Psicológica (2010), 31, 647-657.

The law of elasticity

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Participants estimated the imagined elongation of a spring while they were imagining that a load was stretching the spring. This elongation turned out to be a multiplicative function of spring length and load weight—a cognitive law analogous to Hooke's law of elasticity. Participants also estimated the total imagined elongation of springs joined either in series or in parallel. This total elongation was longer for serial than for parallel springs, and increased proportionally to the number of serial springs and inversely proportionally to the number of parallel springs. The results suggest that participants integrated load weight with imagined elasticity rather than with spring length.

Intuitive physics refers to the cognitive laws of our tacit knowledge of the ordinary physical world (Anderson, 1983; Lipmann & Bogen, 1923; McCloskey, 1983; Shanon, 1976; Smith & Casati, 1994; Wilkening & Huber, 2002). In the following we report an investigation of the intuitive physics related to Hooke's law of linear elasticity. We begin with a description of this law.

HOOKE'S LAW

Consider a close-coiled helical spring with length L and external diameter D, suspended from a fixed support. After an object with weight W is suspended from the lower end of the spring, Hooke's law says that the spring elongation (increment in L) is

$$E = k_0 + k \cdot W \tag{1}$$

with k_0 a measurement error and k a parameter expressing the elasticity of

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the spring (Timoshenko, 1953, pp. 17-20). This parameter increases as L increases according to the law

$$k = k_1 \cdot L \tag{2}$$

with k_1 a parameter representing the effects on elasticity of all factors other than L (Wahl, 1963). Equations 1 and 2 imply

$$E = k_0 + k_1 \cdot L \cdot W\,. \tag{3}$$

EXPERIMENT 1

In the following experiment ten undergraduates produced estimates of imagined elongation of springs with different combinations of values of L, D, and W.

Stimuli. Each stimulus consisted of two objects, a close-coiled helical spring of hardened steel and a non-transparent plastic bottle. Six springs and four bottles were used. The spring wire had circular section with a diameter of 0.6 mm. The number of coils per unit of spring length was the same for all springs. Spring length (L) was 9, 18, or 28 cm. For each L, spring diameter (D) was 7.5 or 9 mm. All bottles had width and height of 9 and 17 cm, respectively. They had a rigid plastic ring on top. For each combination of L and D, bottle weight (W) was 1, 2, 3, or 4 Kg.

To ascertain that the helical springs we used were linearly elastic, we measured E for each different combination of values of L, D, and W—one measurement per combination. Figure 1 shows E plotted against W for each L and each D. The results verify Equation 3.

Each end of each spring had one coil bent obliquely. Each of these two bent coils was linked with one of two rings of rigid steel with thickness and diameter of 1 and 20 mm, respectively. On each trial, the experimenter inserted one of these rings in a hook solidly fixed on a wooden ceiling positioned 1.7 m above the floor. This operation made the spring hang vertically from this ceiling.

The participant was positioned at about 1.6 m from the spring. On the right, about 12 cm from the spring, a vertical 1-m-long measuring tape with markings indicating centimeters and millimeters was constantly presented. One screen was used to prevent the participant from viewing the bottles and the springs before the presentation of the stimulus.

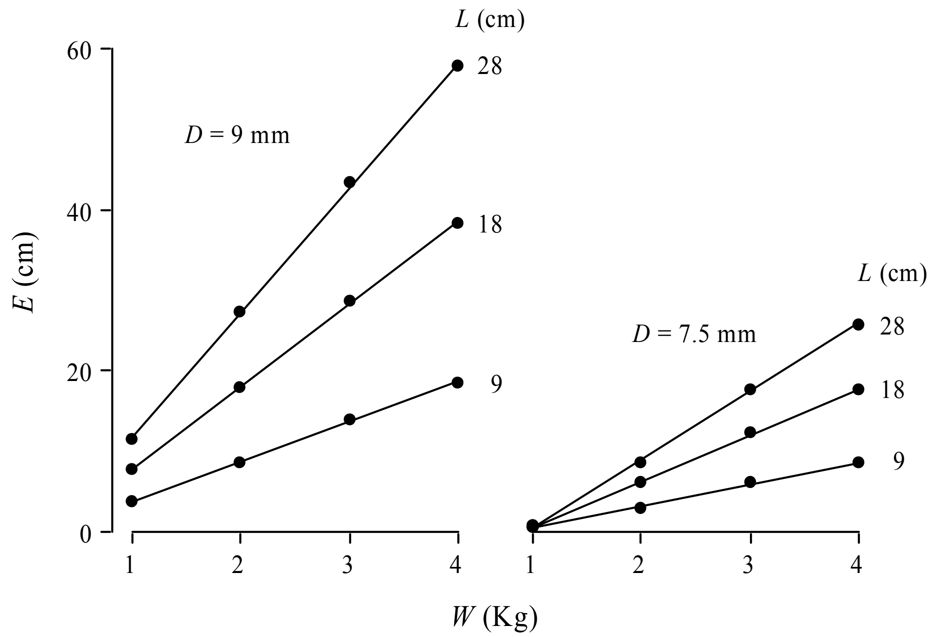

Figure 1. Elongation (*E*) of a vertically suspended close-coiled helical spring plotted against the weight (*W*) of an object hanging from the lower end of this spring. The parameters are the length (*L*) and the diameter (*D*) of the spring. Each combination of *L*, *D*, and *W* defines a different stimulus for Experiment 1.

Procedure. Recall that each spring had two rigid rings. On each trial, the experimenter handed a spring placed horizontally to the participant and asked the participant to insert the participant's left and right index fingers in the left and right rings, respectively. The experimenter then asked the participant to extend the spring by pulling the rings in opposite directions.

When the participant had terminated extending the spring, the participant handed the spring back to the experimenter who hung the spring from the wooden ceiling. The participant saw this operation of hanging. After the spring was hung the experimenter handed a bottle to the participant.

The participant was instructed to estimate the extent of the elongation of the spring hanging from the ceiling in the event that the bottle held in the participant's hands was hanging from the lower end of the spring. The participant had the task to estimate this imagined elongation in centimeters as seen on the measuring tape. The experimenter ensured that the participant clearly understood that "elongation" meant only the extra length the spring would have in the event that the bottle was hanging from the spring, excluding the initial length the spring had when it was at rest.

The entire series of 24 stimuli was presented to each participant two times consecutively, each time with stimuli in random order. The duration of the experiment varied from 16 to 29 min.

Before the experiment, so that the participant realized that the springs were elastic and that the plastic ring of bottles and the rings linked with the end coils of the springs were rigid, the spring with diameter of 7.5 cm and length of 18 cm was hung on the wooden ceiling and the bottle of 3 Kg was hung from the lower end of this spring using a thin hook made of rigid steel.

Immediately after the participant saw that only the spring was extending as a result of the hanging of the bottle, the participant was asked the following questions, one at a time, in this order: (i) Do you remember your studies on elasticity? Is the elongation of a spring increased, decreased, or unchanged when (ii) a weight suspended from the spring, (iii) the spring length, or (iv) the spring diameter is increased?

RESULTS AND DISCUSSION

Recall that E, L, D, and W denote physical variables: spring elongation, spring length, spring diameter, and bottle weight, respectively. Let $\boldsymbol{e}$, $\boldsymbol{l}$, $\boldsymbol{d}$, and $\boldsymbol{w}$ denote imagined spring elongation, perceived spring length, perceived spring diameter, and perceived bottle weight, respectively.

In Figure 2, the left and central diagrams show mean estimated $\boldsymbol{e}$ plotted against the functional measure $\boldsymbol{f_w}$ of $\boldsymbol{w}$, averaged across D and across L, respectively, and the right diagram shows mean estimated $\boldsymbol{e}$ plotted against the functional measure $\boldsymbol{f_l}$ of $\boldsymbol{l}$, averaged across W. The measures $\boldsymbol{f_w}$ and $\boldsymbol{f_l}$ are column means of mean estimated $\boldsymbol{e}$. These measures represent the perceived values of weight and length, respectively. On the horizontal axis the uneven spacing of the $\boldsymbol{f_w}$ measures means that perceived weight is nonlinearly related to objective weight.

The results in the left, central, and right diagrams agree with the laws

$$\boldsymbol{e} = a_0 + a_1 \cdot \boldsymbol{l} \cdot \boldsymbol{w} \tag{4}$$

$$\boldsymbol{e} = b_0 + b_1 \cdot \boldsymbol{d} \cdot \boldsymbol{w} \tag{5}$$

$$e = c_0 + c_1 \cdot l \cdot d\,, \tag{6}$$

respectively, with a_0, b_0, c_0, a_1, b_1, and c_1 parameters. The cognitive law expressed by Equation 4 is formally identical to Hooke's law of elasticity.

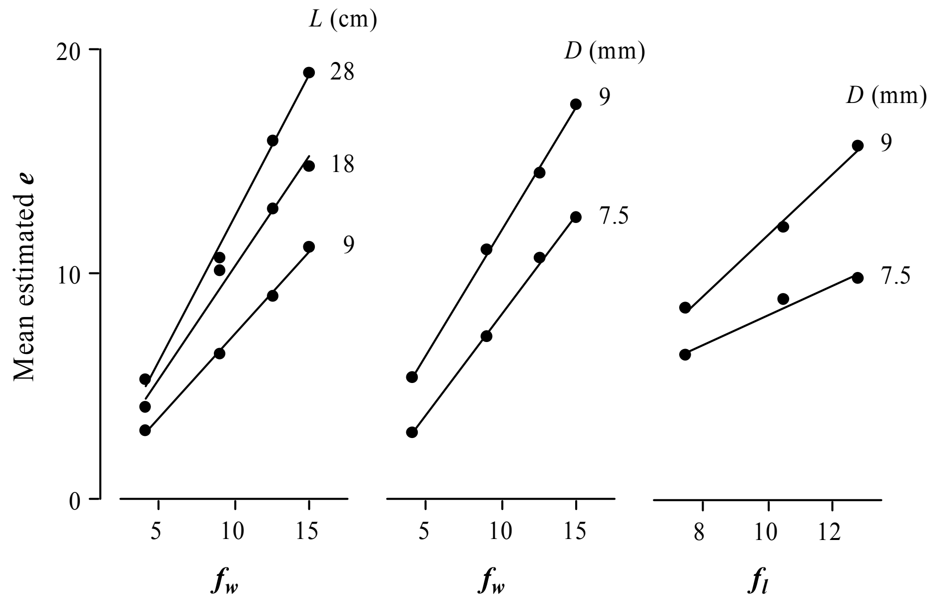

Figure 2. Results of Experiment 1: Mean estimated potential elongation (e, subjective centimeters) of a spring of length L and diameter D plotted against the functional measures of perceived object weight (f_w, left and central diagrams) and perceived spring length (f_l, right diagram). The uneven spacing of the f_w measures means that perceived weight is nonlinearly related to objective weight.

Equations 4–6 are implied by the linear fan theorem of functional measurement (Anderson, 1982, p. 73). When a factorial experimental design is used, this theorem says that the dependent variable (e) is a multiplicative function of the independent variables (l and w, or d and w, or l and d) when the following two premises are met. *Premise 1*: Estimates of the dependent variable are numbers linearly related to the values of the dependent variable. *Premise 2*: Plotting the dependent variable against the functional measure of an independent variable (for example, f_w) for different values of another independent variable (for example of l) yields curves forming a fan of divergent straight lines. Premise 1 is supported by the empirical finding that estimates of linear extent produced by the present method of estimation and by the bisection and rating methods are linearly related to one another (Masin, 2008). Premise 2 is supported when the respective interaction is significant with the linear-linear component of this interaction significant and none of the other components significant.

Equations 4–6 agree with the results of a 3 (L) × 2 (D) × 4 (W) analysis of variance that showed that the two-factor interactions and the respective linear-linear components were significant with the three-factor interaction not significant:

$L \times D$: $F(2,18) = 10.0, p < .005$; $F(1,9) = 11.3, p < .01$

$L \times W$: $F(6,54) = 3.6, p < .005$; $F(1,9) = 16.1, p < .005$

$D \times W$: $F(3,27) = 7.9, p < .001$; $F(1,9) = 29.1, p < .005$

$L \times D \times W$: $F(6,54) = 0.7$.

All the other components of these interactions were not significant [Fs(1,9) = 1.1–3.9].

Inspection of individual data showed that for nine participants the individual patterns of curves were practically the same as the corresponding patterns of curves shown in Figure 3. For the remaining participant, mean estimated ***e*** increased with W and D but was essentially unaffected by L.

The answers to the questions asked to participants were the following.

(i) Eight participants said that they did not remember their studies on elasticity and two that they remembered little about them without knowing Hooke's law.

(ii) All participants said that increasing W increases E.

(iii) Four participants said that increasing L increases E, four said that increasing L decreases E, and two said that increasing L does not alter E.

(iv) Seven participants said that increasing D decreases E and three said that increasing D increases E.

EXPERIMENT 2

The following experiment served to test whether participants imagine the elasticity of springs. Consider springs of equal length and elasticity connected in series or in parallel. The total elasticity of these connected springs is directly proportional to the number of springs when the springs are in series and is inversely proportional to the number of springs when the springs are in parallel (Wahl, 1963). Participants estimated the imagined total elongation of these springs caused by different weights. If participants integrate ***w*** with total spring length, the total imagined elongation of springs in series must be equal to that of these same springs in parallel, since total length of springs is the same in the two cases. If participants imagine the elasticity of springs from the configuration of springs, total imagined elongation must increase proportionally to the number of springs for serial springs and increase inversely proportionally to the number of springs for parallel springs.

Participants. The participants were ten undergraduates. None of them had participated in Experiment 1.

Stimuli. The bottles used for Experiment 1 were used for Experiment 2. Close-coiled helical springs of hardened steel with length, diameter, and wire diameter of 10, 0.7, and 0.05 cm, respectively, were used. Each bottle was combined with one, two or three springs in series (vertical springs aligned vertically) or in parallel (vertical springs aligned horizontally). A rigid ring of steel was linked with each end of each spring. Contiguous springs in series were linked by a single ring. For springs in parallel, a single ring linked all the upper rings and another single ring linked all the lower rings. There were five combinations of springs: one single spring (the combination in series or in parallel of one spring with no other spring), two springs in series, three springs in series, two springs in parallel, and three springs in parallel. Each stimulus was the combination of one of these five combinations of springs with each of the four bottles.

Procedure. The procedure and instructions were the same as those of Experiment 1 with the exception that participants only saw the springs, that is, they did not touch any spring with their hands. A screen was used to hide the operations for the hanging of the springs on the wooden ceiling, in addition to the screen used to prevent the participant from viewing the bottles and the springs before the presentation of the stimulus.

The entire series of 20 stimuli was presented to each participant two times consecutively, each time with stimuli in random order. The duration of the experiment varied from 10 to 15 min.

Before the experiment, a single spring was hung on the wooden ceiling and the bottle of 1 Kg was hung on its lower end. The participant could see that the spring extended as a result of the hanging of the bottle. The participant was then asked the following questions, one at a time, in this order: (i) Do you remember your studies on elasticity? Is total spring elongation increased, decreased, or unchanged when (ii) bottle weight is increased, (iii) springs are in series, or (iv) springs are in parallel? (v) Is the total elongation of springs in series longer, shorter, or the same as that of the same springs in parallel?

RESULTS AND DISCUSSION

Let $\boldsymbol{e}$ be the total imagined elongation of springs, in series or in parallel. Figure 3 shows mean estimated $\boldsymbol{e}$ plotted against the functional measure $\boldsymbol{f_w}$ of $\boldsymbol{w}$ for each number of serial or parallel springs. Had participants integrated $\boldsymbol{w}$ with the total length of springs, mean estimated $\boldsymbol{e}$ would have increased with the number of springs both for serial and parallel springs. Instead, mean estimated $\boldsymbol{e}$ increased proportionally to the total length of serial springs and inversely proportionally to the total length of parallel springs. These results suggest that participants imagined the elasticity of springs.

In Figure 3, least-square straight lines fit the data points. The divergence of these lines agrees with the law

$$\boldsymbol{e} = u_0 + u_1 \cdot \boldsymbol{\varepsilon} \cdot \boldsymbol{w} \tag{7}$$

with u_0 and u_1 constants and $\boldsymbol{\varepsilon}$ the imagined elasticity.

Equation 7 agrees with the results of a 2 (spring configuration, C) × 3 (number of springs, N) × 4 (W) analysis of variance made with the individual estimates of $\boldsymbol{e}$ for the single spring in series being the same as those for the single spring in parallel.

This analysis showed that the effect of N [$F(2,18) = 0.6$] and the $N \times W$ interaction [$F(6,54) = 0.6$] were not significant.

The effect of C [$F(1,9) = 5.8, p < .05$] and the following interactions and respective linear-linear components were significant:

$C \times N$: $F(2,18) = 5.8, p < .05$; $F(1,9) = 5.9, p < .05$

$C \times W$: $F(3,27) = 6.3, p < .005$; $F(1,9) = 7.8, p < .05$

$C \times N \times W$: $F(6,54) = 5.1, p < .005$; $F(1,9) = 8.5, p < .05$.

All the other components of these interactions were not significant [Fs$(1,9) = 0.001$–4.5].

For seven participants, the individual patterns of factorial curves were practically the same as the respective patterns of curves in Figure 3. For the other three participants, estimated ***e*** for serial springs increased with *N* in two participants and was practically constant as *N* increased in the remaining participant, and estimated ***e*** for parallel springs decreased as *N* increased in two participants and was practically constant as *N* increased in the remaining participant. Removal of the data of these three participants produced no essential changes in the results of the above statistical analysis or in the patterns of curves in Figure 3.

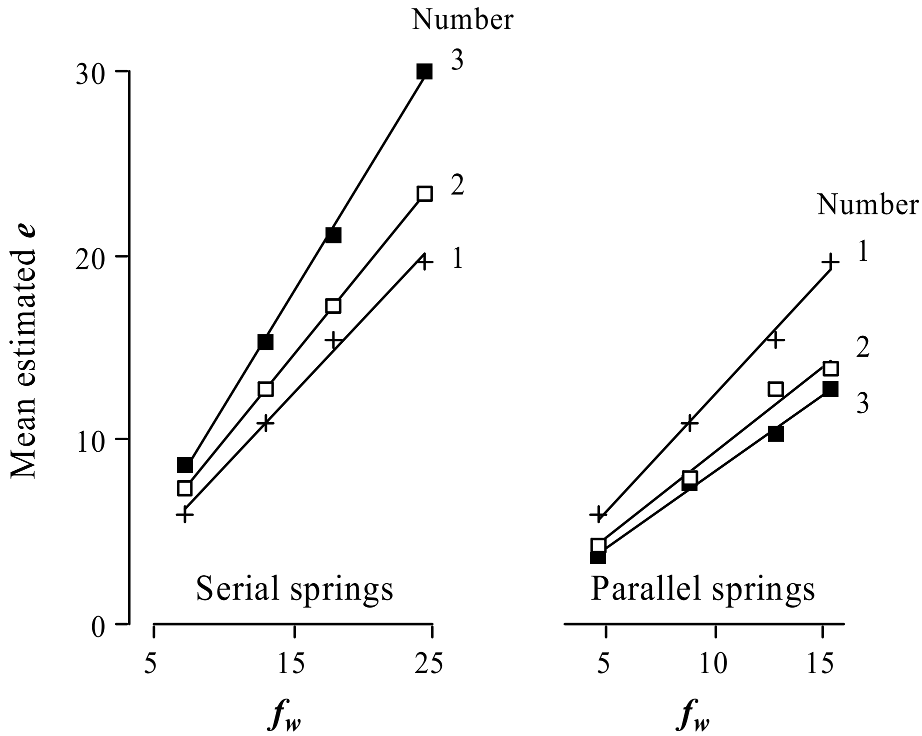

Figure 3. Results of Experiment 2: Mean estimated total imagined elongation (*e*, subjective centimeters) of 1, 2, or 3 springs, joined in series or in parallel, plotted against the functional measure of object weight, f_w.

The answers to the questions asked to participants were the following.

(i) Five participants said that they did not remember their studies on elasticity. The remaining five participants said they remembered little about these studies. When asked further, they said that they did not know about Hooke's law.

(ii) All participants said that increasing W increases E.

(iii) Seven participants said that adding springs in series increases total E, two said that this addition decreases total E, and one said that this addition does not alter total E.

(iv) Seven participants said that adding springs in parallel decreases total E, two said that this addition does not alter total E, and one said that this addition increases total E.

(v) Nine participants said that total E for springs in series is longer and one said that it is shorter than total E for the same springs in parallel.

CONCLUSION

The main finding of Experiment 1 is that the imagined elongation of a spring caused by a load is a multiplicative function of the perceived length of the spring and the perceived weight of the load as expressed by Equation 4, formally identical to Hooke's law of elasticity. The results of Experiment 2 show that individuals can imagine the elasticity of springs, indicating that Equation 7 more properly expresses the cognitive law of elasticity.

It is possible that the cognitive law of elasticity resulted from the participants' past experience with elastic objects of ordinary life. Experiment 2 showed that participants implicitly assumed that the springs were of linear elastic material. However, since these participants did not touch any springs, they could not know whether the springs were linearly or nonlinearly elastic. The unwarranted assumption of linear elasticity suggests that participants manifested the multiplicative law independently of the phenomenon of elasticity. This consideration agrees with the following alternative interpretation of the cognitive law of elasticity (Anderson, 1983).

In a wide variety of judgment tasks in all fields of mental life, there is ample evidence that people integrate information using three main types of rules: adding, multiplying, and averaging (Anderson, 1991, 1996). Participants use these rules in judgment tasks regarding physical objects as in this study and in judgment tasks regarding no physical object as, for example, in social cognition or person perception. This consideration suggests that the cognitive law of elasticity disclosed by the factorial graphs in Figures 2 and 3 is probably merely the manifestation of the particular integration rule the participants selected for the task of the present study (Anderson, 1983). A relevant question of intuitive physics in need of future study is the determination of which factors of a judgment task make particular integration rules supersede other rules (Anderson, 1983; Wilkening, 1982).

Integration rules may assist in the process of scientific investigation.

For example, how did Robert Hooke discover his physical law? The present results suggest that, like the participants of this study, he most probably tacitly knew his law before he discovered it. He may have known it by experience with elastic materials or by automatically using the multiplicative integration rule typically applied in many judgment tasks of ordinary life. Plausibly, other laws of physics were discovered by the same process of focusing on an integration rule and empirically testing its physical consequences.

Teachers of physics should consider the students' automatic use of integration rules (McDermott, 1991). These rules may favor scientific inquiry but may also hinder the understanding of physical phenomena. For example, students may tacitly attribute to elastic objects properties such a linearity which may be inherent in the integration rule rather than in the object.

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(Manuscript received: 15 June 2009; accepted: 24 September 2009)