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Holographic flavor on the Higgs branch

(Based on work with A.V. Ramallo and D. Rodríguez Gómez)
hep-th/0703094 , (hep-th/0609010)

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Valencia, October 2007

- Adding flavor to the AdS/CFT correspondence.
  Dp-Dq brane intersections.

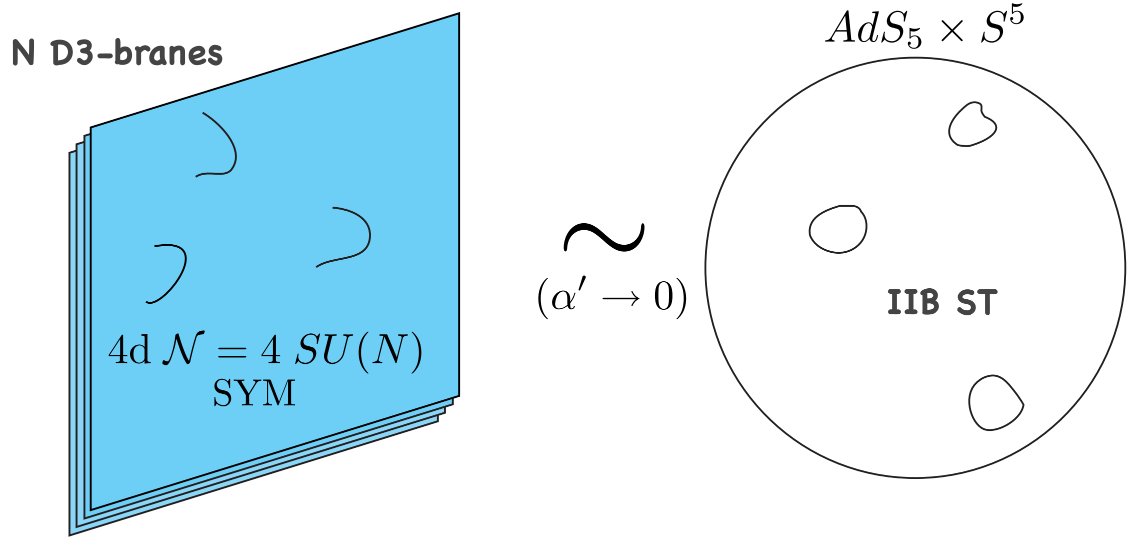
- Holographic dual of the Higgs branch. Macro & micro descriptions.

- Higgs branch of the Dp-D(p+4) system.
  D3-D7: $4\mathrm{d}\ \mathcal{N}=4\ SU(N)\ \mathrm{SYM}$ + N_f fundamental hypermultiplets.

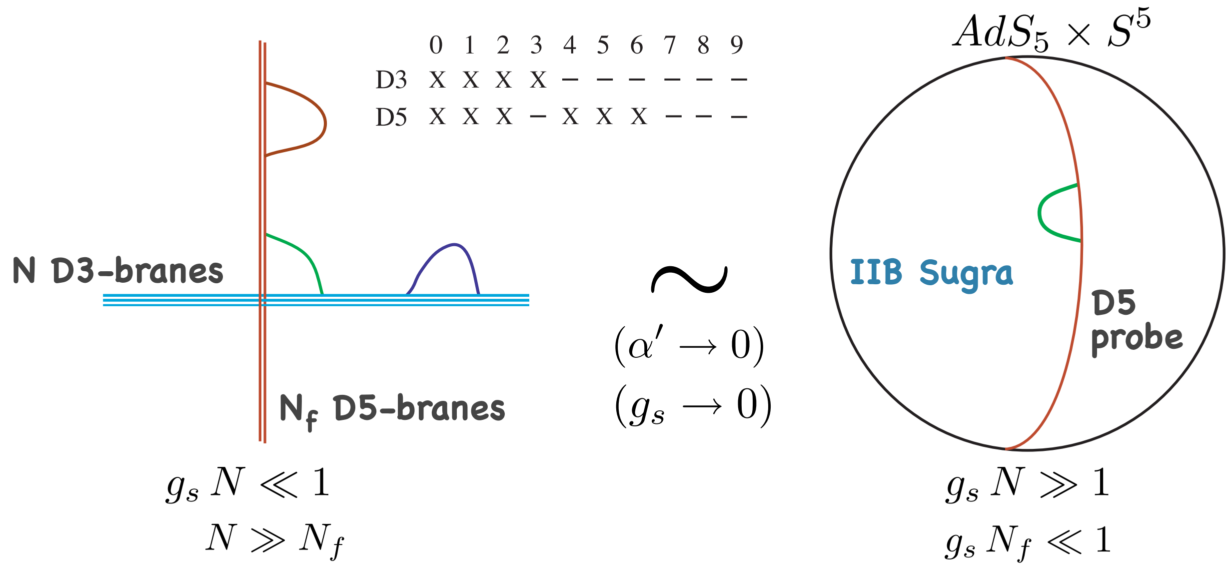
- Higgs branch of the Dp-D(p+2) setup.
  D3-D5: $4\mathrm{d}\ \mathcal{N}=4\ SU(N)\ \mathrm{SYM}$ + 3d fundamental hypermultiplets.

- Dp-Dp, F1-Dp & M-theory M2-M5 intersections.

AdS / CFT Correspondence



Adding Flavor
(living in a defect)



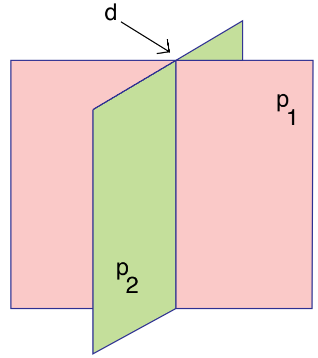
(Karch, Randall)

$(\,d\,|\,p_1 \perp p_2\,)$ INTERSECTIONS

$N_1\ p_1 - branes$

$N_2\ p_2 - branes$

$p_2 \geq p_1$



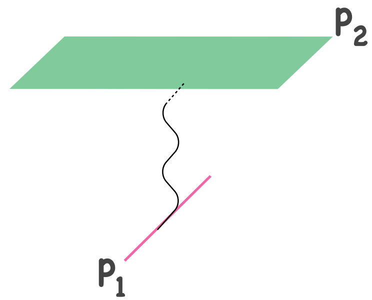
$N_1 \gg N_2$

p_2-brane probe in the p_1-brane background

p_2 -brane probe Fluctuations ⟷ MESONS

[(Karch, Katz) D3-D7]

MASSIVE QUARKS ⟶



- SUSY INTERSECTIONS:
  - $(\,p\,|\,Dp \perp D(p+4))$ [(3 | D3-D7)]
  - $(\,p-1\,|\,Dp \perp D(p+2))$ [(2 | D3-D5)]
  - $(\,p-2\,|\,Dp \perp Dp)$ [(1 | D3-D3)]

Higgs Branch $\longrightarrow$ $\langle$ fundamentals $\rangle \neq 0$

- RECOMBINING COLOR & FLAVOR BRANES (Giveon, Kutasov) $\sim$
  - Change embedding & / or Add WV F_{ab} } FLAVOR BRANE (Macro picture)
  - N_f COLOR BRANES (Micro picture) $\xrightarrow[\text{(Myers Action)}]{\text{Dielectric Effect}}$ FLAVOR BRANES

D3-D7 system $[(p\,|\,Dp \perp D(p+4))]$

		0	1	2	3	4	5	6	7	8	9
N	D3	X	X	X	X	−	−	−	−	−	−
N_f	D7	X	X	X	X	X	X	X	X	−	−

D3 WV: 4d $\mathcal{N}=4$ $SU(N)$ SYM + N_f $\mathcal{N}=2$ fdmtal. hypermultiplets

- mixed COULOMB-HIGGS phase:

F-terms (quarks) $\longrightarrow$

$$(\Phi_3+m)Q^i=0 \qquad \tilde{Q}_i(\Phi_3+m)=0$$

$$\longrightarrow \quad Q^i=\begin{pmatrix}0\\ \vdots\\ 0\\ q_1^i\\ \vdots\\ q_k^i\end{pmatrix} \qquad \tilde{Q}_i=\left(0\cdots 0,\tilde{q}_i^1\cdots,\tilde{q}_i^k\right) \qquad \Phi_3=\begin{pmatrix}\tilde{m}_1 & & & & & \\ & \ddots & & & & \\ & & \tilde{m}_{N-k} & & & \\ & & & -m & & \\ & & & & \ddots & \\ & & & & & -m\end{pmatrix}$$

$$SU(N)\to SU(N-K)$$

(adjoints) $[\Phi_1,\Phi_3]=[\Phi_2,\Phi_3]=0$

$$q^i\tilde{q}_i+[\Phi_1,\Phi_2]=0$$

$$|q^i|^2-|\tilde{q}_i|^2+[\Phi_1,\Phi_1^\dagger]+[\Phi_2,\Phi_2^\dagger]=0 \longleftarrow \text{D-terms}$$

$\mathcal{N}=2$ THEORIES

Dissolved D3-branes: $T_7 \int \mathrm{P}[C^{(4)}] \wedge \mathrm{tr}(F \wedge F)$ ← D7 action WZ term

$\mathcal{M}_{Higgs} \equiv$ 4d Instantons Moduli space

(Douglas; Witten)

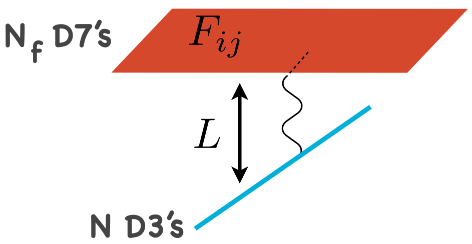
$$\left[\text{D3 WV theory F- \& D-flatness} \equiv \text{ADHM constraints instantons along 4567} \right]$$

- **SUGRA DUAL**

(Macro picture)

	x^μ				$\vec{y}$				$\vec{z}$	
	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X	–	–	–	–	–	–
D7	X	X	X	X	X	X	X	X	–	–

$$\left[\begin{array}{l} r^2 = \vec{y}\cdot\vec{y} + \vec{z}\cdot\vec{z} \\ \rho^2 = \vec{y}\cdot\vec{y} \end{array} \right]$$



$$\sim \quad (\alpha' \to 0) \quad N \gg N_f$$

N_f D7 probes in $AdS_5 \times S^5$

$$\begin{cases} \xi^a = (x^0, \ldots, x^3, \vec{y}) \\ |\vec{z}| = L \\ F_{y^i y^j} \neq 0 \,, \; F_{ij} = {}^*F_{ij} \end{cases}$$

(Guralnik, Kovacs, Kulik; Erdmenger, Große, Guralnik)

$$S^{D7} = -T_7 \int d^8\xi \, e^{-\phi} \, \mathrm{Str}\left[\sqrt{-\det(g+F)}\right] + \underbrace{\frac{T_7}{2} \int \mathrm{Str}\left[P[C^{(4)}] \wedge F \wedge F\right]} = -T_7 \int d^4x \, d^4y \, \mathrm{Str}\,[1] = \boxed{-T_7 \, N_f \int d^4x \, d^4y}$$

WZ term inducing k D3 brane charges

$$T_3 \int d^4x \, d^4y \, C^{(4)}_{x^0x^1x^2x^3} \, \mathcal{P}(y)$$

$$\int d^4y \, \mathcal{P}(y) \; = k$$

$\neq f(L)$

no force

Microscopical description: k dielectric D3-branes fuzzy along 4567

in $AdS_5 \times S^5 \sim$ N-k D3-branes $(N \gg k)$

$$\begin{array}{c|cccccccccc} & \multicolumn{4}{c}{x^\mu} & \multicolumn{4}{c}{\vec{y}} & \multicolumn{2}{c}{\vec{z}} \\ & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \text{D3} & X & X & X & X & \sim & \sim & \sim & \sim & - & - \end{array}$$

$$\begin{bmatrix} r^2 = \vec{y}\cdot\vec{y} + \vec{z}\cdot\vec{z} \\ \rho^2 = \vec{y}\cdot\vec{y} \end{bmatrix}$$

$Y^i : \dfrac{1}{2\pi\,\alpha'}\,[Y^i, Y^j] \equiv i\theta_{ij} \in SU(K)$; $\quad Z^m \longrightarrow$ **abelian**; $\quad {}^*\theta_{ij} = \theta_{ij}$

Match FT Analysis $\longrightarrow$
$$2\pi\alpha'\,\Phi_1 \equiv \frac{Y^1 + iY^2}{\sqrt{2}} \qquad 2\pi\alpha'\,\Phi_2 \equiv \frac{Y^3 + iY^4}{\sqrt{2}} \qquad \Phi_3 \sim Z^1 + iZ^2$$
$\longrightarrow$ **F- & D-equations** $\longrightarrow$
$$\boxed{\begin{aligned} q^i\tilde{q}_i &= \frac{\theta_{23}}{2\pi\alpha'} - i\,\frac{\theta_{13}}{2\pi\alpha'} \\ |\tilde{q}_i|^2 - |q^i|^2 &= \frac{\theta_{12}}{\pi\alpha'} \end{aligned}}$$

$\Rightarrow q\,,\tilde{q} \sim v \neq 0 \leftrightarrow \theta \neq 0$

$$S^{D3} = -T_3 \int d^4\xi\,\mathbf{Str}\left\{\sqrt{-\det\left[P[G + G(Q^{-1}-\delta)G]_{ab}\right]}\sqrt{\det Q}\right\} + T_3 \int d^4\xi\,\mathbf{Str}\left[P\left[C^{(4)}\right]\right] \overset{{}^*\theta_{ij}=\theta_{ij}}{=} -\frac{T_3}{4}\int d^4x\,\mathbf{Str}\left[\theta^2\right]$$
$$= -\pi^2\,T_7(2\pi\alpha')^2 \int d^4x\,\mathbf{Str}\left[\theta^2\right]$$

MAP MICRO $\Longrightarrow$ MACRO description $\quad \mathbf{M}_{k\times k} \Rightarrow f(\vec{y})$

$$\begin{bmatrix} S^{D7}_{WZ} = T_3 \displaystyle\int d^4x\,d^4y \left(\frac{r^2}{R^2}\right)^2 \mathcal{P}(y) \\ S^{D3}_{WZ} = T_3 \displaystyle\int d^4x\,\mathbf{Str}\left[\left(\frac{\hat{r}^2}{R^2}\right)^2\right] \end{bmatrix}$$

$$\boxed{\mathbf{Str}[\,\hat{f}\,] \Rightarrow \int d^4y\,\mathcal{P}(y)\,f(y)}$$

$$(2\pi\alpha')^2\,\mathbf{Str}\left[\theta^2\right] \Downarrow \int d^4y\,\frac{N_f}{\pi^2}$$

$$\boxed{(\,2\pi\alpha')^2\,\theta^2 \Rightarrow \frac{N_f}{\pi^2\,\mathcal{P}(y)}}$$

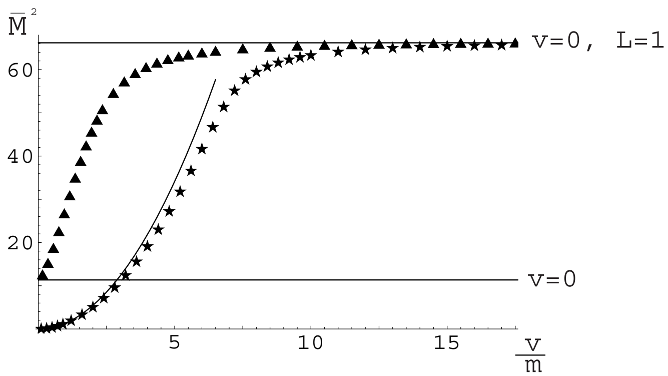
e.g. One SU(2) instanton: $\mathcal{P}(y) = \dfrac{6}{\pi^2}\dfrac{\Lambda^4}{(\rho^2+\Lambda^2)^4}$ **, we recover:** $v \sim \dfrac{\Lambda}{\alpha'}$ **(Guralnik et al) (Erdmenger et al)**

- **Meson spectrum** $(p\,|\,Dp\perp D(p+4))$ **in the Higgs branch**

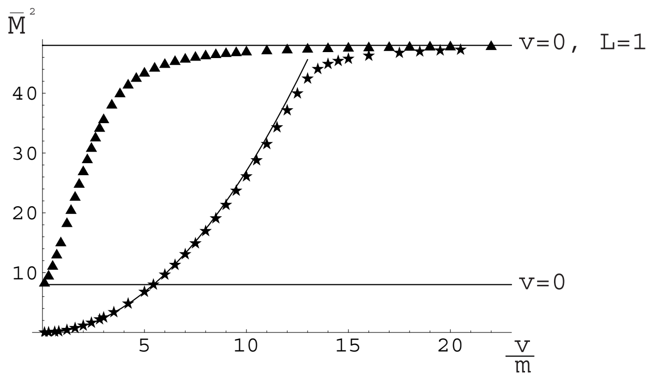
  Only WV gauge field fluctuations: $A = A^{inst} + a \longrightarrow S^{D(p+4)}$ up to order a^2

  Assume one SU(2) instanton & only $a_\mu \neq 0$

D2-D6



D3-D7



★ Our result

▲ $\left[S^{D(p+4)} \sim \sqrt{g}\,F^{ab}F_{ab}\right]$ (Erdmenger et al)

- Spectral flow: $M(v, L=0) \xrightarrow{v\to\infty} M(v=0, L=1)$
- $M \sim \dfrac{m_q}{g_{eff}(m_q)} \quad (v\to\infty)$ (as in D.A. & A.V. Ramallo; R. Myers & R.M. Thomson)
- $M \sim v \quad (v \to 0) \longleftarrow$ WKB approx.

N	D3	X	X	X	X	–	–	–	–	–	–
N_f	D5	X	X	X	–	X	X	X	–	–	–
		0	1	2	3	4	5	6	7	8	9

$0\,1\,2\,3 \to x^\mu$, $\;4\,5\,6 \to \vec{y}$, $\;7\,8\,9 \to \vec{z}$, $\;(\rho, S^2) \longleftarrow \vec{y}$

D3-D5 system $[(p-1\,|\,Dp \perp D(p+2))]$

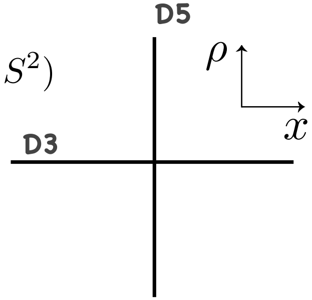
D3 WV: $4\text{d}\ \mathcal{N}=4\ SU(N)$ SYM + N_f 3d fdmtal. hypermultiplets at $x^3 \equiv x = \text{cons} \longrightarrow 0$

(DeWolfe, Freedman, Ooguri)

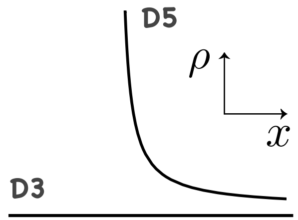
- **SUGRA DUAL** (Macro picture)

N_f D5 probes in $AdS_5 \times S^5$

$$\begin{cases} \xi^a = (x^0, x^1, x^2, \rho, S^2) \\ x^3 = \text{cons} \\ |\vec{z}| = L \end{cases}$$



$$\int_{S^2} F = q = k\pi\alpha' \longrightarrow$$



$$x = -\frac{q}{\rho} + \text{cons}$$

Microscopical description: k dielectric D3-branes fuzzy along the S^2 $\left[d\Omega_2^2 = \sum_{I=1}^{3} dY^I\, dY^I \;;\; \sum_{I=1}^{3} Y^I Y^I = 1 \right]$

k dielectric D3's in $AdS_5 \times S^5$

$$\begin{cases} \xi^a = (x^0, x^1, x^2, \rho) \\ x = x(\rho) \\ |\vec{z}| = L \end{cases} + \quad Y^I = \frac{J^I}{\sqrt{C_2(k)}}, \quad [J^I, J^J] = 2i\,\epsilon_{IJK}\, J^K \longrightarrow \text{SU(2)}$$

$$S^{\text{micro}}_{D3} = S^{\text{macro}}_{D5}$$

(large k)

- **Field Theory**

$456 \longrightarrow X^I_H = 2\pi\alpha' \phi^I_H$

$789 \longrightarrow X^A_V = 2\pi\alpha' \phi^A_V$

Higgs Vacuum

$$\phi_V = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} \qquad q = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \alpha_1 \\ \vdots \\ \alpha_k \end{pmatrix}$$

$[A \longrightarrow$ (N-k) x (N-k) $]$

+

Nahm equations:

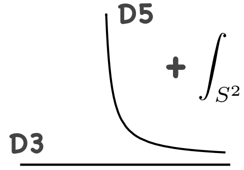
bilinear in q , $\bar{q}$

$$\partial_3 \phi^I_H + \frac{i}{2}\epsilon_{IJK}[\phi^J_H, \phi^K_H] + \alpha^I \delta(x^3) = 0$$

$$\phi^I_H = \frac{f(x)}{\sqrt{C_2(k)}} \begin{pmatrix} 0 & 0 \\ 0 & J^I \end{pmatrix} \quad \xrightarrow{\rho^2 \equiv X^I_H X^I_H} \quad \rho = -\frac{\pi k \alpha'}{x}$$

- **Meson spectrum** $(p-1\,|\,Dp \perp D(p+2))$ **in the Higgs branch**

  Compute the whole set of fluctuations around

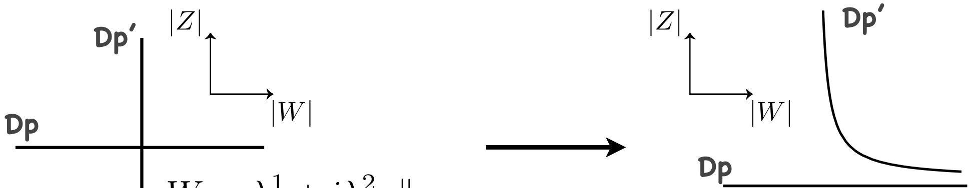


  \+ $\int_{S^2} F = q$

  Full mesonic mass spectrum continuous and gapless

- $(p-2\,|\,Dp \perp Dp)$ **[D3-D3 system:** $4\text{d}\ \mathcal{N}=4\ SU(N)\ \text{SYM}$ **+ 2d fdtal. multiplets]**

  (Constable, Erdmenger, Guralnik, Kirsch)



  $W = \lambda^1 + i\lambda^2 \ \parallel$ Dp

  $Z = Y^1 + iY^2 \ \parallel$ Dp'

  SUSY $\longleftrightarrow$ $\bar{\partial} W = 0$ (holomorphic embeddings)

  $W = \text{cons} \longrightarrow$ localized defect, discrete massive spectrum

  $W \neq \text{cons} \longrightarrow$ brane recombination, continuous & gapless spectrum

  $\left[W = \frac{c}{Z} \leftrightarrow \right.$ Higgs branch, F and D conditions$\left. \right]$ (Guralnik et al)

- **M2-M5 intersection:** M-theory codimension one defect. M5-probe embedding in the M2 background with WV gauge flux and bending along the direction // to the probe were found and shown to be SUSY. Meson spectrum becomes continuous and gapless.

- **F1-Dp intersection:** SUSY embeddings of the Dp in the F1 background with WV gauge flux and bending along the F1 were found. Again the spectrum becomes continuous and gapless.