,1129)	0,2990*** (0,0215)	0,2615*** (0,0188)
a_{11}	0,2934*** (0,0585)	0,2957*** (0,0788)	-0,3466* (0,1935)	-0,2065 (0,1447)	0,5796*** (0,1734)	0,7023 (0,6661)	1,3691** (0,3371)	-0,748*** (0,1309)	0,6349*** (0,0262)	0,4229 (0,2736)	-0,0520 (0,2210)	-0,627*** (0,1270)
a_{12}	-0,0579 (0,0589)	-0,049*** (0,0184)	-0,2796* (0,1894)	-0,1424 (0,1066)	0,0050 (0,0857)	0,3018 (0,6949)	0,9174** (0,3278)	-0,839*** (0,1361)	0,3095*** (0,0775)	-0,0733 (0,2905)	0,0217 (0,2489)	-0,609*** (0,1304)
a_{21}	-0,0589 (0,0584)	-0,065*** (0,0139)	0,7107*** (0,1972)	0,2336 (0,1556)	-0,393** (0,1867)	-0,1483 (0,6619)	-0,702** (0,3132)	0,7702*** (0,1348)	-0,405*** (0,0741)	-0,758*** (0,2567)	0,3903 (0,2769)	0,6744*** (0,1266)
a_{22}	0,2946*** (0,0586)	0,3038*** (0,0837)	0,6578*** (0,1954)	0,1899 (0,1273)	0,2015** (0,0945)	-0,7824 (0,6960)	-0,3027 (0,3006)	0,847*** (0,1405)	-0,075** (0,0312)	-0,2620 (0,2802)	0,2923 (0,3172)	0,6207*** (0,1313)
b_{11}	1,1410*** (0,1714)	1,1020*** (0,2273)	0,6743** (0,3233)	0,3914 (0,4995)	1,0626*** (0,1291)	1,1491*** (0,3789)	0,4974** (0,3047)	1,1819*** (0,1579)	1,2587*** (0,0460)	0,2916 (0,3582)	0,4491 (0,8253)	0,5200 (0,6984)
b_{12}	0,2340 (0,1697)	0,2556 (0,2196)	0,0710 (0,3242)	0,1444 (0,4508)	0,2340 (0,1850)	0,2722 (0,3987)	-0,3481 (0,2861)	0,3373 (0,2191)	0,3823*** (0,0050)	-0,2707 (0,2329)	0,2870 (0,9093)	0,5721 (0,7269)
b_{21}	-0,1649 (0,1715)	-0,1488 (0,2179)	0,2066 (0,3116)	0,5420 (0,4746)	-0,0828 (0,1294)	-0,3639 (0,3643)	0,1307 (0,2739)	-0,645*** (0,2140)	-0,294*** (0,0094)	0,5387 (0,3439)	0,4639 (0,8312)	0,3947*** (0,6883)
b_{22}	0,7457*** (0,1694)	0,7062*** (0,2205)	0,8173*** (0,3100)	0,79375* (0,4324)	0,7540*** (0,1781)	0,52237 (0,3795)	0,97530*** (0,2580)	0,1620 (0,2726)	0,58113*** (0,0041)	1,09757*** (0,2147)	0,64209 (0,9167)	0,35606 (0,7183)
d_{11}		0,2769 (0,2268)		0,4370*** (0,0893)		-0,2224 (0,1225)		0,1985 (0,1172)		0,4223*** (0,0489)		0,4837*** (0,0629)
d_{22}		0,2302** (0,0967)		0,4547*** (0,0911)		-0,271** (0,1158)		0,2154* (0,1268)		0,4220*** (0,0481)		0,4763*** (0,0612)
sc			19,519** (6,2998)	6,1490* (3,7525)			0,1944*** (0,0513)	3,2259*** (0,4429)			8,3639*** (2,1276)	2,8758*** (0,3576)
sa			1,7966*** (0,4837)	5,9437*** (0,0416)			0,5991*** (0,1739)	3,9729*** (1,0768)			1,9292* (1,0491)	4,9120*** (1,0742)
sb			0,5353** (0,2224)	0,8965 (1,6250)			1,8473*** (0,0856)	0,9862*** (0,0702)			0,7523*** (0,1169)	0,8384*** (0,0585)
sd				0,6713** (0,2665)				2,72048 (1,6807)				0,9241*** (0,2646)
p			0,978	0,966			0,976	0,965			0,965	0,966
q			0,972	0,962			0,969	0,962			0,954	0,961

Estimated parameters for all models and indexes (robust standard errors in parenthesis). ***, ** and * represent significance at 1%, 5% and 10% levels

Table 3.- Summary statistics for hedge ratios

EUROPE					
In-sample ***(Out-sample)***					
	Maximum	Minimum	Mean	Variance	Median
BEKK	1,0570 ***(1,0316)***	0,7603 ***(0,8468)***	0,9473 ***(0,9592)***	0,0014 ***(0,0015)***	0,9464 ***(0,9578)***
ASYM-BEKK	1,0676 ***(1,4682)***	0,7460 ***(0,7089)***	0,9415 ***(0,9803)***	0,0017 ***(0,0280)***	0,9447 ***(0,9800)***
MRS-BEKK	1,0660 ***(1,0570)***	0,9357 ***(0,8669)***	1,0157 ***(0,9383)***	0,0003 ***(0,0009)***	1,0206 ***(0,9451)***
MRS-ASYM-BEKK	1,1717 ***(0,9800)***	0,7973 ***(0,8049)***	1,0218 ***(0,8971)***	0,0014 ***(0,0010)***	1,0229 ***(0,8967)***
UK					
In-sample ***(Out-sample)***					
	Maximum	Minimum	Mean	Variance	Median
BEKK	1,2779 ***(1,1621)***	0,8023 ***(0,9078)***	0,9650 ***(0,9698)***	0,0020 ***(0,0027)***	0,9649 ***(0,9504)***
ASYM-BEKK	1,2125 ***(1,3823)***	0,8212 ***(0,7280)***	0,9678 ***(0,9592)***	0,0014 ***(0,0109)***	0,9662 ***(0,9827)***
MRS-BEKK	1,0902 ***(1,4058)***	0,7404 ***(0,7442)***	1,0019 ***(0,9724)***	0,0011 ***(0,0073)***	1,0074 ***(0,9640)***
MRS-ASYM-BEKK	1,1166 ***(1,1649)***	0,7846 ***(0,9262)***	1,0043 ***(0,9716)***	0,0013 ***(0,0017)***	1,0071 ***(0,9579)***
Germany					
In-sample ***(Out-sample)***					
	Maximum	Minimum	Mean	Variance	Median
BEKK	1,0822 ***(1,0650)***	0,8749 ***(0,9408)***	0,9728 ***(0,9927)***	0,0010 ***(0,0006)***	0,9770 ***(0,9933)***
ASYM-BEKK	1,0926 ***(1,1599)***	0,8752 ***(0,7187)***	0,9700 ***(0,9455)***	0,0005 ***(0,0060)***	0,9701 ***(0,9380)***
MRS-BEKK	1,0447 ***(1,0917)***	0,9034 ***(0,8113)***	1,0107 ***(0,9575)***	0,0004 ***(0,0017)***	1,0167 ***(0,9594)***
MRS-ASYM-BEKK	1,0875 ***(0,9803)***	0,8789 ***(0,8757)***	1,0064 ***(0,9473)***	0,0004 ***(0,0004)***	1,0136 ***(0,9522)***

This table presents the summary statistics for the hedge ratio series obtained in each European market for the different models proposed from both in-sample and out-sample analysis.

Table 4.- Effectiveness analysis for the different models proposed

Panel A - In-sample effectiveness									
	Variance reduction (Utility)			Value at Risk (1%) (5%) (10%)			Expected Shortfall (1%) (5%) (10%)		
	UK	Germany	Europe	UK	Germany	Europe	UK	Germany	Europe
Unhedged portfolio	7,1019 (-28.407)	13,0830 (-52.331)	10,7780 (-43.112)	-7,2832 (-4,2444) -3,0231	-10,7111 (-5,6611) -4,4048	-9,2324 (-5,3425) -3,9207	-12,1918 (-6,4098) -5,1242	-15,0682 (-8,3671) -6,8985	-13,5863 (-7,5667) -6,2300
OLS	91,475% (-2.421)	96,668% (-1.743)	94,591% (-2.331)	-2,1358 (-0,8388) -0,5342	-2,8167 -0,9381 -0,6886	-2,0685 (-1,1269) -0,8679	-4,9861 (-1,8553) -1,3192	-3,4327 (-1,6813) -1,3062	-3,3674 (-1,7324) -1,4006
BEKK	89,767% (-2.861)	96,243% (-1.966)	94,146% (-2.523)	-2,1636 (-0,8033) -0,5599	-2,8207 (-0,9672) -0,6658	-2,1794 (-1,2109) -0,8656	-5,8531 (-2,0409) -1,4166	-3,2691** (-1,7500) -1,3241	-3,2092 (-1,7687) -1,4377
ASYM-BEKK	91,735% (-2.221)	96,437% (-1.864)	93,461% (-2.819)	-2,7441 (-0,8415) -0,5297	-2,8124 (-0,9322**) -0,6777	-2,2236 (-1,2219) -0,9012	-4,8536 (-1,8461) -1,3265	-3,2910 (-1,6942) -1,2813	-3,4947 (-1,8587) -1,5003
MRS-BEKK	89,928% (-2.9069)	96,543% (-1.809)	94,245% (-2.481)	-2,1268** (-0,8536) -0,5134**	-2,5475** (-1,0126) -0,6464	-2,0156** (-1,1248)** -0,8582**	-5,6605 (-1,9451) -1,3800	-3,6261 (-1,7061) -1,2896	-3,5958 (-1,7885) -1,4224
MRS-ASYM-BEKK	92,216%** (-2.3476)**	96,701%** (-1.7262)**	94,731%** (-2.271)**	-2,8041 (-0,7884)** -0,5159	-2,6631 (-0,9374) -0,6270**	-2,0687 (-1,1362) -0,8680	-4,508** (-1,6996)** -1,2431**	-3,4202 (-1,6685)** -1,2674**	-2,9199** (-1,6957)** -1,3943**
Panel B -Out-sample effectiveness									
Unhedged portfolio	14,0564 (-56.226)	11,570 (-46.281)	14,0565 (-56.226)	-11,9260 (5,3703) -4,0735	-9,4690 (-5,1069) -3,9729	-11,9260 (-5,3703) -4,0735	-11,0761 (-8,4255) -6,6338	-9,4690 (-7,3291) -5,8716	-11,0761 (-8,4255) -6,6338
OLS	90,280% (-5.461)	95,366% (-2.144)	90,859% (-5.139)	-2,8404 (-1,5730) -1,1495	-1,5904 (-1,1950) -0,8000	-2,5952 (-1,4069) -1,0814	-2,3674 (-1,9360) 1,6502	-1,5904 (-1,3755) -1,1769	-2,4024 -1,8939 -1,5811
BEKK	90,006% (-5.619)	95,280% (-2.184)	90,057% (-5.590)	-2,7503** (-1,6117) -1,2745	-1,6095 (-1,2886) -0,8836	-3,0333 (-1,5012) -1,1619	-2,6391 (-2,0889) -1,7573	-1,6095 (-1,4059) -1,2079	-2,8453 (-2,1588) -1,7536
ASYM-BEKK	88,596% (-6.411)	94,942% (-2.341)	82,678% (-9.739)	-4,2527 (-1,7488) -1,0793	-1,7257 (-1,2620) -0,9324	-4,5691 (-1,9771) -1,1583	-3,9761 -2,6884 -2,0236	-1,7257 -1,3842 -1,2167	-4,2994 (-3,0518) -2,3544
MRS-BEKK	88,882% (-6.251)	95,778%** (-1.953)**	91,116%** (-4.994)**	-4,4355 (-1,5085) -1,3263	-1,5358** (-1,1750) -0,7940	-2,1602** (-1,4208) -1,1823	-3,3495 (-2,4026) -1,9468	-1,5358** -1,3356 -1,1471**	-2,0762** (-1,8293)** -1,5501**
MRS-ASYM-BEKK	90,668%** (-5,257)**	95,635% (-2.019)	87,602% (-6.970)	-2,7115 (-1,4922)** -1,0608**	-1,6755 (-1,0808)** -0,7937**	-3,3978 (-1,3975)** -1,0355**	-2,3596** (-1,9065)** -1,6045**	-1,6755 (-1,3072)** -1,1494	-2,9533 (-2,1160) -1,6983

This table shows the results for the different effectiveness measures in the different countries considered (risk reduction (equation 24), economic viability (25), Value at Risk (26) and Expected Shortfall (27). Panels A and B display the results for the in-sample (01/01/1988-31/12/2008) and out-sample (01/01/2009-30/09/2010) periods. ** represents the model with the best performance for each effectiveness measure considered.

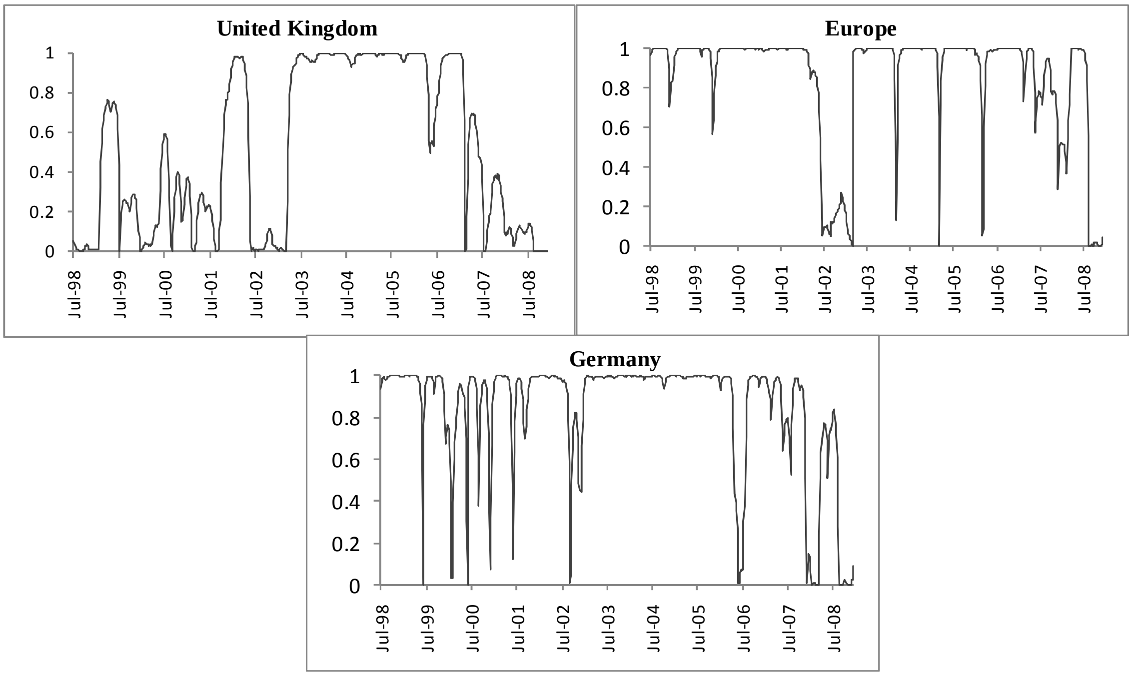
Table 5.- Specification test for the standardized residuals

Panel A. Linear models					
BEKK *(ASYM-BEKK)*	$\hat{\epsilon}_{s,t}$	$\hat{\epsilon}_{f,t}$	$\hat{\epsilon}^2_{s,t}$	$\hat{\epsilon}_{s,t}\hat{\epsilon}_{f,t}$	$\hat{\epsilon}^2_{f,t}$
Mean	-0.0444 ***-0.0021***	-0.0302 ***-0.0338***	0.9949 ***1.0029***	-0.0165 ***-0.0205***	1.0108 ***0.9971***
Std. Dev	0.9948 ***1.0047***	1.0117 ***0.9978***	4.8407 ***2.8840***	2.4644 ***1.6700***	3.3990 ***3.2769***
Skewness	-0.4708 ***-0.4064***	-0.6055 ***-0.4365***	8.1762 ***3.4986***	2.0169 ***-2.0010***	4.3518 ***4.6834***
Kurtosis	5.8089 ***3.8588***	4.2504 ***4.2340***	109.7394 ***18.2919***	64.6949 ***24.3926***	28.2937 ***33.8424***
J–B test	199.67 ***31.81***	68.93 ***51.98***	265281.06 ***6433.77***	86962.68 ***10775.71***	16278.15 ***23637.00***
L–B (6)	17.8264 ***25.8294***	22.8514 ***19.2729***	18.5901 ***24.1604***	28.6156 ***15.0965***	14.3485 ***10.1548***
t-stat for H0:	-0.0446 ***(-0.0021)***	-0.0299 ***(-0.0339)***			
t-stat for H1:			-0.0052 ***(0.0047)***	0.5942 ***(0.4012)***	0.0116 ***(-0.0022)***

Panel B. Non-linear models					
MRS-BEKK *(MRS-ASYM-BEKK)*	$\hat{\epsilon}_{s,t}$	$\hat{\epsilon}_{f,t}$	$\hat{\epsilon}^2_{s,t}$	$\hat{\epsilon}_{s,t}\hat{\epsilon}_{f,t}$	$\hat{\epsilon}^2_{f,t}$
Mean	-0.0090 ***0.0103***	-0.0716 ***-0.0359***	1.0208 ***0.9778***	0.0461 ***0.0070***	0.9740 ***0.9934***
Std. Dev	1.0226 ***0.9795***	0.9707 ***0.9939***	3.2994 ***2.9574***	1.7447 ***1.6894***	2.9268 ***3.2335***
Skewness	-0.3387 ***-0.5074***	-0.4830 ***-0.5088***	4.0646 ***4.1675***	-1.9229 ***-2.3564***	4.3927 ***4.8102***
Kurtosis	4.1488 ***4.1086***	3.9504 ***4.2008***	24.1819 ***26.0987***	23.7857 ***27.0903***	31.3219 ***35.6759***
J–B test	40.47 ***51.39***	41.78 ***56.36***	11710.71 ***13718.81***	10165.51 ***13708.14***	20004.44 ***26396.08***
L–B (6)	28.8913 ***27.0130***	20.1812 ***15.7562***	26.2149 ***24.8734***	15.0549 ***13.1601***	8.5937 ***9.6922***
t-stat for H0:	-0.0088 ***(0.0105)***	-0.0738 ***(-0.0361)***			
t-stat for H1:			0.0221 ***(-0.0209)***	0.4268 ***(0.4081)***	-0.0302 ***(-0.0061)***

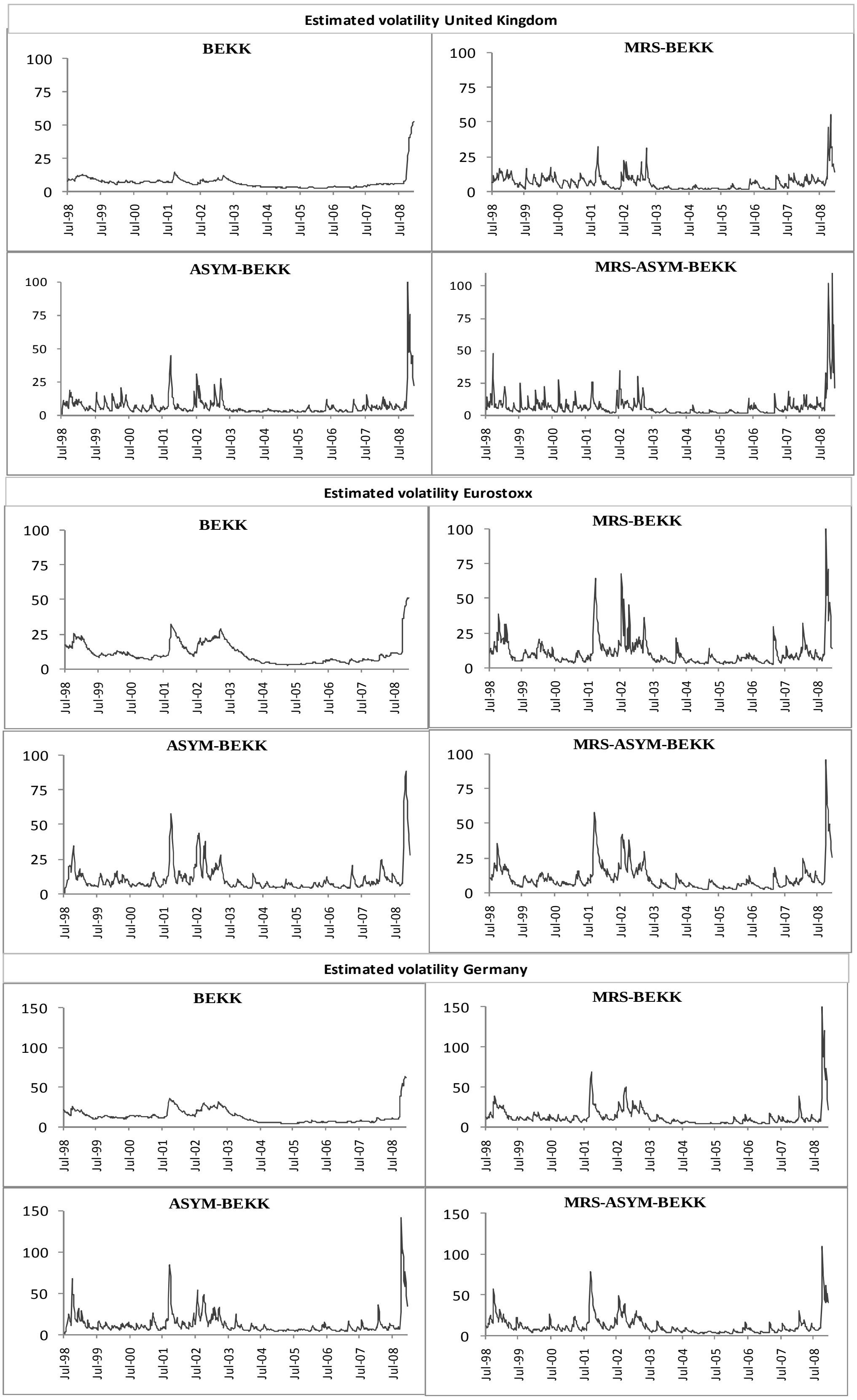
This table shows the statistics for the standardized residuals. Panel A shows the results for the linear models (BEKK and ASYM-BEKK). Panel B displays the results for non-linear models (MRS-BEKK and MRS-ASYM-BEKK). The J–B test is the Jarque–Bera test for normality. L–B (6) is the Ljung–Box autocorrelation test including six lags. It also presents tests about the first two moments of the standardized residuals to validate consistent estimations of the QML procedure from deviations to normality. ***, ** and * represent significance at 1%, 5% and 10% levels. H0 and H1 represent the t-statistic for the two-moment order test developed by Bollerslev and Wooldridge (1992).

Figure 1: Smooth probabilities for low volatility states



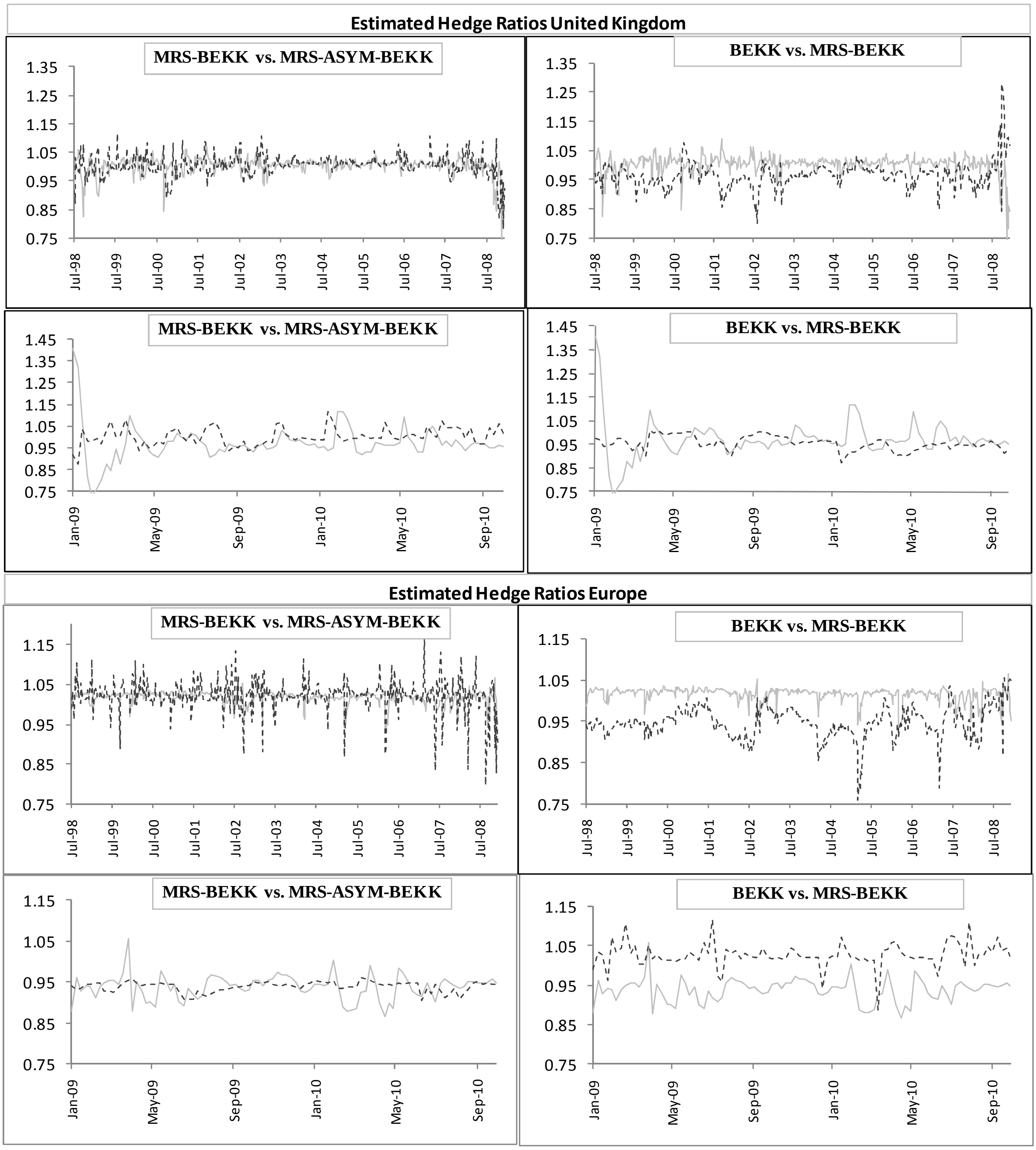
This figure shows the smooth regime probabilities of being in a low variance state (Hamilton and Susmel, 1994) for all the countries considered

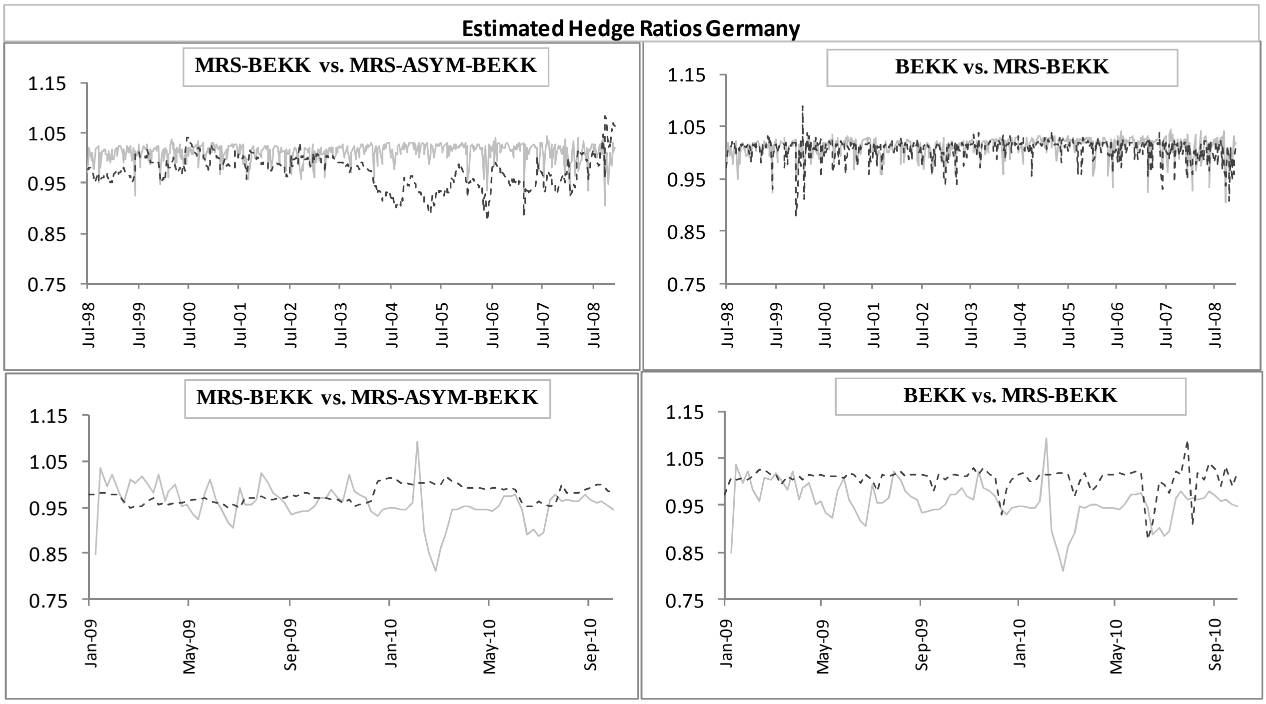
Figure 2.- Estimated weekly volatility



This figure shows the estimated volatilities for the spot market in the different markets considered. Linear GARCH models (symmetric and asymmetric) are displayed on the left-hand side, while the non-linear specifications are on the right-hand side.

Figure 3.- Estimated hedge ratios for in-sample and out-sample periods





This figure shows the estimated time-varying hedge ratios for the in-sample period (top figures) and out-sample period (bottom figures). Continuous lines represent the MRS-BEKK model and dashed lines the other models considered in each case.

GENERAL CONCLUSION

REGIME-SWITCHING VOLATILITY MODELS: APPLICATION TO DYNAMIC HEDGING WITH FUTURE CONTRACTS AND THE ESTIMATION OF THE RISK PREMIUM

In this thesis, two research fields widely discussed in the financial literature (such as the risk-return trade-off and dynamic hedging with futures contracts) are re-examined considering non-linear patterns in financial series modeling. Most of the literature has analyzed empirically these areas from a linear perspective using (multivariate) GARCH models but the evidence obtained is not conclusive at all. Differently from previous works, we adopt Markov Regime Switching GARCH models that allow volatility to have different dynamics according to unobserved regime variables. This methodology let us overcome some of the limitations of traditional GARCH models reflecting potential non-linear patterns in volatility dynamics.

The main purpose of the thesis is to provide new insights in the two fields analyzed using the more complex models presented. First, in the study of the risk-return tradeoff we expect that a positive and significant relationship between return and risk could be obtained against the inconclusive evidence reported in previous studies. Second, we expect a greater effectiveness of the strategies using the non-linear models proposed in the analysis of the dynamic hedging of stock indexes using futures contracts.

In the chapters focused on the risk-return tradeoff we analyze these relationship using data from developed European markets and a wide sample of emerging markets. Besides the differences in the data sample used in each chapter, the methodology also differs in the methodology employed. Univariate specifications are used when a constant set of investment opportunities available to the investor is assumed but when this assumption is relaxed a bivariate framework is employed. Generally, the alternative models used against the MRS-GARCH we proposed are linear GARCH models but we also include alternative methodologies such as the MIDAS regression. We also consider the role of asymmetries in volatility considering the GJR-GARCH specifications and the distributional assumption considered for the models proposed is Normality[94].

Some interesting results are repeated in all these chapters and observed in the different stock markets used. The use of MRS-GARCH models usually reveals the presence of two different volatility regimes in the stock markets analyzed. The unconditional variance in high volatility regime is found to be higher than that in low volatility regimes. Moreover, the graphs of smoothed probabilities being in high volatility regime confirm the existence of two volatility regimes and suggests that periods of high volatility regimes are often associated with international financial crises (such as the dot-com bubble or the last financial crisis). Another improvement of the MRS-GARCH models is that they reduce the high persistence in uni-regime GARCH models during high volatility states. These results are consistent with findings of authors who suggest that regime shifts in volatility can lead to spuriously high levels of volatility persistence. On the other hand, under all distribution assumptions, estimated transition probability of each regime present a high value (superior to 0.9 in most cases) indicating that each regime is quite persistent and the regime transition follow a smooth pattern.

[94] Although the returns series exhibits non-normal patterns, all the QML estimations obtained are consistent even for deviations from normality.

Certainly, one of the most interesting results in this thesis is that a positive and significant risk-return tradeoff is observed in most of the markets considered only during low volatility periods. However, the basic relationship suggested by the theoretical models is not observed during periods of financial turmoil. Moreover, the magnitude of this direct relationship between return and risk (often associated with the risk aversion level) is usually lower during periods of market jitters. This result is repeated for the different stock markets considered and for univariate and bivariate specifications (i.e. assuming constant and stochastic set of investment opportunities). These results suggest a pro-cyclical risk aversion of the investors in all markets analyzed as it noted other authors. Generally, high volatility regimes correspond to periods of recession or low expansion in the country's economy, whereas low volatility regimes correspond with periods of economic expansion. Therefore, during boom periods the investors takes a more conservative position and behaves more risk averse while during high volatility periods the ‘sense’ of risk seems to change. Following this interpretation these results could be related with the investor profile remaining in the market in each market situation. The more risk averse investors tend to leave the market during periods of financial turmoil and let only the less risk averse investors trade during these periods who make the risk aversion level observed in these periods decrease regarding the observed in stable periods.

The analysis of the risk premium evolution in developed European markets reveals that during periods coinciding with high volatility regimes the premium required for investors presents higher values than for the rest of the sample observing an evolution relatively close to volatility evolution. Despite the decrease in the risk price during the high volatility periods, there is an extremely rise in the market risk that lead to higher risk premiums. Moreover, non-linear GARCH models provide slightly higher estimation for the total risk premium during high volatility periods.

There are also other interesting results for each market considered or for a specific methodology which are pointed out below:

1.- The risk premium estimates for Europe are generally higher than that obtained in previous studies for US data, due mainly to the period of financial instability generated by the global crisis of 2007–2009. We obtain an average risk premium between 4% and 8%, depending on the market and the methodology used. Although the risk prices show different patterns depending on the market considered, there is a common and extremely high non-diversifiable risk observed in all European markets during the recent financial crisis period. This is the main cause for the rise of the market risk premium demanded by investors during the financial crisis period.

2.- The role of assuming a stochastic hedge component in the bivariate models analyzing return and risk is not as important as modeling non-linearities in the risk-return relationship. The main results of considering a stochastic investment opportunity set does not change significantly from those of assuming a constant one although a significant impact of the intertemporal component in the risk-return relation is obtained. The evidence is quite similar even using several proxies for this stochastic hedging component obtaining only favorable evidence during low volatility periods using MRS-GARCH models. Moreover, the percentage of the total risk premium corresponding to the premium of the hedging component is relatively small compared to those of the market risk premium, although using non-linear models the differences in the percentages are smaller.

3.- The analysis of the risk return trade-off in the stock emerging markets using MSCI indexes for five of the main stock markets in Latin America (Argentina, Brazil, Chile, Mexico, and Peru), nine Asian markets (such as China, Indonesia, Malaysia, Thailand, India, Korea, Philippines and Taiwan), five Easter European Countries (Czech Republic, Hungary, Poland, Russian and Turkey), three African emerging markets (Morocco, Egypt and South Africa) and the aggregate indexes for Asia, Eastern Europe and Latin America emerging markets support the results observed in developed markets of a pro-cyclical risk aversion behavior (in the sense that during low volatility periods (associated with boom cycles) the investor risk aversion is higher than during high volatility periods (associated with crises periods)).

Finally, in the chapter focused on the effectiveness of non-linear strategies using dynamic hedging with futures contracts we obtain again favorable evidence for the use of these more complex models. The estimation of the models reveals that exist significant differences in the variance equation parameters between states. This may reflect that the volatility process is not defined by a unique process as propose linear GARCH models but by two different volatility processes observed during high and low volatility periods. The consideration of one instead of two volatility processes leads to a poor estimations of volatility, and may have influence in the estimated hedge ratios. The differences in volatility between low and high volatility states are observed both in terms of the (asymmetric) impact of shocks and past variance on the volatility formation in each state.

The volatility estimations and forecast are also different between linear and non-linear models. These differences have effects on the effectiveness reached by each strategy as our empirical results demonstrated. Non-linear models generally outperform the rest of the models both in sample and out-of-sample analysis. The results are robust across the countries and for most of the effectiveness measures proposed. As the out-sample analysis is performed during the last financial crisis period it seems that non-linear models improves the rest of the models during these periods of market jitters. The reason may be due to the consideration of different volatility processes distinguishing between calm and uncertain periods let these models achieve better performance than models which cannot make this distinction.

So, what it seems clear is the important role of non-linearities in the fields analyzed in this thesis. The more complex models presented here reflect more properly the dynamics of financial data than the traditional linear models commonly used in the literature analyzing these topics: 1) it allows more efficient estimations and more accurate forecasts for conditional variance which lead to a higher hedging effectiveness and 2) it allows estimations conditioned on the market state which let us re-evaluate the conclusions on the risk-return trade-off. The main results suggest that modeling volatility through non-linear MRSG models seems more attractive and reveal other interesting results that traditional models cannot show. For future research this methodology may be applied to other fields related with market volatility such as volatility transmission patterns between markets or time-varying betas developed in a non-linear framework distinguishing the conclusions reached in low and high volatility periods.

RESUMEN

MODELOS DE CAMBIO DE RÉGIMEN EN VOLATILIDAD: APLICACIÓN PARA LA COBERTURA CON CONTRATOS DE FUTURO Y LA ESTIMACIÓN DE LA PRIMA DE RIESGO

INTRODUCCIÓN [95]

Desde la aparición de los modelos de volatilidad condicional GARCH (Engle(1982) y Bollerslev(1986)) han sido muchos los trabajos donde se proponen mejoras con la finalidad de incorporar regularidades empíricas presentes en la mayoría de series de carácter financiero (véase: Lien (1996), Maliq (2003), Susmel (2000)).

Uno de las últimas aportaciones nace con los modelos Markov Regimen Switching GARCH (MRSG) (Hamilton, 1989, Gray, 1996, Sarno y Valente, 2000). La novedad básica de dichos modelos es que permiten condicionar las estimaciones realizadas al régimen de volatilidad existente. Son modelos no lineales dependientes del número de regímenes considerados; generalmente dos: alta y baja volatilidad. Esta metodología nos permitirá analizar las conclusiones de distintas teorías económicas, así como evidencias empíricas, diferenciando si dichas conclusiones son las mismas bajo mercados en calma que cuando estos presentan situaciones de alta volatilidad. Este tipo de análisis es especialmente relevante en la actualidad, donde los mercados financieros presentan un grado elevado de inestabilidad y comienzan a surgir una corriente de pensamiento que pone en duda una gran parte del cuerpo teórico sobre el que se asientan la mayor parte de teorías de la moderna economía financiera.

Los MRSG mejoran los modelos de volatilidad GARCH estándar en tres aspectos (Baele, 2005): 1) recogen el hecho que la persistencia de los modelos GARCH es menor en periodos de volatilidad alta que en periodos de calma. No considerar este aspecto provocará sobre estimaciones de la persistencia (Lamoureaux y Lastrapes, 1990; Cai, 1994), lo que tendrá claros efectos sobre la predicción de volatilidad; 2) Las predicciones de este tipo de modelos son mejores que las obtenidas con los modelos más parsimoniosos (Marcucci, 2005); 3) Estos modelos recogen el hecho de que la correlación tiene un comportamiento asimétrico respecto al tamaño de los rendimientos, esto es, tiende a ser superior cuando los rendimientos son bajos que cuando éstos son altos (Ang y Bekaert 2002).

[95] Dado que ninguno de los capítulos han sido redactados en ninguna de las dos lenguas oficiales de la Universidad de Valencia, cumpliendo con su normativa, a continuación se resumen los cuatro capítulos de la tesis. El presente resumen se ha realizado en cumplimiento de la Disposición Adicional cuarta de la Normativa reguladora de los procedimientos de elaboración, autorización, nombramiento del Tribunal, defensa y evaluación de las tesis doctorales de la Universidad de València, aprobada en Consejo de Gobierno el 6 de Junio de 2006.

El objetivo de esta tesis se centra en analizar los resultados obtenidos con esta metodología, y compararlos con los obtenidos con otras metodologías, en dos campos de investigación ampliamente analizados en la literatura financiera, como son: 1) Relación entre el rendimiento esperado y volatilidad; 2) Cobertura dinámica con contratos de futuro.

1. **Relación entre el rendimiento esperado y volatilidad.** El objetivo en este campo de investigación se centrará en analizar la relación intertemporal entre rendimiento condicional (medido sobre el exceso respecto al activo libre de riesgo) y la volatilidad condicional (Merton, 1973), tanto para el mercado español como mercados internacionales. La literatura financiera no presenta resultados concluyentes con respecto a la relación entre ambos aspectos ni su significatividad. En este sentido, los resultados han sido diversos según el período muestral estudiado, la periodicidad de los rendimientos utilizados y la metodología empleada. Sin la finalidad de ser exhaustivos, las principales metodología son: GARCH (French et al. (1987), Campbell (1987), Glosten et al. (1993), Scrugg (1998), Engle y Lee(1999), Scrugg y Glabadanidis (2003); MRSG (Chauvet y Potter (2001), Whitelaw(2000), Mayfield (2004)); MIDAS (Ghysels, et. al., 2005; León, et. al.(2007)) o modelos que utilizan variables que recogen la evolución del ciclo económico para la predección del rendimiento y volatilidad condicional (Fama and French, 1988,1989). Una extensión de estos últimos modelos es la de considerar adicionalmente factores obtenidos a partir de series de carácter económico o financiero (Ludvigson, Ng (2007).

   En nuestro trabajo se contrastarán los resultados obtenidos con las distintas metodologías, analizando si dichos resultados pueden estar condicionados al estado (volatilidad alta o baja) de los mercados. La consideración de distintos estados puede arrojar conclusiones sobre el cumplimiento de esta relación fundamental en contextos diferentes. Además la consideración de una volatilidad lineal (en lugar de una no lineal) puede ser la causa de que la evidencia sobre el tema resulte inconcluyente.

2. **Cobertura dinámica**. El objetivo que nos proponemos es analizar cómo afecta a las coberturas realizadas con contratos de futuro considerar cambios de régimen en la varianza. La estimación de estos ratios de cobertura se realizará utilizando distintos modelos GARCH multivariantes. Concretamente, desde modelos más utilizados en

la literatura como el GARCH BEKK a modelos más novedosos donde se consideren los siguientes aspectos: a) las relaciones de equilibrio a largo plazo incluyendo un Término de Corrección de Error (TCE) en la modelización de los momentos de primer orden (Alizadeh et al. 2008; Lien, 1996); b) la existencia de distintos tipos de régimen en la volatilidad, utilizando MRSG (Lee y Yoder 2007a; Alizadeh et. al 2008); c) el comportamiento asimétrico de la volatilidad (Brooks et. al, 2003). Todo ello nos llevará a estimar modelo TCE-MRSG-BEKK.

El estudio se realizará tanto para coberturas dentro de la muestra (*in the sample*) como fuera de la muestra (*out of the sample*) más ajustadas al verdadero proceso de decisión seguido por cualquier inversor. Para medir la efectividad de las distintas aproximaciones se estudiará la disminución del riesgo de la cartera cubierta y la viabilidad económica de una política de cobertura dinámica donde se considerarán los costes de transacción en los que se incurriría.

Tras esta descripción más general, a continuación se detalla más en profundidad cada uno de los cuatro capítulos en los que se ha dividido esta tesis (los tres primeros referentes a la relación rendimiento-riesgo y el último centrado en la cobertura dinàmica con contratos de futuros).

Capítulo 1: Reexaminando la relación rendimiento-riesgo: la influencia de la crisis financiera del 2007-2009

Uno de los temas más debatidos en economía financiera es el que trata de establecer una relación entre rendimiento y riesgo. Han habido numerosos intentos para explicar y entender cuáles son las dinámicas y las interacciones que siguen estas 2 variables fundamentales. Desde un punto de vista teórico, uno de los trabajos más citados analizando la relación rendimiento-riesgo es el que presenta Merton (1973) en su modelo ICAPM. Merton demuestra que existe una relación lineal entre el rendimiento en exceso del mercado y su varianza condicional y su covarianza con el conjunto de oportunidades de inversión.

A pesar del rol tan importante que esta relación presenta en la literatura financiera, no existe un consenso claro sobre su evidencia empírica. En un marco teórico, todos los parámetros y las variables del modelo pueden ser variantes en el tiempo. Sin embargo, para hacer este modelo tratable empíricamente se deben hacer distintos supuestos; el más común es considerar precios del riesgo constantes. Otro supuesto bastante común es el considerar un conjunto de oportunidades de inversión contante a lo largo del tiempo, permaneciendo el riesgo de mercado como única fuente de riesgo. También es necesario hacer distintos supuestos sobre las dinámicas que siguen los segundos momentos condicionales. Finalmente, el modelo empírico se establece en una economía de tiempo discreto en lugar en lugar de la economía de tiempo continuo usada en el modelo teórico de equilibrio.

Dados estos supuestos hay distintos trabajos que proponen modelos empíricos alternativos para obtener una evidencia favorable como sugieren los modelos teóricos. La metodología más ampliamente utilizada para analizar la relación rendimiento-riesgo es el enfoque GARCH-M. Este enfoque es sencillo de implementar pero los resultados obtenidos son generalemnte pobres y en muchos casos contradictorios.

Por tanto, se necesitan enfoques alternativos a la usual metodología GARCH-M. De entro los más destacados existentes en la literatura financiera nos centramos en aquellas alternativas que tratan de obtener evidencia favorable usando sólo la información de la cartera de mercado. Estas principales alternativas son la incusión de cambios de régimen en el modelo empírico (RS-GARCH) y el uso de regresiones con datos de distinta frecuencia (MIDAS). El primero propone una relación no lineal entre rendimiento y riesgo el cuál está basado en el marco teórico desarrollado en el trabajo de Whitelaw (2000). Este marco teórico es ligeramente diferente del enfoque de Merton porque se obtiene una función compleja, no lineal y variante en el tiempo para explicar la relación entre rendimiento y riesgo. El segundo presenta un especificación alternativa, la regresión MIDAS, para modelizar los segundos momentos condicionales enfrente de los modelos GARCH.

Utilizando estos tres modelos empíricos (GARCH, RS-GARCH y MIDAS), pasamos a analizar las series financieras europeas seleccionadas. Sólo en el caso no lineal (cuando el mercado se encuentra en contextos de baja volatilidad) se obtiene una evidencia favorable según sugieren los modelos teóricos. Sin embargo en contextos de alta volatilidad esta relación se torna no significativa. Además, ninguno de las dos especificaciones lineales es capaz de reflejar una evidencia positiva y significativa.

Adicionalmente en este primer anàlisis, también se considera la posible influencia que la reciente crisis financiera del 2007 pueda tener sobre dicha relación. Para ello, se incluyen en los modelos empíricos una variable que controle por este periodo. Los principales resultados se repiten ya que sólo en el caso no lineal para estados de mercados en calma se obitene una evidencia positiva y significativa. La influencia que tiene la crisis financiera en la relación rendimiento-riesgo es común en todos los mercado en lo que se refiere a un aumento considerable del riesgo de mercado, pero dependiendo del mercado analizado los resultados diferen en la variación del precio del riesgo. A pesar de estos resultados distintos en el precio del riesgo, el aumento del riesgo no diversificable es tan alto, que la prima de riesgo exigida en este periodo llega a valores extremos en todos los países analizados.

Capítulo 2: El tradeoff rendimiento-riesgo en los mercados emergentes

A pesar de la gran cantidad de literatura que se ha centrado en la relación redimiento-riesgo que se ha centrado en los mercados desarrollados, hay pocos trabajos que analizan esta relación en mercados emergentes. Los trabajos que estudian este tema son pocos y en todos ellos se utiliza como modelo empírico una metodología GARCH-M obteniéndose en todos ellos una relación débil entre rendimiento y riesgo. Por tanto, en ninguno de ellos se considera la posibilidad de relaciones no lineales entre rendimiento y riesgo.

En este contexto, la contribución más importante que se realiza en este capítulo es el estudio de la relación rendimiento-riesgo en un marco no lineal. Como se ha comentado, la cuestión a tratar está casi sin explorar ya que, a nuestro entender, es la primera aplicación de este tipo de modelización en mercados emergentes.

Concretamente, se utilizan dos modelos empíricos que analizan una relación lineal (GARCH-M) y no lineal (RS-GARCH-M) a un conjunto de 25 mercados emergentes según la clasificación establecida por Morgan Stanley Capital International. Los principales resultados que se obtienen son que es necesario una especificación no lineal para capturar adecuadamente una relación positiva y significativa entre rendimiento y riesgo, ya que cuando se utilizan modelos lineales en ningún caso se obtiene evidencia favorable. Además, los resultados también nos muestran que existe una relación entre los regímenes de volatilidad y el nivel de aversión al riesgo.

En los mercados emergentes el nivel de aversión al riesgo es más elevado en estados de baja volatilidad y más bajo en momentos de alta volatilidad. Este resultado apoya la tesis de la existencia de un nivel de aversión al riesgo pro-cíclico documentado para los mercados desarrollados.

Para dotar de robustez a los resultados obtenidos se repite el análisis para el caso del comportamiento asimétrico de la volatilidad obteniéndose en esencia resultados similares. También se realiza un segundo estudio omitiendo el término constante del modelo empírico, ya que algunos autores sugieren que en el modelo teórico no aperece y su inclusión no está justificada. En este caso la evidencia que obtenemos es más pobre y se observa que la omisión de la constante puede llevar a resultados más débiles. De cualquier forma, como no sabemos exactamente cuál es el verdadero proceso generador de datos, con los modelos restringidos se pueden estar estimando modelos mal especificados y, por tanto, incluimos el término constante.

Capítulo 3: Relaciones no lineales entre rendimiento y riesgo: un enfoque multi factor con cambios de régimen

En este apartado se relaja uno de los supuestos realizados en la construcción de los modelos empíricos de los dos capítulos anteriores. En este capítulo se supone un conjunto de oportunidades de inversión estocástico por lo que los modelos univariantes de los capítulos anteriores resultan insuficientes. Se debe desarrollar un nuevo marco que sea capaz de recoger las dos fuentes de riesgo en este nuevo marco de anàlisis de la relación rendimiento-riesgo: riesgo de mercado y riesgo del componente intertemporal.

La literatura previa en modelos multifactor es de nuevo inconcluyente. La mayoría de trabajos empíricos que consideran modelos multifactor analizan relaciones lineales entre rendimiento y riesgo y en algunos casos se obtiene evidencia favorable, pero en otros estudios no se observa esta relacion positiva y significativa.

Este capítulo se motiva a partir de la introducción de no linealeades en un modelo multifactor (Whitelaw, 2000). Es decir, además de incluir las dos fuentes de riesgo que sugieren los modelos teóricos (riesgo de mercado y componente intertemporal) se introduce una relación no lineal entre los rendimientos esperados y las fuentes de riesgo dependiendo del régimen de volatilidad que domine el mercado en cada momento.

El estudio empírico en este capítulo se realiza para el mercado español. Para recoger cada una de las fuentes de riesgo contruimos las series de rendimintos del principal índice bursátil del mercado (IBEX-35) y para las variables que recogerán el componente intertemporal (conjunto de oportunidades de inversión) utilizamos una batería de alternativas, ya que no hay consenso en cuál es la proxy que debe recoger el conjunto intertemporal, entre las que se incluyen Letras del Tesoro a 1 año, Bonos a 3, 5 y 10 años, una cartera equiponderada con los 3 bonos y el spread entre los bonos a 10 y 3 años.

Los principales resultados vuelven a ser similares que en capítulos anteriores. Los resultados significativos en cuanto a la relación rendimiento-riesgo sólo se obtienen en los modelos multi-factor no lineales y sólo para estados de baja volatilidad. Además el nivel de aversión al riesgo de los inversores es mayor en estados de baja volatilidad que en estados de alta volatilidad, apoyando la aversión al riesgo procíclica de los inversores.

La representación gráfica de la prima de reisgo del mercado español nos permite observar que a pesar de que el nivel de aversión al riesgo durante los periodos de alta volatilidad es menor, la prima de riesgo durante estos periodos se incremente notablemente debido esencialmente a los extremados niveles de riesgo no diversificable. Básicamente, la prima de riesgo total viene definida esencialemente por el riesgo de mercado, siendo la prima exigida por el riesgo intertemporal bastante menor. No obstante, el peso del componente intertemporal en la prima de riesgo total es mayor en los modelos no lineales que en los modelos lineales.

Capítulo 4: Midiendo la efectividad de la cobertura en futuros sobre índices: Superan los modelos dinámicos a los estáticos? Un enfoque con cambios de régimen.

Durante las últimas dos décadas con el desarrollo de los mercados de derivados, una gran cantidad de literatura se ha centrado en analizar distintas técnicas para reducir el riesgo de las inversiones. Una técnica simple para este propósito es la cobertura con contratos de futuros, que a pesar de su sencillez ha recibido una gran atención por parte de la investigación académica. La literatura sobre este tema es muy amplia y en gran parte se centra en determinar el índice de cobertura óptimo. El método más común es aquel que minimiza la varianza de los rendimientos de una cartera con posiciones en los mercados de contado y futuro.

El trabajo pionero en ratios de cobertura constantes se debe a Ederington (1979). En este enfoque, el índice de cobertura es $\left(RC = {\sigma_{sf}}/{\sigma_f^2} \right)$. Este ratio de cobertura se estima a través de la pendiente de la regresión por mínimos cuadrados ordinarios (OLS) entre los rendmimientos de contado y de futuro.

Sin embargo, este enfoque presenta varios problemas. Uno de ellos es que no tiene en cuenta el desequilibrio a largo plazo entre los mercados spot y de futuros (Ghosh, 1993; Lien, 1996).Otro problema es que se suponen segundos momentos condicionales constantes y, por tanto, cobertura estática no condicionada a la llegada de nueva información al mercado. Hay esencialmente dos métodos para obtener ratios de cobertura dinámica. El primero consiste en permitir que los ratios de cobertura sean coeficientes que varíen con el tiempo y estimarlos de forma directa (Alizadeh y Nomikos, 2004;. Lee et al, 2006).

El segundo enfoque (Kroner y Sultan, 1991;. Brooks et al, 2002) utiliza los momentos condicionales de segundo orden de los rendimientos spot y futuro a partir de modelos GARCH multivariantes, que permiten la estimación de los ratios de cobertura en el periodo t ajustado al conjunto de información disponibles para el inversor en t-1.

La mayor parte de la literatura se ha centrado en este segundo enfoque, proponiendo modelos cada vez más completos que capturen con mayor precisión las características de los datos financieros y, de ese modo, superar las limitaciones de los modelos GARCH más simples. Una de las limitaciones de los modelos GARCH es que son incapaces de capturar de forma fiable las características de las series financieras, específicamente el impacto asimétrico de las noticias. Se sabe que los shocks negativos tienen un mayor impacto en las series financieras que los shocks positivos. Este hecho debería ser tenido en cuenta al estimar ratios de cobertura. Ya que algunos autores afirman que la efectividad de la cobertura es mayor cuando este comportamiento asimétrico se considera.

Otra de las limitaciones de los modelos GARCH es que el alto grado de persistencia de la volatilidad que se obtiene de forma generalizada y con indepnedencia de las series financieras consideradas al estimarlos. Este alto nivel de persistencia sugiere la presencia de varios regímenes en el proceso de la volatilidad (Marcucci, 2005). Ignorar estos cambios de régimen podría dar lugar a estimaciones ineficiente de la volatilidad. Por lo tanto, la consideración de varios regímenes en el proceso de volatilidad podría dar lugar a estimaciones más precisas y, por lo tanto, a un mejor funcionamiento de las estrategias de cobertura.

En los últimos años, los modelos de cambio de régimen han adquirido una nueva dimensión con el desarrollo de los modelos Markov Regime-Switching (MRS). Estos estudios proponen un método recombinativo para las matrices de covarianzas condicionales que permiten a los modelos ser tratables econométricamente. Algunos de los estudios previos con cambios de régimen se centran en modelizar la ecuación de varianza pero descuidan la ecuación de la media. Algunos autores incorporan un término de corrección de error (ECT) que permite a las características de la series ser relacionadas en el corto y largo plazo.

La evidencia de estudios con cambios de régimen muestra estimaciones más robustas si se permite a la volatilidad seguir distintos regímenes en función de las condiciones del mercado, con el resultado de que la efectividad de la cobertura será mayor.

El principal objetivo de este capítulo es analizar la influencia de los patrones no lineales y los cambios de régimen en la efectividad de las estrategias de cobertura dinámicas y evaluar si estos modelos muestran una mejora con respecto a los modelos más simples más comúnmente utilizados en la literatura. Se comparan los resultados de los ratios de cobertura estimados y de la efectividad asumiendo una dinámica lineal y no lineal entre los rendimientos de los mercados spot y futuro. El estudio se realiza para los principales índices bursátiles de varios mercados europeos (FTSE para el Reino Unido, el DAX para Alemania y Eurostoxx50 para Europa) y los contratos de futuro asociados a estos índices, teniendo en cuenta un análisis ex post y ex ante, siendo esta último más cercano el proceso de decisión seguido por un inversor / coberturista.

El periodo muestral analizado también incluye la última crisis financiera para mostrar cuáles son los modelos que mejor funcionan en periodos de mercados convulsos.

En nuestro estudio empírico, utilizamos varios modelos multivariantes GARCH. Más específicamente, se utiliza el modelo tradicional BEKK y su variante asimétrica. Por otra parte, la existencia de relaciones de cointegración entre spot y los mercados de futuros nos lleva a la incorporación de un Término de Corrección de Error en la ecuación de la media. Por último, también se proponen modelos más complejos que consideran relaciones no lineales mediante el uso de una especificación con cambios de régimen, lo que permite la obtención de ratios de cobertura que dependen de la situación del mercado y así analizar si el uso de estos modelos más complejos conduce a una mejora significativa de la estrategia de cobertura. Esto nos permite comparar la efectividad de los modelos GARCH lineales con los de los modelos GARCH no lineales.

La efectividad de las estrategias de cobertura se miden a través de varios enfoques. En primer lugar, se calcula la reducción de la varianza de la cartera de cobertura utilizando las diferentes estrategias respecto a la cartera descubierta. En segundo lugar, se analiza la significatividad económica de la reducción del riesgo en términos de la utilidad de los inversores . Finalmente, también se estiman medidas de efectividad alternativas basadas en las colas de la distribución de pérdidas, tales como Valor en Riesgo (VaR) y el Expected Shortfall (ES) enfrente de las dos primeras basadas en la varianza de la función de pérdidas.

Los principales resultados que se obtienen en este capítulo muestran que teniendo en cuenta las no linealidades en la especificación de la volatilidad da lugar a diferencias en las estimaciones y en las predicciones de la volatilidad. Estas diferencias tienen un impacto en los ratios de cobertura obtenidos y la efectividad alcanzada, haciendo que los modelos no lineales logren una mayor efectividad.

Otro resultado interesante se obtiene al comparar las estrategias dinámicas lineales con las estáticas. En la literatura previa no hay consenso en cuanto si los modelos dinámicos ofrecen mejores coberturas que los estáticos; no hay pruebas contundentes en la capacidad de estos modelos para mejorar la efectividad obtenida con los modelos más simples, incluso el modelo estático MCO. Se observa que los modelos estáticos superan a los dinámicos lineales en todos los casos; sin embargo, cuando consideramos no linealidades los modelos dinámicos superan al resto de modelos. Por tanto, el capítulo muestra que los modelos dinámicos superan a los estáticos siempre que se consideren no linealidades en los rendimientos y varianzas de los mercados spot y futuro mientras que la consideración de dinámicas lineales puede llevar a peores coberturas en terminos de la efectividad alcanzada. Este resultado es robusto para los diferentes países e independiente de la medida de la efectividad utilizada.

CONCLUSIONES

En esta tesis, dos campos de investigación ampliamente tratados en la literatura financiera (tales como la relación entre rendimiento y riesgo y la cobertura dinámica con contratos de futuros) se reexaminan considerando patrones no lineales. La mayoría de la literatura previa ha analizado empíricamente estas áreas desde una perspectiva lineal utilizando modelos GARCH (uni y multivariantes), pero la evidencia obtenida no es del todo concluyente. De forma diferente a trabajos anteriores, utilizamos modelos Markov Regime Switching GARCH que permiten a la volatilidad seguir diferentes dinámicas acorde a variables de estado ocultas. Esta metodología nos permite superar ciertas limitaciones de los modelos GARCH tradicionales y reflejan posibles patrones no lineales en las dinámicas de la volatilidad.

El principal objetivo de estas tesis es proporcionar nuevos puntos de vista en los dos campos analizados utilizando los modelos no lineales. En primer lugar, en el estudio de la relación rendimiento riesgo esperamos que una relación positiva y significativa entre rendimiento y riesgo sea obtenida en contra de la evidencia poco concluyente recogida en anteriores estudios. En segundo lugar, esperamos una mayor efectividad de las estrategias de cobertura usando los modelos no lineales propuestos en el análisis de la cobertura dinámica de índices bursátiles con contratos de futuros.

En los capítulos dedicados a la relación rendimiento-riesgo analizamos estas relaciones utilizando datos de mercados europeos desarrollados y una amplia muestra de mercados emergentes. Además de las diferencias en la muestra usada en cada capítulo, la metodología también difiere entre capítulos. Se utilizan especificaciones univariantes cuando se supone un conjunto de oportunidades de inversión constante pero cuando este supuesto se relaja utilizamos un marco bivariante. Generalmente, los modelos utilizados como alternativa a los modelos MRS-GARCH propuestos son modelos GARCH lineales pero también incluimos metodologías alternativas como la regresión MIDAS. También consideramos el rol de las asimetrías en volatilidad considerando la especificación GJR y asumiendo distribuciones normales[96] para las innovaciones de los modelos.

[96] Aunque todas las series de rendimientos muestran patrones no normales, todas las estimaciones QMV son consistentes incluso frente a desviaciones de la normalidad.

Algunos resultados interesantes se repiten en todos estos capítulos y se observan en los distintos mercados analizados. El uso de modelos MRS-GARCH revela la presencia de dos regímenes de volatilidad diferentes en los mercados analizados. La varianza incondicional en los regímenes de alta volatilidad es mayor que en los estados de baja volatilidad. Además, los gráficos de probabilidades suavizadas del estado de alta volatilidad confirman que los periodos de alta volatilidad suelen coincidir con crisis financieras internacionales (como la burbuja punto-com o la última crisis financiera). Otra mejora de los modelos MRS-GARCH es que reducen la alta persistencia de los modelos GARCH univariantes durante periodos de alta volatilidad. Estos resultados son consistentes con una amplia literatura donde se sugiere que los cambios de régimen pueden llevar a al estimación de un alto grado de persistencia de la volatilidad por motivos espúreos.

Por otra parte, bajo todos los supuestos distribucionales, la probabilidad de transición estimada en cada régimen presenta un alto valor (superior a 0.9 in la mayoría de casos) indicando que cada régimen es bastante persistente y la transición entre regímenes sigue un patrón suave.

Sin duda, uno de los resultados más interesantes en esta tesis es la relación positiva y significativa entre rendimiento y riesgo observada en la mayoría de mercados considerados durante periodos de baja volatilidad. Sin embargo, esta básica relación sugerida en modelos teóricos no se observa durante periodos de inestabilidad financiera. Este resultado se repite para los distintos mercados analizados considerando especificaciones univariantes y bivariantes (es decir, suponiendo conjuntos de oportunidades de inversión constantes y estocásticos). Estos resultados sugieren una aversión al riesgo pro-cíclica de los inversores en todos los mercados como ya notaron otros autores. Generalmente, los regímenes de alta volatilidad corresponden con periodos de recesión o lenta expansión de la economía del país, mientras que los periodos de baja volatilidad corresponden a periodos de expansión. Por tanto, durante periodos de auge los inversores toman una posición más conservadora y se comportan más adversos al riesgo mientras que durante periodos de alta volatilidad este ‘sentido’ del riesgo parece cambiar. Siguiendo esta interpretación estos resultados pueden ser relacionados con el perfil del inversor que permanece en el mercado en cada contexto. Los inversores más adversos al riesgo abandonan el mercado durante periodos de inestabilidad y dejan a los inversores menos adversos negociar durante estos periodos quienes hacen que el nivel de aversión al riesgo observado en estos periodos decrezca con respecto al observado en periodos estables.

El análisis de la evolución de la prima de riesgo en mercados europeos desarrollados revela que durante periodos coincidentes con regímenes de alta volatilidad la prima requerida por los inversores presenta valores mayores que para el resto de la muestra observándose una evolución similar a la volatilidad. A pesar de la disminución del precio del riesgo durante periodos de alta volatilidad, existe un aumento extremo del riesgo de mercado que lleva a primas de riesgo más elevadas durante estos periodos. Además, los modelos GARCH no lineales proporcionan estimaciones ligeramente superiores para la prima de riesgo total durante periodos de alta volatilidad.

Hay otros resultados interesantes para cada Mercado considerado o para cada metodología específica las cuáles se señalan a continuación:

1. Las primas de riesgo estimadas para Europa son generalmente más elevadas que las obtenidas en estudios previos para Estados Unidos, debido principalmente al periodo de inestabilidad financiera generado por la crisis global de 2007-2009. Obtenemos una prima de riesgo promedio entre un 4% y un 8% anual dependiendo del mercado y de la metodología utilizada. Aunque los precios del riesgo muestran diferentes patrones dependiendo del mercado considerado, hay un común y extremadamente elevado riesgo no diversificable en todos los mercados europeos durante el reciente periodo de crisis financiera. Esta es la principal causa del incremento de la prima de riesgo requerida por los inversores durante este periodo.

2. El rol del conjunto de oportunidades alternativas de inversión estocástico (factor de riesgo intertemporal) en los modelos bivariantes no es tan importante como la modelización de las no linealidades en la relación rendimiento-riesgo. Los resultados al considerar un conjunto de oportunidades de inversión estocástico no difieren significativamente de aquellos que asumen uno constante, aunque se obtiene un impacto significativo del componente intertemporal en la relación rendimiento-riesgo. La evidencia es bastante similar incluso usando distintas proxies para el componente de cobertura estocástico obteniendo sólo evidencia favorable durante periodos de baja volatilidad en los modelos MRS-GARCH. Además, el porcentaje de la prima de riesgo total correspondiente a la prima del componente de cobertura (conjunto alternativas de inversión) es relativamente pequeña comparada

con aquella asociada a la prima de riesgo de mercado, aunque en los modelos no lineales las diferencias son menores.

3. El análisis de la relación rendimiento riesgo en mercados emergentes utilizando índices MSCI para cinco de los principales índices en Latino América (Argentina, Brasil, Chile, Méjico y Perú), nueve mercados asiáticos (China, Indonesia, Malasia Tailandia India, Corea del Sur, Filipinas y Taiwán), cinco países de Europa del Este (República Checa, Hungría, Polonia, Rusia y Turquía), tres mercados emergentes africanos (Marruecos, Egipto y Suráfrica) y los índices agregados de mercados emergentes de Asia, Europa del Este y Latino América apoyan los resultados observados en mercados desarrollados de una aversión al riesgo pro-cíclica (en el sentido de que durante periodos de baja volatilidad (asociados con periodos de auge) la aversión al riesgo del inversor es mayor que durante periodos de alta volatilidad (asociados con periodos de crisis).

Finalmente, en el capítulo centrado en la efectividad de estrategias de cobertura no lineales con contratos de futuros obtenemos nuevamente evidencia favorable para los modelos más complejos. La estimación de los modelos revela que existen diferencias significativas en las estimaciones de los parámetros estimados entre estados. Esto puede reflejar que el proceso de volatilidad no está definido únicamente por un único proceso como proponen los modelos GARCH lineales sino por dos procesos de volatilidad diferentes observados durante periodos de baja y alta volatilidad. La consideración de uno en lugar de dos procesos lleva a pobres estimaciones de la volatilidad y puede tener influencia en los ratios de cobertura óptimos estimados. Las diferencias en volatilidad entre periodos de alta y baja volatilidad se observan en el impacto (asimétrico) de las innovaciones y de las varianzas retardadas en la composición y evolución de la volatilidad en cada estado.

Las estimaciones y predicciones de la volatilidad también difieren entre modelos lineales y no lineales. Estas diferencias tienen efectos sobre la efectividad alcanzada por cada estrategia como nuestros resultados empíricos muestran. Los modelos no lineales generalmente superan el resto de modelos para análisis dentro y fuera de la muestra. Los resultados son robustos entre países y para la mayoría de las medidas de efectividad propuestas. Como el análisis fuera de muestra se realiza durante el periodo de crisis financiera, parece que los modelos no lineales mejoran el resto de modelos durante periodos de inestabilidad financiera.

La razón puede ser que la consideración de distintos procesos de volatilidad distinguiendo entre periodos de calma e incertidumbre permiten a estos modelos alcanzar una mejor efectividad que modelos que no realizan esta distinción.

Así que, lo que parece claro es el importante rol de las no linealidades en los campos analizados en esta tesis. Los modelos más complejos presentados aquí reflejan mejor las dinámicas de las series financiera que los tradicionales modelos lineales comúnmente usados en la literatura que analiza estos temas: 1) permite estimaciones más eficientes y predicciones más precisas para la varianza condicional lo que lleva a una mayor efectividad de las coberturas realizadas y 2) permite estimaciones condicionadas al estado de la economía lo que permite re-examinar las conclusiones sobre la relación rendimiento-riesgo. Los principales resultados sugieren que la modelización de la volatilidad a través de modelos no lineales MRSG parece más atractivo y revela otros resultados interesantes los modelos tradicionales no pueden mostrar. Para investigaciones futuras esta metodología puede ser aplicada a otros campos relacionados con la volatilidad del mercado como son la transmisión de volatilidad entre mercados o el análisis de betas variantes en el tiempo desarrollados en un contexto no lineal distinguiendo las conclusiones alcanzadas en estados de alta y baja volatilidad.