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Perception of acceleration in motion-in-depth with only monocular and both monocular and binocular information

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Observers are often required to adjust actions with objects that change their speed. However, no evidence for a direct sense of acceleration has been found so far. Instead, observers seem to detect changes in velocity within a temporal window when confronted with motion in the frontal plane (2D motion). Furthermore, recent studies suggest that motion-in-depth is detected by tracking changes of position in depth. Therefore, in order to sense acceleration in depth a kind of second-order computation would have to be carried out by the visual system. In two experiments, we show that observers misperceive acceleration of head-on approaches at least within the ranges we used [600-800 ms] resulting in an overestimation of arrival time. Regardless of the viewing condition (only monocular or monocular and binocular), the response pattern conformed to a constant velocity strategy. However, when binocular information was available, overestimation was highly reduced.

In many actions requiring adjustment to moving objects, the observer is often confronted with targets that change their speed[1]. In physics, the rate of change in speed is defined as acceleration and the situation of constant velocity is a particular case. This conception raised the issue of whether or not visual acceleration has the same status as in physics. That is to say, if the visual system is tuned to acceleration and we perceive constant velocity just as the case of null acceleration. Pioneer studies showed that humans respond to smoothly accelerated motion as if the velocities were constant but they could detect high rates of changes in speed (see Gottsdanker 1956, for a review). However, these experiments failed to shed more light into the question because important parameters did not deserve consideration (see

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[1] Usually in vision research, "velocity" refers to the motion vector, which is the speed in a certain direction.

Regan, Kaufman, and Lincoln, 1986). The range of velocity, which was frequently too fast (e.g. Kaufman, Cyrulnick, Kaplowitz, Melnick, and Stoff, 1971; Runeson, 1975) or the dependence on motor skill (Gottsdanker, 1952), among others parameters, made difficult to lead to more general conclusions.

Except for the "never-replicated" results reported by Rosenbaum (1975), who concluded that constant acceleration and velocity are perceived accurately and directly, no other work provides data for supporting the idea of direct computation of acceleration. Instead, more recent work showed convincingly that acceleration is only perceived via changes in velocity (Werkhoven, Snippe, and Toet, 1992). Werkhoven et al. used the same paradigm that allowed Nakayama and Tyler (1981) to demonstrate that humans are able to directly sense visual motion and they do not perceive it from change of object's position over time. Hogervorst and Eagle (2000) reported that acceleration plays an important role in recovering three-dimensional structure from motion. Brouwer, Brenner and Smeets (in press) showed that humans can detect changes of velocity even with short presentation times (300 ms), although they conclude that acceleration is not used to initiate locomotion in catching balls.

However, most of the studies mentioned above have addressed the perception of acceleration within a 2-D space. As far as we know, situations where the observer has to face an accelerating object on a head on collision path have not been systematically studied. Motion in depth describes a motion pattern of the retinal image that is different from that generated by motion in the fronto-parallel plane (see figure 1). Furthermore, there is strong psychophysical evidence for independence of motion-in-depth channels. For example, changing-size channels do respond when the target motion is along the line-of-sight only (Regan and Beverly, 1978). A difference between motion in the frontal plane and motion in depth that is relevant to us is related to the velocity-position debate. As mentioned above, there is empirical evidence that the visual system infers 2D motion via velocity detectors (Nakayama and Tyler, 1981; Seiffert and Cavanagh, 1998). Conversely, detection of different kinds of second-order motion, included motion in depth, seems to be achieved with a mechanism sensitive to change in position. For example, motion in stereo-defined stimuli that oscillated in depth was recovered by tracking position (near, far) instead of velocity (Seiffert and Cavanagh, 1998). Such a kind of mechanism would be attention-modulated (Cavanagh, 1992) and when attention is distracted away from the moving target, motion in depth processing would decline. Some studies (e.g. Gray, 2000) support this hypothesis.

As long as motion in depth involves a kind of feature (e.g. position) tracking system, the following question arises: could acceleration in motion in depth be detected by such system? If acceleration in the fronto-parallel plane is sensed, as evidence to date suggest, from comparing speeds at different times, then it is likely that first-order derivatives of motion can be extracted. In other words, acceleration would be perceived indirectly from tracking changes of velocity. However, to the extent that motion in depth is based on changes of position, any sense of acceleration in depth would have to be extracted from changes of position. It would be needed, therefore, a kind of second-order derivative[2] of position. In this paper, we address this issue in two experiments.

In a first experiment, observers had to estimate the arrival time of approaching synthetic objects by using monocular information only, thereby removing stereo-based distance cues that could feed the motion-in-depth mechanism. Changing optical size will be the only available cue for an accurate temporal response to be the expansion pattern. This would feed the changing-size channel of the motion-in-depth mechanism. Since optical size confounds larger objects that are further away with nearer smaller objects, no information on relative position in depth is provided in Experiment 1. We, hence, expect a lack of sense of acceleration in this experiment. Figures 1a and 1b illustrate the similarity of the available monocular visual information between constant velocity and accelerated situations.

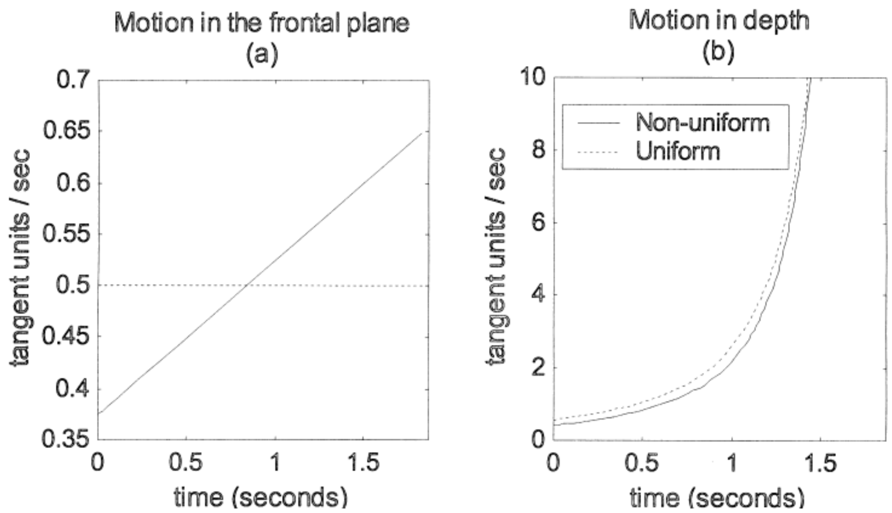

Figure 1. (a) Velocity information in 2D motion. (b) Rate of expansion for an object that is approaching an observer. Note that speed profiles

[2] Acceleration can be defined as the second order temporal derivative of the position function.

for constant velocity and acceleration are very different in 2D motion, while information on rate of expansion is hardly distinguishable between non-uniform and uniform approaching velocity in the case of motion in depth.

In Experiment 2, we introduce stereo-motion by changing relative disparity. There is strong evidence (Cumming and Parker, 1994) that stereo-motion is mainly detected by means of temporal changes in binocular disparity instead of inter-ocular velocity differences. Some authors had the necessity of postulating the existence of two distinct stereoscopic systems (see Regan, 1991 for a revision): a position-in-depth system that would respond to static disparity and a motion-in-depth system, which would detect relative disparity. Therefore, following this hypothesis, stereo-motion would not be detected by tracking changes in position. However, more recent studies (e.g. Cumming and Parker, 1994; Seiffert and Cavanagh, 1998) provide compelling arguments against that. Assuming that stereo-motion is perceived via changes in position-in-depth, any sense of acceleration would be based on the extraction of a second-order derivative of position. Since information on relative position in depth is available in Experiment 2, we expect a more accurate estimation of the arrival time, thereby supporting second-order computations (see general discussion).

EXPERIMENT 1

The geometric layout of the simulated situation is illustrated in figure 2. In this experiment, only the monocular variables were considered. Monocular variables include the visual angle (θ) subtended by the object and its first temporal derivative (θ') : the rate of expansion. It is well known that the ratio θ/θ', known as τ (Lee, 1976), signals the time to contact (e.g. López-Moliner, Maiche and Estaún., 2000; Regan and Hamstra, 1993). However, its use has recently been questioned (Maiche, López-Moliner and Estaún, 2000; López-Moliner and Bonnet, 2002; Smith, Flach, Dittman and Stanard, 2001). In order to perform the task, the knowledge resulting from θ and θ' or some combination of them, is the only available source of information in Experiment 1.

However, τ signals time-to-contact accurately if approaching velocity is constant. If the movement is accelerated or decelerated, the computation of τ would either overestimate or underestimate respectively the arrival time. Figure 1 shows the temporal course of monocular variables for both uniform and non-uniform motion in the 2D and motion-in-depth situations.

As can be noted, on the basis of rate of expansion (figure 1b), accelerated objects are almost indistinguishable from the constant velocity case. Conversely, note the difference between constant velocity and acceleration in the 2D situation (figure 1a).

The aim of this experiment is to examine whether the observers' responses took into account future changes of velocity when only monocular cues of motion in depth are available.

METHOD

Subjects. 8 subjects with normal (4) or corrected-to-normal (4) vision participated in experiment 1. Three subjects are authors of this paper and had foreknowledge of the aim of the experiment.

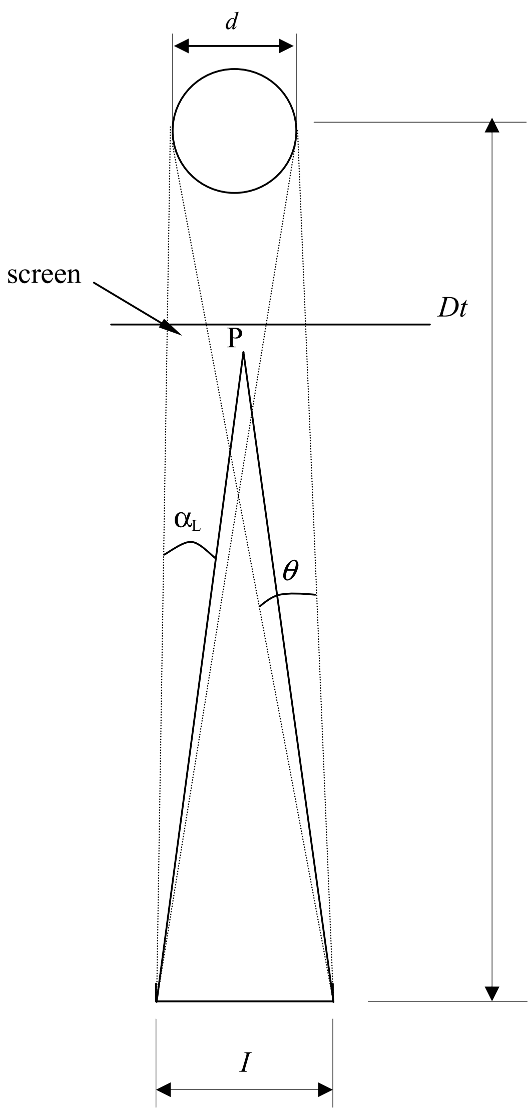

Figure 2. Geometrical layout of the experimental setup. *I* denotes the inter-ocular distance, P is a fixation point on the screen, *d* is the diameter of the simulated object, θ is the angle subtended by the object and it varies as a function of time. Binocular disparity δ relative to P equals α_L + α_R, only α_L is shown. *Dt* denotes the distance from the object to the observer at time *t*.

Apparatus and Stimuli. The stimuli were generated by our own software in a PC (Pentium-II 400 Mhz) and were displayed on a high-resolution monitor (1024 x 768 pixels) at a frame rate of 120 Hz in synchrony with the monitor refresh rate. The screen (EIZO FlexScan F77 21-in.) was viewed monocularly from 60 cm and the unused eye was patched. At that distance the display subtended 36.92 x 27.69 deg. The luminance of the stimuli (solid circles) was 20 cd·m^{-2} and they were superimposed on a black background (0.3 cd·m^{-2}). When constant velocity was simulated, the circle's angular subtense was varied through time according to the following expression (Regan and Hamstra, 1993):

$$\tan\theta_t = \frac{\tan\theta_0}{1 - t/T_0} \qquad (1)$$

where θ_0 is the starting angle and T_0 the designated time-to-contact. However, for accelerated objects is different from zero, we varied the semiangular subtense according to (see mathematical appendix A for a complete derivation):

$$tan\frac{\theta_t}{2} = \frac{d}{(T_0 - t)(at + 2v_0 + a\ T_0)} \qquad (2)$$

where d denotes the physical diameter, v_0 is the initial velocity and a is the constant acceleration.

In order to measure time estimates accurately, an external high-resolution timer incremented a counter every 848 ηsec from the beginning of the animation.

Procedure. Three different values of initial velocity and four proportion of acceleration were used to create the moving stimuli. The three values of initial velocity were as follows: 30, 35 and 40 cm·s^{-1}. Detection of constant acceleration seems to follow a kind of Weber law (Calderone and Kaiser, 1989); that is to say, the amount of acceleration that can be detected is defined as a percentage of the initial velocity. So as to compare acceleration across initial velocities, the different proportions of acceleration (relative to the initial velocity) were set as follows: 0, 20, 40 and 80% of the initial velocity, resulting in different, but comparable, values for each velocity. The percentage of 80% is above the largest threshold reported by Calderone and Kaiser (1989), which is 72%. In order to eliminate rate of expansion as a cue for temporal proximity, we simulated the approaching objects as having two different physical diameters: 2 and 4 cm resulting in 24 (3x4x2) distinct stimuli. Similarly, the presentation duration was uniformly set at random in the interval [600-800 ms] on a trial-to-trial basis to uncorrelate size increment ($\Delta\theta$) and arrival time. This interval is far above 100-140 ms, which is reported to be the temporal window within which the motion system integrates the velocity vector signal (Werkhoven et al. 1992). The simulated starting distance was always set to 75 cm resulting in 12 (3×4) different arrival times ranging from 1.25 sec to 2.5 s. Since this procedure did not preserve orthogonality between acceleration and arrival time, dummy trials were randomly inserted with a probability of 0.2 between the experimental trials. These dummy trials simulated objects approaching at constant velocity with the same arrival times as accelerating objects.

Observers had to press one button to signal the time at which they thought the simulated object would arrive to the point of observation, had it continued on its previous trajectory. Each session consisted of 20 training trials followed by two blocks of 24 experimental trials. The different 24 stimuli were presented once within each block and the order was set at random. Each observer undertook four sessions.

As in Freeman, Harris and Tyler (1994), feedback was given in the form of two sounds in the training trials only. The first sound had a fixed frequency (10 kHz), while the frequency of the second tone varied proportionally to the estimation error. If time was underestimated the second tone had a lower frequency and vice versa. Subjects were accordingly informed of the meaning of the two tones.

RESULTS AND DISCUSSION

Figure 3a shows the mean estimated time under different initial velocities (v_0) as a function of proportion of acceleration (*acc*). As can be seen, estimated time appears to be independent of acceleration. In order to test for a flat curve, we fitted a line to each velocity condition. For all three initial velocities the slope of the fitted lines was not statistically different from zero (v_o=30: t=1.13, p=0.377; v_o=35: t=0.23, p=0.836; v_o=40: t=0.23, p=0.837). However, after conducting a repeated measures ANOVA, the v_0 had a significant effect on the estimated time ($F(2,14)=40.15$; $p<0.001$). Pair comparisons yielded significant differences between the different levels of v_0 (minimum $t(7)$ was 5.69 , p<0.001). In figure 3c, we show the mean estimated time under different object's diameters (s). The same ANOVA on diameter size yielded a significant main effect (F=44.61, $p<0.001$), but neither acceleration nor any interaction ($v_0 * acc$, $v_0 * s$, $s * acc$ and $v_0 * acc * s$) yielded a significant effect.

Figure 3b, shows the mean error of the estimated time. All observers overestimated the arrival time. This pattern conforms to a constant velocity-based strategy, so that the subjects responded as whether the object was moving at constant velocity. On average, the estimated time was consistent with an object traveling at 28 cm·s^{-1}. The overestimation in the constant velocity condition is reported elsewhere (e.g. Freeman et al., 1994). These authors found that observers tended to overestimate short temporal proximities (1-2 s) and underestimate long temporal proximities (4-5 s). Our range of temporal proximity was 1.25-2.5 s, so it was very similar to that used by Freeman et al. Most important for the goal of this experiment is that observer's responses were determined by initial velocity and size instead of temporal proximity. Table 1 shows the summary of a multiple linear regression on data averaged over all observers. Since observers did not take physical acceleration into account, the fit of the model was accordingly poor. The model accounted for 11.8% of the variance. As can be seen, only initial velocity and size contributed to the model in a significant way. Unlike in Freeman et al. (1994) that did not use acceleration, temporal proximity had no predictive value on time. It is worth mentioning at the effect of size on the estimation time. Although time-to-contact was always overestimated, the error was lower for the larger size (4cm). Since relative size can be a cue for distance, and in the absence of other cues, subjects could have interpreted it as an informative source on proximity, thereby considering larger objects as being nearer that smaller objects. This explanation would be consistent with the use of cognitive operations, which have been reported elsewhere in prediction motion tasks (e.g. DeLucia, Tresilian and Meyer, 2000).

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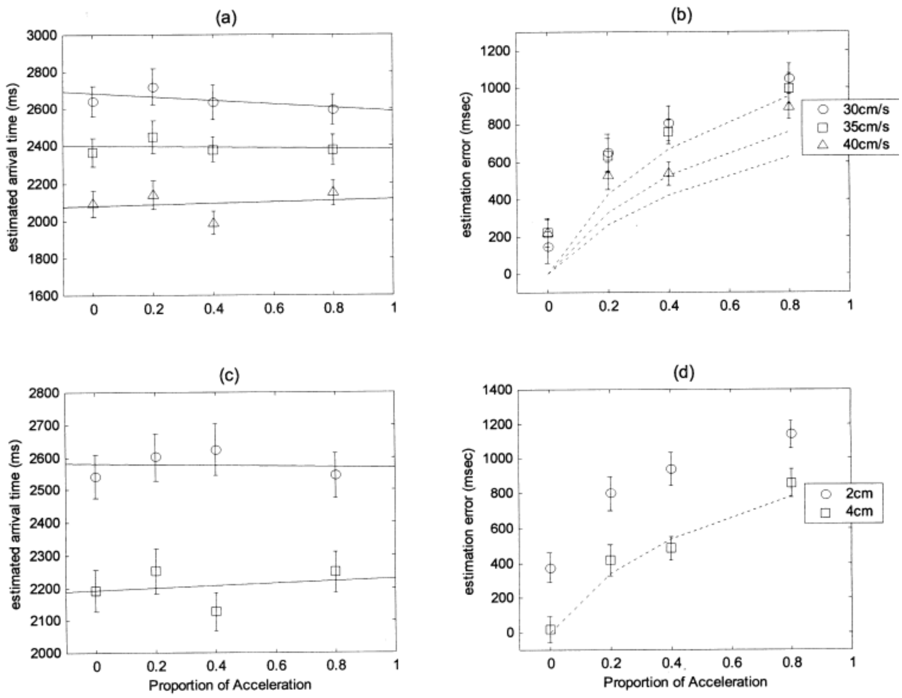

Figure 3. Results of experiment 1 (monocular viewing). (a) Estimated time as a function of acceleration under different initial velocities. (b) Estimation error as a function of acceleration for the different velocities. (c) Estimated time as a function of acceleration under different size conditions. (d) Estimation error as a function of acceleration for the two different sizes. Dashed lines in panels b and d denote the error pattern predicted by a constant velocity strategy, given that the object is moving at the initial velocity. In panel b, we plotted one dashed line for each initial velocity. In panel d, the expected pattern is the mean of the three initial velocities.

Table 1. Summary of Stepwise multiple regression for experiment 1.

Source	Coefficient	*t* ratio	*p*
Intercept	4668.042	*t*=4.552	.000
Acceleration	68.912	*t*=.254	.800
Initial velocity	-49.719	*t*= -3.676	.000
Size	24.539	*t*= -7.861	.000
Temporal proximity	0.03608	*t*= .131	.896

EXPERIMENT 2

METHOD

Subjects. The same 8 observers who participated in Experiment 1 served as subjects in Experiment 2.

Apparatus and Stimuli. We used the same stimuli and apparatus as in experiment 1. The only difference is the creation of the stereo-motion by changing relative disparity through time. Different images were presented to both eyes by using LDC shutter spectacles (ASUS VR100). Each eye received new images at a frame rate of 60 Hz. Since monitor was viewed from 60 cm and simulated starting distance was 75 cm. initial disparity (uncrossed) was 1.24 deg. The maximum final relative disparity could range from 1.09 to 5.13 deg (crossed). As in Experiment 1, presentation time was uniformly set randomly within the interval [600, 800 ms], thereby removing final relative disparity as a cue for temporal proximity. All the subjects could fusion both images along the displayed trajectory. Relative disparity at any time *t* is defined as:

$$\delta_t = \alpha_{L,t} + \alpha_{R,t} \qquad (3)$$

where α is the angle subtended between the object and the fixation point (P) (see figure 2) and in our case is the same for both eyes. α is updated according to:

$$\alpha_t = \left[\frac{(T_0 + t)(at + 2v_0 + aT_0)}{I}\right] - C \qquad (4)$$

where I is the inter-ocular distance, v_0 is the initial velocity, a denotes the acceleration and T_0 the time to contact. Finally, C is a constant term, in our case C =1.5167 rad (see mathematical appendix B for details).

Procedure. Exactly the same procedure as in Experiment 1 served for carrying out this experiment.

RESULTS AND DISCUSSION

As in Experiment 1, acceleration did not influence estimations of TTC. As can be seen in Figure 4a, the mean estimated time-to-contact under different initial velocities as a function of acceleration described a flat curve, (slope=0, v_0=30: t=0.44, p=0.702; v_0=35: t=0.697, p=0.558; v_0=40:

t=0.707, p=0.553). A repeated measure ANOVA on initial velocity had a significant effect on the estimated time ($F(2,14)=54.32$, $p<0.001$). As before, there was a significant difference between distinct levels of initial velocity (minimum $t(7)=5.69$, p<0.001). Unlike Experiment 1, object's size did not had a significant effect ($F(1,7)=4.68$, $p=0.07$), compare figure 4c with figure 3c (significant effect of size in experiment 1). As before, neither acceleration nor any interaction had a significant effect. We performed a multiple linear regression on averaged data and table 2 shows the contribution of the different variables to the estimated time. Again, only initial velocity and size contributed to the model in a significant way. However, the percentage of variance accounted for by size declined from 5.67% in Experiment 1 to 0.61% in Experiment 2. Like in the monocular viewing experiment, the percentage of variance accounted for by the model was poor (6.2%) due to the fact that the relevant physical feature of the stimuli (acceleration) could not be taken into account.

Figures 4b and 4d show the estimation error. Time is clearly underestimated for the constant velocity condition and for the lowest level of acceleration (20%). However the pattern of the estimation error resembled that obtained in experiment 1. The mean estimated arrival time conforms to a constant velocity of 34 $cm \cdot s^{-1}$, so a little bit larger than in Experiment 1 resulting in less error. The fact that size did not contribute in this experiment as did in the monocular condition, could be explained by the use of disparity-based position information instead of any relative distance cue provided by size. Since in this experiment binocular relative disparity provided the subjects with knowledge of relative distance in depth, it did not make sense to use relative size as relevant source for distance. This result is consistent with the findings reported by Gray and Regan (1998). These authors showed that size was much less used when binocular information was available. They did not use accelerated stimuli, though.

The pattern of results could conform to the strategy of detecting changes in position in depth at different rates specified by the distinct initial velocities. The strategy also applies to the monocular situation. Had the subject responses been based on changes of velocity, they would not have been affected by size, since for a given constant velocity the relative rate of expansion is the same for both sizes. Therefore, their response pattern was consistent with the use of changing distance position in depth.

Finally, it seems that such strategy is unable to detect acceleration from tracking changes of position, at least in the ranges we have used.

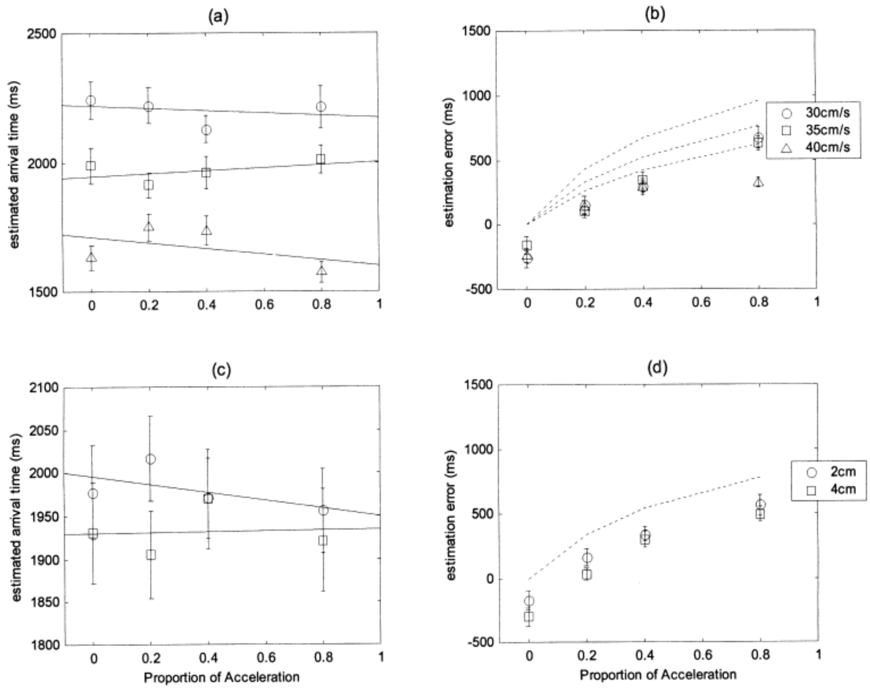

Figure 4. Results of experiment 2 (binocular viewing). (a) Estimated time as a function of acceleration under different initial velocities. (b) Estimation error as a function of acceleration for the different velocities. (c) Estimated time as a function of acceleration under different size conditions. (d) Estimation error as a function of acceleration for the two different sizes. The dashed line denotes the same as in figure 3.

Table 2. Summary of Stepwise multiple regression for experiment 2.

Source	Coefficient	*t* ratio	*p*
Intercept	4197.551	*t*= 3.391	.001
Acceleration	-12.538	*t*=-.038	.969
Initial velocity	-50.946	*t*=-3.120	.002
Size	-70.052	*t*= -2.365	.018
Temporal proximity	0.02405	*t*=.072	.943

GENERAL DISCUSSION

Different sources of information can potentially have a relevant role in visually guided actions. From a Gibsonian perceptual framework, it has often been argued that detecting invariants, which specify properties of the external environment, would be sufficient for action control. Detecting invariants is assumed to be immune to either cognitive operations or any other heuristic strategy. Probably τ is the most known example of invariant that has been put forward in order to account for the control of action. This parameter, however, cannot account for future changes of velocity providing that its computation assumes constant velocity for the approaching object. Regardless of this limitation, our data clearly suggest that some cognitive heuristics take place in order to achieve the TTC of an accelerated object, even for short time spans like those used in our experiments.

Results from Experiment 1 tell us about the implicit use of relative distance inferred from the two different simulated physical sizes. Had the subjects tried to use an invariant function (e.g. τ), we would not have found differences between the two simulated object's diameters. Note that τ is invariant across different physical sizes. The use of object's size as a source of information seemed to be overridden by relative disparity in Experiment 2. Relative disparity is by no means invariant to absolute distance. Therefore, its use in Experiment 2 turns out to be a similar position-based strategy as in the monocular viewing condition (experiment 1).

Nevertheless, some differences between both experiments should be pointed out. Using position in depth knowledge based on size could likely imply more cognitive operations than using binocular relative disparity. The angle's subtense of larger objects further away can be the same as the angle's subtense of nearer smaller object, given the synthetic nature of the simulated objects.

Although both are cues for relative position in depth, relative disparity unambiguously signals which object is further away from the observer. It makes sense, hence, that binocular information overrides information coming from size processing. Overall, our data are reflecting the ability of the visual system for shifting from one source of information to another (Rushton and Wann, 1999).

Yet another question, which we previously raised, concerns the privileged use of changing position instead of velocity in motion in depth. Although acceleration in the fronto-parallel plane can be detected via changes of velocity (e.g. Werkhoven et al. 1992), this does not seem to apply to the motion in depth case. Our results suggest that observers can

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differentially respond to distinct velocity patterns. Note that initial velocity yielded a significant effect on temporal estimates. However they failed to sense changes of this velocity that is acceleration. We argue, according to reported data (e.g. Seiffert and Cavanagh, 1998), that observers can successfully react to different approaching velocities because they do sense velocity in depth via object's changing position. The one-steps further necessary to sense acceleration from changing position seems to be difficult to achieve by the visual system. However, we must keep in mind that we used a relatively short time span (up to 800 ms). While this temporal window would suffice for detecting acceleration from changes of velocity in the fronto-parallel plane (Werkhoven at al. 1992, Brower et al. in press) it might not for sensing acceleration from changing position. Larger temporal windows should be used in future research to address this question.

RESUMEN

Percepción de la aceleración en el movimiento en profundidad con información monocular y con información monocular y binocular. En muchas ocasiones es necesario adecuar nuestras acciones a objetos que cambian su aceleración. Sin embargo, no se ha encontrado evidencia de una percepción directa de la aceleración. En su lugar, parece ser que somos capaces de detectar cambios de velocidad en el movimiento 2-D dentro de una ventana temporal. Además, resultados recientes sugieren que el movimiento en profundidad se detecta a través de cambios de posición. Por lo tanto, para detectar aceleración en profundidad sería necesario que el sistema visual lleve a cabo algun tipo de cómputo de segundo orden. En dos experimentos, mostramos que los observadores no perciben la aceleración en trayectorias de aproximación, al menos en los rangos que utilizados [600-800 ms] dando como resultado una sobreestimación del tiempo de llegada. Independientemente de la condición de visibilidad (sólo monocular o monocular más binocular), la respuesta se ajusta a una estrategia de velocidad constante. No obstante, la sobreestimación se reduce cuando la información binocular está disponible.

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Mathematical appendix A

We need to derive the relationship between time t and the angular subtense (θ_t) of an object of diameter d that approaches an observer with an initial velocity v_0 and constant acceleration a. Using basic trigonometry, from figure 1, at any time t, being t>0, we have that:

$$\tan\frac{\theta_t}{2} = \frac{d/2}{D_t} \qquad \text{(a.1)}$$

Where D_t is the distance from the object to the observer at time t.

Since

$$D_t = D_0 - \left[\frac{1}{2}at^2 + v_0 t\right] \qquad \text{(a.2)}$$

where D_0 is the starting distance from the object to the observer, and

$$D_0 = \frac{1}{2}aT_0^2 + v_0 T_0 \qquad \text{(a.3)}$$

where T_0 is the arrival time or time-to-contact at time $t = 0$. Combining (a.2) and (a.3), we have:

$$D_t = \left[\frac{1}{2}aT_0^2 + v_0 T_0\right] - \left[\frac{1}{2}at^2 + v_0 t\right] \qquad \text{(a.4)}$$

Simplifying (a.4) and then factorizing we have:

$$D_t = -\frac{1}{2}\left(-T_0 + t\right)\left(at + 2v_0 + aT_0\right) \qquad \text{(a.5)}$$

Thus (a.1) can be rewritten as

$$\tan\frac{\theta_t}{2} = \frac{d}{(T_0 - t)(at + 2v_0 + aT_0)} \qquad \text{(a.6)}$$

Mathematical appendix B

We need to derive how α varies with time. Since same procedure applies to both eyes (α_L and α_R), we will drop eye subindex . From figure 2 we have that at any time *t*:

$$\alpha_t = \mathrm{atan}(D_t / (I/2)) - \mathrm{atan}\,(D_m / (I/2)) \qquad \text{(b.1)}$$

where I/2 denotes the semi-interocular distance, D_t the distance from the object to the observer at time *t* and D_m the distance from the monitor to the observer. Since D_m is a constant, we will denote the second member of the subtraction by C, in our case C ≈ 1.5167 rad. Therefore,

$$\alpha_t = \mathrm{atan}(D_t / (I/2)) - C \qquad \text{(b.2)}$$

and combining (b.2) and (a.5), we have

$$\alpha_t = \left[\frac{(T_0 + t)(at + 2v_0 + aT_0)}{I}\right] - C \qquad \text{(b.3)}$$