

# Backreacting Flavors in the Klebanov-Witten Model via D7-branes

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(based on: [hep-th/0612118](#), [arXiv:0706.1238](#), [arXiv:0710.0374](#))

in collaboration with

F. Canoura, S. Cremonesi, C. Nuñez and A.V. Ramallo)

# Gauge/Gravity with Flavors

- Different large  $N_c$  expansions:

## t'Hooft limit

$$N_c \rightarrow \infty$$

$$g_{YM}^2 N_c, \quad N_f \text{ fixed}$$

Planar diagrams *without*  
quark loops

Topological expansion in  
*closed* Riemann surfaces

Gauge/Gravity framework:

Karch,Katz 02, Kruczenski et al. 03, ...

## Veneziano limit

$$N_c \rightarrow \infty$$

$$g_{YM}^2 N_c, \quad \frac{N_f}{N_c} \text{ fixed}$$

Planar diagrams *with* quark  
loops

Topological expansion in  
surfaces with *boundaries*

Graña,Polchinski 01,

Bertolini et al. 01, ...

Some phenomena visible with fundamentals:

- screening of color charges, breaking of flux tubes
- Seiberg duality
- meta-stable SUSY breaking vacua

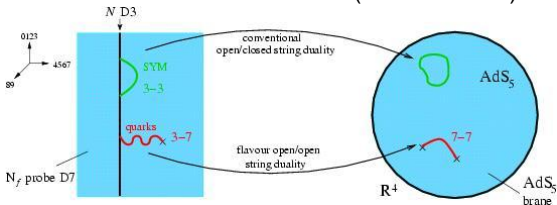
# non-AdS/non-CFT with Flavors

Add flavors to the Gauge/Gravity correspondence as *higher dimensional flavor branes*: **D7-branes**

Karch, Katz 02

Field Theory side:  $g_s N_c \ll 1$

Gravity side: (near horizon)  $g_s N_c \gg 1$



D7-branes are *not* replaced by flux:  
 Flavor symm in Field Theory  $\longleftrightarrow$  Gauge symm on D7-branes

t'Hooft limit:  $N_f$  fixed  $g_s N_f \rightarrow 0$

Veneziano limit:  $N_f/N_c$  fixed  $g_s N_f \gg 1$  fixed

# Backreacting flavors on the KW background

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Gauge/Gravity  
with Flavors

Outline

D3 + D7 on  
Conifold

Field Theory

Fractional  
Branes

Conclusions

## Outline:

- ▶ D3-branes on a *conifold* geometry
- ▶ Procedure for adding *backreacting* D7-branes
- ▶ The supergravity solution
- ▶ Comparison with field theory
- ▶ Addition of *fractional branes*
- ▶ Conclusions

# D3-branes on the Conifold

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Gravity side:

$N_c$  D3-branes  
at the tip  
(near horizon)

 $\Rightarrow$ 

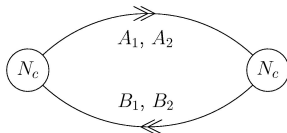
- $AdS_5 \times T^{1,1}$
- $N_c$  units of RR 5-form flux on both factors
- constant dilaton

Field Theory side:

Gauge theory:  $SU(N_c) \times SU(N_c)$   $\mathcal{N} = 1$  SuperconformalMatter: bifundamental  $A_{1,2}$  and  $B_{1,2}$ 

Superpotential:

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$



Flavor symmetries:

$$SU(2)_\ell \times SU(2)_r \times U(1)_R \times U(1)_B$$

We are going to **add flavor D7-branes** to this background

# D7-branes: Computation of the Backreaction

Procedure:

Casero, Nunez, Paredes 05

▶ Find SUSY D7 probes

The background is  $\mathcal{N} = 1$  superconformal: 8 supercharges

The D7's preserve only 4 supercharges

▶ Compute the backreaction of the D7's

D7-branes provide energy (Einstein and dilaton eqns)

and  $F_1$  flux (also EOMs for fluxes)

▶ The solution would have a non-trivial angular profile

▶ Smearing

Distribute the D7's along the angular directions ( $N_f$  large)

Kill the angular dependency

⇒ Ansatz with *only radial functions*

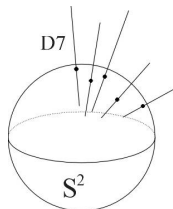
▶ Solve the SUSY equations and check the EOMs

# The Smearing Procedure

With (magnetic) sources

**Bianchi identities** are violated:

$$dF_1 = - \sum_{i=1}^{N_f} \delta^{(2)}(D7_{(i)})$$



Homogeneous angular distribution (at large  $N_f$ ):

$$dF_1 = - \sum_{i=1}^{N_f} \delta^{(2)}(D7_{(i)}) \rightarrow -\Omega_2$$

This comes from the DBI and WZ actions:

$$S_{WZ} = \sum_{i=1}^{N_f} \int_{D7_{(i)}} C_8 \rightarrow \int C_8 \wedge \Omega_2$$

$$S_{DBI} = \sum_{i=1}^{N_f} \int_{D7_{(i)}} e^\phi \sqrt{-\hat{g}} d^8 \xi \rightarrow \dots$$

# Ansatz for the Backreacted Solution

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In our setup,  $\Omega_2$  is computed to be:

$$\Omega_2 = \frac{N_f}{4\pi} \left( \sin \theta_1 d\theta_1 \wedge d\varphi_1 + \sin \theta_1 d\theta_1 \wedge d\varphi_1 \right)$$

Ansatz:  $SU(2)_\ell \times SU(2)_r \times U(1)_R$  isometry

$$ds_{10}^2 = h^{-\frac{1}{2}} dx_{1,3}^2 + h^{\frac{1}{2}} ds_6^2$$

$$ds_6^2 = dr^2 + \frac{e^{2g}}{6} \sum_{i=1,2} (d\theta_i^2 + \sin^2 \theta_i d\varphi_i^2) + \frac{e^{2f}}{9} (d\psi + \sum_{i=1,2} \cos \theta_i d\varphi_i)^2$$

$$e^\phi = e^{\phi(r)}$$

$$F_5 = (1 + *) d^4x \wedge dh^{-1}$$

$$F_1 = \frac{N_f}{4\pi} (d\psi + \sum_{i=1,2} \cos \theta_i d\varphi_i)$$

Unknow functions:  $h(r)$ ,  $g(r)$ ,  $f(r)$ ,  $\phi(r)$ .

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# The Supergravity Solution

The system can be **analytically integrated** (many integr const)

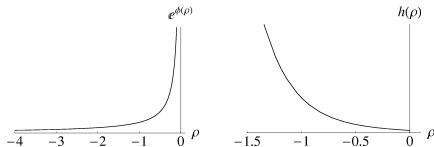
Change radial variable  $r \rightarrow \rho$ :  $e^{-f} dr = d\rho$

$$e^\phi = \frac{4\pi}{3N_f} \frac{1}{(-\rho)}$$

$$e^g = (1 - 6\rho)^{1/6} e^\rho$$

$$e^f = (-6\rho)^{1/2} (1 - 6\rho)^{-1/3} e^{\rho}$$

$$h = -27\pi N_c \int_0^\rho e^{-4g} + c_2$$



- ▶ The warp factor  $h(\rho)$  can be analytically integrated
- ▶ The solution is defined in:  $-\infty < \rho < 0$
- ▶  $\rho = 0$  (**UV**): the dilaton blows up – Landau pole
- ▶  $\rho \rightarrow -\infty$  (**IR**): up to logarithmic correction of order  $1/|\log(r)|$  the geometry approaches  $AdS_5 \times T^{1,1}$  with  $e^\phi(r) \rightarrow 0$   
The Einstein frame curvature is finite

# The Dual Field Theory

Gauge theory:  $SU(N_c) \times SU(N_c)$   $\mathcal{N} = 1$

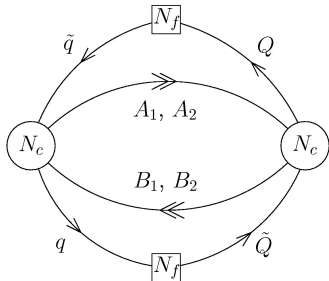
Flavor symmetry:  $U(N_f) \times U(N_f)$  (partially anomalous)

Matter:  $A_{1,2}$  and  $B_{1,2}$  **bifundamentals**

D7 matter:  $q, \tilde{q}, Q, \tilde{Q}$  **fundamentals**

Superpotential (localized D7's):

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1 + \tilde{q} A_1 Q + \tilde{Q} B_1 q$$



**Smearing:** only the superpotential (cubic coupling) is affected  
Appears a linear combination of  $A_i$  and  $B_j$

Having added flavors to a conformal theory, we expect *positive*  $\beta$ -function

# Comparison with Field Theory and Extensions

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- Matching of  $\beta$ -functions and  $U(1)_R$  anomaly

Compute:                      gauge couplings  $g_i$     (from  $e^\phi$ )  
    YM theta angles  $\theta_i^{YM}$     (from  $C_0$ )

$\beta$ -functions:                       $\beta_{g_i}$  are positive  
    Match  $\gamma_A$  and  $\gamma_q$  at almost conformal point

$U(1)_R$  anomaly:                       $U(1)_R \times U(1)_B \rightarrow \mathbb{Z}_{N_f} \times U(1)_B$

- Extensions:

- ▶ Massive flavors:                      SUGRA solution for massive flavors  
    Check of holomorphic decoupling

- ▶ Generic Calabi-Yau singularities  
    SUGRA solution with backreacting D7's  
    for every  $AdS_5 \times M_5$  (Sasaki-Einstein)

# Addition of Fractional Branes

## ★ Field Theory side

Realize different gauge ranks:  $G_1 > G_2$

**RG flow:** cascade of Seiberg dualities  
both ranks and their difference reduce

Extra **gauge singlet** fields involved

## ★ Gravity side

Add 3-form flux:  $F_3$  and  $H_3$

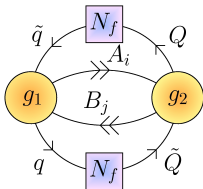
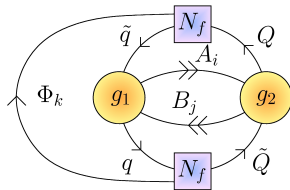
$P_{D7}[H_3] \neq 0 \Rightarrow$  forced to put **gauge flux  $\mathcal{F}$**  on the D7's

SUGRA solution with 3-form flux + backreacting D7's with  $\mathcal{F}$

- Matching of the cascade:

Page charges  $\rightarrow$  matching of gauge rank cascade

Intersecting D7 with flux  $\mathcal{F} \rightarrow$  extra gauge singlets



# Conclusions

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- System of D3 + D7 backreacting branes on Conifold (*flavored* KW model)
- SUGRA solution (in terms of analytic functions)
- Dual field theory
- Perfect matching of:
  - ▶ gauge and flavor symmetries
  - ▶  $\beta$ -functions
  - ▶ anomalies
- System of fractional D3 + D7 with flux  $\mathcal{F}$  on Conifold (*chirally flavored* KT model)
- SUGRA solution (in terms of analytic functions)  
Matching with the field theory cascade